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Macroeconomic regularities in the growth of nations : an empirical inquiry

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This paper analyses empirically the process of economic development from what may be called a system-theory point of view. The four macroeconomic variables primarily considered are the population, the gross national product (GNP), the value of foreign trade and the total receipt of the government's budget, which for a given country grossly characterize its extent, its economic activity, its interactions with the international environment and its centralization level, respectively. The question is whether stable relations hold between these variables in the course of economic development. Two different and complementary viewpoints are taken : the first is an international comparison for almost all (non-communist) countries in the world, the second is an historical analysis for industrial countries. The conclusions are (i) for countries at the same level of development (i.e. with the same GNP/capita) but with different population levels, foreign trade will be proportional to population to the power 3/4, at least statistically (with a correlation of 0·95), and (ii) for countries with the same population but different GNP/capita, the ratio of foreign trade to GNP will slowly increase with GNP. That conclusion which holds statistically for a collection of countries still holds for a given country in the course of its economic development during the last hundred years.

1. Introduction

1.1. *Averaging methods in economics*

The ever-changing nature of the economic world is probably one of the major challenges for the science of economics. Obviously it would be very desirable to have at least a few firm landmarks in that moving landscape.

Two ways, at least, of achieving this are offered to us. The first originates in the observation that long-term evolutions are usually smoother than short-term ones. Of course, short-range fluctuations are still present in long-term data. However, if there is a marked (upward or downward) trend, then this trend will mask the fluctuations. Let us propose a naive comparison. If recorded every ten seconds, the movements of an ant will appear rather chaotic ; but they will usually reveal a definite route, from nest to food for instance, if recorded every two minutes. By using a moving-average method, as is common practice in economic history, the fluctuations become even less apparent because positive fluctuations may cancel negative ones.

The second method may also be illustrated by our ant comparison. Instead of just observing one ant, which could possibly lose its way, let us observe several ants (with the same kind of activity) and perform an average in some sense. This way, we might get rid of the few spurious individual behaviours. In economics this implies observation of many similar countries and considering them as realizations of one and the same development process.

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In mathematical terms, these two approaches correspond to the ways in which a stochastic process (which is here non-stationary) may be averaged, i.e.

(i) time-averaging for one realization of the process over a sufficiently (in principle infinitely) long span,
(ii) ensemble-averaging over several realizations of the process.

Both methods will be used in this paper. The ensemble-averaging method will be used in § 2 where all (non-communist) countries in the world will be considered as black boxes experiencing an economic growth process between 1960 and 1976. The time-averaging method will be used in § 3 in a modified form ; the moving average is performed implicitly by using logarithmic scales.

1.2. *The ensemble-averaging method*

In contrast to time-averaging, ensemble-averaging does not seem to have been much employed so far in economics. Let us therefore discuss further the conditions under which this method will be used. In particular, the point that the ensemble should consists of ' similar ' countries (in respect of the process under consideration) has to be examined.

Although a few countries may not have experienced any growth, there is no doubt about the general upward trend during the period 1960–1976. Growth in India and growth in the U.S., for instance, may appear as qualitatively different processes if one considers such sectorial variables as technology investments, education, transportation facilities, etc. Despite these conspicuous differences, there are obviously also basic similarities. In every country :

—Goods are produced, distributed and purchased.
—Some goods have to be imported and others can be exported.
—A government procures support for transportation, public education and security etc.

To be meaningful, the ensemble-averaging approach should then concentrate on basic variables such as gross national product (GNP), foreign exchanges or government expenses and receipts. Presumably a few other basic variables could be considered, but these, which will be analysed in this paper, have the advantage that the related data are easily available.

Remark

Most econometric models introduce many structural variables (related by a system of simultaneous equations) providing a almost complete description of an economy. The purpose of this paper is to describe a few basic features common to all economics or, stated differently, we are trying to move back far enough to get an overall view of the economic landscape.

1.3. *Logarithmic scales and the power law*

In this paper only logarithmic scales will be used. Let us briefly explain why. Whenever the range of a variable extends over about three (or more) orders of magnitude (i.e. intervals larger than $(x_0, 10^3 x_0)$) graphical representation requires a logarithmic scale. For instance, the population P of the (non-communist) countries in the world ranges (for the year 1970) over the interval (in millions)

$$(0{\cdot}153,\ 539)$$

the lower bound corresponding to the Barbados islands and the upper bound to India.

There are, however, much deeper reasons for the use of logarithmic scales in economics than these practical considerations. They are related to the two following observations.

Firstly, the basic growth law of many economic variables is the exponential law. In other words, a new production or a newly settled population develops with a constant *relative* growth rate, i.e. according to the equation

$$\frac{1}{x}\frac{dx}{dt}=\lambda$$

where λ is a positive constant. After a while, some kind of saturation may occur. Nevertheless, due to the rapid increase of the exponential, the growth during that first period usually accounts for the largest part of the final size. This was first recognized by Gibrat (1931).

Secondly, individual economic agents (whether consumers or firms) are sensitive to *relative* variations, not to absolute ones; a price increase from $1 to $2 will, for instance, be experienced in the same way as an increase from $100 to $200 (for another product), whereas an increase from $100 to $101 will be almost unnoticeable.

For these reasons, relative variations displayed by logarithmic scales are often much ' better ' variables than absolute numbers.

These considerations are closely connected with the concept of ' power-law growth ', i.e.

$$y=ax^b \tag{1}$$

If the variables x and y both experience exponential growth:

$$x=x_0 \exp(\alpha t), \quad y=y_0 \exp(\beta t)$$

relation (1) will hold with $b=\beta/\alpha$. This is referred to by biologists as ' allometric ' growth (Bertalanffy 1968). We shall return to this concept in § 3.

1.4. *Four basic (national) macroeconomic variables*

For a given country four (extensive) variables will be considered:

(1) The population P which gives a measure of the human extent of that country.
(2) The GNP G which gives a measure of the country's economic activity.
(3) The sum of the imports and the exports C, which gives a measure of the interaction of the country with the rest of the world.
(4) The total federal budget receipts R which gives a measure of the economic weight of the central government in the nation's economy.

Remark

In short, we shall as a rule speak of the GNP although the variable actually considered may be a variant of the GNP. In § 2, for instance, we shall consider the gross domestic product (GDP) which differs from the GNP in so far as the income arising from investments and possessions owned abroad is not included. Differences in the figures are relatively small.

In § 2 the three variables P, G and C for all non-communist countries are considered. It is shown that they are related in the following way

$$\frac{C}{P^{3/4}} \simeq A g^{1 \cdot 1}$$

where $g = G/P$ is the GNP per inhabitant and A is a constant. This relation is of course of a statistical nature ; the dispersion is, however, very reasonable. In § 3 the functions

$$C = f_1(G), \quad R = f_2(G)$$

are considered for a number of countries, for the largest G-interval permitted by available data. These functions prove to be power functions, i.e.

$$C = C_0 G^{\gamma}, \quad R = R_0 G^{\rho}$$

the exponents γ and ρ being characteristic of a given country. First, let us discuss the availability and accuracy of the data we shall need.

1.5. *On the availability and comparability of data*

From the four variables, the one which is best known (and known for the longest period) is of course the population P. The variables C and R have also been recorded in developed countries at least since the beginning of the 19th century. However, the accuracy of R is much lower than that of C, mainly because the very definition of R is subject to continuous modifications, especially in countries having a federal government like the U.S.

The GNP was introduced in national accounts after World War II. The United Nations' recommendations for definition and reckoning of the GNP came into use in a large number of countries in the 1960s. Thus, the period for which there are GNP data to some degree comparable covers, at the moment, two decades. This is the minimum period allowing the kind of analysis undertaken in § 2.

