Ergodicity Breaking in Disordered Matter: Novel Methods Across Physical Systems

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Summary of contents

Theoretical Framework

Statistical Mechanics and the Ergodic Hypothesis

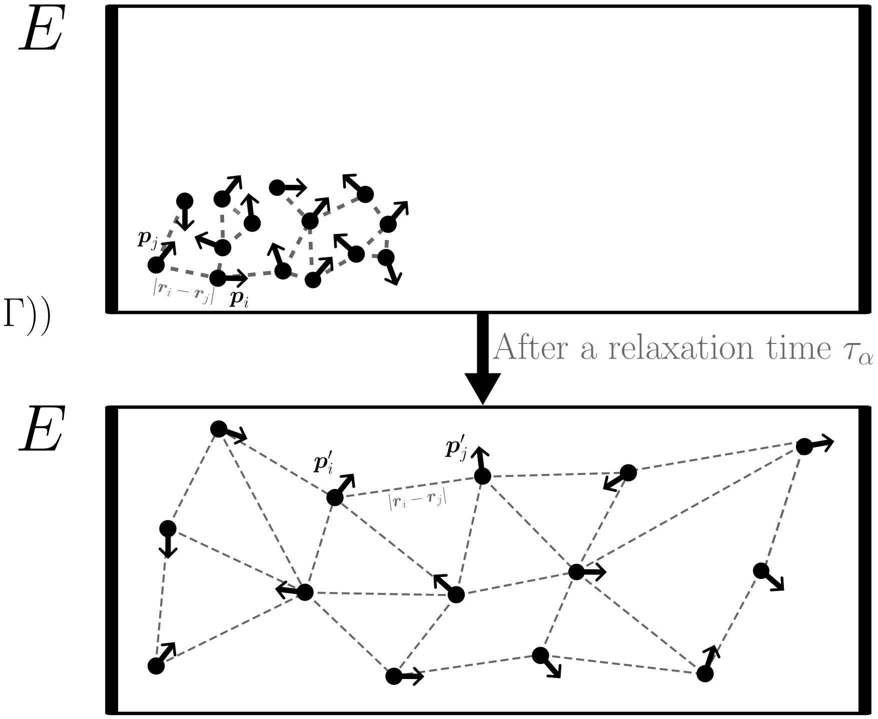
Microscopic models

$$\mathcal{H} = \sum_{i=1}^{N} \frac{\boldsymbol{p}_i^2}{2m_i} + \sum_{i>j} V(|\boldsymbol{r}_i - \boldsymbol{r}_j|)$$

$\Omega_N(E,V) = \frac{1}{N!h^{3N}} \int d\Gamma \delta(E - \mathcal{H}(\Gamma))$
with $\Gamma = (\boldsymbol{r}^N, \boldsymbol{p}^N)$

Macroscopic observables

$$P = k_B T \left(\frac{\partial \ln \Omega_N}{\partial V}\right)_{E,N}$$

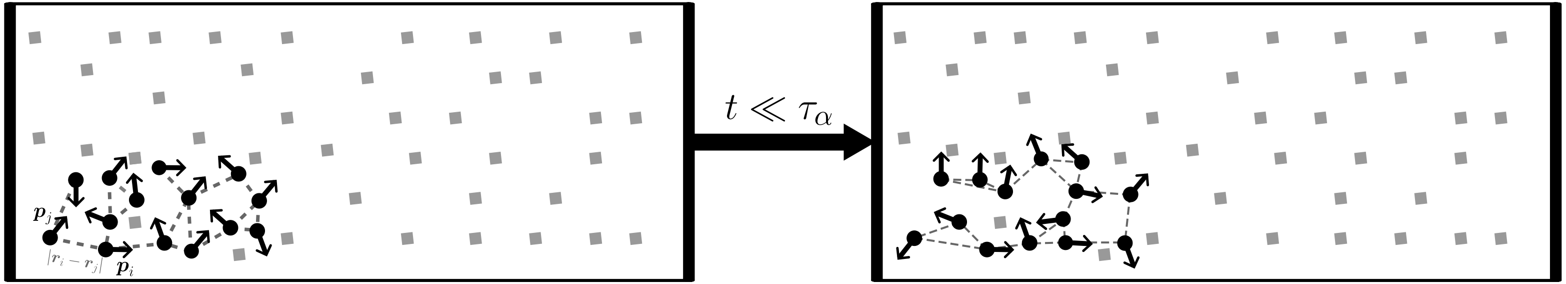


Observables characterized by $\langle \mathcal{O} \rangle_{\text{eq}}$

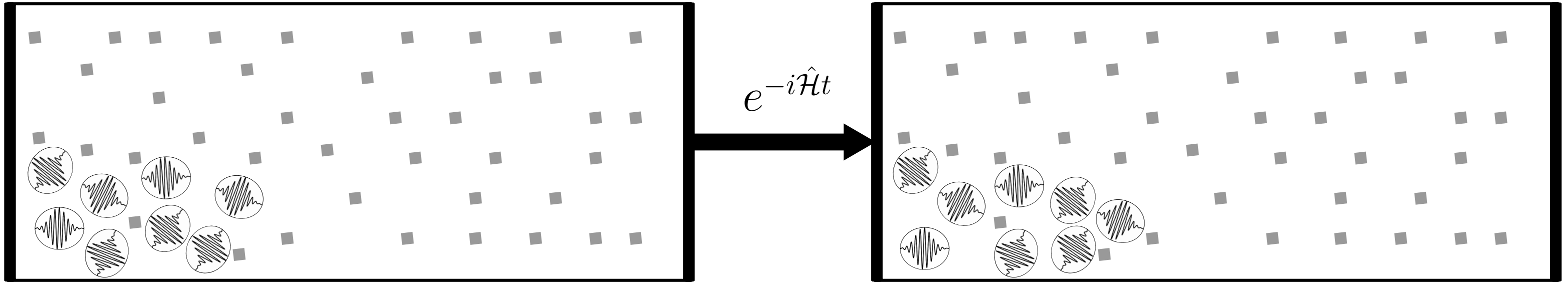
Ergodicity breaking

We can have out-of-equilibrium situations when...

τ_α is very large



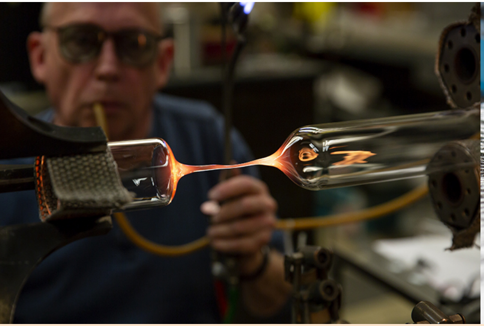
...or purely quantum effects

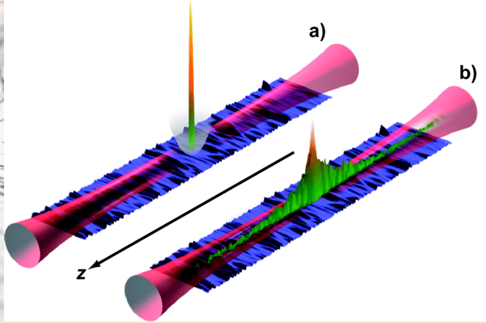


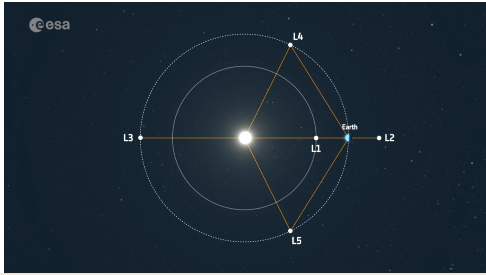
Ergodicity breaking

$$\boxed{\langle \mathcal{O} \rangle_{\mathrm{eq}} \neq \overline{\mathcal{O}}}$$

Ergodicity breaking signals the emergence of rich, interesting and complex phenomena (Phase transitions)







