

NEUTRINOPRODUCTION OF PIONS NEAR THE FIRST RESONANCE

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Abstract: Inelastic scattering of neutrinos in the region of the $N^*(1236)$ resonance was calculated, using a model of the type "isobar plus Born terms". Applying CVC and PCAC in a strict way, no free parameter was left. In our analytic calculation, the generalized helicity method was used, in order to separate the various polarization terms in the cross section. Numerical results are shown separately for the vector part, the axial part and the vector-axial interference in both the Born term contribution and the isobar contribution. Finally, a comparison is made with experimental data obtained at CERN.

1. INTRODUCTION

Inelastic scattering of high-energy neutrinos has become a popular subject among high-energy physicists since the time the first experiments on neutrinos were started at CERN [1]. In connection with these and some more recent experiments [2, 3], a large number of theoretical calculations has also been performed [4-17], mainly on neutrino production in the region of the first N^* resonance. The model presented here, based upon summing up Feynman diagrams for the isobar term plus the Born terms (fig. 1) is not essentially different from some others, in particular from that of Salin [14]. However, our work contains some distinctive features, which are the following:

The general structure of the cross section, as a combination of various polarization terms, is shown by using the generalized helicity method [18].

The consequences of a strict application of PCAC (partial conservation of the axial current) are stressed; in particular, this application is shown to require the addition of a contact term to the axial Born terms.

We give some numerical indications on the relative contributions of the vector part, the axial part and the vector-axial interference in both the isobar term and the Born terms.

Some additional characteristics of our model are the following:

We neglect the interference between resonant term and Born terms, by using the following qualitative argument: On the mass shell of the N^* , this interference term vanishes; on either side of the mass shell, it takes oppo-

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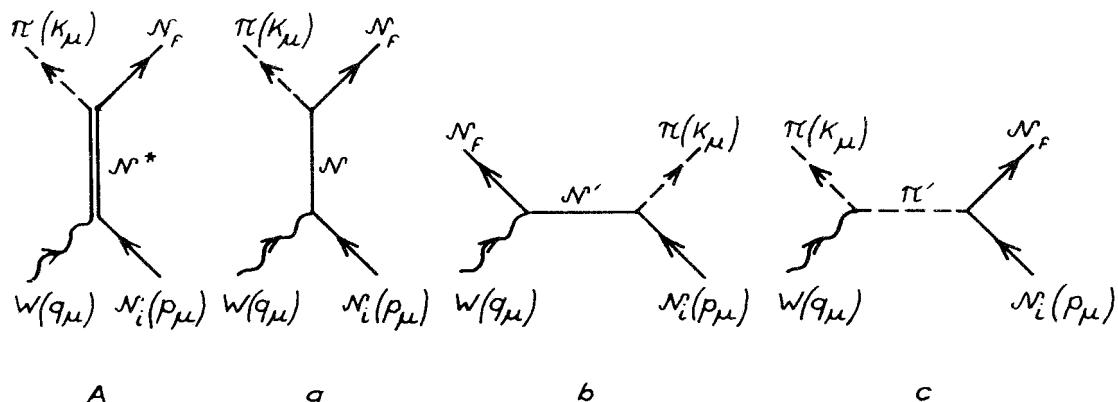


Fig. 1. Feynman diagrams for neutrinoproduction of a single pion. (W represents the vector plus axial current; the ν W vertex was removed.) A) Resonant term; a), b), c) Born terms.

site signs, so that approximate cancellation may be assumed when one integrates over a mass region approximately centered at 1236 MeV.

For the vector part, we use the CVC (conserved vector current) hypothesis, together with the results of Cochard and Kessler [19] who used a similar model for electro- and photoproduction.

For the axial part, we use, in both the isobar term and the Born terms, one-pion exchange plus PCAC. This determines the four coupling constants we need.

Using a universal form factor of the "dipole" type, there is no free parameter left in our calculation.