On the other hand, to study the evolution of a *given* country we may use the GNP series which has been calculated for years prior to World War II by the statistical agencies of all major countries. Needless to say, the accuracy of such calculations decreases for early periods, since the majority of the many variables forming the GNP have to be estimated. A general account of the problems raised by such economic measurements can be found in Morgenstern (1963).

2. Relation between population, national product and foreign trade

2.1. *Qualitative discussion*

The population P, the GNP G and the sum C of imports and exports are, for any country, three important extensive variables. We shall denote by g and c the intensive variables with respect to P, that is

$$g=\frac{G}{P}, \quad c=\frac{C}{P}$$

In this section we shall address the problem of their interdependence. Does, for example, a relation of the form

$$f(P, G, C)=0 \tag{2}$$

hold, at least statistically ? Two observations give us a first insight into the problem.

Observation 1

The larger a country, the smaller the importance of its foreign trade relative to its national product.

Consider a large country A and a small one B, differing in the size of their territory and in their population, but alike in all other respects (in particular their GNP per inhabitant g should be the same).

In B, home harvesting of many crops is impossible because of climatic limitations, and B will probably also be short of many useful minerals. If the consumption per inhabitant is qualitatively the same as in A, then the products which are lacking must be imported and those imports must be compensated by equivalent exports (at least on average over a large period). Hence, we should expect that :

$$\frac{C(\mathrm{B})}{C(\mathrm{A})}>\frac{G(\mathrm{B})}{G(\mathrm{A})}=\frac{P(\mathrm{B})}{P(\mathrm{A})}$$

This is indeed supported by facts. As a typical example, take the United States as country A and Denmark as country B. Then (for 1976 data),

$$\frac{G(\mathrm{B})}{G(\mathrm{A})}\simeq\frac{P(\mathrm{B})}{P(\mathrm{A})}=\frac{5{\cdot}1}{215}=0{\cdot}024$$

whereas

$$\frac{C(\mathrm{B})}{C(\mathrm{A})}=\frac{21532}{242155}=0{\cdot}089$$

Observation 2

The foreign trade of a country increases faster than its economic development (measured by the GNP/inhabitant g).

Considering two countries A and B with same area and population, this means that, if $g(\mathrm{A})>g(\mathrm{B})$,

$$\frac{C(\mathrm{A})}{C(\mathrm{B})}=\frac{c(\mathrm{A})}{c(\mathrm{B})}>\frac{g(\mathrm{A})}{g(\mathrm{B})}$$

This property is not easy to explain intuitively ; we may just guess that the complexity of industrial production and the correlative international division of labour (itself favoured by strong productivity gains in long-range transportation) increases faster than g. It is, however, strongly supported by facts. Taking, for instance, (with 1970 data) France as A and Mexico as B,

$$\frac{c(\mathrm{A})}{c(\mathrm{B})}=9{\cdot}6, \quad \frac{g(\mathrm{A})}{g(\mathrm{B})}=4{\cdot}2$$

or Uruguay as A and Paraguay as B,

$$\frac{c(\mathrm{A})}{c(\mathrm{B})}=3{\cdot}6, \quad \frac{g(\mathrm{A})}{g(\mathrm{B})}=3{\cdot}2$$

It is the aim of this section to provide a quantitative and statistical statement for Observations 1 and 2.

2.2. *The statistical approach*

To be able to carry out a statistically significant analysis, data must be collected for as many countries as possible. The *United Nations World Statistics Yearbooks* provide the required data for all nations in the world. We leave communist countries aside because they use a different concept for the GNP, namely the net material product. We are then left with 108 countries.

Remark

It was not possible to draw curves which are independent of the value of money, because volume estimations of C and G are not yet available. Hence, since C and G are evaluated in U.S. dollars the figures will depend upon exchange rate fluctuations.

Let us now analyse the three-dimensional set of points

$$(P_i, G_i, C_i), \quad 1 \leqslant i \leqslant 108$$

If these points were more or less uniformly distributed in some volume, a very bad correlation would result. On the other hand the correlation would be very strong if the points gathered around a two-dimensional plane represented by eqn. (2). Intercepting that surface by planes which are parallel to the coordinates plane (P, C) would, for instance, result in a family of curves labelled by different values of G. We preferred, however, to label them by the value of g instead of G for the following reasons :

(i) The variable g is intuitively more appealing ; it represents the level of development of the country. Thus on each graph we have only countries with a similar production system.

(ii) g has a smaller range of variation than G (10^4 for G, but only 10 for g). This makes the distribution of the data into only a few groups more meaningful.

If plotted, the data points do indeed gather (for a given range g) along definite curves, of the form

$$C=BP^{\alpha}$$

where B depends upon g.

Year or period	Interval for g (U.S. dollars)	Number of countries in the class	α	B	Correlation coefficient ρ
1960	< 150	11	0·63	76·7	0·85
	(150, 400)	26	0·68	141	0·93
	(400, 800)	13	0·72	3440	0·90
	> 800	18	0·83	7440	0·97
1970	< 150	26	0·76	42·9	0·82
	(150, 650)	43	0·71	196	0·88
	(650, 2000)	23	0·76	618	0·89
	> 2000	16	0·80	1940	0·96
1976	< 700	33	0·63	311	0·68
	(700, 1700)	21	0·68	852	0·95
	(1700, 4000)	11	0·63	3490	0·80
	> 4000	18	0·87	4290	0·96
1960–1976	< 150	43	0·73	50·0	0·82
	(150, 350)	61	0·80	130	0·92
	(350, 800)	61	0·73	378	0·90
	(800, 3000)	66	0·76	1030	0·88
	> 3000	28	0·79	3950	0·93

Table 1.

For brevity the curves are omitted but the results (for 1960, 1970 and 1976) are given in Table 1. It can be seen that the correlation is in general reasonably high. This corroborates the existence of a relation of the form (2). As could be expected the dispersion is larger for the developing countries of the first g-class, since those countries are still characterized by rather different traditional productive systems.

Instead of plotting the data for different years on distinct curves, we could tentatively lump them together on the same curve. The underlying assumption in that procedure is that, in the course of time, the countries basically go through the same development levels. Table 1 shows that the correlation of the combined data is as good as that for separate years, and the large number of points makes the result statistically more significant.

Remark

One drawback in so doing is that we have to mix data expressed in 1960, 1970 or 1976 dollars, which may not be comparable if the fluctuations of the dollar are very large. However, as we noted before, the fluctuations of the exchange rates already affect the curves for separate years.

2.3. *The dependency between P, C and G*

The results in Table 1 strongly support a functional relation of the form

$$C = B(g)P^{\alpha}$$

where α is close to $\frac{3}{4}$. From the values of $B(g)$, we can moreover infer that $B(g)$ is approximately proportional to g.

For an overall check of this dependency, we shall represent the variable C/P^α as a function of g, the exponent α being adjusted to give the best correlation. The value

$$\alpha = 0{\cdot}75$$

turns out to be the right one and we thus get

$$\ln\left(\frac{C}{P^{3/4}}\right) \simeq 1{\cdot}11 \ln g + \ln 0{\cdot}32 \tag{3}$$

with a correlation $\rho = 0{\cdot}95$. The number of points here is 259 and they are plotted in Fig. 1.

