

## QCD at moderately large distances

Yuri Dokshitzer\*

LPTHE, Université Pierre et Marie Curie,  
4, Place Jussieu, 75252 Paris, France

### 1. Introduction

Phenomenology of scattering processes involving hadrons always was, and still is, providing puzzles and inspiration. If 30–40 years ago quantum field theory (QFT) had been kept in higher respect, the most general phenomenological features of hadron interactions that were known then could have already hinted at QCD as a possible underlying microscopic theory of hadrons.

Hints from the past:

- The fact that in high energy hadron interaction processes *inelastic* breakup typically dominates over elastic scattering hinted at proton being a loosely bound compound object:

$\implies$      *Constituent Quarks*

- Constancy of transverse momenta of produced hadrons, rare appearance of large- $k_{\perp}$  fluctuations, was signaling the weakness of interaction at small relative distances:

$\implies$      *Asymptotic Freedom*

- The last but not the least;
  - The total hadron interaction cross sections turned out to be practically *constant* with energy. *If* we were to employ the standard quantum field theory (QFT) picture of a particle exchange between interacting objects,

$$\sigma_{\text{tot}} \propto s^{J-1} \simeq \text{const},$$

*then* this called for a spin-one elementary field,  $J = 1$ , to be present in the theory.

- *Uniformity in rapidity* of the distribution of produced hadrons (Feynman plateau) pointed in the same direction, *if*, once again, we were willing to link final particle production to accompanying QFT “radiation”.

$\implies$      *Vector Gluons.*

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\*On leave from PNPI, 188350 Gatchina, St. Petersburg, Russia.

Nowadays the dossier of puzzles & hints that the hadron phenomenology has accumulated is very impressive. It includes a broad spectrum of issues ranging from unexplained regularities in hadron spectroscopy to soft “forceless” hadroproduction in hard processes. To locate and formulate a puzzle, to digest a hint, — these are the road-signs to the hadron chromodynamics construction site. We are learning how to listen. And to hear.

The reason why one keeps talking, for almost 30 years now, about puzzles and hints, about *constructing* QCD rather than *applying* it, lies in the conceptually new problem one faces when dealing with a non-Abelian theory with unbroken symmetry (like QCD). We have to understand how to master QFTs whose dynamics is intrinsically unstable in the infrared domain: the objects belonging to the physical spectrum of the theory (supposedly, colourless hadrons, in the QCD context) have no direct one-to-one correspondence with the fundamental fields the microscopic Lagrangian of the theory is made of (coloured quarks and gluons).

In these circumstances we don’t even know how to formulate at the level of the microscopic fields the fundamental properties of the theory, such as conservation of probability (unitarity) and analyticity (causality):

- What does *Unitarity* imply for confined objects?
- How does *Causality* restrict quark and gluon Green functions and their interaction amplitudes?
- What does the *mass* of an INFO (Identified Non-Flying Object) mean?

The issue of quark masses is especially damaging since a mismatch between quark and hadron thresholds significantly affects predicting the yield of heavy-flavoured hadrons in hadron collisions.

Understanding confinement of colour remains an open problem. Given the present state of ignorance, one has no better way but to circle along the *Guess-Calculate-Compare* loop. There are, however, *guesses* and *guesses*.

## 2. Words, words, ...

Speaking of “perturbative QCD” (pQCD) can have two different meanings.

- In a narrow, strict sense of the word, *perturbative approach* implies representing an answer for a (calculable) quantity in terms of series in a (small) expansion parameter  $\alpha_s(Q)$ , with  $Q$  the proper hardness scale of the problem under consideration.
- In a broad sense, *perturbative* means applying the language of quarks and gluons to a problem, be it of perturbative (short-distance, small-coupling) or even non-perturbative nature.

The former definition is doomed: the perturbative series so constructed are known to diverge. In QCD these are asymptotic series of such kind that cannot be “resummed” into an analytic function in a unique way. For a given calculable (collinear-& -infrared-safe; CIS) observable [1] the nature of this nasty divergence can be studied and quantified as an intrinsic uncertainty of pQCD series, in terms of the so-called *infrared renormalons* [2].