Ergodicity breaking

$$\boxed{\langle \mathcal{O} \rangle_{\text{eq}} \neq \overline{\mathcal{O}}}$$

...but it comes with its fair share
of complications when we attempt to study it

Slow relaxational dynamics

Finite-size effects

Long-range correlations and persistent memory

Sensitivity to rare-events

Metastability

The Critical Clusters for Frustrated Spin Systems

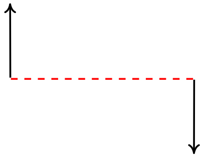
The main classical model - Ising spins

$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij}\sigma_i\sigma_j - h_{\text{ext}}\sum_{i=1}^{N}\sigma_i$$

with $\sigma_i = \uparrow$ or $\sigma_i = \downarrow$ with $i = 1, \cdots, N$

- J_{ij} interaction between σ_i and σ_j
- h_{ext} external magnetic field

---- AFM bond



—— FM bond

Global magnetization density

$$m = \frac{1}{N}\sum_{i=1}^{N}\langle\sigma_i\rangle$$

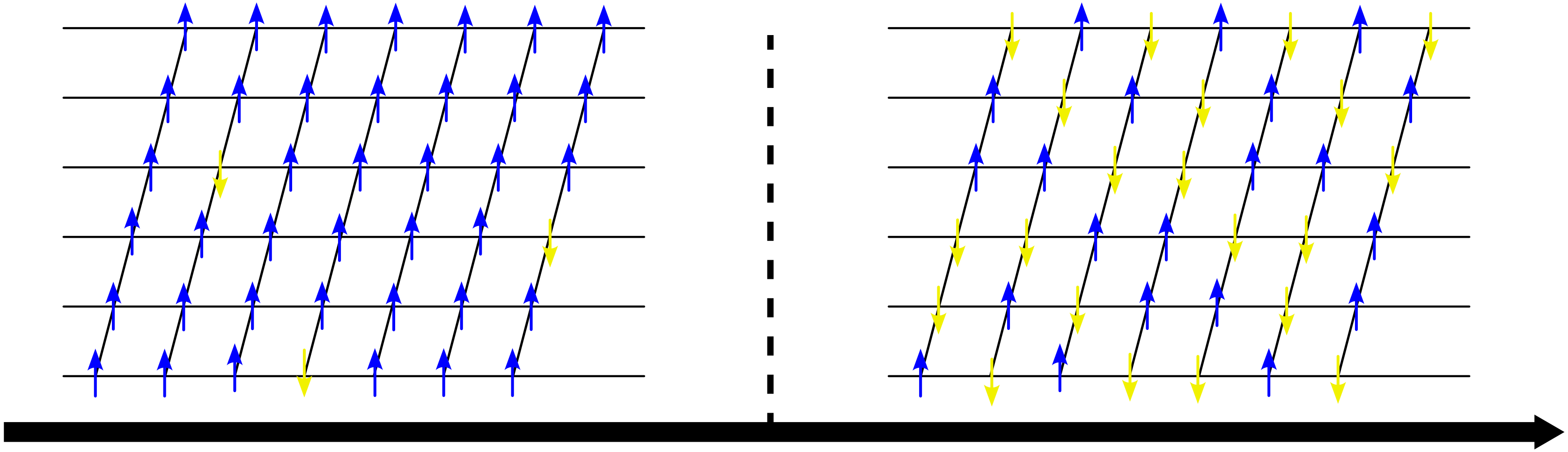
Correlation function

$$G(r) = \langle\sigma_i\sigma_j\rangle_{|\boldsymbol{r}_i - \boldsymbol{r}_j| = r}$$

The Ferromagnetic Ising model

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

$G(r) \sim e^{-r/\xi(T)} + m^2$ $\qquad$ $G_c(r) \sim \frac{1}{r^{d-2+\eta}}$ $\qquad$ $G(r) \sim e^{-r/\xi(T)}$



$G(r) \xrightarrow[r\to\infty]{} m^2$ $\qquad$ T_c $\qquad$ $G(r) \xrightarrow[r\to\infty]{} 0$

The Ferromagnetic Ising model

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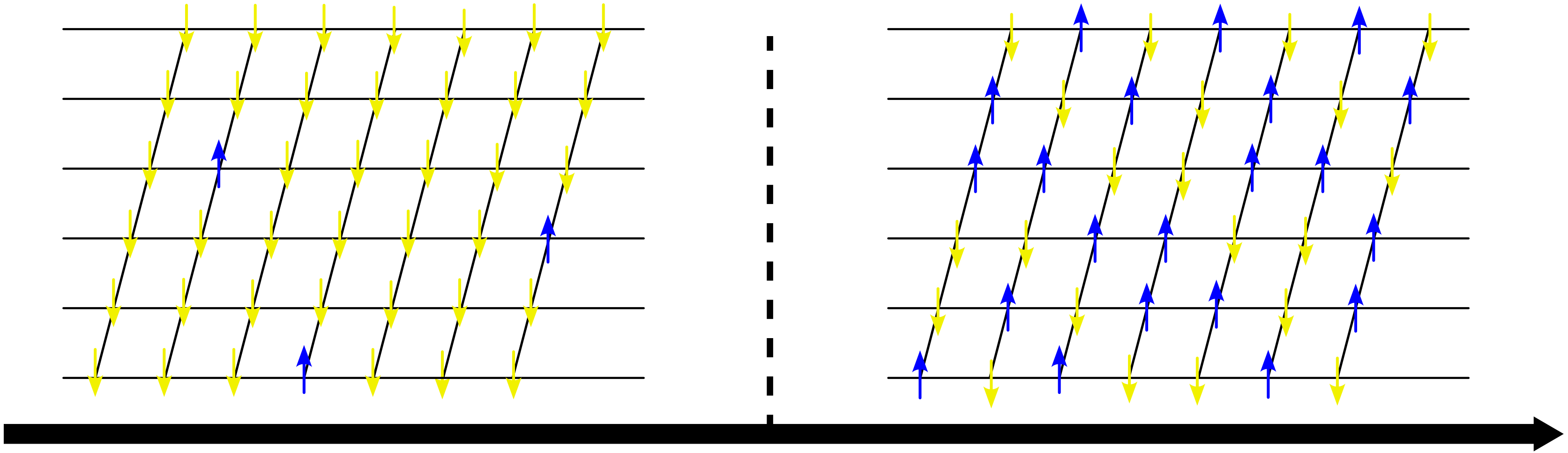


$G(r) \xrightarrow[r\to\infty]{} m^2$ $\quad$ T_c $\quad$ $G(r) \xrightarrow[r\to\infty]{} 0$

Critical slowing down

At the critical point

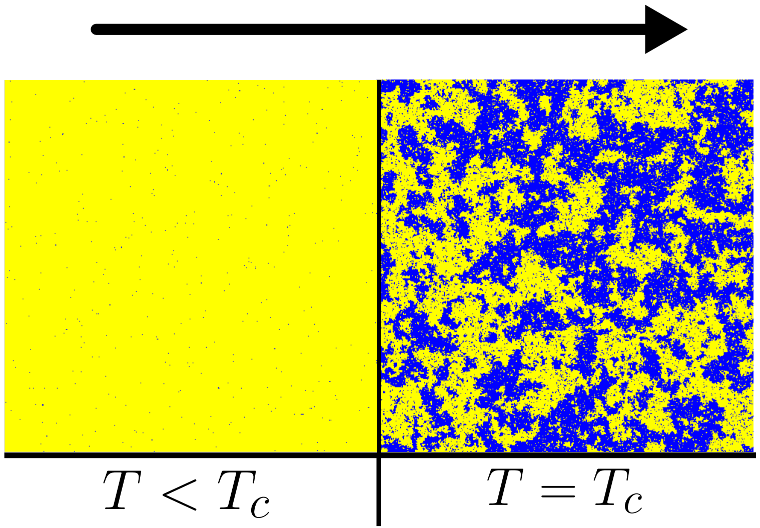
$$G_c(r) \sim \frac{1}{r^{d-2+\eta}}$$

Relaxation time

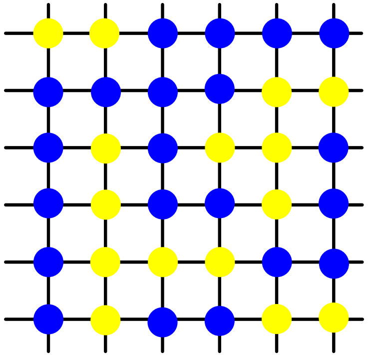
$$\tau_\alpha \sim L^{z_c}$$

with $z_c \simeq 2.17$ for $d = 2$

In a numerical simulation, an initial random configuration acquires the spatial correlations slowly