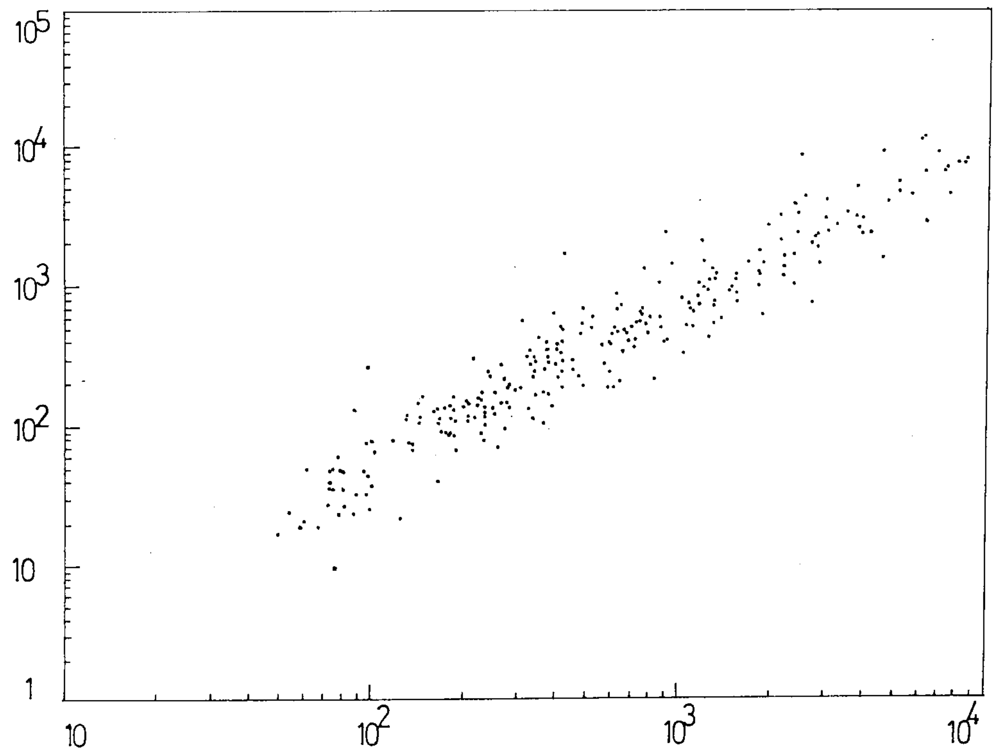


Figure 1. Value of the ratio $C/P^{3/4}$ versus GDP per capita for 108 countries in years 1960, 1970, 1976 (*World Statistics in Brief: United Nations* 1977, 1978).

Remark

To get an appreciation of the significance of this correlation, let us quote a result of the same kind. It is commonly stated that there is a correlation between the GNP/capita and the level of education of the population. If the data for various countries are plotted on a graph, the correlation turns out to be only $\rho = 0{\cdot}83$ (Le Thanh Khoi 1973, p. 51).

Relation (3) can also be written

$$\frac{C}{P^{3/4}} \simeq A g^{1{\cdot}11} \tag{4}$$

Note that the constant A need not be equal to 0·32. It actually depends upon the particular distribution function of the random variables C, P and

g. We may obtain the value of A as the average of the numbers $C/(P^{3/4}g^{1\cdot 11})$, which gives $A \simeq 0{\cdot}37$.

We may summarize the analysis as follows :

Proposition

For the non-communist countries in the world (between 1960 and 1976) the dependency between the value C of foreign exchange, the population P and the GNP/capita g has (statistically) the following form :

$$C = 0{\cdot}37 P^{3/4} g^{1\cdot 11}$$

C and g being expressed in dollars.

What is the relevance of this formula ?

(i) Although it was established for a given span, we hold it to be true over a long period. This is confirmed (at least as far as the dependence upon g is concerned) by the analysis of § 3.

(ii) Sometimes the ratio C/G is considered as a measure of the opening of a nation's economy onto the world. This could be quite misleading ; indeed according to the Proposition, this ratio may be high (low) for two different reasons ; because the nation is small (large) or because it is very developed (underdeveloped)†.

To get a better measure than the ratio C/G, compute the ratio

$$r = \frac{C}{P^{3/4} g^{1\cdot 11}}$$

If it is greater (less) than 0·37, the nation has a larger (smaller) opening onto the world than the ' average nation ' of that size and development level.

In order to illustrate this, Table 2 gives the value of the ratio r for a number of typical countries for the year 1975. The results agree with intuition ; the only surprising result is the one for Japan, which is, however, corroborated by § 3 (Table 3).

	$r = C/(P^{3/4}g^{1\cdot 11})$
Malaysia	0·77
Saudi Arabia	0·73
Netherlands	0·65
Great Britain	0·51
France	0·36
Switzerland	0·31
Japan	0·30
U.S.A.	0·20

Table 2.

† The same kind of bias is met in biology when the ratio of the weight of the brain to the total weight is used as an evaluation of ' intelligence '. This ratio is, for instance, about 1/700 for a cat, and about 1/400 for a lion, although both animals have the same kind of behaviour. Here too a corrective factor has to be introduced, taking the animal's size into account.

Relation between intensive variables

As explained above, the exponent α was adjusted to give the best possible correlation. How sensitive is the correlation coefficient ρ to the value of α? The following test will give a partial answer and is of independent interest.

Let α be equal to 1; then C/P^{α} becomes simply c and we get the relation

$$\ln c = 1{\cdot}11 \ln g + \ln 0{\cdot}92$$

with a correlation coefficient $\rho = 0{\cdot}92$.

This correlation, although notably lower than before (particularly if we remember that it is ρ^2 rather than ρ which is meaningful) remains quite high, leading to the conclusion that per capita variables, which we called intensive variables, should be preferred to extensive ones (the correlation between $\ln C$ and $\ln G$ is, needless to say, very bad).

It should be noticed also that the exponent of g turns out to be the same as before.

3. Global economic variables as a function of the GNP

3.1. *The distinction between global and sectorial economic variables*

A global economic variable is here understood as an economic variable which is representative of the whole economic activity of a country. Such are, for instance, the total budget revenue R (there are fixed sources of revenue in the budget, but basically it depends upon tax revenues and these are modulated by economic activity) or the value of external trade C.

We may, in contrast, consider sectorial variables (such as wheat, timber or steel production or prices), which are representative only of a specific sector of the total economy.

This distinction is not always an obvious one, since a given sector is related to others and thus, *indirectly*, to the whole economy. If these relations are weak enough, we have a true sectorial variable; if they are rather strong, the variable may be considered as a global economic variable. Examples are given below in § 3.5.

3.2. *The observation of stable long-term trends*

Our aim in analysing the functions $C = C(G)$ and $R = R(G)$ is to show that for every country, in the course of its economic development, some specific ratios (such as C/G^{γ} and R/G^{ρ}) remain in general unchanged. This is very similar to what can be observed during the growth of a living organism. To illustrate this, Fig. 2 shows graphs taken from Iob and Swanson (1938). See also in this respect Brody (1945). From an economic point of view, the investigation of long-term price series or production data for sectorial outputs also reveals slow secular movements and in some cases definite and stable trends (Janossy 1966, Kuznets 1967). We are here, however, in a better position to observe such stable long-term trends, mainly for the following two reasons:

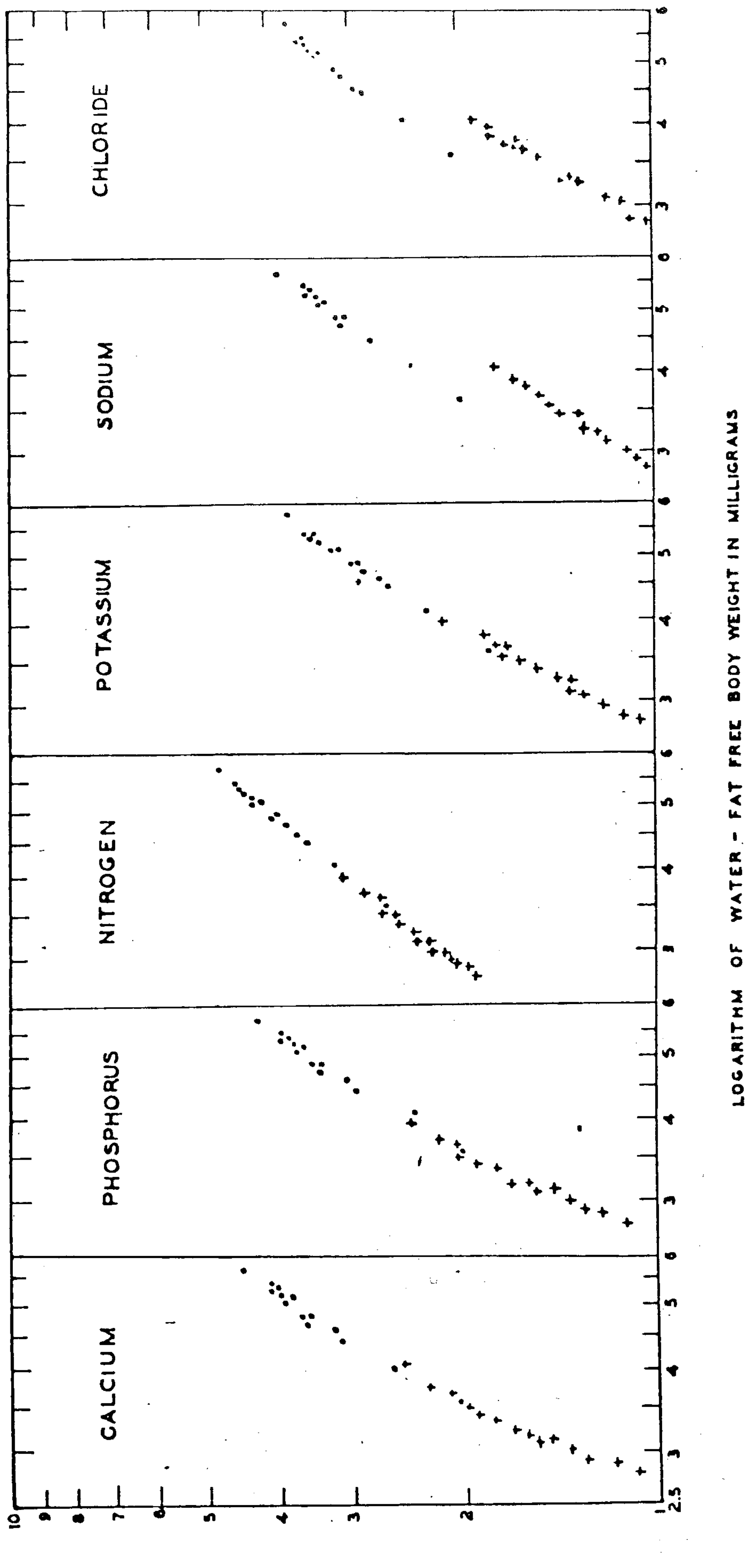


Figure 2. Relative growth of human foetus (dots) and young rat (crosses); weight of chemical constituent against water–fat free body weight minus weight of constituent (after Iob and Swanson (1938)).

(1) Needless to say, the larger the fluctuations of a variable, the longer the observation period required for the determination of possible long-term trends. Sectorial variables are much more subject to fluctuations than global ones because of climatic hazards (especially in the case of agricultural products), discrepancies between production and demand, and long-term technological drifts.
(2) The representation of global variables as a function of G, instead of time, more or less removes the ups and downs due to cyclical recessions. This enables us to concentrate on stable underlying structures.

3.3. *Qualitative discussion*

As already noted, we are primarily interested in demonstrating the existence of some invariant relations among (global) economic variables. As a by-product, we shall be able to partly answer the following two questions.

In § 2.1 we assumed that production specialization and commercial interactions among countries increase. Mathematically, this would mean that for a given country the ratio C/G has a tendency to become larger. Is this assumption supported by statistical data ? The answer will be yes.

Is the ratio R/G of the total government budget receipts to the GNP steadily increasing ? The answer will be yes. It has, however, to be qualified. First, as already noted, the statistical definition of R may vary from time to time according to new accounting rules edicted by governments. Second, the ratio R/G, considered as a function of G, is anyway a slowly varying function. For France, for instance, the ratio R/G grows as $G^{0.07}$. Nevertheless, because of the dramatic increase of G during the last hundred years, the growth of $G^{0.07}$ will be substantial. In the case of France, R/G will rise from 11% in 1830 to 23% in 1979.

Before coming to results for various countries, let us explain how we selected those countries. Firstly, data over a long period are only available for ' old ' industrial countries. Secondly, we tried to consider large countries as well as small ones to examine the influence of size. Thirdly, we omitted (i) communist countries because of their different statistical conventions, and (ii) countries having experienced major border modifications implying additional statistical difficulties.

3.4. *Results*

3.4.1. *Relation between foreign trade and GNP*

Table 3 indicates the values of the coefficient γ of the power law $C = C_0 G^\gamma$ and Figs. 3 (*a*), (*b*), (*c*) and (*d*) show the curves of some of the countries. γ is the slope of the straight lines of the graphs ; when there are two straight lines (as for France, Fig. 3 (*a*)) they have been drawn parallel. We did not try to compute γ systematically by a least-square method because the few odd points corresponding to wars would have been troublesome.

As can be observed, in recent times all the coefficients γ have been larger than one, except (surprisingly enough) that for Japan, which is however very close to one. As a consequence, the ratio C/G is everywhere increasing, slowly but steadily, reflecting the common idea of a greater economic interdependence of nations.

	Period	γ	Source
Belgium	1953–1980	1·31	Annuaire Statistique de la Belgique
France	1830–1980	1·22	Annuaire Statistique de la France
Great Britain	1856–1967 1968–1980	0·83 1·18	Annual Abstract of Statistics
Italy	1871–1980	1·22	Statistiche Storiche dell' Italia
Japan	1949–1980	0·98	Japan Statistical Yearbook
Netherlands	1920–1980	1·06	Tachtig Jaren Statistick in Tijdrecksen 1899–1979
Switzerland	1938–1980	1·11	Annuaire Statistique de la Suisse
U.S.A.	1879–1915 1945–1965 1965–1979	0·75 0·95 1·50	Historical Statistics of the U.S.A. Colonial Times to 1970 and Statistical Abstract of the U.S.A.

Table 3.

Remark

With the results of § 2 in mind, it would seem more natural to draw the curves $\ln c = f(\ln g)$. Economic development, however, outran demographic development by far (when G is multiplied by 100, P is multiplied by 3 in the case of the United States). Thus, results would be the same.

Great Britain and the United States deserve special comment since their trends do not remain constant, in contrast to the other studied countries. Great Britain experienced a modification of its trend in the 1960s. The evolution of the United States is even more interesting. In Table 3 three successive power law coefficients are given ; however, it is hardly possible to speak of a power law : the curve of Fig. 3 (*d*) looks more like a parabola than a straight line. This shows, *a contrario*, that the power law is not merely an artifice resulting from the use of a logarithmic axis.

3.4.2. *Relation between receipts and GNP*

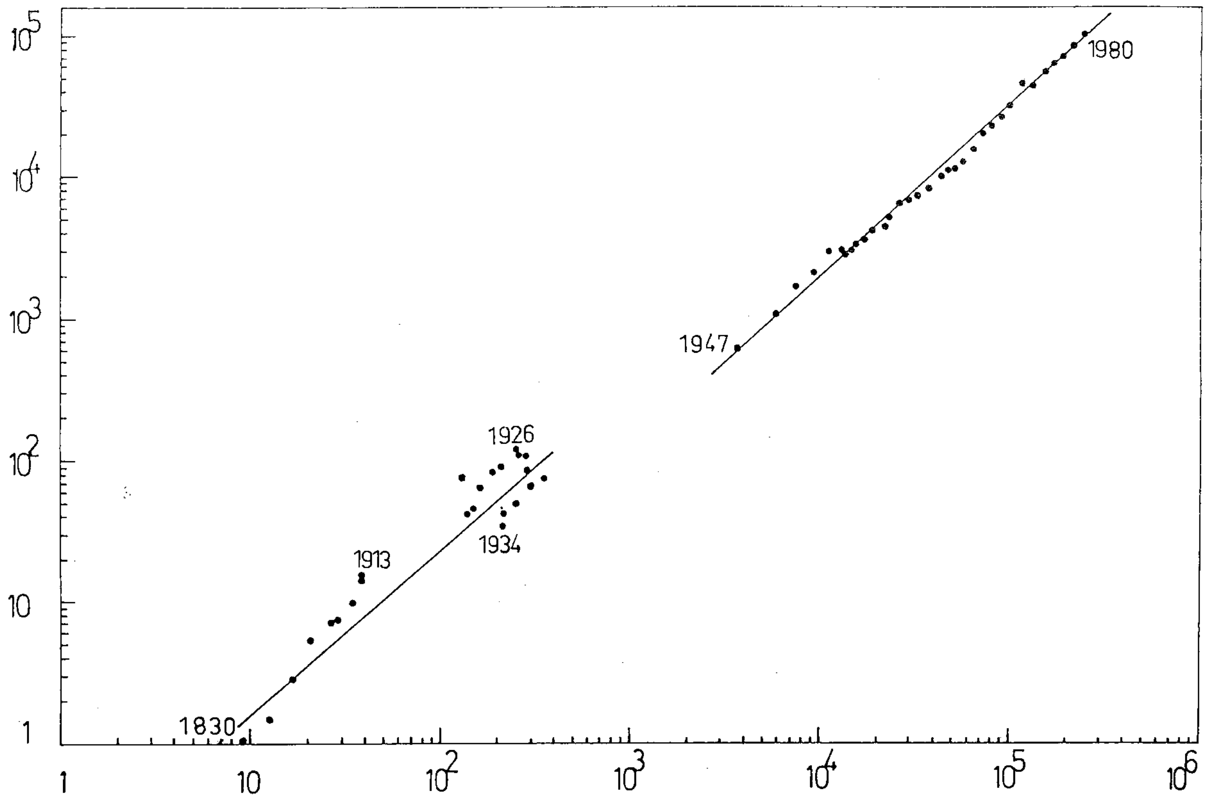
Only those countries with a centralized government are considered. For federate countries, the definition of the federal budget may vary according to the respective attributes of local and federal authorities.

Table 4 indicates the coefficient ρ of the power law

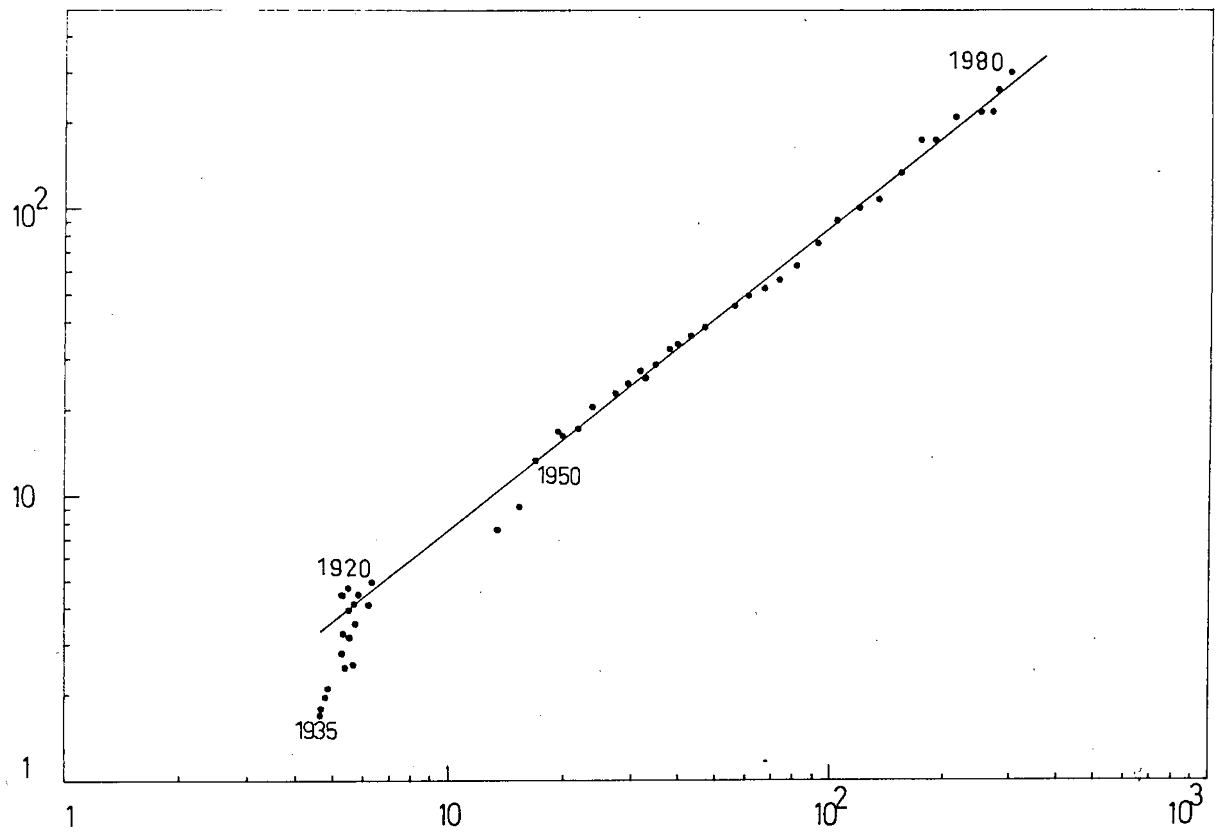
$$R = R_0 G^{\rho}$$

and Figs. 4 (*a*) and (*b*) show the curves for France and Japan.

Out of sheer curiosity (and keeping in mind our former reserves) let us see how the ratio R/G varied for federate countries. For the Netherlands it increased from 9% in 1900 to 38% in 1980, for the U.S.A. it increased from 3% in 1900 to about 20% in 1980, and for Switzerland, it *decreased* from 11% in 1945 to 9% in 1980.

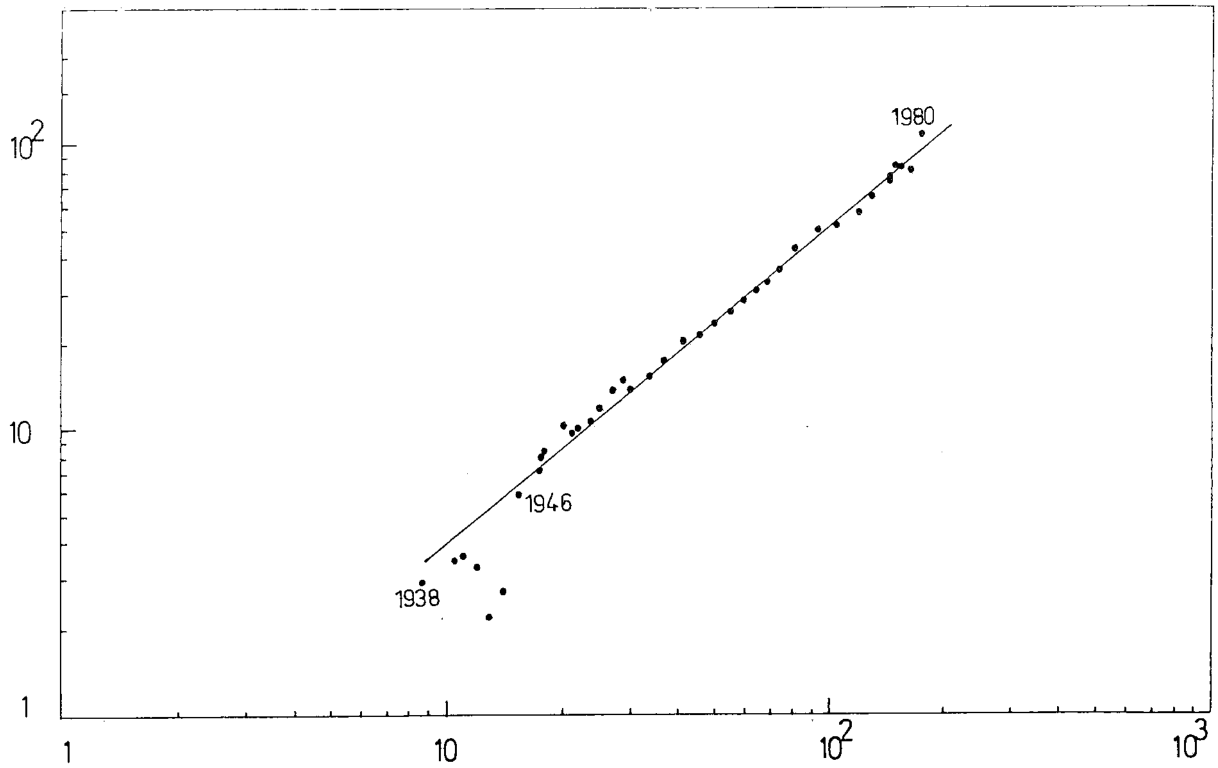


(*a*)

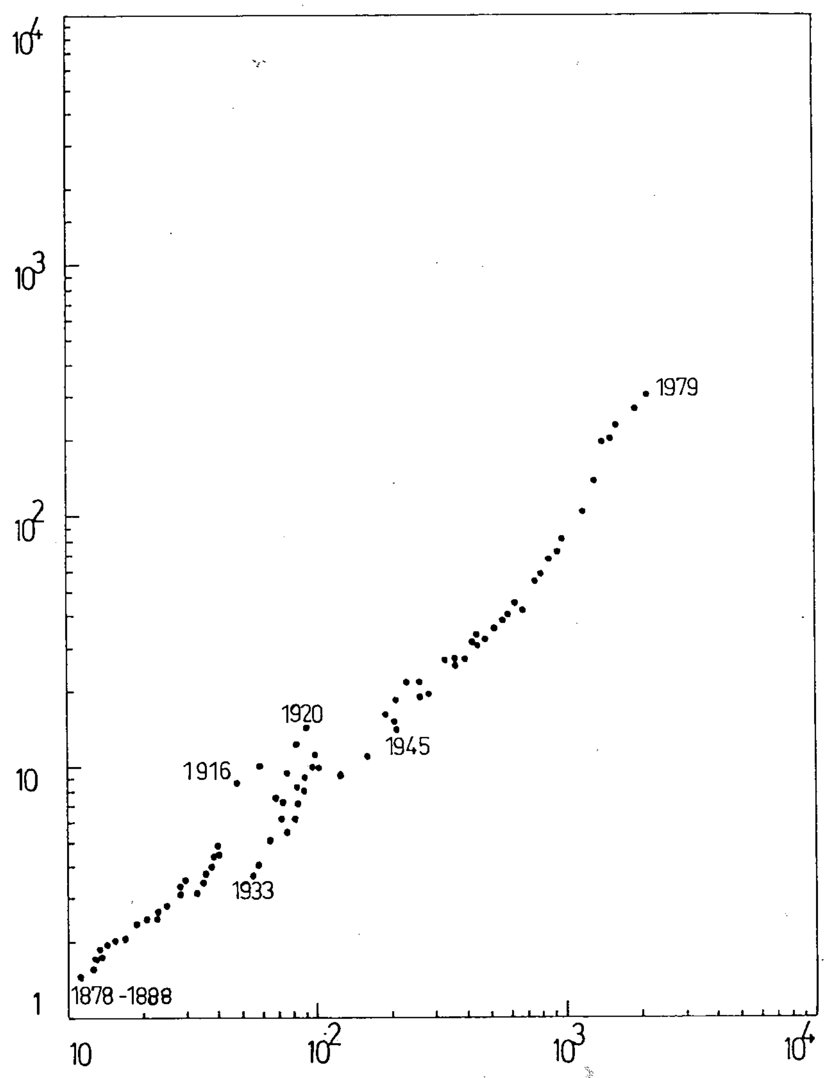


(*b*)

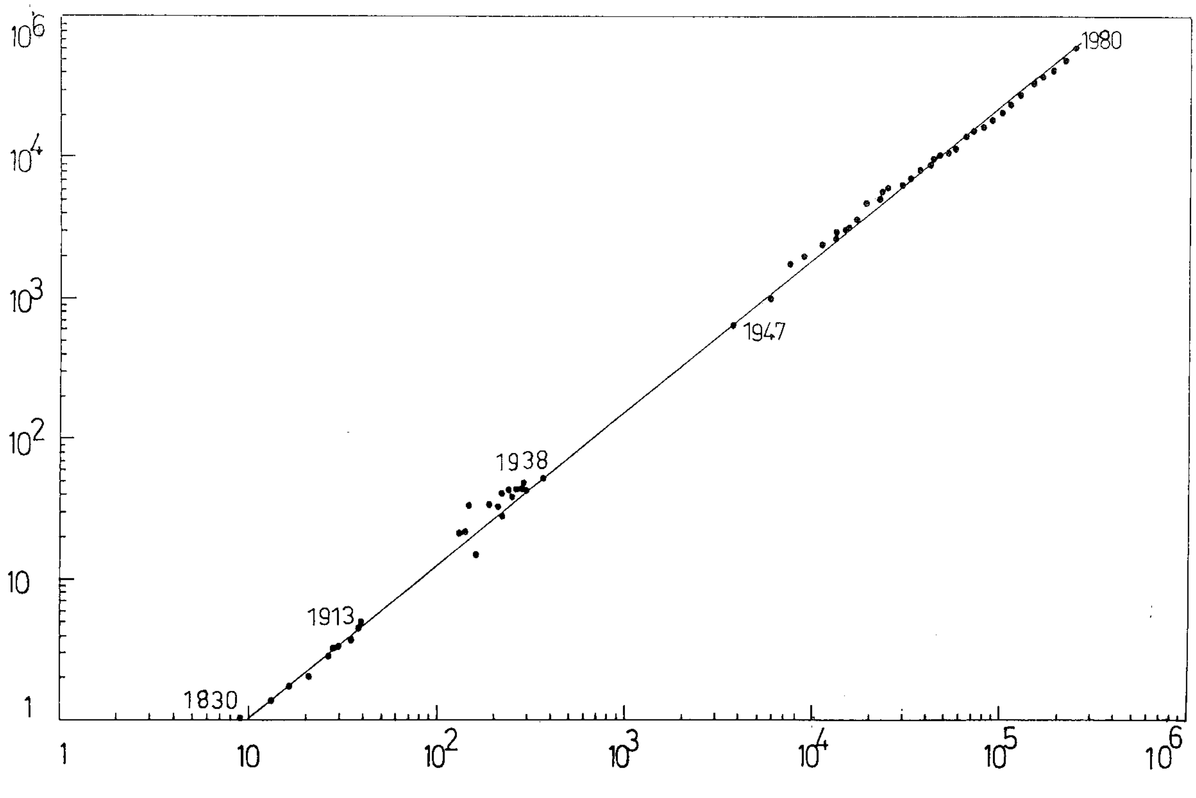
Figure 3. (*a*) Foreign trade versus net national product (both in billions of centimes) for France. (*b*) Foreign trade versus net national income (both in billions of gulden) for The Netherlands. (*c*) Foreign trade versus GNP (both in billions of francs) for Switzerland. (*d*) Foreign trade versus GNP (both in billions of dollars) for the U.S.A.


1980
10^2
10
1946
1938
1
1
10
10^2
10^3

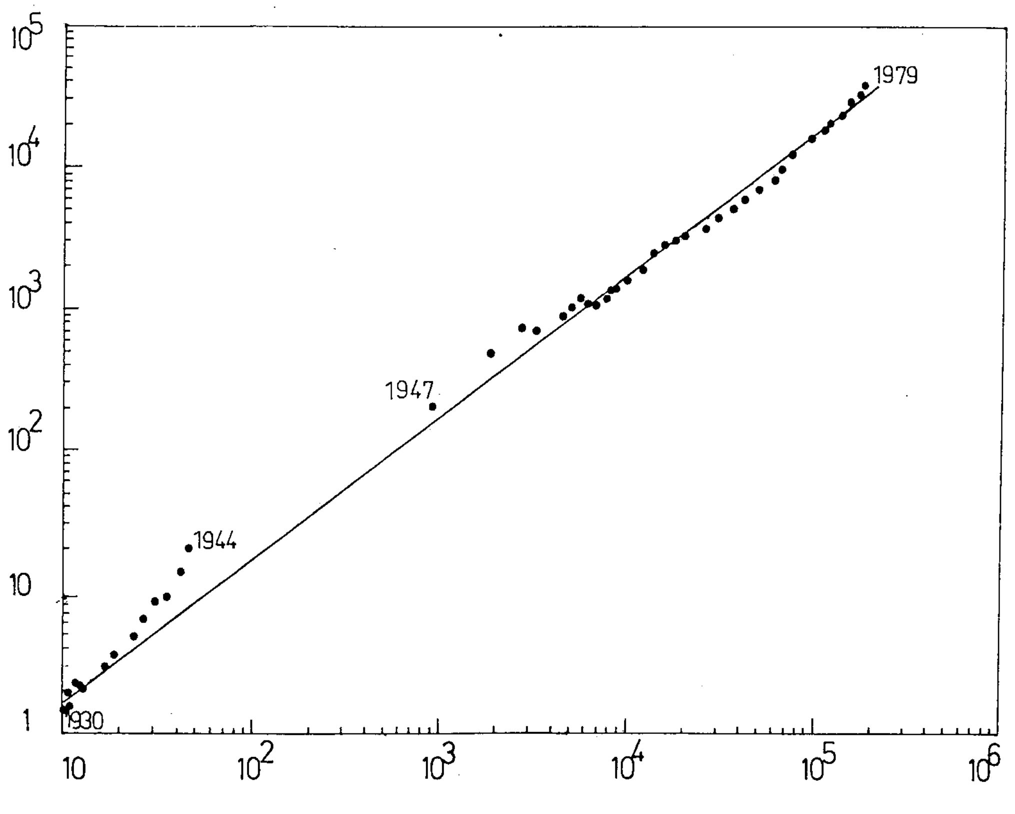
(*c*)


10^4
10^3
1979
10^2
1920
1945
1916
10
1933
1878 -1888
1
10
10^2
10^3
10^4

(*d*)



(*a*)



(*b*)

Figure 4. (*a*) Budget receipts versus net national income (both in billions of centimes) for France. (*b*) Budget receipts versus national income (both in billions of yen) for Japan.

	Period	ρ	Initial and final values of the ratio R/G
Belgium	1953–1980	1·07	23% →27%
France	1830–1980	1·07	11% →23%
Italy	1860–1980	1·06	13% →30%
Japan	1930–1979	1·01	17% →22%

Table 4.

3.4.3. *Relation between foreign trade and budget receipts*

The relation $C = \text{cte}\ R^{\beta}$ is of course simply a consequence of the two relations above for C and R, with $\beta = \gamma/\rho$, but for some countries long series exist for C and R, whereas the GNP figures are only known for a rather short period. Examining the relation between C and R thus gives, indirectly, the possibility to test the two power laws for C versus G and R versus G.

Figures 5 (*a*) and (*b*) for Belgium and Japan show that the power laws should hold, even for remote periods. The exponent β is given in Table 5.

	Period	β
Belgium	1859–1980	1·12
Japan	1882–1979	0·97

Table 5.

3.5. *Power laws for other global economic variables*

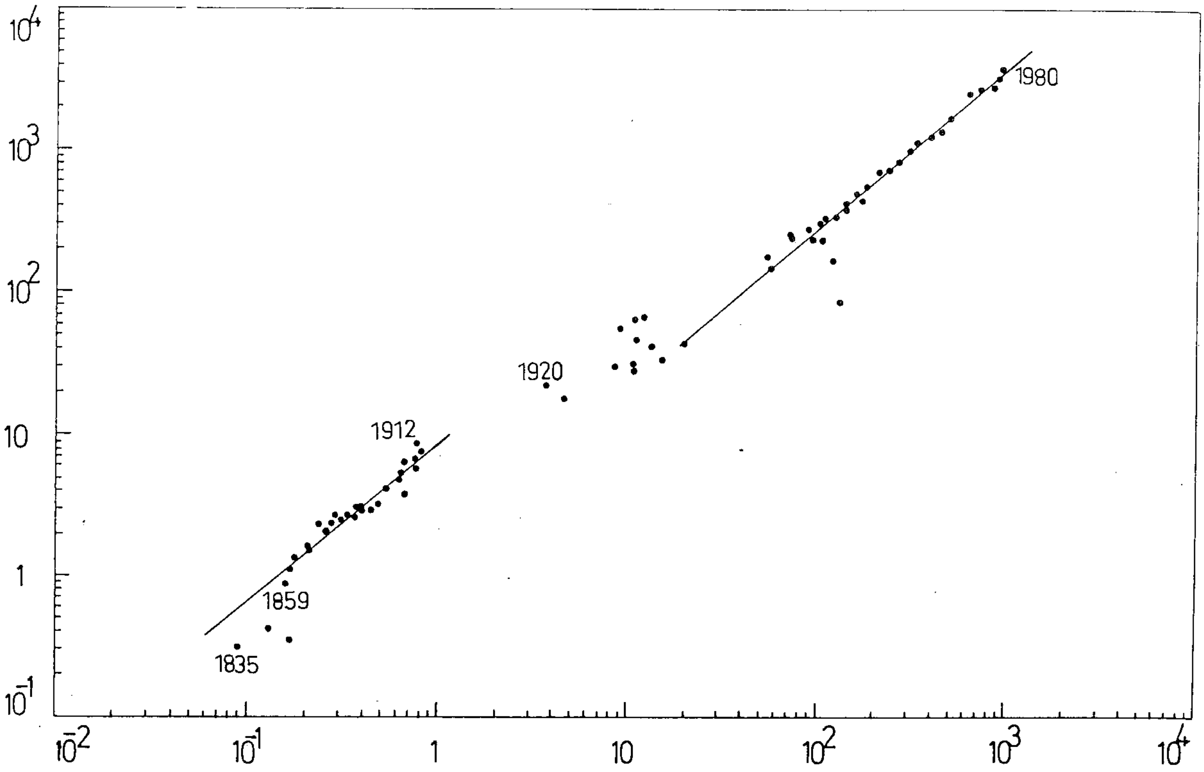
One could expect the same kind of power law to hold for other global variables ; for instance broad sectors such as wholesale and retail trade, services, capital goods are probably representative of the activity of the whole economy. The graph of Fig. 6 shows that this intuition is indeed confirmed by the statistical material.

Such data are, however, only available for recent years (i.e. for about 20 years, except for the U.S.A. where they have been recalculated from 1929 on). It is hazardous to try to obtain a trend from such limited data. It is mainly for this reason that we concentrated on foreign trade and budget data, the other reason being that differences in the definition of sectors from one country to another makes comparisons difficult.