Such uncertainties are non-analytic in the coupling constant and signal the presence of non-perturbative (large-distance) effects. For a CIS observable, non-perturbative physics enters at the level of power-suppressed corrections  $\exp\{-c/\alpha_s(Q)\} \propto Q^{-\gamma}$ , with  $\gamma$  an observable-dependent positive integer<sup>2</sup> number.

On the contrary, the broader definition of being “perturbative” is bound to be right. At least as long as we aim at eventually deriving the physics of hadrons from the quark-gluon QCD Lagrangian.

To distinguish between the two meanings, in what follows we will supply the word *perturbative* with a superscript <sup>{1}</sup> or <sup>{2}</sup>. Thus, when discussing the strong interaction domain in terms of quarks and gluons, we will be able to speak about non-perturbative<sup>{1}</sup> perturbative<sup>{2}</sup> effects.

### 3. Probing confinement with perturbative tools

Let us discuss the test case of the total cross section of  $e^+e^-$  annihilation into hadrons as an example.

To predict  $\sigma_{\text{hadr}}^{\text{tot}}$  one calculates instead the cross sections of quark and gluon production,  $(e^+e^- \rightarrow q\bar{q}) + (e^+e^- \rightarrow q\bar{q} + g) + \text{etc.}$ , where quarks and gluons are being treated *perturbatively* as real (unconfined, flying) objects. The *completeness* argument provides an apology for such a brave substitution:

Once instantaneously produced by the electromagnetic (electroweak) current, the quarks (and secondary gluons) have nowhere else to go but to convert, *with unit probability*, into hadrons in the end of the day.

This *guess* looks rather solid and sounds convincing, but relies on two hidden assumptions:

1. The allowed hadron states should be numerous as to provide the quark-gluon system the means for “regrouping”, “blanching”, “fitting” into hadrons.
2. It implies that the “production” and “hadronization” stages of the process can be separated and treated independently.

1. To comply with the first assumption the annihilation energy has to be taken large enough,  $s \equiv Q^2 \gg s_0$ . In particular, it fails miserably in the resonance region  $Q^2 \lesssim s_0 \sim 2M_{\text{res}}^2$ . Thus, the point-by-point correspondence between hadron and quark cross sections,

$$\sigma_{\text{hadr}}^{\text{tot}}(Q^2) \stackrel{?}{=} \sigma_{q\bar{q}+X}^{\text{tot}}(Q^2),$$

cannot be sustained except at very high energies. It can be traded, however, for something better manageable.

Invoking the dispersion relation for the photon propagator (causality  $\implies$  analyticity) one can relate the *energy integrals* of  $\sigma^{\text{tot}}(s)$  with the correlator of electromagnetic currents in a deeply Euclidean region of large *negative*  $Q^2$ . The latter corresponds to small space-like distances between interaction points, where the perturbative<sup>{2}</sup> approach is definitely valid.

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<sup>2</sup>usually, though not necessarily [3]

Expanding the answer in a formal series of local operators, one arrives at the structure in which the corrections to the trivial unit operator generate the usual perturbative<sup>{1}</sup> series in powers of  $\alpha_s$  (logarithmic corrections), whereas the vacuum expectation values of dimension-full (Lorentz- and colour-invariant) QCD operators provide non-perturbative<sup>{1}</sup> corrections suppressed as powers of  $Q$ .