The critical clusters of the Ising model



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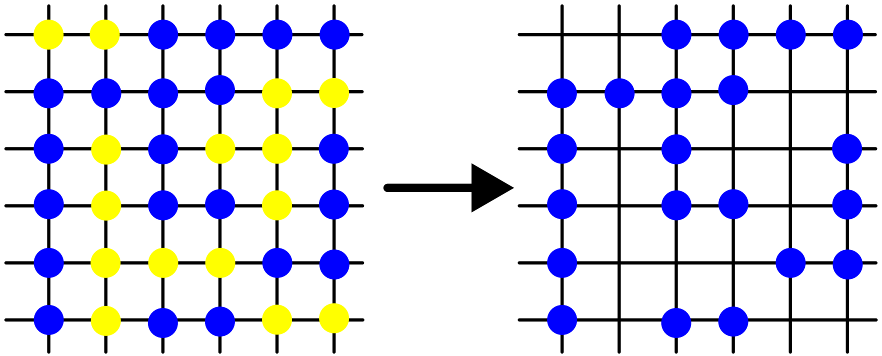

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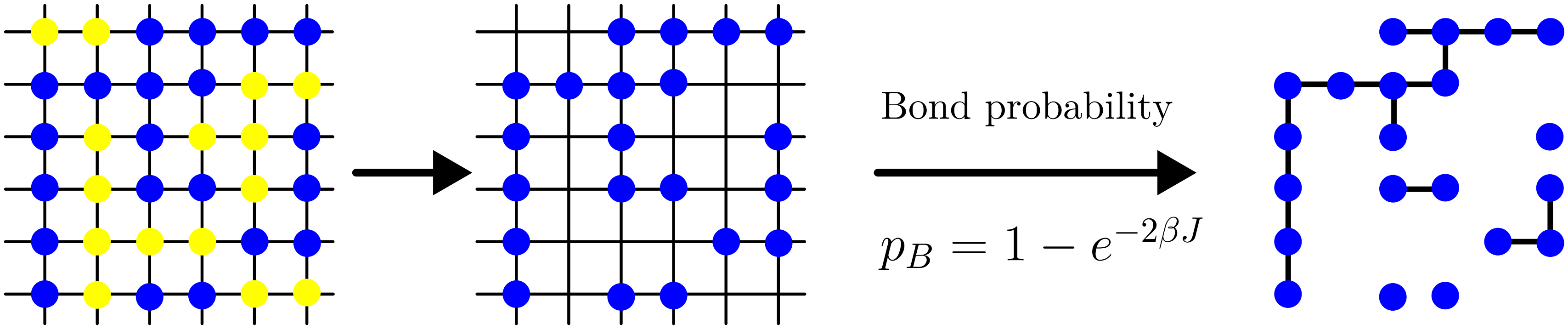


$$\langle \sigma_i \sigma_j \rangle \sim \mathbb{P}(i \leftrightarrow j)$$

[Fortuin, Kasteleyn (1969); Coniglio, Klein (1980)]

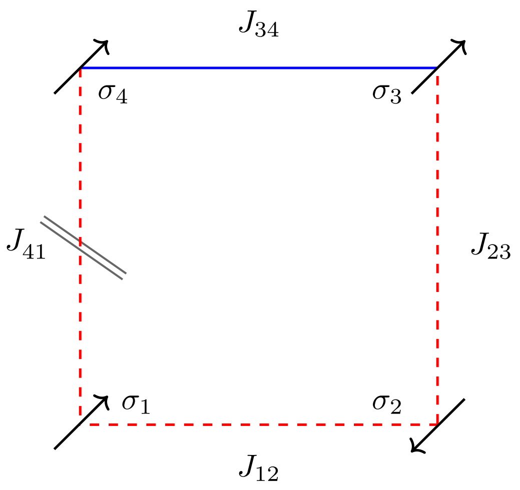
They serve as the basis of efficient Monte Carlo algorithms!

[Swendsen, Wang (1986); Wolff (1989)]

The critical clusters of frustrated models

$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij}\sigma_i\sigma_j$$

$$\mathcal{P}(J_{ij}) = \underbrace{\rho\delta(J_{ij}-J)}_{\text{FM bond}} + \underbrace{(1-\rho)\delta(J_{ij}+J)}_{\text{AFM bond}}$$



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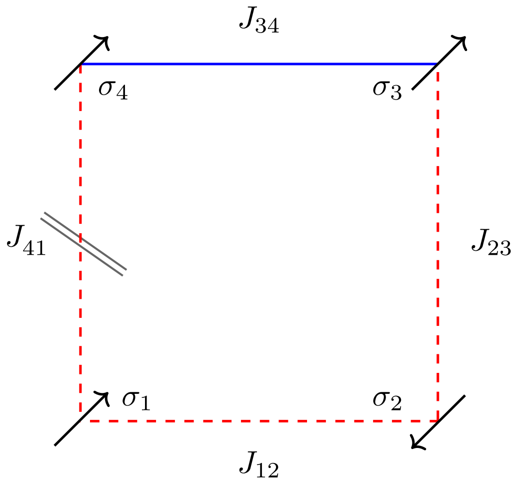


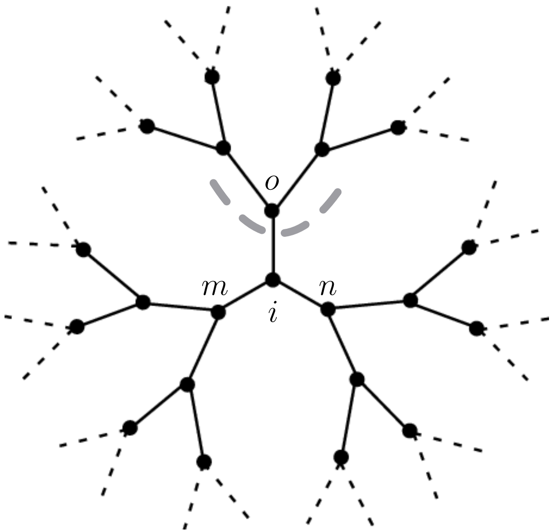
$$\boxed{p_B^{(ij)} = 1 - e^{-2\beta J_{ij}}}$$

[Coniglio et al. (1991)]
[Houdayer et al. (2001)]
[Chayes, Redner, Machta (1998)]
[Newman, Stein (2007)]

Can become negative!

Percolation on the Bethe lattice

Analytically continued $p_B^{(ij)} < 0$



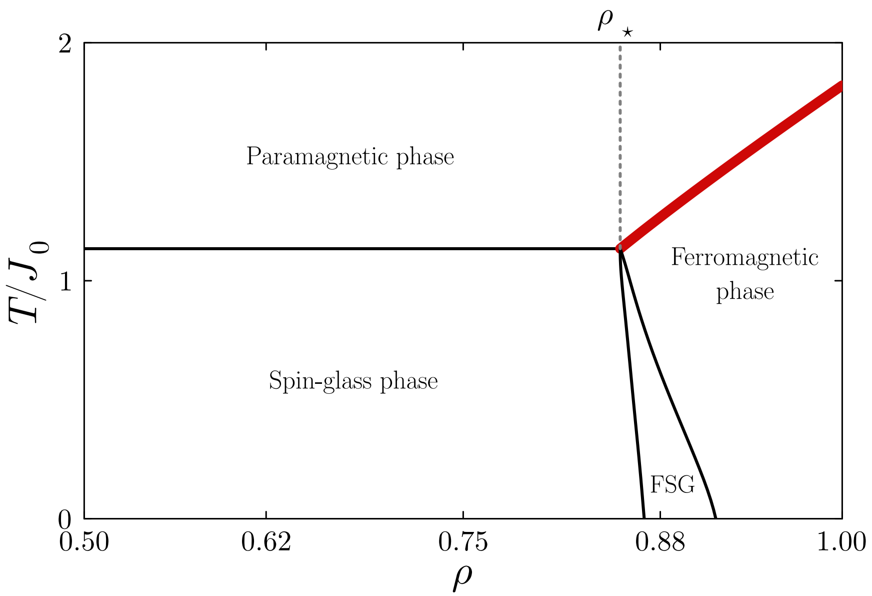
fixed connectivity $\kappa + 1 = 3$

Exact solution via the cavity method:

