Field theoretic approach to flat polymerized membranes

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Mainly based on PRE Letter [**105** (2022) 1, L012603] with
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LPTHE
LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

Outline

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Statistical mechanics of membranes

Random D-surfaces embedded in d-dimensional space

Complex systems whose physical properties are dominated by the entropy of thermal fluctuations (case of interest: $D = 2$ in $d = 3$ space)

Distinct types of microscopic orders

- crystalline (or tethered or polymerized): fixed-connectivity,
- other: fluid (vanishing shear modulus) and hexatic membranes

Very rich set of universality classes

Extensive studies since the 80s: discovery of the (low-temperature) flat phase of crystalline membranes [Nelson and Peleti '87, Aronovitz and Lubensky '88, David and Guitter '88, Le Doussal and Radzihovsky '92, ...]

Interest considerably boosted by the discovery of graphene (2004)
clean free-standing graphene at room T
should exhibit a critical flat phase behaviour

Physical examples of "flat" $D = 2$ crystalline membranes

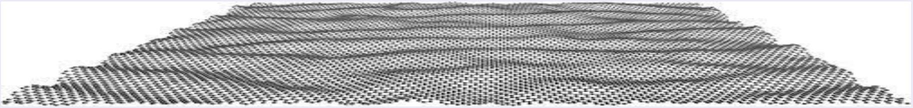
Long-range orientational order vs anomalous elasticity / rigidity
(in order to evade the Mermin-Wagner theorem)

Cytoskeletons of red blood cell membranes

Early light scattering experiments (roughness exponent measurement) give evidence for a critical flat phase behaviour [Schmidt et al. '93]

Free-standing graphene (subject to weak tension)

Ripples as out-of-plane deformations that stabilize the flat phase (possibly not fully thermal in origin, effects of disorder, boundaries, electrons ...)



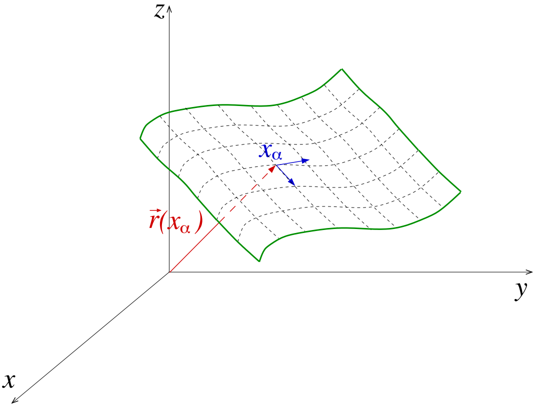
Evidence for anomalous effects [Nicholl et al. '15, '17; Colangelo et al. '19; Lopez-Polin et al. '15, '21]

Theoretical model of a (flat) crystalline membrane

D-dimensional manifold in *d*-dimensional space described by:

- *D*-dimensional (rest) coordinates $\mathbf{x}$,
- *d*-dimensional (embedding) vector $\mathbf{r}(\mathbf{x})$.

For an undeformed (flat) structure: $\mathbf{r}(\mathbf{x}) = \mathbf{x}$



[Image from the review [M. Bowick and A. Travesset '00]]

Theoretical model of a (flat) crystalline membrane

D-dimensional manifold in d-dimensional space described by:

- D-dimensional (rest) coordinates $\mathbf{x}$,
- d-dimensional (embedding) vector $\mathbf{r}(\mathbf{x})$.

Landau action functional

Global translational invariance of S: depends only on derivatives $\partial_i \mathbf{r}$ $(i = 1, \cdots, D) \Rightarrow$ derivative field theory

$$
\begin{aligned}
S[\mathbf{r}] &= \int \mathrm{d}^D x \left[\frac{\kappa}{2} \left(\partial_i^2 \mathbf{r} \right)^2 + \frac{t}{2} \left(\partial_i \mathbf{r} \right)^2 + u \left(\partial_i \mathbf{r} \, \partial_j \mathbf{r} \right) + v \left(\partial_i \mathbf{r} \, \partial_i \mathbf{r} \right) \right] \\
&+ \frac{b}{2} \int \mathrm{d}^D x \int \mathrm{d}^D y \, \delta^{(d)} \left(\mathbf{r}(\mathbf{x}) - \mathbf{r}(\mathbf{y}) \right)
\end{aligned}
$$

- κ: bending rigidity (extrinsic curvature)
- t, u, v: elastic constants (related to Lamé coefficients)
- b: excluded volume (no self-avoidance: $b = 0$, phantom membrane)