This is the realm of the famous ITEP sum rules [4] which proved to be successful in linking the parameters of the low-lying resonances in the Minkowsky space with expectation values characterising a non-trivial structure of the QCD vacuum in the Euclidean space. The leaders among them are the gluon condensate  $\alpha_s G^{\mu\nu} G_{\mu\nu}$  and the quark condensate  $\langle\psi\bar{\psi}\rangle$  which contribute to the total annihilation cross section, symbolically, as

$$\sigma_{\text{had}}^{\text{tot}}(Q^2) - \sigma_{q\bar{q}+X}^{\text{tot}}(Q^2) = c_1 \frac{\alpha_s G^2}{Q^4} + c_2 \frac{\langle\psi\bar{\psi}\rangle^2}{Q^6} + \dots \quad (1)$$

2. Validating the second assumption also calls for large  $Q^2$ . To be able to separate the two stages of the process, it is *necessary* to have the production time of the quark pair  $\tau \sim Q^{-1}$  to be much smaller than the time  $t_1 \sim \mu^{-1} \sim 1 \text{ fm}/c$  when the first hadron appears in the system. Whether this condition is *sufficient*, is another valid question. And a tricky one.

Strictly speaking, due to gluon bremsstrahlung off the primary quarks, the perturbative production of secondary gluons and  $q\bar{q}$  pairs spans an immense interval of time, ranging from a very short time  $t_{\text{form}} \sim Q^{-1} \ll t_1$  all the way up to a macroscopically large time  $t_{\text{form}} \lesssim Q/\mu^2 \gg t_1$ .

This accompanying radiation is responsible for formation of hadron jets. It does not, however, affect the total cross section. It is the rare hard gluons with large energies and transverse momenta,  $\omega \sim k_{\perp} \sim Q$ , that only matter. This follows from the famous Bloch-Nordsieck theorem which states that the logarithmically enhanced (divergent) contributions due to real production of *collinear* ( $k_{\perp} \ll Q$ ) and *soft* ( $\omega \ll Q$ ) quanta cancel against the corresponding virtual corrections:

$$\sigma_{q\bar{q}+X}^{\text{tot}} = \sigma_{\text{Born}} \left( 1 + \frac{\alpha_s}{\pi} [\infty_{\text{real}} - \infty_{\text{virtual}}] + \dots \right) = \sigma_{\text{Born}} \left( 1 + \frac{3 C_F \alpha_s(Q^2)}{4 \pi} + \dots \right).$$

The nature of the argument is purely perturbative. Can the Bloch-Nordsieck result hold beyond pQCD?

Looking into this problem produced an extremely interesting result that has laid a foundation for the development of perturbative<sup>{2}</sup> techniques aimed at analysing non-perturbative<sup>{1}</sup> effects.

V. Braun, M. Beneke and V. Zakharov have demonstrated that the real-virtual cancellation actually proceeds much deeper than was originally expected.

Let me briefly sketch the idea.

- First one introduces an infrared cutoff (non-zero gluon mass  $m$ ) into the calculation of the radiative correction.
- Then, one studies the dependence of the answer on  $m$ . A CIS quantity, by definition, remains finite in the limit  $m \rightarrow 0$ . This does not mean, however, that it is insensitive

to the modification of gluon propagation. In fact, the  $m$ -dependence provides a handle for analyzing the *small transverse momenta* inside Feynman integrals. It is this region of integration over parton momenta where the QCD coupling gets out of perturbative<sup>{1}</sup> control and the genuine non-perturbative physics comes onto the stage.

- Infrared sensitivity of a given CIS observable is determined then by the first non-vanishing term which is *non-analytic* in  $m^2$  at  $m = 0$ .

In the case of one-loop analysis of  $\sigma^{\text{tot}}$  that we are discussing, one finds that in the sum of real and virtual contributions not only the terms singular as  $m \rightarrow 0$ ,

$$\ln^2 m^2, \quad \ln m^2,$$

cancel, as required by the Bloch-Nordsieck theorem, but that the cancellation extends [5,6] also to the whole tower of *finite* terms

$$m^2 \ln^2 m^2, \quad m^2 \ln m^2, \quad m^2, \quad m^4 \ln^2 m^2, \quad m^4 \ln m^2.$$

In our case the first *non-analytic* term appears at the level of  $m^6$ :

$$\frac{3 C_F \alpha_s}{4 \pi} \left( 1 + 2 \frac{m^6}{Q^6} \ln \frac{m^2}{Q^2} + \mathcal{O}(m^8) \right).$$

It signals the presence of the non-perturbative  $Q^{-6}$  correction to  $\sigma^{\text{tot}}$ , which is equivalent to that of the ITEP quark condensate in (1). (The gluon condensate contribution emerges in the next order in  $\alpha_s$ .)