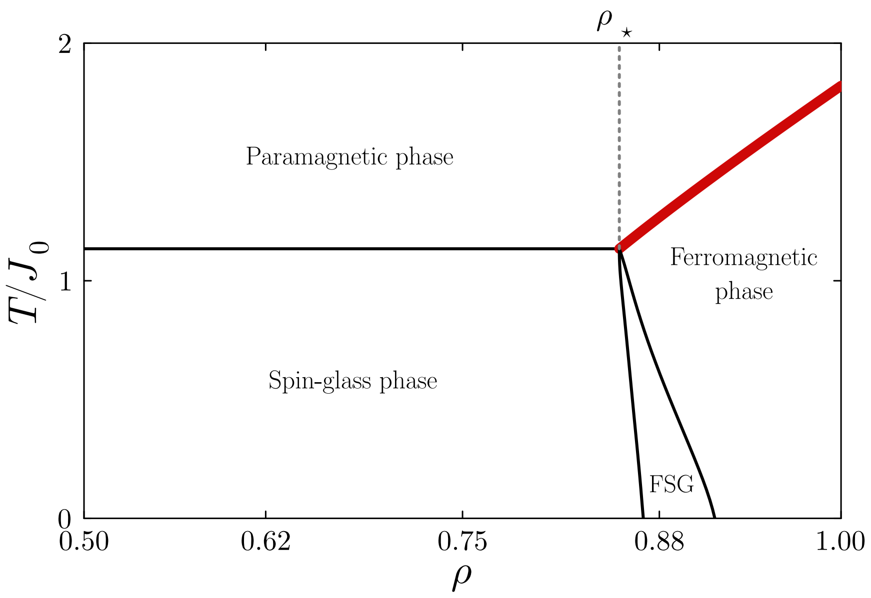
$$\pi_{i\to o} = \eta_{i\to o}(\uparrow)\frac{\pi_{n\to i}\,\psi_{in}[\eta_{m\to i}(\uparrow)\,\psi_{im}+1]+\pi_{m\to i}\,\psi_{im}[\psi_{in}(\eta_{n\to i}(\uparrow)-\pi_{n\to i})+1]}{[\eta_{m\to i}(\uparrow)\,\psi_{im}+1][\eta_{n\to i}(\uparrow)\,\psi_{in}+1]}$$

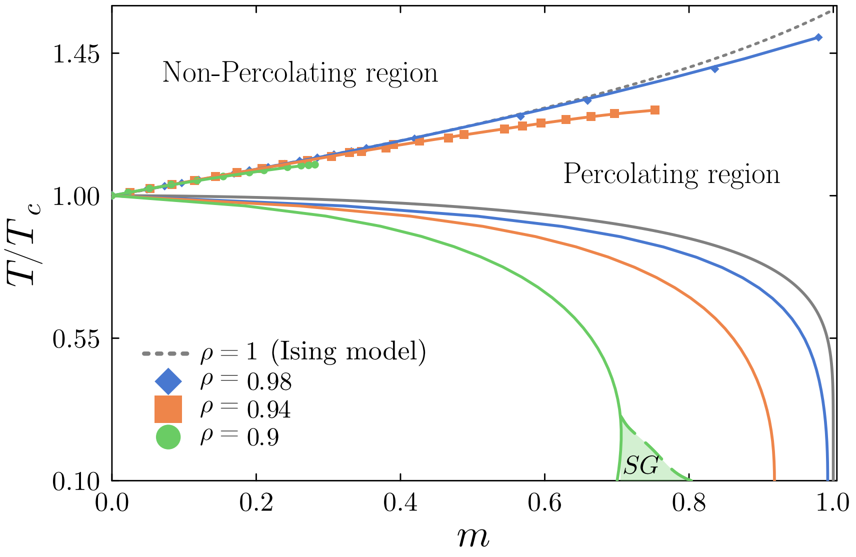
$\pi_{i\to o}$ probability that spin i belongs to the percolating cluster in absence of o,
with $\eta_{i\to o}$ the configuration probability that i is up in absence of o,
and $\psi_{im} = e^{2\beta J_{im}} - 1$.

The critical clusters of frustrated models



The critical clusters of frustrated models





The critical transition and the exponents coincide!

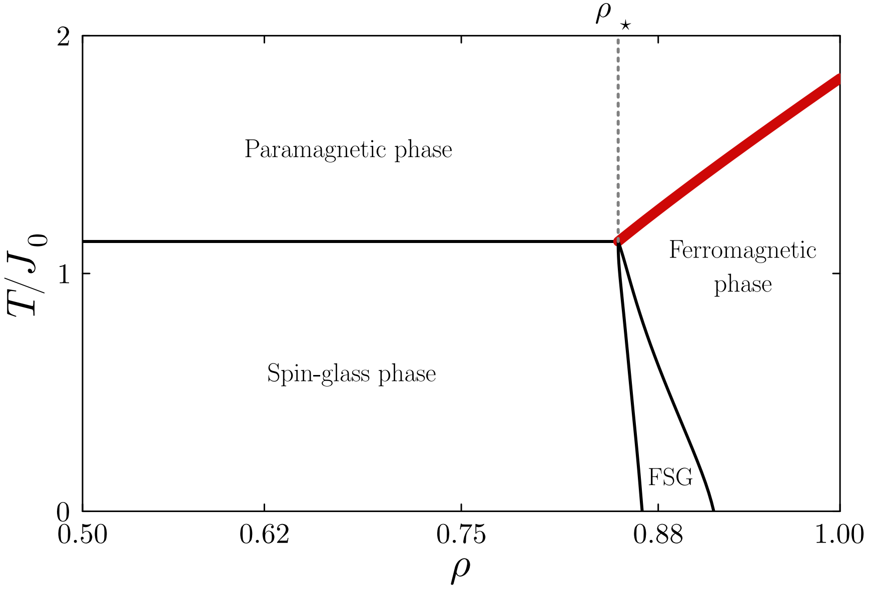
$$\langle \sigma_i \sigma_j \rangle \sim \mathbb{P}(i \leftrightarrow j)$$

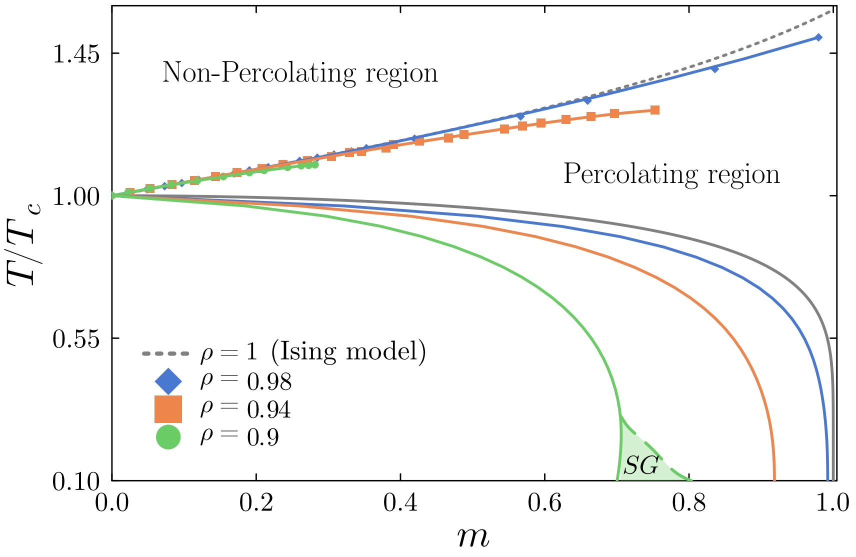
...also for other frustrated models

Percolation and criticality of systems with competing interactions on Bethe lattices: limitations and potential strengths of cluster schemes

Greivin Alfaro Miranda,[1] Mingyuan Zheng,[2,3,*] Patrick Charbonneau,[2,4]
Antonio Coniglio,[5] Leticia F. Cugliandolo,[6] and Marco Tarzia[7]

The critical clusters of frustrated models





The critical transition and the exponents coincide!

...but probabilities are negative!