Theoretical model of a (flat) crystalline membrane

Two-field model [Nelson and Peleti '87, Aronovitz and Lubensky '88]

Low-T expansion: $\mathbf{r}(\mathbf{x}) = \big(\mathbf{x} + \mathbf{u}(\mathbf{x}),\, \mathbf{h}(\mathbf{x})\big)$

- $\mathbf{u}$ is the D-dimensional (in-plane) phonon field,
- $\mathbf{h}$ the $d - D$-dimensional (out-of-plane) flexural field.

$$S[\mathbf{u}, \mathbf{h}] = \int \mathrm{d}^D x \left[\frac{\kappa}{2} \left(\partial_i^2 \mathbf{h}\right)^2 + \frac{\lambda}{2}\, u_{ii}^2 + \mu\, u_{ij}^2\right] + \text{irrelevant}$$

with strain tensor $u_{ij} = \partial_i u_j + \partial_j u_i + \partial_i \mathbf{h} \cdot \partial_j \mathbf{h} + \text{irrelevant}$

Dimensional analysis: $[u] = D - 3$, $[h] = (D-4)/2$, $[\lambda] = [\mu] = 4 - D$.

Upper critical dimension: $D_{\mathrm{uc}} = 4$

Massless fields: long-range elastic interactions (evade Mermin-Wagner)

$S[\mathbf{u}, \mathbf{h}]$ is quadratic in $\mathbf{u}$: possible to integrate $\mathbf{u}$ exactly

Theoretical model of a (flat) crystalline membrane

Effective model [Nelson and Peleti '87, Aronovitz and Lubensky '88]

Non-trivial tensor structure with non-local coupling

$$\begin{aligned} S_{\text{eff}}[\mathbf{h}] &= \frac{\kappa}{2}\int_{\mathbf{k}} k^4\,|\mathbf{h}(\mathbf{k})|^2 \\ &+ \frac{1}{4}\int_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4} \mathbf{h}(\mathbf{k}_1).\mathbf{h}(\mathbf{k}_2)\, R_{ab,cd}(\mathbf{q})\, k_1^a\, k_2^b\, k_3^c\, k_4^d\; \mathbf{h}(\mathbf{k}_3).\mathbf{h}(\mathbf{k}_4) \end{aligned}$$

where $R_{ab,cd}(\mathbf{q}) = b\; N_{ab,cd}(\mathbf{q}) + \mu\, M_{ab,cd}(\mathbf{q})$

$$\begin{aligned} N_{ab,cd}(\mathbf{q}) &= \frac{1}{D-1}\, P^T_{ab}(\mathbf{q})\, P^T_{cd}(\mathbf{q}), \quad P^T_{ab}(\mathbf{q}) = \delta_{ab} - \frac{q_a q_b}{\mathbf{q}^2} \\ M_{ab,cd}(\mathbf{q}) &= \frac{D-1}{2}\left[N_{ac,bd}(\mathbf{q}) + N_{ad,bc}(\mathbf{q})\right] - N_{ab,cd}(\mathbf{q}) \end{aligned}$$

"New" (D-dependent) coupling: $b = \mu\,(D\lambda + 2\mu)/(\lambda + 2\mu)$

The 2-field model and the EFT are completely equivalent.

Anomalous elasticity in the flat phase from renormalization

Scaling behaviour of correlation functions

$$G_{uu} \sim q^{-(2+\eta_u)}, \qquad G_{hh} \sim q^{-(4-\eta)}$$

with anomalous dimensions η_u and η such that

$$\eta_u = 4 - D - 2\eta \quad \text{(from Ward identities)}$$

$\Rightarrow$ a single exponent $\eta > 0$ (e.g., roughness: $\zeta = (4 - D - \eta)/2$)

Strong renormalizations [Nelson and Peleti '87, Aronovitz and Lubensky '88, David and Guitter '88]

- enhanced (length-scale dependent) rigidity: $\kappa_R(q) \sim q^{-\eta}$
- softened elastic constants: $\mu_R(q) \sim \lambda_R(q) \sim q^{\eta_u}$

Long-range orientational order (normals to the membrane)
Lower critical dimension $D_{\mathrm{lc}} < 2$ [Aronovitz, Golubović and Lubensky '89]

Note: auxetic behaviour characterized by a negative Poisson ratio

Flat phase: quick review of 3 decades of results

- ϵ-expansion ($D = 4 - 2\epsilon$):

$$\eta^{(\text{1loop})} = 0.96 \quad \text{[Aronovitz and Lubensky '88; David, Guitter et al. '89]}$$
$$\eta^{(\text{2loop})} = 0.795 \quad \text{[Mauri and Katsnelson '20]}$$