2. DETAILS OF CALCULATION

2.1. Application of the generalized helicity method

We extend here the treatment shown in ref. [18] for electron scattering to the case of (elastic or inelastic) neutrino scattering. We thus consider the graph of fig. 2, where W represents the vector plus axial "particle" exchanged; \mathcal{N}' is the final hadronic state, i.e. either the nucleon (in the case of elastic scattering) or some baryonic resonance, or a pion + nucleon system (it is easily shown that, for such a system, the helicity treatment is exactly the same as for a single particle, once one assumes that no angular distribution of the hadrons is measured; see ref. [20] for proof).

Here we must divide the lepton current j_μ and the hadron current j'_μ into a vector and an axial part [‡]:

$$j_\mu = v_\mu + a_\mu, \quad j'_\mu = v'_\mu + a'_\mu. \quad (1)$$

Introducing here again, as in ref. [18], the systems of unit four-vectors ϵ_m and ϵ'_m (see in particular fig. 6 of ref. [18]), defining here again the helicity amplitudes:

[‡] For simplicity, and without loss of generality, we shall assume all current components to be real.

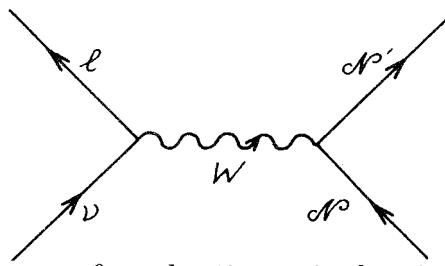


Fig. 2. Feynman diagram for elastic or inelastic neutrino scattering.

$$j_m = (\epsilon_m, j) / |\epsilon_m|, \quad j'_m = (\epsilon'_m, j') / |\epsilon'_m|, \quad (2)$$

and applying this definition to both the vector and axial parts, we get:

$$n_i I = \sum (v_m + a_m)(v_n + a_n) r^{mm'} r^{nn'} (v'_{m'} + a'_{m'}) (v'_{n'} + a'_{n'}), \quad (3)$$

where I is the interaction rate (dynamic factor in the cross section), n_i is the number of initial spin states, and Σ means summation over all spin states of external particles (summation over m, m', n, n' is implied by using covariant and contravariant indices); $r^{mm'}$ is the rotation matrix defined by eq. (4) in ref. [18]; let us recall that $m, m' = +, 0, -, \parallel$.

Introducing new subscripts k, k' ($= +, 0, -$), and using angular momentum conservation just as in ref. [18], we get:

$$n_i I = (V_k + A_k + I_k) R^{kk'} (V'_{k'} + A'_{k'} + I'_{k'}) + 2JR^0"J' + SR" "S', \quad (4)$$

with the definitions:

$$V_k = \sum (v_k)^2, \quad A_k = \sum (a_k)^2, \quad I_k = 2 \sum v_k a_k, \\ J = \sum (v_0 + a_0)(v_{\parallel} + a_{\parallel}), \quad S = \sum (v_{\parallel} + a_{\parallel})^2, \quad (5)$$

and similar definitions for the primed quantities; we also defined:

$$R^{kk'} = (r^{kk'})^2, \quad R^0" = r^{00} r" ", \quad R" " = r" " r" ". \quad (6)$$

Using now the fact that vector and axial amplitudes show an opposite behaviour under the parity transformation, we get:

$$V_+ = V_-, \quad A_+ = A_-, \quad I_+ = -I_-, \quad I_0 = 0,$$

$$\sum v_0 a_{\parallel} = \sum v_{\parallel} a_0 = \sum v_{\parallel} a_{\parallel} = 0. \quad (7)$$

Expliciting now the matrix elements of $r^{mm'}$, we obtain the formula:

$$n \cdot I = T T' (1 + \cos^2 \theta) + (T L' + L T') \sin^2 \theta + L L' \cos^2 \theta + 2(\bar{T} \bar{T}' + J J') \cos \theta + S S', \quad (8)$$

which is the analogue of eq. (15) of ref. [18], and where we defined:

$$T = V_+ + A_+, \quad \bar{T} = I_+, \quad L = V_0 + A_0, \quad J = \sum (v_0 v_{||} + a_0 a_{||}),$$

$$S = \sum [(v_{||})^2 + (a_{||})^2], \quad (9)$$

and similar definitions were used for the primed quantities. Further simplifications occur if we assume, according to CVC, that $v_{||} = 0$.