...also for other frustrated models

Percolation and criticality of systems with competing interactions on Bethe lattices: limitations and potential strengths of cluster schemes

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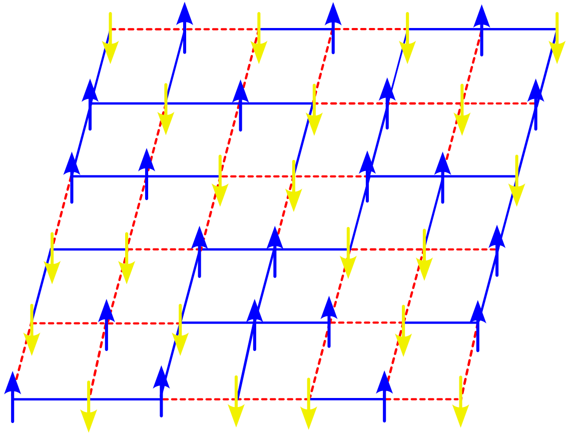
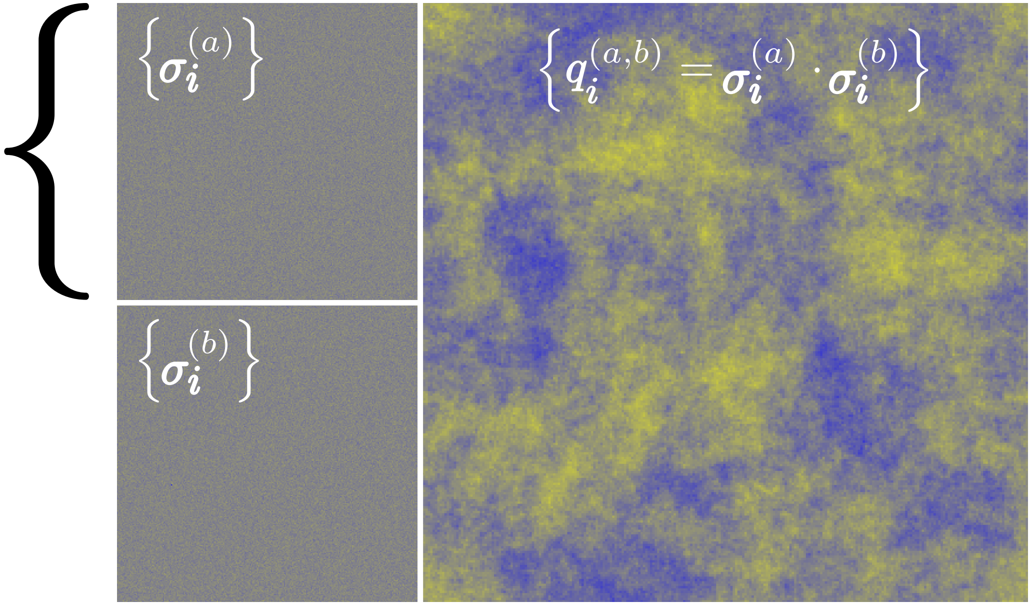
The spin glass phase

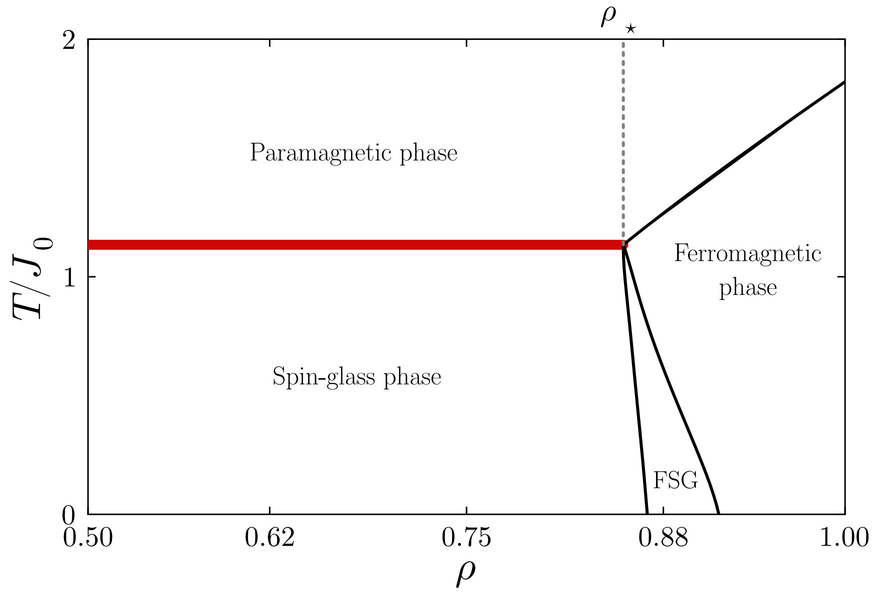
There is no (trivial) symmetry breaking field!

[Baity-Jesi et al. (2019)]





The critical clusters for the spin glass transition



Two replica clusters

$$p_B^{(ij)} = \left(1 - e^{-\beta|J_{ij}| - \beta\sigma_i\sigma_j J_{ij}}\right)^2$$

[Newman, Stein (2007)]

Multiple replica clusters

$$p_B^{(ij)} = \left(1 - e^{-\beta|J_{ij}| - \beta\sigma_i\sigma_j J_{ij}}\right)^I$$

[Münster, Weigel (2023)]

> [69] Note that using more replicas in Eq. (15) notably reduces the occupation probability and thus lowers the onset temperature for percolation. As a consequence, it might be interesting to see if for a certain I the percolation transition is directly linked to the finite temperature spin-glass transition for $d > 2$.

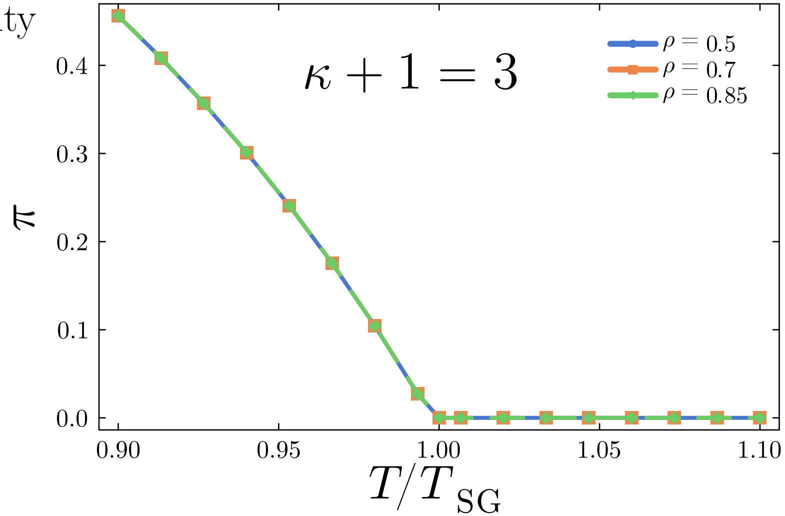
The critical clusters for the spin glass transition

Multiple replica clusters

$$p_B^{(ij)} = (1 - e^{-\beta|J_{ij}| - \beta\sigma_i\sigma_j J_{ij}})^I$$

$$I \approx 2.86$$

π: average percolation probability



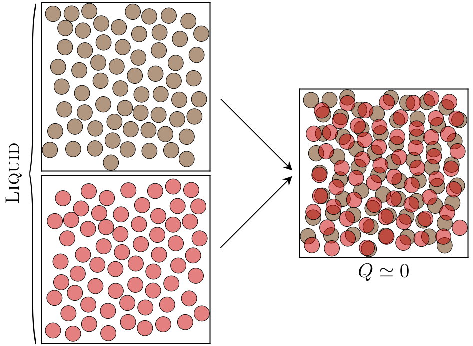
Percolation and thermodynamic transitions coincide, for all ρ within the paramagnetic to spin glass transition

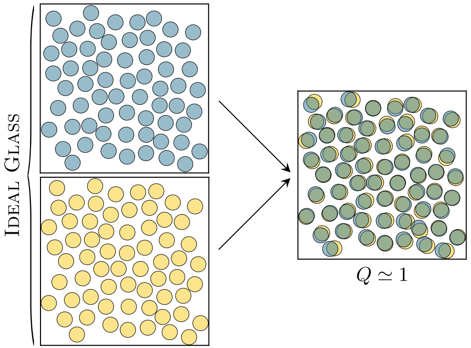
Is this parameter "I" physical?
Does "I" depend on κ?

The SWAP Method for Frustrated Spin Systems

Structural glasses

Is there a underlying growing correlation?