A similar program can be carried out for other CIS quantities as well, including intrinsically Minkowskian observables which address the properties of the final state systems and, unlike the total cross sections, do not have a Euclidean image.

The most spectacular non-perturbative<sup>{1}</sup> results were obtained for a broad class of *jet shape variables* (like thrust,  $C$ -parameter, broadenings, and alike). As has long been expected [7–10], these variables possess relatively large  $1/Q$  confinement correction effects.

Employing the “gluon mass” as a large-distance trigger was formalized by the so-called dispersive method [11]. There it was also suggested to relate new non-perturbative<sup>{1}</sup> dimensional parameters with the momentum integrals of the effective QCD coupling  $\alpha_s$  in the infrared domain. Though it remains unclear how such a coupling can be rigorously defined from the first principles, the *universality* of the coupling makes this guess verifiable and therefore legitimate. All the observables belonging to the same class  $1/Q^p$  with respect to the nature of the leading non-perturbative<sup>{1}</sup> behaviour, should be described by the same parameter.

In particular, the extended family of jet shapes (including energy-energy correlations [3], out-of-plane transverse momentum flows [12] etc.) can be said to “measure” the first moment of the perturbative<sup>{2}</sup> non-perturbative<sup>{1}</sup> coupling,

$$\alpha_0 \equiv \frac{1}{\mu_I} \int_0^{\mu_I} dk \alpha_s(k^2), \quad \mu_I = 2 \text{ GeV}, \quad (2)$$

where the choice of the “infrared” boundary value  $\mu_I$  is a matter of convention.

The interested reader will find a detailed discussion of the method, of the guesses made and the problems faced, as well as the turbulent history of its application, in review talks [13]. Here I will only report the new spectacular results of the perturbative<sup>{2}</sup> study of jet shape variables in DIS carried out recently by M. Dasgupta and G. Salam [14].

#### 4. Intermediate distances in DIS

In Fig. 1 the results are shown of the two-parameter fits to the means of jet shapes in  $e^+e^-$  annihilation together with the fits to the jet shape distributions in DIS [14]. Consistency among the same-family variables is quite impressive.

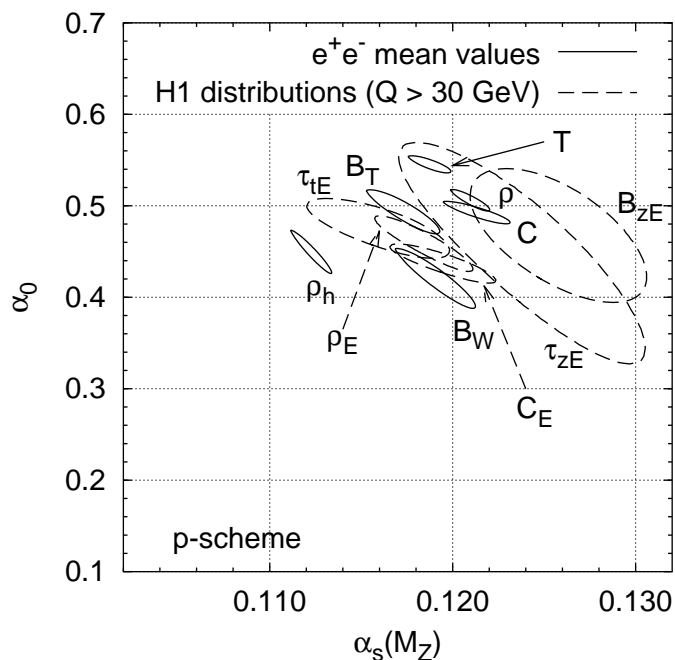


Figure 1.  $\alpha_s(M_Z^2)$  and  $\alpha_0$  from jet shapes.  $1-\sigma$  contours for the means in  $e^+e^-$  annihilation (solid) and for the shape distributions in the current fragmentation jet in DIS (dashed).