At the $\nu \ell W$ vertex, using the expression of the current:

$$j_\mu = \bar{u}_\ell \gamma_\mu (1 - i\gamma_5) u_\nu, \quad (10)$$

we get

$$T = -\bar{T} = \frac{t + m^2}{m m_\nu}, \quad L = -S = iJ = -\frac{(t + m^2)m}{t m_\nu}, \quad (11)$$

where the time-like unit $\hat{1}$, used in ref. [18], was replaced by the symbol i , more familiar to the reader; we defined: $t = -q^2$, calling q_μ the four-momentum of W (we take $q^2 = q_0^2 - \mathbf{q}^2$, so that t is positive); m is the mass of the charged lepton; m_ν is a fictitious mass for the neutrino, which vanishes once the kinematic factor is included into the cross section. We finally get (dropping now the primes using for all quantities pertaining to the hadron vertex):

$$\frac{d^2\sigma}{dM' dt} = K \int \{ T[\cos^2 \theta + 1 + \frac{m^2}{t} (\cos^2 \theta - 1)] - L(\cos^2 \theta - 1 + \frac{m^2}{t} \cos^2 \theta) + 2(-\bar{T} + \frac{m^2}{t} iJ) \cos \theta + \frac{m^2}{t} S \} dx, \quad (12)$$

where M' is the invariant mass of \mathcal{N}' ; x is the cosine of the pion emission angle in the pion-nucleon c.m. frame; the expression of $\cos \theta$ (where θ is an imaginary angle) is derived from ref. [18]:

$$\cos \theta = \frac{4M t E - (t + m^2)(t + M'^2 - M^2)}{(t + m^2)[t^2 + 2(M'^2 + M^2)t + (M'^2 - M^2)]^{\frac{1}{2}}}. \quad (13)$$

Here, M is the nucleon mass, and E the neutrino lab energy.

Finally, K is defined differently for the two types of terms considered:

(i) Isobar term. Here, \mathcal{N}' is identified with N^* (so that j'_μ refers to the process $WN \rightarrow N^*$), and we get:

$$K = \frac{G^2}{16\pi^2} \frac{t + m^2}{E^2} \frac{M'^3 \Gamma}{M[(M'^2 - M_0^2) + M'^2 \Gamma^2]}, \quad (14)$$

where $G/\sqrt{2}$ is the universal constant of weak interactions ($G \simeq 10^{-5} M^{-2}$) M'_0 is the on-shell mass of the resonance and Γ its decay width. Notice that integration over x gives simply a factor 2.

(ii) Born terms. \mathcal{N} is identified with the pion-nucleon system (so that j'_μ refers to the process $WN \rightarrow \pi N$), and we get:

$$K = \frac{G^2}{64\pi^3} \frac{t+m^2}{E^2} \chi, \quad (15)$$

where χ is the magnitude of the pion's three-momentum in the pion-nucleon c.m. frame.

2.2. Currents and coupling coefficients

Our problem is now reduced to properly defining the four currents involved

$$v_\mu^{(N^*)}, \quad v_\mu^{(\text{Born})}, \quad a_\mu^{(N^*)}, \quad a_\mu^{(\text{Born})},$$

and correctly choosing the coupling coefficients (coupling constants times form factors).