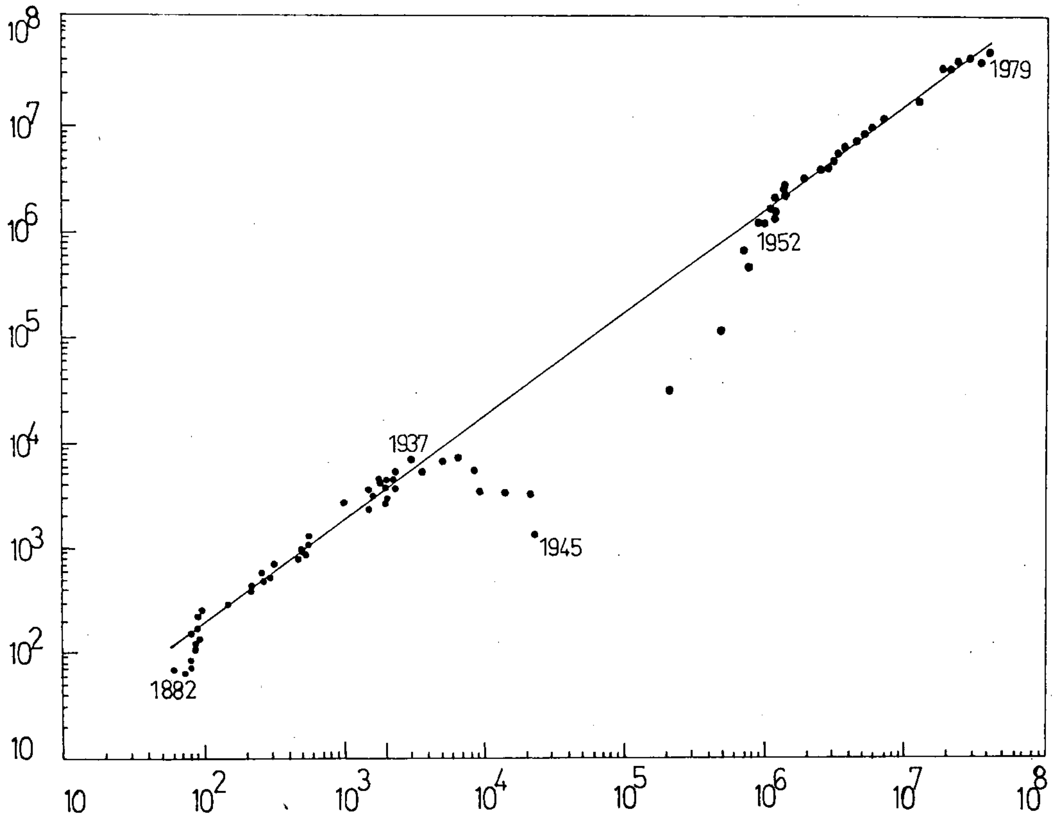
4. Conclusion

The search for empirical laws undertaken in this paper may appear at first sight rather disappointing for at least two reasons.

(1) The dispersions away from the proposed average laws are fairly large. This is, however, inherent to social sciences as opposed to natural sciences.



(a)



(b)

Figure 5. (*a*) Foreign trade versus budget receipts (both in billions of francs) for Belgium. (*b*) Foreign trade versus budget receipts (both in billions of yen) for Japan.

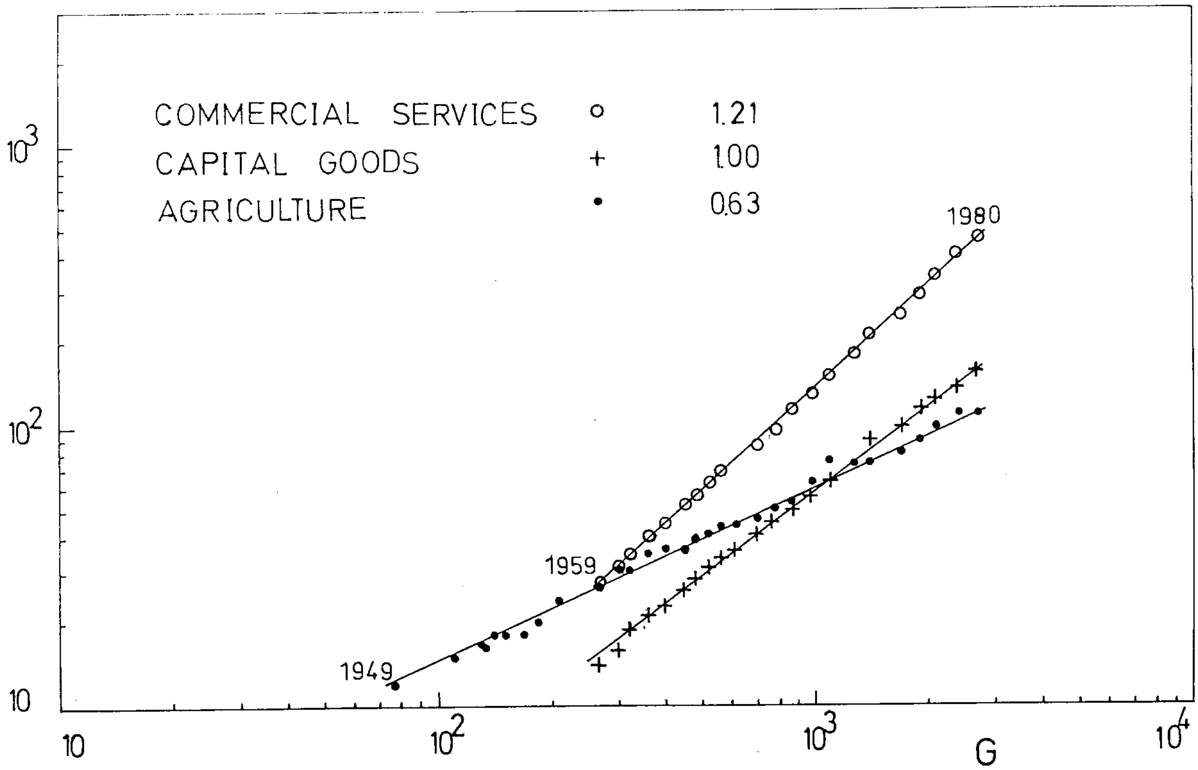


Figure 6. Sectors of national income versus net interior product (both in billions of francs) for France. *Source:* Annuaires Statistiques de la France.

(2) The statement of an empirical law does not by itself provide any further comprehension of the reality of economic development. We believe, however, that it may greatly facilitate forthcoming, more accurate, explanations.

In short, we may summarize our observations by the two following propositions (the constant k should not be the same in the different relations):

Proposition 1

(a) For countries having same GNP/capita, the ratio C/P is (presently) a decreasing function of P, namely

$$\frac{C}{P} \simeq \frac{k}{P^{1/4}}$$

(b) For countries of same population, but different GNP/capita, the ratio C/G is (presently) an increasing function of G, namely

$$\frac{C}{G} \simeq kG^{0 \cdot 1}$$

Proposition 2

For almost all 'old' industrial countries, the ratio C/G was an increasing function of G during the last hundred years, namely

$$\frac{C}{G} \simeq kG^{\gamma - 1}$$

The exponent γ (which is characteristic of a given country) ranges from 1·06 to 1·77 and its mean value is 1·14 (which is consistent with Proposition 1 (*b*)).

The consistency between Proposition 1 (*b*) for constant population and Proposition 2 may at first look strange, since industrial countries also experienced population growth. However, as already noted, the population increase is almost two orders of magnitude smaller than the GNP (or the foreign trade) variation.

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