It is important to stress that prior to hunting for non-perturbative<sup>{1}</sup> effects, the state-of-the-art perturbative<sup>{1}</sup> predictions have to be derived and implemented. In the case of distributions this involves resummation of logarithmically enhanced contributions in all order of perturbation theory. Having addressed this problem in the DIS environment, Dasgupta and Salam have found a new set of log-enhanced terms that has been previously overlooked in the literature. These corrections (dubbed “non-global” by the founders) only affect the observables that are based on a measurement restricted to a *fraction* of the total phase space available for gluon radiation. For example, restricted to one hemisphere, or to any limited angular region. (In particular, among the observables that suffer from

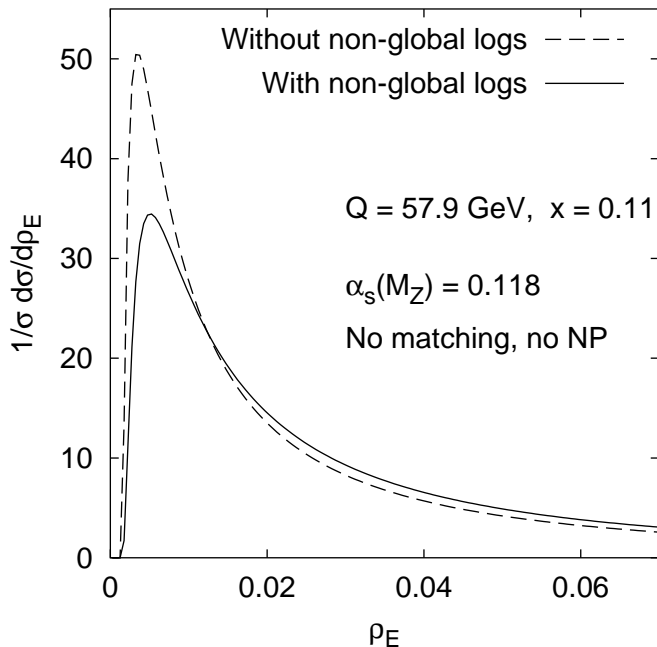


Figure 2. Normalised invariant mass distribution with and without non-global single-logarithmic corrections [15].

this newly discovered effect is the Sterman-Weinberg jet cross section,— the first classical example of a CIS quantity.)

Being subleading (single logarithmic) in nature, these corrections nevertheless modify the log-resummed perturbative predictions quite significantly, as shown in Fig. 2.

## 5. Conclusions

Deep inelastic scattering phenomena always were on the QCD forefront. Exploring quark-gluon dynamics in the DIS environment becomes even more important nowadays.

While we concentrated on *verifying* perturbative<sup>{1}</sup> QCD predictions for multiple hadroproduction, DIS was handicapped as lacking a manageable hadron-free initial state, as compared with the “clean”  $e^+e^-$  annihilation. Now that one aims at understanding an interface between hard and soft physics, this is no longer a disadvantage, and DIS should take the lead.

The main *advantage* of DIS is that the *energy* of the process,  $s$ , is not kinematically equated to its hardness,  $Q^2$ , as in the case of annihilation ( $s = Q^2$ ). Thus, in DIS one can study intermediate and small hardness scales while staying away from the difficult resonance region, without restricting the phase space for multiparticle production.

On the other hand, the study of quasi-diffractive phenomena in lepton-hadron scattering offers a variety of hardness handles ( $Q^2$ ,  $t$ ,  $J/\psi$  and  $\Upsilon$  masses). Diffraction is interesting on its own as a non-linear phenomenon closely linked to unitarity. Moreover, it can be looked upon as a first step towards understanding multi-gluon exchange, which is necessary for

uncovering the perturbative<sup>{2}</sup> non-perturbative<sup>{1}</sup> physics of lepton/hadron-nucleus and heavy ion scattering.

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