For the two first currents written above, we used the model proposed by Cochard and Kessler [19] for the analysis of electro- and photoproduction in the $N^*(1236)$ region, together with the CVC hypothesis. We recall that in this model the WN^* coupling is assumed to be pure magnetic dipole in its vector part, i.e.:

$$v_\mu^{(N^*)} = f(q^2) \bar{u}_\rho (\gamma_\sigma - \frac{t_\sigma}{M}) (-i\gamma_5) u(q^\rho g_\mu^\sigma - q^\sigma g_\mu^\rho) M^{-1}, \quad (16)$$

where u is the spinor for the ingoing nucleon, u_ρ the spinor-vector for the N^* and t_σ the four-momentum of the latter. For the coupling coefficient $f(q^2)$, we obtain, using the fit of ref. [19] [‡] and CVC:

$$|f(q^2)| = 1.7 \left(1 + \frac{|q^2|}{0.71}\right)^{-2} \quad (q^2 \text{ in } \text{GeV}^2/c^2)$$

for the channel $W^+ n N^{*+}$ (we must multiply by $\sqrt{3}$ for the channel $W^+ p N^{*++}$).

The most general form of the axial isobar current is:

$$a_\mu^{(N^*)} = \bar{u}_\rho \left\{ g_1(q^2) g_\mu^\rho + \frac{q^\rho}{M} \left[g_2(q^2) \gamma_\mu + g_3(q^2) \frac{q_\mu}{M} + g_4(q^2) \frac{p_\mu + t_\mu}{M} \right] \right\} u. \quad (17)$$

The coupling coefficients g_2 and g_4 may be neglected on the basis of static model arguments [12]. For g_3 , the one-pion exchange gives ^{††}:

[‡] Correction terms involving ρ - and ω -exchange which were used in that work are left out here.

^{††} Here, $g_3(0)$ stands for $g_3(|q^2| \lesssim m_\pi^2)$.

$$g_3(0) = \left(\frac{G}{\sqrt{2}}\right)^{-1} g_{\pi NN}^* g_{\pi^- \nu} M^2 / (q^2 - m_\pi^2)$$

with: $|g_{\pi \ell \nu}| = 1.48 \times 10^{-7} / m_\pi$; $|g_{\pi^+ p N^* \ell \nu}| = 2.26 / m_\pi$ (see ref. [21]). Then g_1 is derived from g_3 by PCAC (namely, the requirement that $q^\mu a_\mu \sim m_\pi^2 / (m_\pi^2 - q^2)$), and we find:

$$g_1(0) = - \left(\frac{G}{\sqrt{2}}\right)^{-1} g_{\pi NN}^* g_{\pi \ell \nu}.$$

Finally, using the universal form factor of the dipole type, we get:

$$\frac{g_1(q^2)}{g_1(0)} = \frac{g_3(q^2)}{g_3(0)} = \left(1 + \frac{|q^2|}{0.71}\right)^{-2}, \quad (q^2 \text{ in } \text{GeV}^2/c^2).$$

We now go over to the Born part of the axial current. This part is written (noticing that graph c) of fig. 1 does not contribute, because of G -invariance):

$$a_\mu^{(\text{Born})} = g_{\pi NN} \bar{u}' \{ a \gamma_5 (\gamma^\rho p_\rho + \gamma^\rho q_\rho + M) [h_1(q^2) \gamma_\mu + h_2(q^2) \frac{q_\mu}{M}] + b [h_1(q^2) \gamma_\mu + h_2(q^2) \frac{q_\mu}{M}] \gamma_5 (\gamma^\rho p_\rho - \gamma^\rho k_\rho + M) \} \gamma_5 u, \quad (18)$$

with: $a = [(p+q)^2 - M^2]^{-1}$, $b = [(p-k)^2 - M^2]^{-1}$.

Eq. (18) can easily be rewritten (using in particular the Dirac equation) in the simpler form:

$$a_\mu^{(\text{Born})} = -g_{\pi NN} \bar{u}' [h_1(q^2) (a k_\rho \gamma^\rho \gamma_\mu + b k_\rho \gamma_\mu \gamma^\rho) + h_2(q^2) (b - a) \frac{q_\mu}{M} k_\rho \gamma^\rho] u. \quad (19)$$

Identifying the term in q_μ with that given by one-pion exchange, we get:

$$h_2(0) = \left(\frac{G}{\sqrt{2}}\right)^{-1} g_{\pi \ell \nu} g'_{\pi NN} \frac{M}{(q^2 - m_\pi^2)},$$

where $g'_{\pi NN}$ differs from $g_{\pi NN}$ only by a Clebsch-Gordan coefficient.