→ Random First Order Transition (RFOT)

[Kirkpatrick et al. (1989)]

[Lubchenko, Wolynes (2007)]

Or is it just a dynamical effect?
(No thermodynamics)

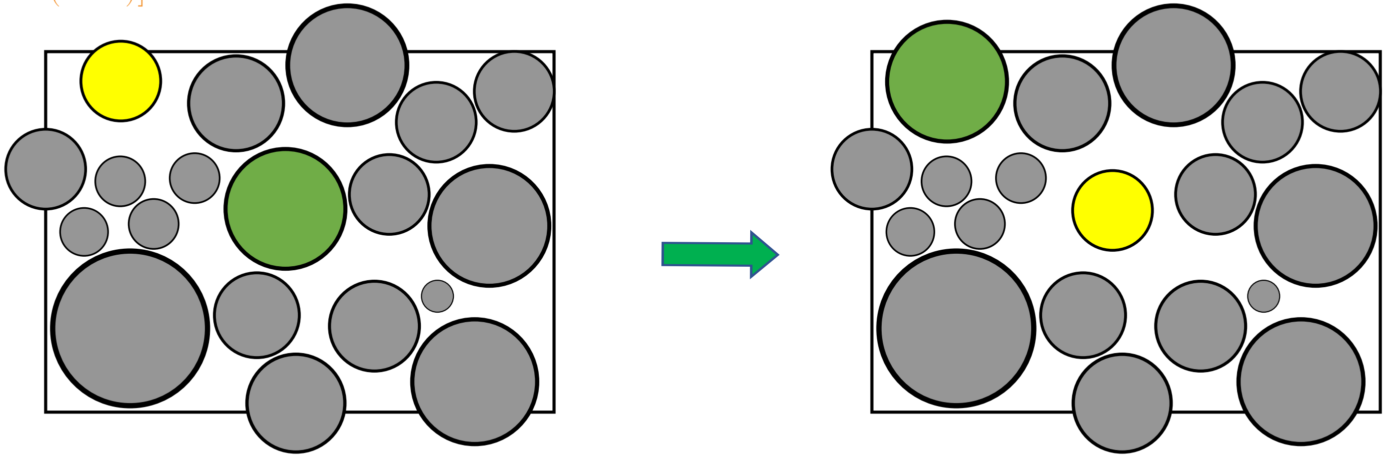
[Guiselin, Tarjus, Berthier (2022)]

Kinetically Constrained Models (KCM) → Dynamical Facilitation

[Garrahan, Chandler (2003)]

The SWAP method for particle systems

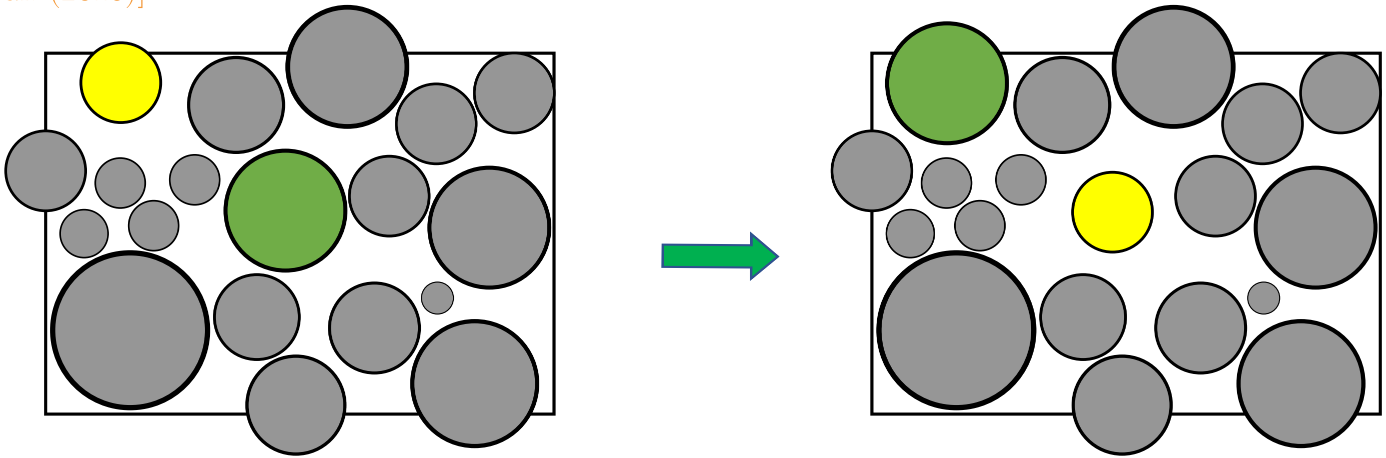
[Berthier et al. (2016)]



For some models there is a tremendous acceleration!

The SWAP method for particle systems

[Berthier et al. (2016)]

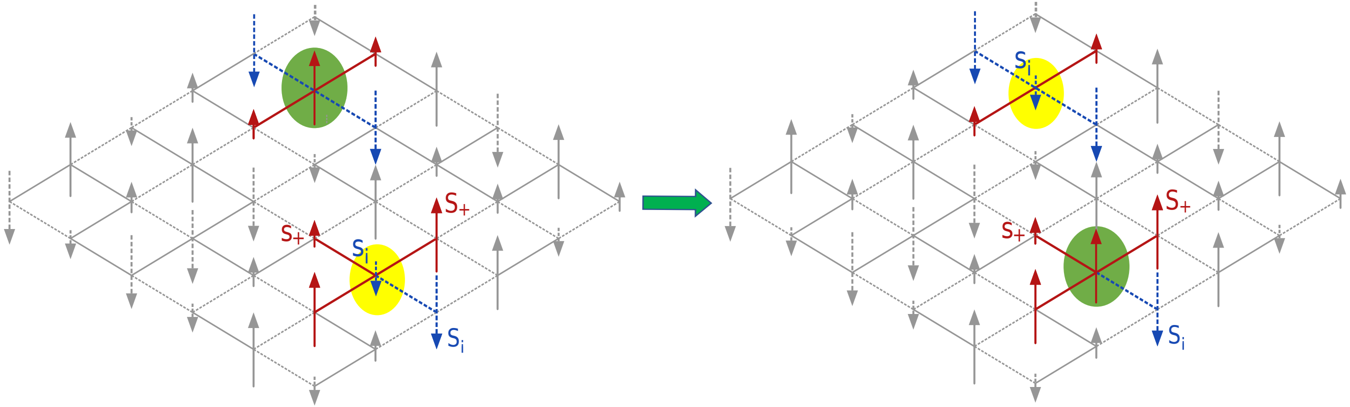


For some models there is a tremendous acceleration!

[Wyart, Cates (2017)] ⟶ Then the slowing down is mainly a dynamical effect

[Gutiérrez, Garrahan, Jack (2019)] ⟶ It also accelerates a KCM!

The SWAP method for spin systems



The 2d (soft) Edwards-Anderson model

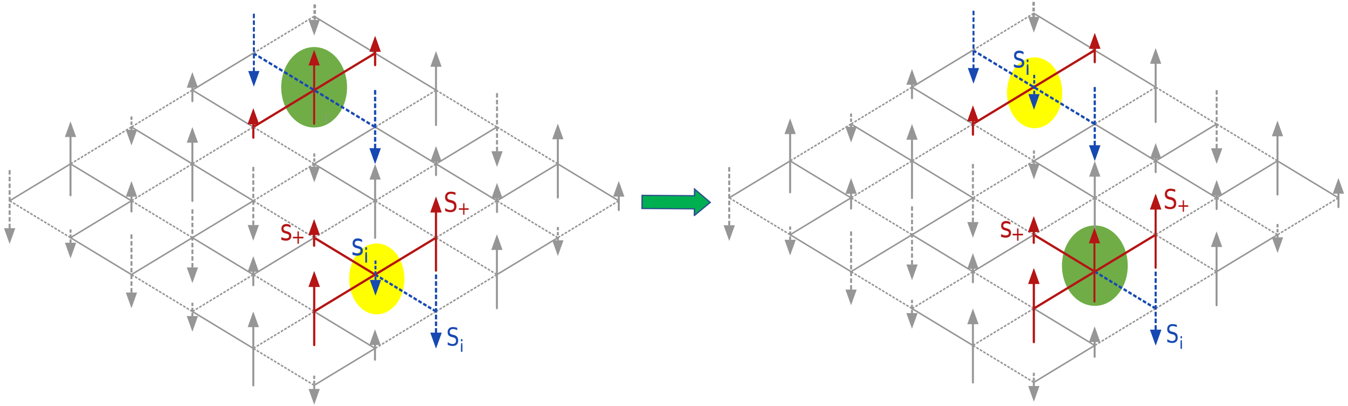
$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} s_i s_j = -\sum_{\langle ij \rangle} J_{ij} \tau_i \tau_j \sigma_i \sigma_j$$

$$\mathcal{P}(J_{ij}) = \frac{1}{2}\delta(J_{ij} - J) + \frac{1}{2}\delta(J_{ij} + J)$$

$\tau_i \in [1 - \Delta/2, 1 + \Delta/2]$

$\Delta \in [0, 2)$

The SWAP method for spin systems



The 2d (soft) Edwards-Anderson model

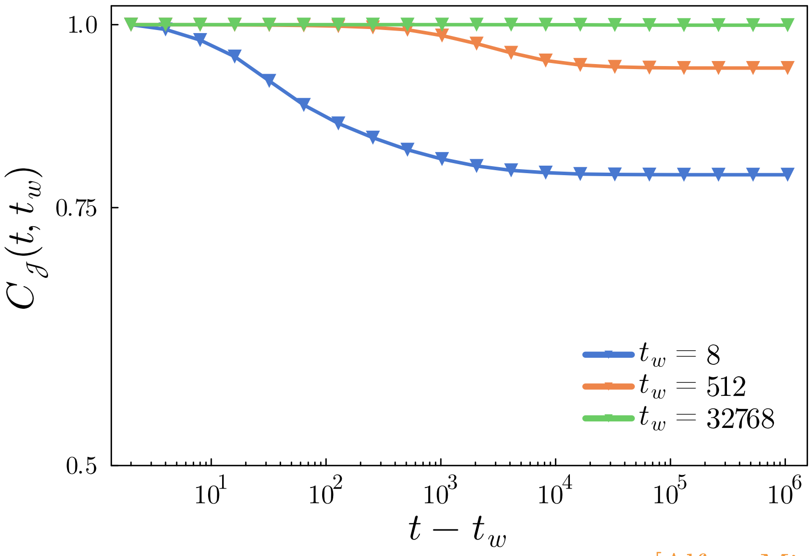
$$\mathcal{H} = -\sum_{\langle ij \rangle} \underbrace{\mathcal{J}_{ij}}_{\text{partially annealed}} \sigma_i \sigma_j$$

controlled by the parameter p_{swap}

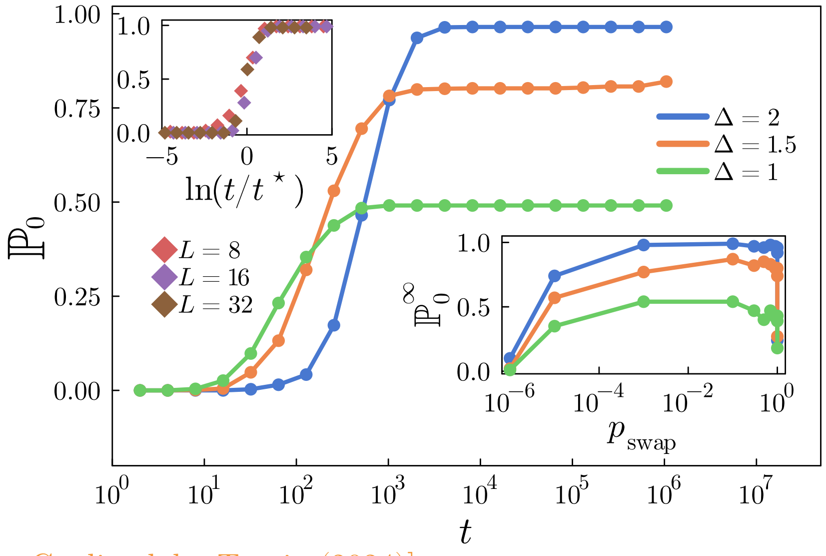
$$\begin{cases} \text{with probability } p_{\text{swap}} & \rightarrow N \text{ (non-local) exchange attempts } (\sigma_i, \tau_i) \leftrightarrow (\sigma_j, \tau_j) \\ \text{with probability } 1 - p_{\text{swap}} & \rightarrow N \text{ single spin flip attempts } \sigma_i \rightarrow -\sigma_i \end{cases}$$

2d spin glass ground state sampling

Autocorrelation of the effective bonds ($\mathcal{J}_{ij} = J_{ij}\tau_i\tau_j$)



Probability of finding a ground state



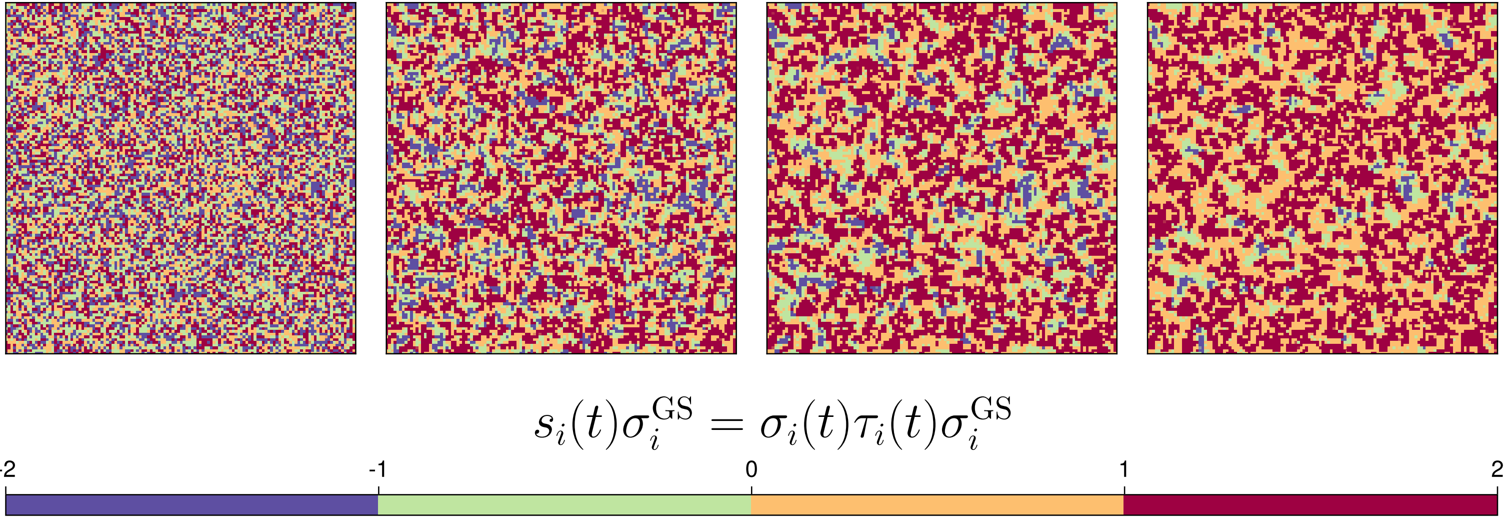
[Alfaro Miranda, Cugliandolo, Tarzia (2024)]

How does it work?

Zero temperature quench for $\Delta = 2$, $p_{\mathrm{swap}} = 0.1$ and $L = 128$

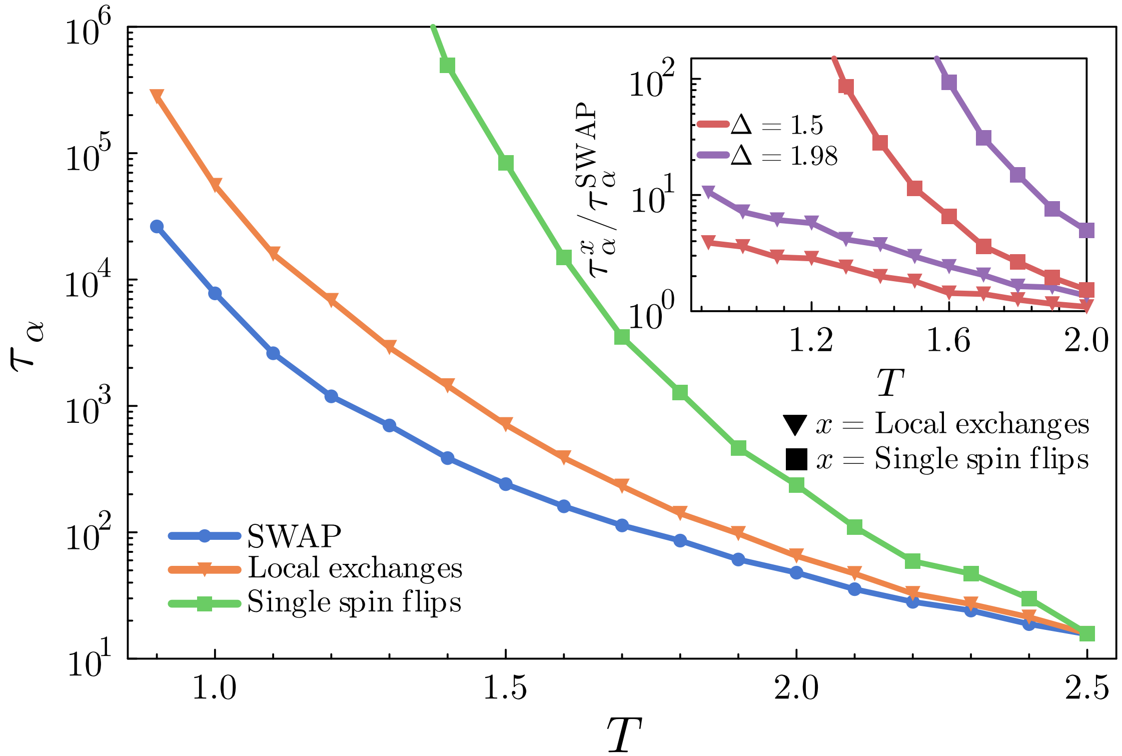
[Alfaro Miranda, Cugliandolo, Tarzia (2024)]

$t = 0$ $\quad$ $t = 128$ $\quad$ $t = 2048$ $\quad$ $t = 32768$



$$s_i(t)\sigma_i^{\mathrm{GS}} = \sigma_i(t)\tau_i(t)\sigma_i^{\mathrm{GS}}$$

Relaxation time speed up



We define this τ_α as the time for which the self-correlation has become age independent (i.e. $C(t, t_w) = \hat{C}(t - t_w)$) and has decayed to 20% of its original value (i.e. $\hat{C}(\tau_\alpha) = 0.2\ \hat{C}(0)$)

2-3 orders of magnitude acceleration!

Maybe dynamical facilitation plays an important role in the efficiency of the SWAP algorithm for particles

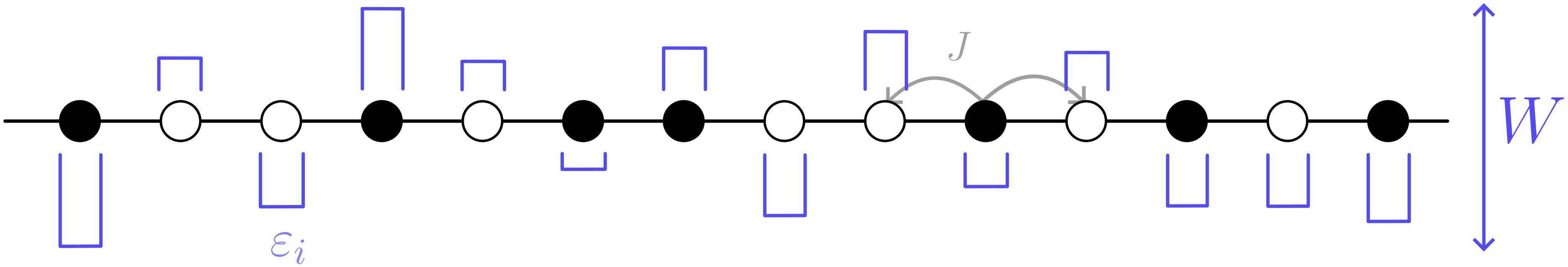
[Alfaro Miranda, Cugliandolo, Tarzia (2024)]

The Importance of Rare Events in Many-Body Localization

Anderson localization

Non-interacting quantum particles in a disordered potential

$$\hat{\mathcal{H}} = -J \sum_{\langle ij \rangle} \left(\hat{c}_i^\dagger \hat{c}_j + \text{h.c.} \right) + \sum_{i=1}^{N} \varepsilon_i \hat{c}_i^\dagger \hat{c}_i \qquad \varepsilon_i \in [-W/2, W/2]$$



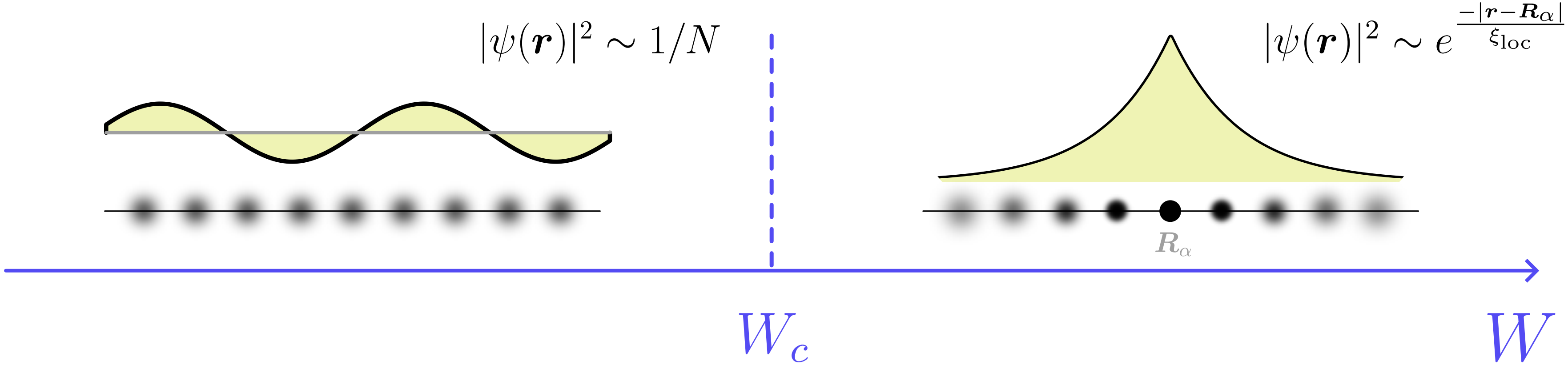
[Anderson (1958)]

Anderson localization

Non-interacting quantum particles in a disordered potential

$$\hat{\mathcal{H}} = -J \sum_{\langle ij \rangle} \left(\hat{c}_i^\dagger \hat{c}_j + \text{h.c.} \right) + \sum_{i=1}^{N} \varepsilon_i \hat{c}_i^\dagger \hat{c}_i \qquad \varepsilon_i \in [-W/2, W/2]$$

For $d > 2$, there is a W_c such that



Many-body localization (MBL)

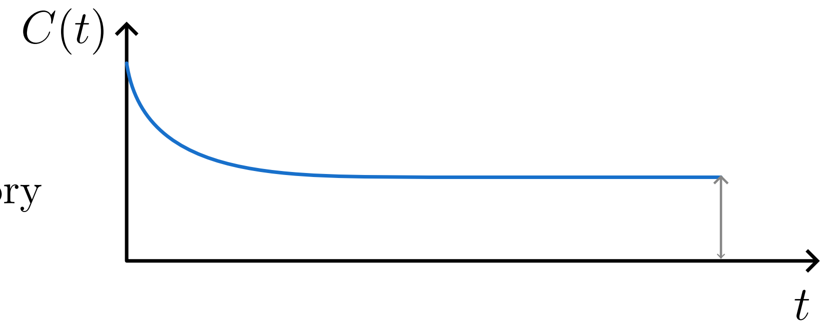
Interacting quantum particles in a disordered potential

$$\hat{\mathcal{H}} = -J\sum_{i=1}^{L}\left(\hat{c}_i^\dagger \hat{c}_{i+1} + \text{h.c.}\right) + \sum_{i=1}^{L}\varepsilon_i\,\hat{c}_i^\dagger \hat{c}_i + \Delta\sum_{i=1}^{L}\hat{n}_i\hat{n}_{i+1}$$

Can the localized phase survive interactions?

[Fleishman, Anderson (1980); Gornyi, Mirlin, