Perturbative QCD and Power Corrections

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A short review is given of the present status of the studies of genuine confinement effects in multiple hadron production in hard processes.

1 Beware of Soft Confinement

☞ What I tell you three times is true.
"*The Hunting of the Snark*". Lewis Carroll

Perturbative QCD (pQCD) covers orders of magnitude in the basic hard cross sections. Scattered clouds on the pQCD horizon (presently, a "high-p_T anomaly" and some problems with hadroproduction of heavy quarks and direct photons) don't define the weather. However, viewing the QCD landscape it is essential to remember that its "peace and quite" is deceptive. The closest analogy which comes to mind comparing QCD with its electroweak SM-counterpart is that of hell and heaven: the former being scary but entertaining, the latter — perfect but boring (for a detailed review see [1]).

1.1 The name of the game: MLLA

☞ Although this may seem a paradox, all exact science is dominated by the idea of approximation.
Bertrand Russell

In spite of the smallness of the coupling at small distances, $\alpha_s \ll 1$, quarks and gluons willingly multiply in hard interactions. This happens because the actual parameter of the PT expansion gets enhanced by the log of the scale of the large momentum Q applied to the system: $\alpha_s(Q) \to \alpha_s(Q)\log Q \sim 1$. Such log-enhanced contributions have to be taken care of in all orders, giving rise to "leading-log resummations". The structure of these contributions allows for a *probabilistic parton interpretation.* Parton multiplication in jets is described by the so-called Modified Leading Logarithmic Approximation (MLLA) which embodies the exact angular ordering resulting from coherence in multiple soft gluon radiation. As an approximation, MLLA is necessary for deriving asymptotically correct PT predictions.

QCD coherence is crucial for treating particle multiplication **inside** jets, as well as for hadron flows **in-between** jets. Moreover, it allows the prediction of the *shape* of the inclusive energy distributions practically from the "first principles", apart from an overall unknown normalization **constant**.

1.2 Inclusive energy spectra

It is well known that the DIS structure functions cannot be calculated perturbatively. What pQCD controls is the scaling violation pattern, governed by the QCD parton evolution equation which describes how the parton densities change with changing the scale of the transverse-momentum probe:

$$\frac{\partial}{\partial \ln k_\perp} D(x,k_\perp) = \frac{\alpha_s(k_\perp)}{\pi} P(z) \otimes D(x/z,k_\perp)\,. \tag{1}$$

In the Mellin moment space, $f_\omega \equiv \int_0^1 f(x) x^{\omega-1} dx$, the equation becomes algebraic, yielding

$$D_\omega(k_\perp) \equiv D_\omega(Q_0)\cdot \exp\left\{\int_{Q_0}^{k_\perp} \frac{dk}{k}\gamma_\omega(\alpha_s(k))\right\}, \quad \gamma_\omega(\alpha_s) = \frac{\alpha_s}{\pi} P_\omega\,. \tag{2}$$

It is the ω-dependence of the input function $D_\omega(Q_0)$ ("initial parton distributions") that limits predictability of the Bjorken-x dependence of DIS cross sections.

In the time-like jet evolution, due to the Angular Ordering, the evolution equation becomes non-local in the $k_\perp$ space:

$$\frac{\partial}{\partial \ln k_\perp} D(x,k_\perp) = \frac{\alpha_s}{\pi} P(z) \otimes D(x/z,\, z\cdot k_\perp)\,; \qquad [\,\mathbf{AO:}\ \Theta' = k'_\perp/zE \ \ge\ \Theta = k_\perp/E]\,. \tag{3}$$

Using the Taylor expansion trick,

$$D(x/z,\, z\cdot k_\perp) = \exp\left\{\ln z \frac{\partial}{\partial \ln k_\perp}\right\} D(x/z,k_\perp) \;=\; z^{\frac{\partial}{\partial \ln k_\perp}}\cdot D\,, \tag{4}$$

the solution in the moment space comes out similar to that for the DIS case, Eq. 2, but now with an *operator* as an argument of the splitting function P:

$$\hat{d}\cdot D_\omega = \frac{\alpha_s}{\pi} P_{\omega+\hat{d}}\cdot D_\omega\,, \quad \hat{d} \equiv \frac{\partial}{\partial \ln k_\perp}\,. \tag{5}$$

This leads to the differential equation

$$\left(P^{-1}_{\omega+\hat{d}}\,\hat{d} \;-\; \frac{\alpha_s}{\pi} - \left[P^{-1}_{\omega+\hat{d}}, \frac{\alpha_s}{\pi}\right] P_{\omega+\hat{d}}\right)\cdot D = 0\,. \tag{6}$$

Since we are interested in the small-x region, the essential moments are small, $\omega \ll 1$. For the sake of illustration let us keep only the most singular piece of the splitting function (DLA),

$$P_\omega \simeq \frac{2N_c}{\omega}\,. \tag{7}$$

Then Eq. 6 immediately gives a quadratic equation for the anomalous dimension,

$$(\omega+\gamma)\gamma \;-\; \frac{2N_c\alpha_s}{\pi} + \mathcal{O}\left(\frac{\alpha_s^2}{\omega}\right) \;=\; 0\,. \tag{8}$$

NB: It suffices to use the next-to-leading approximation to the splitting function, $P_\omega \simeq 2N_c/\omega - a$, with $a = 11N_c/6 + n_f/(3N_c^2)$, and to keep the leading correction coming from differentiation of the running coupling in Eqs. 6, 8, to get the more accurate MLLA anomalous dimension γ_ω.

The leading anomalous dimension following from Eq. 8 is

$$\gamma = \frac{\omega}{2}\left(-1 + \sqrt{1 + 8N_c\alpha_s/\omega^2}\right). \tag{9}$$

When expanded to the first order in α_s, it coincides with that for the space-like evolution, $\gamma \simeq \alpha_s/\pi \cdot P_\omega$, with P given in Eq. 7.

The time-like DLA anomalous dimension Eq. 9 (as well as its MLLA improved version) has a curious property. Namely, in a sharp contrast with the DIS case, it allows the momentum integral in Eq. 2 to be extended to very small scales. Even integrating down to $Q_0 = \Lambda$, the position of the "Landau pole" in the coupling, one gets a finite answer for the distribution (the so-called *limiting spectrum*), simply because the $\sqrt{\alpha_s(k)}$ singularity happens to be integrable!

It would have been a bad taste to actually trust this formal integrability, since the very perturbative approach to the problem (selection of dominant contributions, parton evolution picture, etc) relied on α_s being a numerically small parameter. However, the important thing is that, due to time-like coherence effects, the (still perturbative but "smallish") scales, where $\alpha_s(k) \gg \omega^2$, contribute to γ basically in a ω-*independent* way, $\gamma + \omega/2 \propto \sqrt{\alpha_s(k)} \neq f(\omega)$. This means that "smallish" momentum scales k affect only an overall *normalization* without affecting the *shape* of the x-distribution. Since such is the rôle of the "smallish" scales, it is natural to expect the same for the truly small — non-perturbative — scales where the partons transform into the final hadrons. This idea has been formulated as a hypothesis of local parton-hadron duality (LPHD).[2,3]

According to LPHD, the x-shape of the so-called "limiting" spectrum which one obtains by formally setting $Q_0 = \Lambda$ in the parton evolution equations, should be mathematically similar to that of the inclusive hadron distribution. Another essential property is that the "conversion coefficient" should be a true constant independent of the hardness of the process producing the jet under consideration. Starting from the LEP-I epoch, this "prediction" stood up to scrutiny by e^+e^-, DIS and Tevatron experiments.

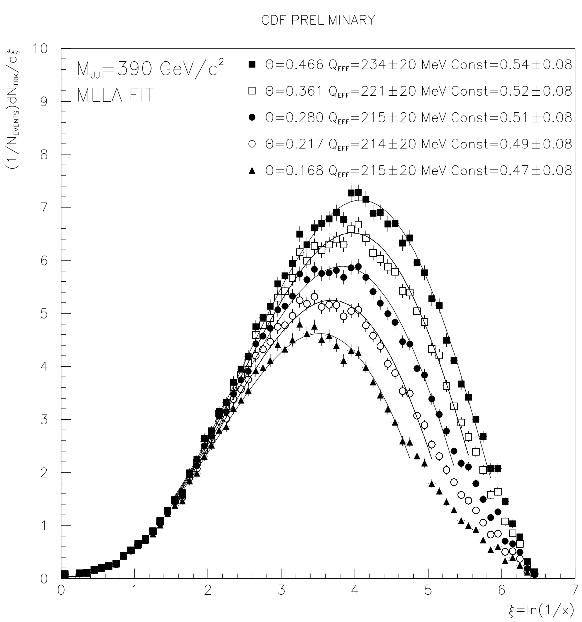

Fig.1: CDF hump-backed plateau versus an analytic MLLA prediction for the yield of secondary partons (soft gluons).[4]

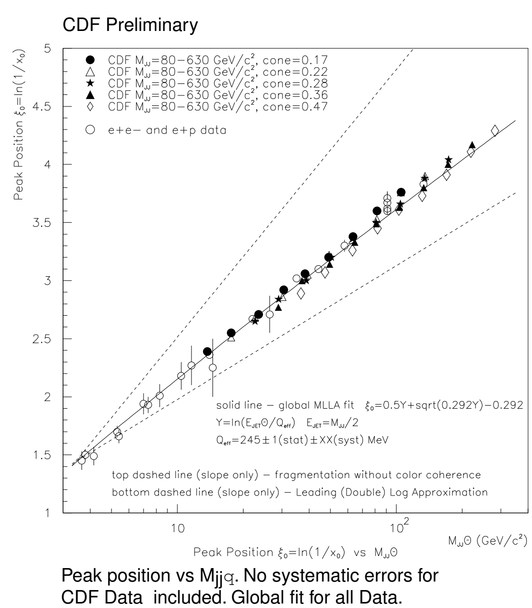

Fig.2: Position of the maximum in the inclusive energy spectra versus a parameter-free MLLA prediction.[4]

The message is, that "brave gluon counting", that is applying the pQCD language all the way down to very small transverse momentum scales, indeed reproduces the x- and Q-dependence of the observed inclusive energy spectra of charged hadrons (pions) in jets.

Even such a tiny (subleading) effect as an envisaged difference in the position of the maxima in quark- and gluon-initiated humps seems to have been verified, 15 years later, by the recent DELPHI analysis.[5,6]

1.3 Inter-jet particle flows

☞ "Can you do addition?" the White Queen asked. "What's one and one and one and one and one and one and one and one and one and one?" "I don't know," said Alice. "I lost count."
Through the Looking Glass

Even more striking is *miraculously* successful rôle of gluons in predicting the pattern of hadron multiplicity flows in the inter-jet regions — realm of various *string/drag* effects.It isn't strange at all that with *gluons* one can get, e.g., $1+1 = 2$ while $1+1 + \frac{9}{4} = \frac{7}{16}$, which is a simple *radiophysics* of composite antennas, or quantum mechanics of conserved colour charges.

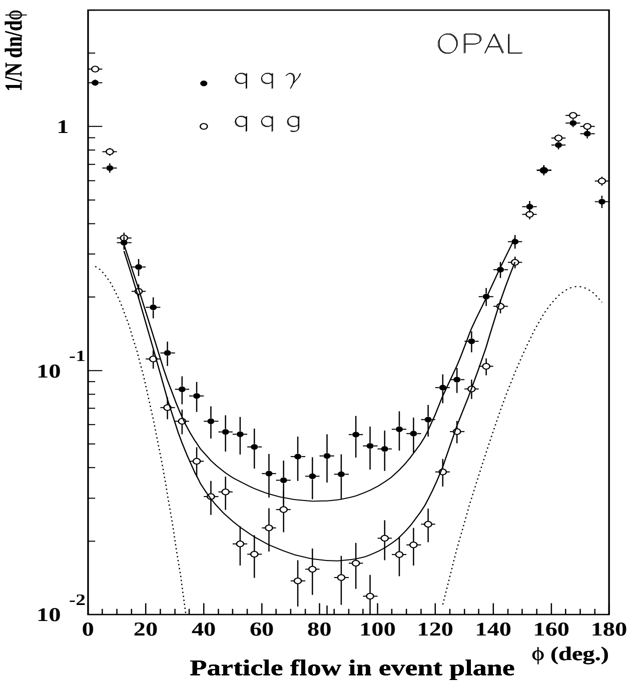

Fig.3: Comparison of particle flows in the $q\bar{q}$ valley in $q\bar{q}\gamma$ and $q\bar{q}g$ 3-jet events versus a parameter-free analytic prediction based on the soft gluon radiation pattern.[7]

This particular example of "quantum arithmetics" has to do with comparison of hadron flows in the inter-quark valleys in $q\bar{q}\gamma$ and $q\bar{q}g$ (3-jet) events. The first equation describes the density of soft gluon radiation produced by two quarks in a $q\bar{q}\gamma$ event, with 1 standing for the colour quark charge.

Replacing the colour-blind photon by a gluon one gets an additional emitter with the relative strength 9/4, as shown in the l.h.s. of the second equation. The resulting soft gluon yield in the $q\bar{q}$ direction, however, *decreases* substantially as a result of destructive interference between three elements of a composite colour antenna.

Nothing particularly strange, you might say. What **is** rather strange, though, is that this naive perturbative wisdom is being impressed upon junky 100-200 MeV pions which dominate hadron flows between jets in the present-day experiments such as the OPAL study shown in Fig. 3.

Another amazing test of this sort was provided by the DELPHI measurement of the multiplicity of "(low energetic) tracks emitted perpendicular to the eveny plane" in 3-jet events[8] which has been found to obediently follow a simple PT prediction based on coherent soft gluon radiation.[9]

- The *colour field* that an ensemble of hard primary **partons** (parton antenna) develops, determines, on the one-to-one basis, the structure of final flows of **hadrons**.
- The Poynting vector of the colour field gets translated into the hadron pointing vector without any visible reshuffling of particle momenta at the "hadronisation stage".

When viewed *globally*, confinement is about *renaming* a flying-away quark into a flying-away pion rather than about forces *pulling* quarks together.

1.4 *Gluons and Gluers*

Definition: a *Gluer* is a miserable *gluon* which hasn't got enough time to truly behave like one because its hadronization time is comparable with its formation time, $t_{\text{form.}} \simeq \omega/k_\perp^2 \sim t_{\text{hadr.}} \simeq \omega R^2_{\text{conf.}}$. Contrary to respectful PT gluons born with small transverse size, $k_\perp \gg R^{-1}_{\text{conf.}}$, gluers are not "partons": they do not participate in perturbative cascading (don't multiply). According to the above definition, gluers have *finite* transverse momenta (though may have arbitrarily large *energies*). Having transverse momenta of the order of inverse confinement scale puts gluers on the borderline of applicability of PT language, since their interaction strength is potentially large, $\alpha_s(R^{-1}_{\text{conf.}}) \sim 1$. Rôle of gluers is to provide comfortable conditions for *blanching* colour parton ensembles (jets) produced in hard interactions, *locally* in the configuration space. Gluer formation is a signal of hadronization process taking place in a given space-time region. A label to put on the gluer concept might be — "*A gluer formed $\simeq$ a hadron born*". An Idea emerges: To relate (uncalculable) Non-Perturbative corrections to (calculable) Perturbative cross sections/observables with intensity of *gluer* emission (α_s in the infrared domain).

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PT-calculable observables are Collinear-and-InfraRed-Safe (CIS) observables, those which can be calculated in terms of quarks and gluons without encountering either collinear (zero-mass quark, gluon) or soft (gluon) divergences. Gluers' contributions to such observables are suppressed and are being rightfully neglected in the pure PT ("logarithmic") approximation. These contributions are inversly proportional to a certain *power* of the hardness scale (modulo logs), $\delta\sigma^{\text{NP}}/\sigma \propto \log^q Q/Q^{2p}$. The corresponding observable-dependent exponents can be inferred from the analysis of an intrinsic uncertainty in summing up the PT series (infra-red renormalons, for an extensive review see [10]).

Adopting the concept of *universality* of NP phenomena one can *predict* the ratio of the magnitudes of power corrections to different observables belonging to the same $\{p, q\}$ class.

The PT-approach exploiting gluers allows to go one step further, namely to relate *absolute magnitudes* of genuine NP contributions to CIS observables with the intensity of gluer radiation, i.e. the "QCD coupling" at small transverse momentum scales.

2.1 *Phenomenology*

For example, DIS structure functions are expected to deviate from their perturbative Q^2 dependence by terms generally behaving like $1/Q^2$ ("twist 4"):

$$F_2(x,Q^2) \simeq F_2^{\text{PT}}(x,Q^2)\left[1 + D_2(x,Q^2)/Q^2\right].$$

Comparison of the Power Game prediction[11] with the data[12] allows one to extract the value of the characteristic NP parameter

$$\mathcal{A}_2 = \frac{C_F}{2\pi}\int_0^\infty dk^2\, \delta\alpha_s^{(\text{NP})}(k) \;\simeq\; 0.2\;\text{GeV}^2.$$

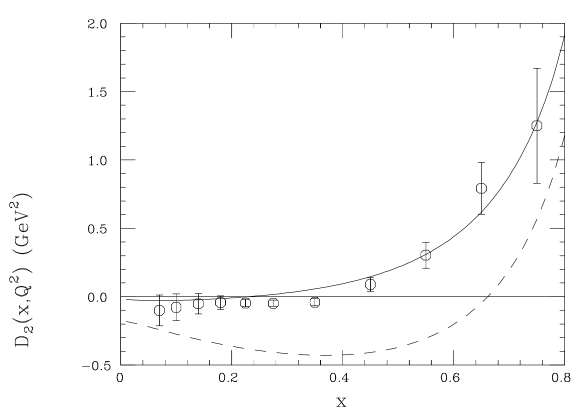

Fig.4: x-dependence of the $1/Q^2$ contribution to F_2

$\mathcal{A}_2$ being fixed, a parameter-free prediction emerges then for the $1/Q^2$ suppressed contribution to F_3 shown by the dashed curve.