Now, in order to apply PCAC, we take the divergence of the above current; after some simplifications (using again the Dirac equation), we get:

$$q^\mu a_\mu^{(\text{Born})} = g_{\pi NN} \{ h_2(q^2) (a - b) \frac{q^2}{M} q_\rho \gamma^\rho + 2h_1(q^2) [M(a - b) q_\rho \gamma^\rho - 1] \}. \quad (20)$$

We see that the application of PCAC requires on the one hand:

$$h_1(q^2) = (2M^2)^{-1} (m_\pi^2 - q^2) h_2(q^2),$$

i.e.:

$$h_1(0) = -(2M)^{-1} \left(\frac{G}{\sqrt{2}}\right)^{-1} g_{\pi\ell\nu} g'_{\pi NN};$$

and on the other hand the addition of the term $-g_{\pi NN} q^2 M^{-2} h_2(q^2)$ to the above divergence, which means that we must add to the current $a_\mu^{(\text{Born})}$ a contact term $-g_{\pi NN} M^{-2} h_2(q^2) q_\mu$.

Here again, we use the universal form factor:

$$\frac{h_1(q^2)}{h_1(0)} = \frac{h_2(q^2)}{h_2(0)} = \left(1 + \frac{|q^2|}{0.71}\right)^{-2}, \quad (q^2 \text{ in } \text{GeV}^2/c^2).$$

The value taken for $g_{\pi NN}$ was: $|g_{\pi^+ \text{np}}| = 19.5$.

All our coupling coefficients are now determined without any free parameter, except for the relative sign of the vector and axial coupling coefficients. In the Born terms, this relative sign is shown (from the analysis of β -decay) to be such as to produce constructive V-A interference. By analogy (which may be based upon a static model), we also suppose constructive interference in the isobar term.

For all details of the calculation of T, L, J, S, \bar{T} (to be used in eq. (12)) from the above expressions of the currents, we refer the reader to ref. [20].

Three reaction channels are to be considered:

$$(\alpha) W^+ + p \rightarrow \pi^+ + p, \quad (\beta) W^+ + n \rightarrow \pi^+ + n, \quad (\gamma) W^+ + n \rightarrow \pi^0 + p.$$

For each diagram of fig. 1, these channels differ from each other only by a trivial isospin coefficient (it should be noticed that this coefficient is zero for diagram a) in channel (α) and diagram b) in channel (β) .

3. RESULTS

We first considered channel (α) , i.e. the reaction $\nu + p \rightarrow \mu^- + p + \pi^+$. In figs. 3a and 3b, we show the vector, axial and interferent (vector-axial) contributions to the resonant term and the Born terms respectively. These curves were obtained by integration over the pion-nucleon invariant mass (between threshold and 1400 MeV) and over q^2 . Fig. 3c shows the total cross section, obtained by summing up all these curves, and compared with the experimental curve [2, 3] and with the theoretical results of other authors [13, 14, 16].

We then considered channels (β) and (γ) , and also the "average channel" for neutrinoproduction on complex nuclei (where the number of protons and neutrons is taken as equal); the latter is defined by summing up channels (α) , (β) and (γ) and dividing by 2.

For the isobar term, the relative contributions (V, A, V-A) are still the same as those shown in fig. 3a); they are only to be multiplied by a factor $\frac{1}{9}$ for channel (β) , $\frac{2}{9}$ for channel (γ) , and $\frac{2}{3}$ for the "average channel" $[(\alpha) + (\beta) + (\gamma)]/2$. As to the Born terms, the relative contributions (V, A, V-A) are shown in fig. 4a for channel (β) , in fig. 4b for channel (γ) and in fig. 4c

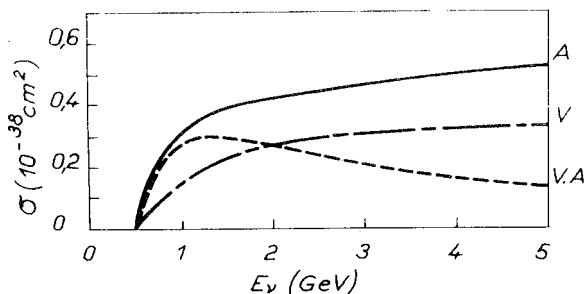


Fig. 3a. Vector, axial and vector-axial resonant contributions to the total cross section for $\nu + p \rightarrow \mu^- + \pi^+ + p$, as functions of the neutrino lab energy.

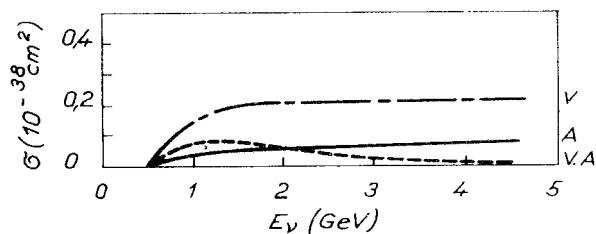


Fig. 3b. Vector, axial and vector-axial Born term contributions to the total cross section for $\nu + p \rightarrow \mu^- + \pi^+ + p$, as functions of the neutrino lab energy.

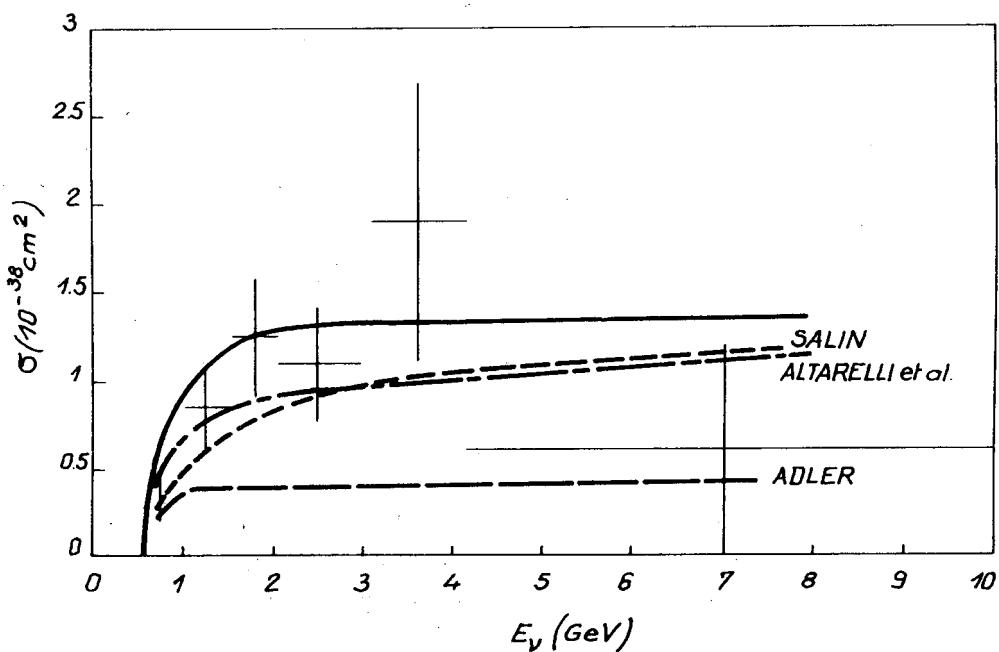


Fig. 3c. Total cross section of $\nu + p \rightarrow \mu^- + \pi^+ + p$, as a function of the neutrino lab energy. — our calculation; --- Salin; -·- Altarelli et al.; - - Adler. Experimental points from refs. [2, 3].

for the "average channel". For that last channel, the contribution of all curves was summed up, and the total cross section obtained was compared with an experimental result [22] in fig. 4d.