- Self Consistent Screening Approximation (SCSA):

$$\eta^{(\text{SCSA})} = 0.821 \quad \text{[Le Doussal and Radzihovsky '92]}$$
$$\eta^{(\text{improved SCSA})} = 0.789 \quad \text{[Gazit '09]}$$

- Non Perturbative Renormalization Group (NPRG):

$$\eta^{(\text{NPRG})} = 0.849 \quad \text{[Kownacki and Mouhanna '09]}$$

- Monte Carlo simulations:

$$\eta^{(\text{sim})} = 0.750(5) \quad \text{[Bowick et al. '96]} \quad = 0.795(10) \quad \text{[Tröster '13]}$$

- Experiments: $\eta^{(\text{blood cells})} = 0.70(20)$ [Schmidt et al. '93]
  $\eta^{(\text{graphene})} \approx 0.82$ [Lopez-Polin et al. '15]

Flat phase: quick review of 3 decades of results

Other methods:

- Large-d approach ($d = 3$):

$$\eta^{(\text{Large-}d)} = \frac{2}{d} + O(1/d^2) = 0.667$$ [Aronovitz, Golubović and Lubensky '89]

[Aronovitz, Golubović and Lubensky '89]

- Large-d_c ($d_c = d - D$) approach:

$$\eta^{(\text{Large-}d_c)} = \frac{2}{d_c} - \frac{68\zeta_3 - 73}{27 d_c^2} + O(1/d_c^3) = 1.676$$ [Saykin et al. '20]

- modern amplitude techniques (bootstrap) ?

  [Mauri and Katsnelson '20]: membrane models are scale but not conformal invariant (and possibly non-unitary)

  modern amplitude techniques (bootstrap) do not apply

Flat phase: quick review of 3 decades of results

This talk: higher orders of the ϵ-expansion

- More accuracy on the value of η
- Better understanding of the perturbative structure of the flat phase
- Comparison with NPRG and SCSA

More than 3 decades between the first (1-loop) result and the 2-loop one...

2-loop: [O. Coquand, D. Mouhanna and ST, PRE '20]

3-loop: [S. Metayer, D. Mouhanna and ST, PRE Letter '22]

Note: 4-loop in 2-field model [Pikelner '22]

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Anomalous elasticity, fluctuations and disorder in elastic membranes

Check for updates

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on the underlying embedding-space rotational invariance. The results of the two-loop calculation are in progress, and already indicate that the SCSA is not exact, with the deviations appearing at the two-loop order [79].

We also observe from (81) that the solution for $d_c = 0$ is $\eta(D, d_c = 0) = \frac{4-D}{2}$, i.e., $\eta_u = 0$, which is

[78] P.M. Chaikin, T.C. Lubensky, Principles of Condensed Matter Physics, Cambridge University Press, Cambridge, 1995.
[79] L. Radzihovsky, B. Jacobsen, Ph.D. Thesis, University of Colorado at Boulder (2003), unpublished.
[80] M. Warner, E.M. Terentjev, Liquid Crystal Elastomers, Oxford Science Publications, 2005.

Outline

Perturbative setup in the 2-field model

$$S[\mathbf{u},\, \mathbf{h}] = \int \mathrm{d}^D x \left[\frac{\kappa}{2} \left(\partial_i^2 \mathbf{h}\right)^2 + \frac{\lambda}{2}\, u_{ii}^2 + \mu\, u_{ij}^2 \right] + \text{irrelevant}$$

with strain tensor $u_{ij} = \partial_i u_j + \partial_j u_i + \partial_i \mathbf{h} \cdot \partial_j \mathbf{h} + \text{irrelevant}$
Feynman rules:

- Free massless flexuron propagator ($\alpha, \beta = 1, \cdots, d_c$):

$$S^{(0)}_{\alpha\beta}(\mathbf{k}) = \frac{\delta_{\alpha,\beta}}{k^4} \quad = \quad \alpha \xrightarrow[\mathbf{k}]{} \beta$$

- Free massless phonon propagator ($i, j = 1, \cdots, D$):

$$D^{(0)}_{ij}(\mathbf{q}) = \frac{1}{q^2} \left(\frac{1}{\mu} P^{(\perp)}_{ij}(\mathbf{q}) + \frac{1}{2\mu + \lambda}\, P^{(\|)}_{ij}(\mathbf{q}) \right) \quad = \quad i \underset{\mathbf{q}}{\sim\sim\sim} j$$

Perturbative setup in the 2-field model

$$S[\mathbf{u},\, \mathbf{h}] = \int \mathrm{d}^D x \left[\frac{\kappa}{2}\left(\partial_i^2 \mathbf{h}\right)^2 + \frac{\lambda}{2}\, u_{ii}^2 + \mu\, u_{ij}^2\right] + \text{ irrelevant}$$

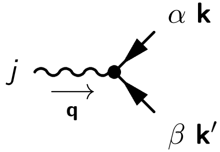
with strain tensor $u_{ij} = \partial_i u_j + \partial_j u_i + \partial_i \mathbf{h} \cdot \partial_j \mathbf{h} + \text{irrelevant}$

Feynman rules:

- 3-point phonon-flexuron vertex:

$$\Gamma^{j\,(0)}_{\alpha\beta}(\mathbf{k}, \mathbf{k}', \mathbf{q}\,) \;\; = \;\; -\frac{\mathrm{i}}{2}\,\delta_{\alpha,\beta}\,\left[\mu\left(\mathbf{q}\cdot\mathbf{k}\;\, k'_j + \mathbf{q}\cdot\mathbf{k}'\;\, k_j\right) + \lambda\,\mathbf{k}\cdot\mathbf{k}'\;\, q_j\right]$$

$=$



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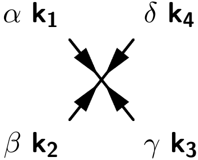
with strain tensor $u_{ij} = \partial_i u_j + \partial_j u_i + \partial_i \mathbf{h} \cdot \partial_j \mathbf{h} + \text{irrelevant}$
Feynman rules:

- 4-point (fully symmetrized) flexuron vertex:

$$\Gamma^{(0)}_{\alpha\beta\gamma\delta}(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3},\mathbf{k_4}) = \frac{\lambda}{24}\,[\ \delta_{\alpha,\beta}\,\delta_{\gamma,\delta}\,\mathbf{k_1}\cdot\mathbf{k_2}\ \mathbf{k_3}\cdot\mathbf{k_4} + \cdots] + \\ + \frac{\mu}{24}\,[\ \delta_{\alpha,\beta}\,\delta_{\gamma,\delta}\,\mathbf{k_1}\cdot\mathbf{k_3}\ \mathbf{k_2}\cdot\mathbf{k_4} + \cdots\ ]$$

$=$



Perturbative setup in the EFT

$$
\begin{aligned}
S_{\text{eff}}[\mathbf{h}] \;&=\; \frac{\kappa}{2}\int_{\mathbf{k}} k^4\,|\mathbf{h}(\mathbf{k})|^2 \\
&+\; \frac{1}{4}\int_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4} \mathbf{h}(\mathbf{k}_1).\mathbf{h}(\mathbf{k}_2)\,R_{ab,cd}(\mathbf{q})\,k_1^a\,k_2^b\,k_3^c\,k_4^d\;\mathbf{h}(\mathbf{k}_3).\mathbf{h}(\mathbf{k}_4)
\end{aligned}
$$

Feynman rules:

- Free massless flexuron propagator ($\alpha,\beta = 1,\cdots,d_c$):

$$
S^{(0)}_{\alpha\beta}(\mathbf{k}) = \frac{\delta_{\alpha,\beta}}{k^4} \quad = \quad \alpha \;\xrightarrow[\mathbf{k}]{}\; \beta
$$

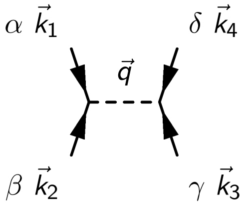
Perturbative setup in the EFT

$$
\begin{aligned}
S_{\text{eff}}[\mathbf{h}] &= \frac{\kappa}{2}\int_{\mathbf{k}} k^4\,|\mathbf{h}(\mathbf{k})|^2 \\
&+ \frac{1}{4}\int_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4} \mathbf{h}(\mathbf{k}_1).\mathbf{h}(\mathbf{k}_2)\, R_{ab,cd}(\mathbf{q})\, k_1^a\, k_2^b\, k_3^c\, k_4^d\ \mathbf{h}(\mathbf{k}_3).\mathbf{h}(\mathbf{k}_4)
\end{aligned}
$$

Feynman rules:

- 4-point flexuron vertex:

$$
V^{(0)}_{\alpha\beta\gamma\delta}(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3},\mathbf{k_4}) = -\frac{R_{abcd}(\mathbf{k_1}+\mathbf{k_2}\,)}{4}\,\delta_{\alpha,\beta}\,\delta_{\gamma,\delta}\,k_1^a\,k_2^b\,k_3^c\,k_4^d
$$

$=$ 

Dyson equations in 2-field model and EFT

In the 2-field model:

$$S_{\alpha\beta}(\mathbf{k}) = S^{(0)}_{\alpha\beta}(\mathbf{k}) + S^{(0)}_{\alpha\gamma}(\mathbf{k})\,\Sigma_{\gamma\delta}(\mathbf{k})\,S_{\delta\beta}(\mathbf{k})$$
$$D_{ij}(\mathbf{q}) = D^{(0)}_{ij}(\mathbf{q}) + D^{(0)}_{ik}(\mathbf{q})\Pi_{kl}(\mathbf{q})\,D_{lj}(\mathbf{q})$$

$\Rightarrow$ compute the self-energies Σ (flexuron) and Π (phonon)

Parametrization for the flexuron self-energy (IR safe correlator despite $1/k^4$):

$$S_{\alpha\beta}(\mathbf{k}) = \frac{\delta_{\alpha,\beta}}{k^4}\,\frac{1}{1-\tilde{\Sigma}(k^2)}\,, \qquad \tilde{\Sigma}(k^2) = k^{-4}\,\Sigma(\mathbf{k})$$

Parametrization for the phonon self-energy:

$$D_{\parallel}(\mathbf{q}) = \frac{1}{(2\mu+\lambda)\,q^2}\,\frac{1}{1-\tilde{\Pi}_{\parallel}(q^2)}\,, \qquad \tilde{\Pi}_{\parallel}(q^2) = \frac{1}{(2\mu+\lambda)\,q^2}\,\Pi_{\parallel}(q^2)$$
$$D_{\perp}(\mathbf{q}) = \frac{1}{\mu\,q^2}\,\frac{1}{1-\tilde{\Pi}_{\perp}(q^2)}\,, \qquad \tilde{\Pi}_{\perp}(q^2) = \frac{1}{\mu\,q^2}\,\Pi_{\perp}(q^2)$$

Dyson equations in 2-field model and EFT

In the EFT:

$$S_{\alpha\beta}(\mathbf{k}) = S^{(0)}_{\alpha\beta}(\mathbf{k}) + S^{(0)}_{\alpha\gamma}(\mathbf{k})\,\Sigma_{\gamma\delta}(\mathbf{k})\,S_{\delta\beta}(\mathbf{k})$$

$$R_{abcd}(\mathbf{q}) = R^{(0)}_{abcd}(\mathbf{q}) + R^{(0)}_{abef}(\mathbf{q})\Pi_{efgh}(\mathbf{q})\,R_{ghcd}(\mathbf{q})$$

$\Rightarrow$ compute the self-energies Σ (flexuron) and Π (vertex)

Parametrization for the vertex self-energy:

$$R_{abcd}(\mathbf{q}) = R^N(\mathbf{q})\,N_{abcd}(\mathbf{q}) + R^M(\mathbf{q})\,M_{abcd}(\mathbf{q})\,,$$

$$\Pi_{abcd}(\mathbf{q}) = \Pi^N(\mathbf{q})\,N_{abcd}(\mathbf{q}) + \Pi^M(\mathbf{q})\,M_{abcd}(\mathbf{q})\,,$$

$$R^N(\mathbf{q}) = \frac{b}{1-\tilde{\Pi}^N(\mathbf{q})}\,, \qquad \tilde{\Pi}^N(\mathbf{q}) = b\,\Pi^N(\mathbf{q})\,,$$

$$R^M(\mathbf{q}) = \frac{\mu}{1-\tilde{\Pi}^M(\mathbf{q})}\,, \qquad \tilde{\Pi}^M(\mathbf{q}) = \mu\,\Pi^M(\mathbf{q})\,.$$

Self-energy computations

Perturbative solution of the Dyson equations in 2-field model and EFT

$\Rightarrow$ compute the self-energies Σ and Π

Efficient use of massless Feynman diagram technics [Kotikov and ST '19]

Fully automated computations (by Simon Metayer):

- Feynman diagrams generated using QGRAF [Nogueira '93]

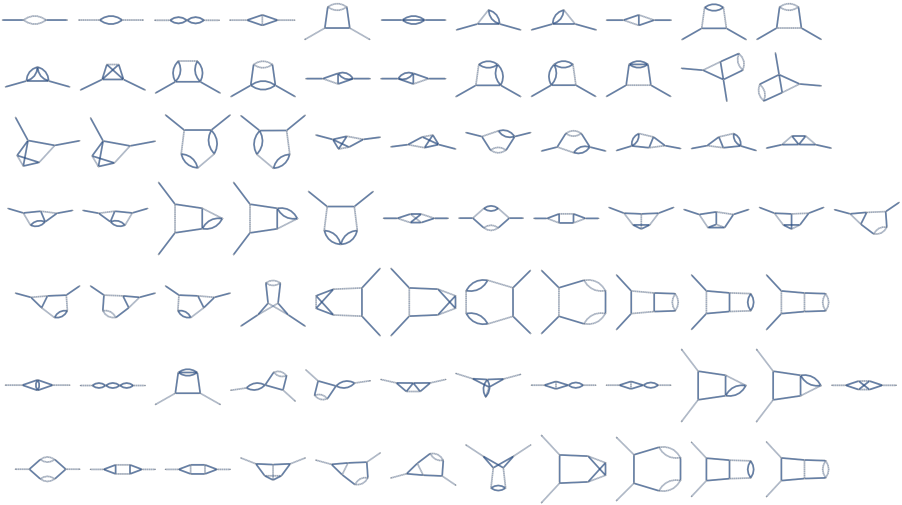
  Total number of diagrams at 3 loop:

  - 2-field model: 61 diagrams
  - EFT: 32 diagrams

- import to MATHEMATICA and perform numerator and tensor algebra
- reduction to master integrals using LITERED [Lee '14]
- compute renormalization group (RG) flows and functions

Our approach is exact order by order in perturbation theory

View of 1, 2 and 3-loop self-energy diagrams in the 2-field model:



Renormalization conventions

Computations in dimensional regularization ($D = 4 - 2\varepsilon$)

Renormalization constants computed in the $\overline{\text{MS}}$ scheme $\overline{M}^2 = 4\pi e^{-\gamma_E} M^2$

$$\mathbf{u} = Z\,\mathbf{u_r}, \quad \mathbf{h} = Z^{1/2}\,\mathbf{h_r}, \quad \mu = Z_\mu\,\mu_r\,M^{2\varepsilon}, \quad \lambda = Z_\lambda\,\lambda_r\,M^{2\varepsilon}, \quad b = Z_b\,b_r\,M^{2\varepsilon}$$

where they take the form of Laurent series in ε

Approach is algebraic (as in QCD):

- 2-field model:

$$\text{finite} = (p^4 - \Sigma)\,Z, \quad \text{finite} = (p^2\,Z_\mu\,\mu_r - \Pi_\perp)\,Z^2,$$
$$\text{finite} = (p^2(Z_\lambda\,\lambda_r + 2\,Z_\mu\,\mu_r) - \Pi_{||})\,Z^2$$

- EFT:

$$\text{finite} = (p^4 - \Sigma)\,Z, \quad \text{finite} = ((Z_\mu\,\mu_r)^{-1} - \Pi_M)\,Z^{-2},$$
$$\text{finite} = ((Z_b\,b_r)^{-1} - \Pi_N)\,Z^{-2}$$

Renormalization group functions

Beta functions and field anomalous dimension extracted from the Zs

$$\beta_x = M\,\partial_M Z_x \qquad (x = \{\lambda, \mu, b\}),$$
$$\eta = M\,\partial_M \log Z$$

Perturbative solution of system of equations $\Rightarrow$ fixed points

- 2-field model:

$$\beta_\lambda(\lambda^*, \mu^*) = 0\,, \qquad \beta_\mu(\lambda^*, \mu^*) = 0$$

- EFT

$$\beta_\mu(\mu^*, b^*) = 0\,, \qquad \beta_b(\mu^*, b^*) = 0$$

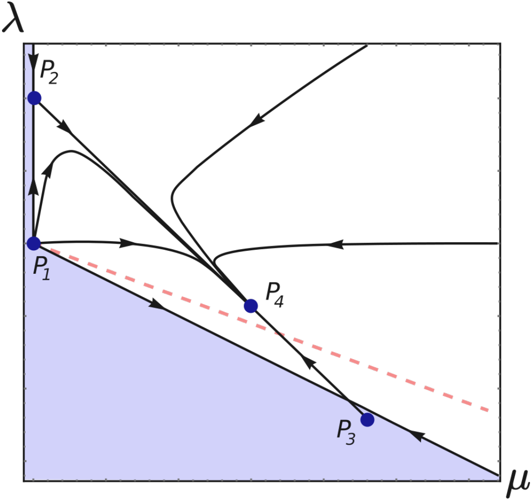
Derive scheme-independent and universal $\eta(\lambda^*, \mu^*)$ and $\eta(\mu^*, b^*)$

strong check: $\quad \eta(\lambda^*, \mu^*) = \eta(\mu^*, b^*)$

Outline

Fixed points and anomalous dims in the 2-field model

We find four fixed points (λ_i^*, μ_i^*) $(i = 1, \cdots, 4)$



Mechanical stability delimited by $\mu = 0$ and $2\lambda + \mu = 0$

Special line (NPRG, SCSA, 1-loop): $3\lambda + \mu = 0$

Four fixed points (λ_i^*, μ_i^*) $(i = 1, \cdots, 4)$:

- P_1: unstable gaussian FP ($\lambda_1^* = \mu_1^* = 0$) with $\eta_1 = 0$
- P_2: unstable shearless FP ($\lambda_2^* = 32\pi^2\epsilon/d_c$, $\mu_2^* = 0$) with $\eta_2 = 0$

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- P_2: unstable shearless FP $(\lambda_2^* = 32\pi^2\epsilon/d_c,\ \mu_2^* = 0)$ with $\eta_2 = 0$
- P_3: mechanically unstable at 2 and 3-loop (negative bulk-modulus $B_3^* = \lambda_3^* + 2\mu_3^*/D < 0$)

$$\eta_3 = 0.9524\,\epsilon - 0.0711\,\epsilon^2 - 0.0698\,\epsilon^3 - 0.0750\,\epsilon^4 + O(\epsilon^5)$$

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- P_4: IR-stable non-trivial fixed point $(\lambda_4^*$ and $\mu_4^*)$

$$\eta \equiv \eta_4 = \frac{24}{25}\,\epsilon - \frac{144}{3125}\,\epsilon^2 - \frac{4(1286928\,\zeta_3 - 568241)}{146484375}\,\epsilon^3 + O(\epsilon^4)$$

P_4 contols the physics of the flat phase

1-loop: [Aronovitz and Lubensky '88]
2-loop: [Coquand, Mouhanna and ST, PRE '20]
3-loop: [Metayer, Mouhanna and ST, PRE Letter '22]
4-loop: [Pikelner, EPL '22]

Four fixed points (λ_i^*, μ_i^*) $(i = 1, \cdots, 4)$:

- P_1: unstable gaussian FP $(\lambda_1^* = \mu_1^* = 0)$ with $\eta_1 = 0$
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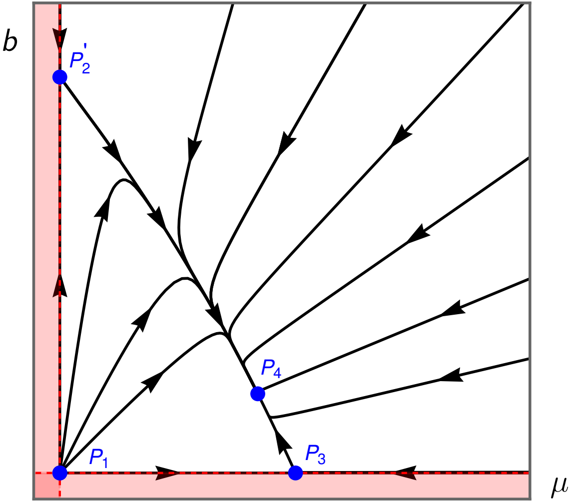
1-loop: [Aronovitz and Lubensky '88]
2-loop: [Coquand, Mouhanna and ST, PRE '20]
3-loop: [Metayer, Mouhanna and ST, PRE Letter '22]
4-loop: [Pikelner, EPL '22]

Fixed points and anomalous dims in the EFT

We find four fixed points (b_i^*, μ_i^*) $(i = 1, \cdots, 4)$



Mechanical stability delimited by $\mu = 0$ and $b = 0$

Four fixed point (b_i^*, μ_i^*) $(i = 1, \cdots, 4)$

- P_1: unstable gaussian FP $(b_1^* = \mu_1^* = 0)$ with $\eta_1 = 0$
- P'_2: unstable shearless FP $(\mu_2^* = 0,\ b_2^* \neq 0)$ with

$$\eta'_2 = 0.8000\,\varepsilon - 0.0053\,\varepsilon^2 + 0.0110\,\varepsilon^3 + O(\varepsilon^4)$$

Note that at P'_2: $\eta^{(2\mathrm{loop})} = 0.795$ [Mauri and Katsnelson '20]

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Same result as in the 2-field model !

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$$\eta \equiv \eta_4 = \frac{24}{25}\,\epsilon - \frac{144}{3125}\,\epsilon^2 - \frac{4(1286928\,\zeta_3 - 568241)}{146484375}\,\epsilon^3 + O(\epsilon^4)$$

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Structure of the perturbative series and comparison

From our results (+ Pikelner's 4-loop) we find ($d_c = 1$ and $D = 4 - 2\varepsilon$):

- P_2': $\eta_2' = 0.8000\,\varepsilon - 0.0053\,\varepsilon^2 + 0.0110\,\varepsilon^3 + O(\varepsilon^4)$
- P_3: $\eta_3 = 0.9524\,\epsilon - 0.0711\,\epsilon^2 - 0.0698\,\epsilon^3 - 0.0750\,\epsilon^4 + O(\epsilon^5)$
- P_4: IR-stable non-trivial fixed point

$$
\begin{aligned}
\eta \equiv \eta_4 &= 0.9600\,\epsilon - 0.0461\,\epsilon^2 - 0.0267\,\epsilon^3 - 0.0200\,\epsilon^4 + O(\epsilon^5) \\
\eta^{\mathrm{SCSA}} &= 0.9600\,\epsilon - 0.0476\,\epsilon^2 - 0.0280\,\epsilon^3 - 0.0177\,\epsilon^4 + O(\epsilon^5) \\
\eta^{\mathrm{NPRG}} &= 0.9600\,\epsilon - 0.0367\,\epsilon^2 - 0.0266\,\epsilon^3 - 0.0178\,\epsilon^4 + O(\epsilon^5)
\end{aligned}
$$

Asymptotic series but small (and essentially decreasing) coefficients

Case $D = 2$ ($\varepsilon = 1$) and at P_4:

$$\eta^{\text{1-loop}} = 0.96\,,\ \eta^{\text{2-loop}} = 0.9139\,,\ \eta^{\text{3-loop}} = 0.8872\,,\ \eta^{\text{4-loop}} = 0.8670$$

$$\eta^{\mathrm{SCSA}} = 0.8209\,,\ \eta^{\mathrm{NPRG}} = 0.8491\,,\ \eta^{\mathrm{sim}} = 0.795(10)\,,\ \eta^{[2/2]} = 0.806$$

Beyond the rainbow (cancellations)

Factor $\mathcal{V}$ (our previous results correspond to $\mathcal{V}=1$)

SCSA exact in the rainbow approximation ($\mathcal{V}=0$ no vertex corrections)

- P_2':

$$\eta_2' = 0.8\,\varepsilon - 0.00533\,\varepsilon^2 + (0.02478 - 0.01376 \times \mathcal{V})\,\varepsilon^3 + O(\varepsilon^4)$$

$$\eta^{\text{SCSA}} = 0.8\,\epsilon - 0.00533\,\epsilon^2 + 0.02478\,\epsilon^3 + O(\epsilon^4)$$

$$\eta^{\text{NPRG}} = 0.8\,\epsilon + 0.0347\,\epsilon^2 + 0.0098\,\epsilon^3 + O(\epsilon^4)$$

- P_3:

$$\eta_3 = 0.9524\,\epsilon - (0.067 + 0.0043 \times \mathcal{V})\,\epsilon^2 - (0.056 + 0.014 \times \mathcal{V})\,\epsilon^3 + O(\epsilon^4)$$

$$\eta^{\text{SCSA}} = 0.9524\,\epsilon - 0.067\,\epsilon^2 - 0.056\,\epsilon^3 + O(\epsilon^4)$$

$$\eta^{\text{NPRG}} = 0.9524\,\epsilon - 0.054\,\epsilon^2 + 0.052\,\epsilon^3 + O(\epsilon^4)$$

- P_4:

$$\eta_4 = 0.96\,\epsilon - (0.0476 - 0.0015 \times \mathcal{V})\,\epsilon^2 - (0.0280 - 0.0012 \times \mathcal{V})\,\epsilon^3 + O(\epsilon^4)$$

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Outline

Conclusion

Field theoretic approach to the flat phase of polymerized membranes:

- highly derivative field theories
- intricate tensor structure
- requires state of the art automation of calculations

From our exact results:

- $\eta^{\text{3-loop}} = 0.8872$ in the range of values obtained by other methods
- 2-field model and EFT have identical final results (strong check) despite the fact that intermediate steps differ
- asymptotic perturbative series have remarkably small (and mostly decreasing) coefficients (very rare in theoretical physics)
- this peculiar structure partly arises from strong cancellations, e.g., of vertex corrections $\Rightarrow$ explains the success of SCSA (and NPRG)

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