Another example is provided by a variety of observables including jet shapes (Thrust, C-parameter, jet Broadenings, Oblateness), energy-energy correlation (EEC), σ_L, etc. which belong to the $p = 1/2$ class and thus exhibit numerically large NP deviations.

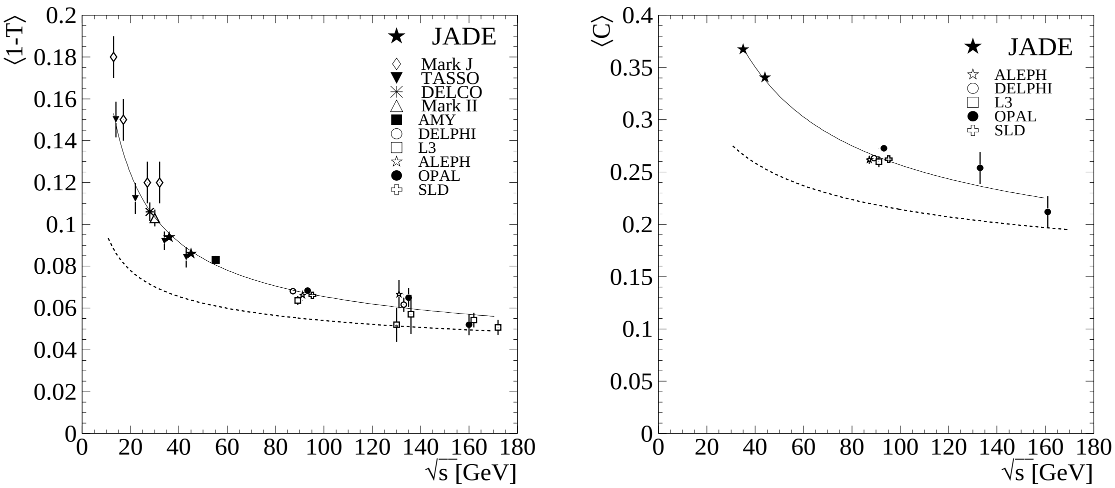

Fig.5: Mean Thrust and C-parameter in e^+e^- annihilation.[13]

A pure phenomenological study of the deviation of the mean Thrust and C-parameter values from the corresponding two-loop PT predictions shown by dotted lines in Fig. 5 hints at

$$\delta \langle 1-T \rangle^{(\mathrm{NP})} = \langle 1-T \rangle - \langle 1-T \rangle^{(\mathrm{PT})} \quad \simeq \quad 1\,\mathrm{GeV}/Q\,,$$
$$\delta \langle C \rangle^{(\mathrm{NP})} = \quad \langle C \rangle \;-\; \langle C \rangle^{(\mathrm{PT})} \quad \simeq \quad 4\,\mathrm{GeV}/Q\,,$$

with the power game bet being instead $\quad \delta \langle C \rangle^{(\mathrm{NP})} / \delta \langle 1-T \rangle^{(\mathrm{NP})} \;=\; 3\pi/2.$

2.2 Universality of confinement effects in jet shapes

The Power Game grew muscles when it was realised that it can be played not only with the Q-dependence of the means at stake. The *distributions* of shape variables were shown [14] to be subject to a $1/Q$ *shift*, by that very amount that describes the genuine NP contribution to the *mean* value of the corresponding jet shape variable.

For example, the C-parameter distribution (for the values of C not too close to zero) can be obtained by simply shifting the corresponding all-order-resummed purely perturbative spectrum by an amount inverse proportional to Q,

$$\frac{1}{\sigma}\frac{d\sigma}{dC}(C) \simeq \left(\frac{1}{\sigma}\frac{d\sigma}{dC}\right)^{\mathrm{PT}} \left(C - \frac{D_C}{Q}\right).$$

The corresponding result of a recent JADE analysis is shown in Fig. 6.

The same *shift* prescription, and similar high quality description, hold for other CIS jet observables like Thrust.

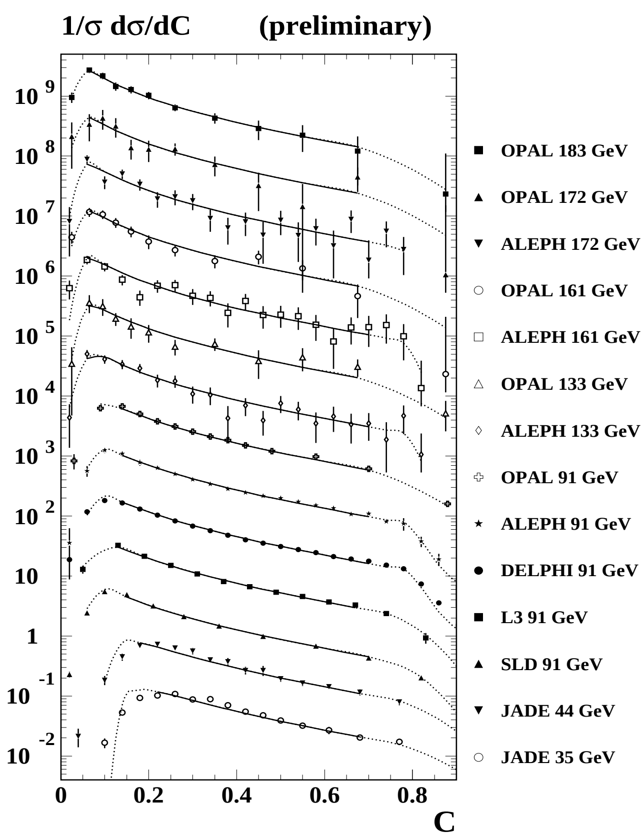

Fig.6: C-parameter distributions[15] versus PT-spectra *shifted* by $\delta C \simeq 4\,\mathrm{GeV}/Q$

A thrilling story of one important exception was told in Vancouver.[16] *Jet Broadening(s)* defined as a sum of the moduli of transverse momenta of particles in jet(s) (wrt the Thrust axis) was first predicted to have a $\log Q$-enhanced NP shift, since this NP contribution to B was naturally thought to accumulate gluers with rapidities up to $\log Q$.

The data however simply could not stand it.[17,18] Fits based on the $\log Q$-enhanced shift were bad and produced too small a value of $\alpha_s(M_Z)$, and the NP parameter α_0 inconsistent with that extracted from analyses of the Thrust and C-parameter means and distributions.

Tragic consequences for the universality belief seemed imminent.

2.3 Broadening: tragedy, catharsis, lessons

☞ Raffiniert ist der Herrgott, aber nicht boesartig
A. Einstein

Catharsis came with recognition of the fact that the Broadening measure (B) is more sensitive to quasi-collinear emissions than other jet shapes, and is therefore strongly affected by an interplay between PT and NP radiation effects. With account of the omnipresent PT gluon radiation, the *direction of the quark* that forms the jet under consideration can no longer be equated with the direction of the Thrust axis (employed in the definition of B). As a result of this interplay, the hadron distribution was found to be not only shifted but also *squeezed* with respect to its PT counterpart.

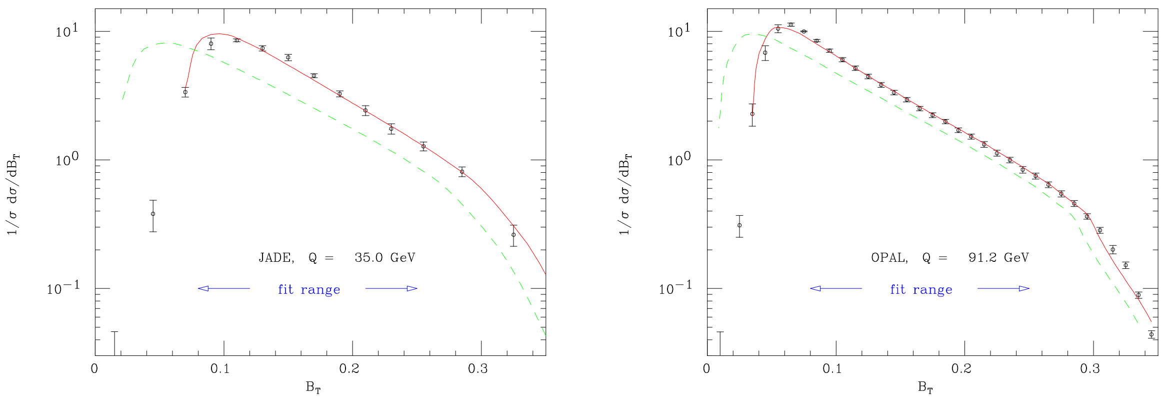

Fig.7: Perturbative (dashed) and NP-shifted/squeezed Total Broadening distributions.[19]

Three lessons can be drawn from the Broadening drama.

Pedagogical lesson the Broadenings taught, was that of the importance of keeping an eye on PT gluons when discussing effects of NP gluers. An example of a powerful interplay between the two sectors was recently given by the study of the energy-energy correlation in e^+e^- in the back-to-back kinematics.[20] The leading $1/Q$ NP contribution was shown to be promoted by PT radiation effects to a much slower falling correction, $Q^{-0.32\text{-}0.36}$.

Physical output of the proper theoretical treatment was restoration of the universality picture: within a reasonable 20% margin, the NP parameters extracted from T, C and B means and distributions were found to be the same.

Gnostic output was also encouraging. Phenomenology of NP contributions to jet shapes has shown that it is a robust field with a high discriminative power: it does not allow one to be misled by theorists.

2.4 A step forward: "shape functions"

A strong push is being given to the Power Game by the notion of Shape Function(s) introduced by G. Korchemsky and G. Sterman.[21]

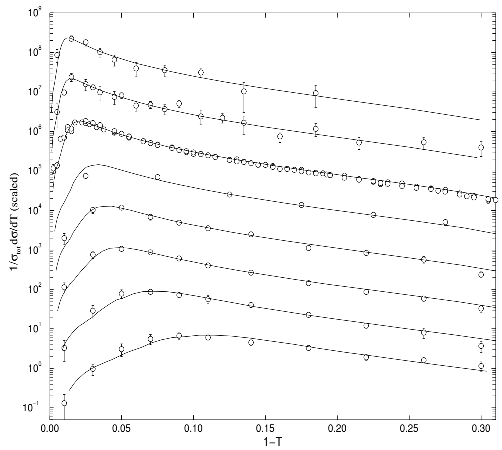

Fig.8: Thrust distributions at Q=14, 22, 35, 44, 55, 91, 133 and 161 GeV versus Power Game prediction [21]

Introducing the *distribution* describing the power shift on an event by event basis (shape function) makes it possible to lift off the condition $1-T \gg \langle 1-T\rangle^{(\mathrm{NP})}$. A simple physically motivated ansatz for such a distribution for the Thrust case,

$$\frac{1}{\sigma_{tot}}\frac{d\sigma(1-T)}{dT} = \int_0^{Q(1-T)} d\epsilon\, f(\epsilon) \frac{d\sigma\left(1-T-\frac{\epsilon}{Q}\right)}{dT}$$

with

$$f(\epsilon) = \frac{2\,(\epsilon/\Lambda)^{a-1}}{\Lambda\Gamma\left(\frac{a}{2}\right)} \exp\left(-\frac{\epsilon^2}{\Lambda^2}\right),$$

produces a remarkable fit to hadron data shown in Fig. 8.

Shape functions for different jet shapes can be related with certain characteristics of the energy-momentum flow at the hadronization stage, specific for a given observable.

2.5 Universality problem

A detailed discussion of the main problems one faces in establishing the rules of the Power Game can be found in the proceedings of the 1998 ICHEP.[16] These problems include separation of power corrections coming from the infrared region from those determined by the ultraviolet physics, merging (in a renormalon-free manner) the PT and NP contributions to the full answer, the problem of splitting the magnitude of the power term into an observable-dependent PT-*calculable* factor and a *universal* NP parameter.

The key question is whether the latter is really universal. The whole game would have little sense if it were not. Allowing each observable to have a private fitting parameter we would not learn much about the way confinement acts in hadronizing ensembles of partons produced in hard interactions.

Reasonable doubt was expressed in a seminal paper by Nason and Seymour [22] as to whether universality can be expected to hold for jet shapes which are not truly inclusive observables. The configuration of offspring partons in the gluon decay matters for jet shapes, so that the value of the power term may be affected, in an observable-dependent way, beyond the leading level in α_s (which a priori is no longer a small parameter since the characteristic momentum scale is low).

Analyses of two-loop effects in $1/Q$ suppressed contributions have been carried out for jet shapes in e^+e^- annihilation and DIS. The output proved to be surprisingly simple. It was shown that there exists a definite prescription for defining the so-called “naive” one-loop estimate of the magnitude of the power contribution, such that the two-loop effects of *non-inclusiveness* of jet shapes reduce to a universal, *observable-independent*, renormalisation of the “naive” answer by the number known as the “Milan factor”.[23,24] This is true for the NP contributions in the thrust, invariant jet mass, C-parameter and broadening distributions, for the energy-energy correlation measure, as well as for other observables subject to linear in $1/Q$ confinement effects.

It is probably the striking simplicity of the resulting prescription to be blamed for apparently cold reception the “Milan factor” enjoyed among theoreticians.

Verification of the Milan factor prescription is underway. M. Dasgupta, L. Magnea and G. Smye have undertaken the project of explicitly calculating the two-loop effects in the NP contribution to the C-parameter distribution.[25] The analytical result they are coming up with has verified the key simplification used in the original derivation of the Milan factor namely, the soft gluon approximation. This is good news. The not-so-good news is that the final expressions for $\mathcal{M}$ do differ...

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The Power Game as a new theoretical instrument emerged from its toddler years but has not yet reached respectable teens. It is understandable that, being both *predictive* and *verifiable* (the qualities almost extinct nowadays), it attracted a lot of attention and was developing, in its early days, on a week-to-week (if not a day-to-day) basis. Accelerated childhood tends to be marked by bruises, on the child's part, and by troubles on the parents'.

A partial history of *misconceptions* the advocates of the Power Game had to muddle through can be found in [16]. Now we are in a position to enrich this history with a *miscalculation*. An unfortunate omission of a trivial factor in the two-parton phase space resulted in a wrong value originally derived for $\mathcal{M}$: the so-called "non-inclusive" contribution to the Milan factor, r^{ni}, has to be multiplied by a factor of 2. As a result,

$$\begin{aligned} \mathcal{M} &= 1 + r_{in} + r_{ni} \Longrightarrow 1 + 3.299 C_A/\beta_0 + \mathbf{2} \times (-0.862 C_A - 0.052 n_f)/\beta_0 \\ &= 1 + (1.575 C_A - 0.104 n_f)/\beta_0 \;=\; 1.49 \quad \text{for } n_f = 3\,, \qquad (\text{instead of } = 1.8)\,. \end{aligned} \tag{10}$$

The n_f-part of the corrected Milan factor Eq. 10 agrees with [25] and, as the authors point out, also solves the longstanding discrepancy with the explicit two-loop calculation of the "Abelian" (n_f-dependent) correction to singlet e^+e^- fragmentation functions (σ_L) which was carried out by M. Beneke, V. Braun, and L. Magnea.[26]

Refitting jet shape data with the corrected $\mathcal{M}$ lies ahead. It will drive up the NP parameter α_0 by about 10% but will change neither α_s nor the present status of the universality pattern.

The situation with universality these days can be viewed as satisfactory. It is far from perfect, however. In particular, there seems to be a conceptual problem with describing the means and distributions of those specific jet variables that deal with a certain single jet rather than the event as a whole. The known cases this remark applies to, are the *Heavy* jet mass and the *Wide* jet broadening. An adequate game strategy for dealing with such (*less inclusive*) observables remains to be found.

A last remark is due concerning the title "Power Games". An ideology and technologies are being developed for describing genuine confinement effects in various global characteristics of multi-particle production. I believe there was a good reason for calling it a "game". To really enjoy playing one has to follow the rules (which, by the way, does not contradict the fact that some entertaining games intrinsically embody bluff).

In the present context, "the rules" means equating "PT" with the two-loop prediction and looking upon the rest as being "NP". The boundary between PT and NP physics is, to a large extent, a matter of convention. In particular, including an additional loop into a "PT prediction" (see, e.g. [27]) or redefining it, say, with use of the Borel wisdom,[28] inevitably affects the magnitude of a "genuine NP contribution". Such an elusive behaviour of NP effects may appear especially confusing in jet shape phenomenology where, according to the Sterman's lemma,[29] the NNLO (α_s^3) effects are perfectly capable of mimicking the $1/Q$ behaviour.

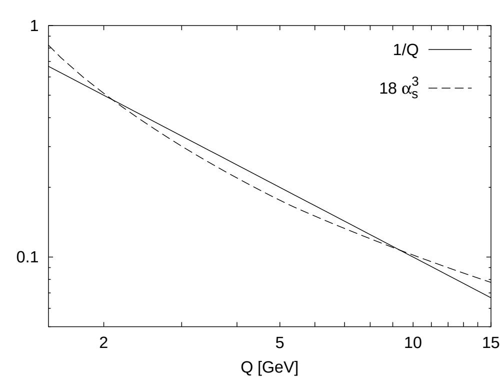

Illustration of the Sterman's lemma

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