Figs. 5a and 5b show for channel (a) the various contributions (V, A, V-A) for both the resonant term and the Born terms) to the differential cross section $d\sigma/d(q^2)$, obtained by integrating over the pion-nucleon invariant mass (between threshold and 1400 MeV) and over the energy spectrum of the neutrinos used at CERN [2]. In fig. 5c, this cross section is compared with the experimental data [2] (our curve was normalized arbitrarily in order to fit the experimental number of events). We also show the theoretical result we would obtain by modifying the axial form factor, namely by taking $(1 + |q^2|/M_A^2)^{-2}$ with $M_A^2 = 1.21$ (instead of 0.71) GeV^2 .

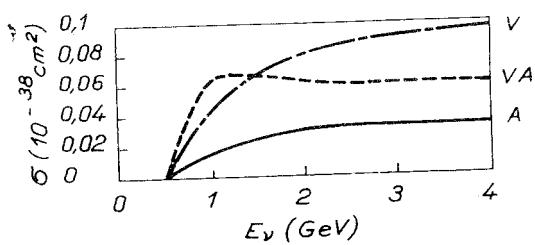


Fig. 4a. Vector, axial and vector-axial Born term contributions to the total cross section for $\nu + n \rightarrow \mu^- + \pi^+ + n$, as functions of the neutrino lab energy.

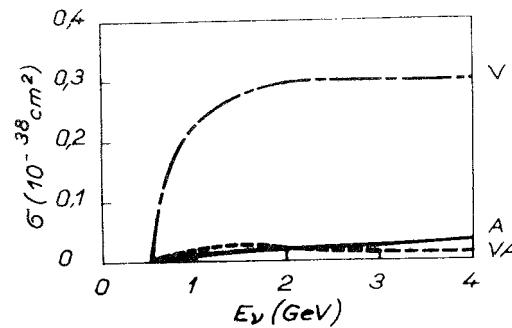


Fig. 4b. Vector, axial and vector-axial Born term contributions to the total cross section for $\nu + n \rightarrow \mu^- + \pi^0 + p$, as functions of the neutrino lab energy.

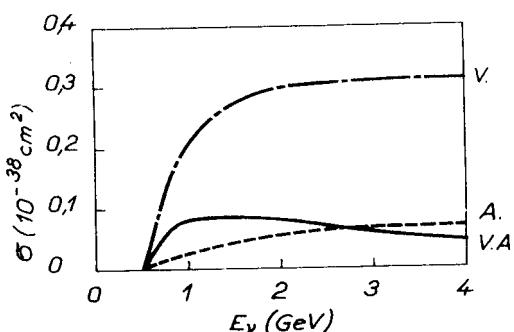


Fig. 4c. Vector, axial and vector-axial Born term contributions to the total cross section for neutrino production of a single pion on a complex nucleus with equal number of neutrons and protons, as functions of the neutrino lab energy.

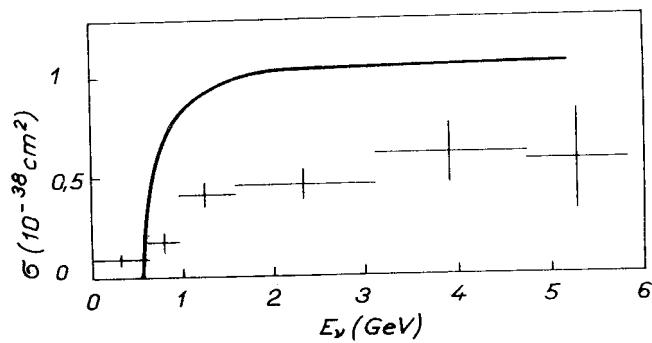


Fig. 4d. Total cross section for neutrino-production of a single pion on a complex nucleus, as a function of the neutrino lab energy. Experimental points are from ref. [22].

5. DISCUSSION AND CONCLUSION

The detailed analysis of the various contributions involved shows that in our model:

the resonant term dominates over the Born terms;

the axial part appears to dominate in the resonant term, whereas the vector part is generally predominant in the Born terms;

both in the resonant term and in the Born terms, the axial part shows a peak at low transfers (Adler's effect [23]) and then decreases rapidly with increasing transfer.

The comparison of our results with the experimental data shows that:

the total cross section for neutrino production on hydrogen is well fitted by our model;

the agreement is less satisfactory for neutrino production on complex nuclei; however, it is well known that there the experimental points are somewhat less reliable;

our transfer curve decrease faster than the (surprisingly flat) experimental one; a modification of the axial form factor (using a larger value for M_A) would make the agreement somewhat better, without producing any drastic change in the total cross section.

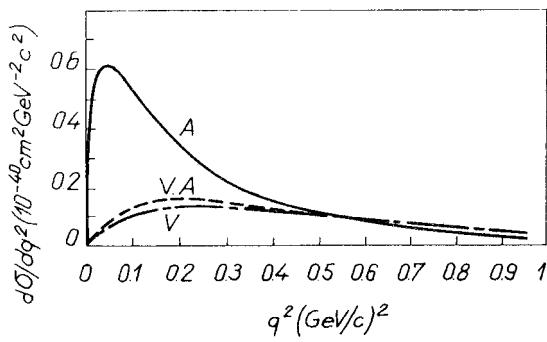


Fig. 5a. Vector, axial and vector-axial resonant contributions to the differential cross section $d\sigma/d(q^2)$ for $\nu + p \rightarrow \mu^- + \pi^+ + p$.

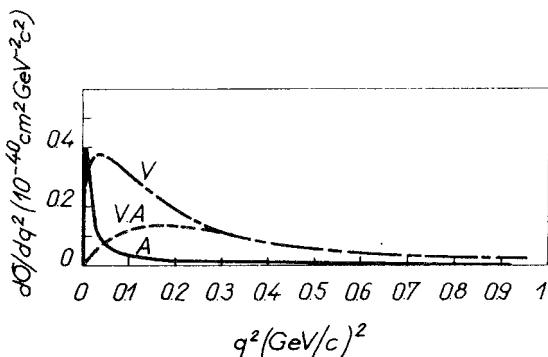


Fig. 5b. Vector, axial and vector-axial Born term contributions to the differential cross section $d\sigma/d(q^2)$ for $\nu + p \rightarrow \mu^- + \pi^+ + p$.

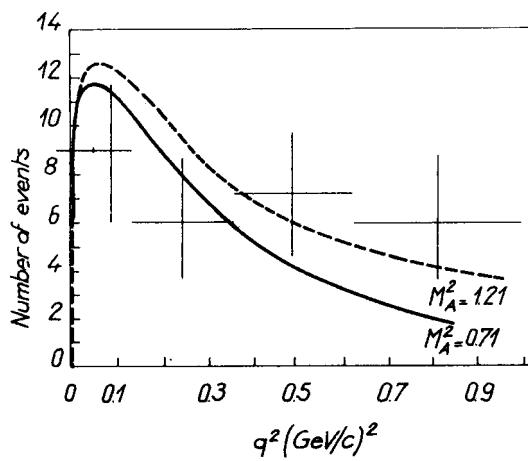


Fig. 5c. Number of events as a function of q^2 for $\nu + p \rightarrow \mu^- + \pi^+ + p$.
 $\text{--- } M_A^2 = 0.71 \text{ GeV}^2$; $\text{--- } M_A^2 = 1.21 \text{ GeV}^2$. Experimental points are from ref. [2].

It is to be hoped that in a near future new high-energy neutrino scattering experiments, performed with more intense beams, will allow us to choose in a more critical way between the various theoretical models proposed. In any case, we think that the polarization structure shown in subsect. 2.1 above - a structure which was first exhibited in ref. [24] and which is entirely model-independent - may serve usefully as a guiding tool for the analysis of further experiments.

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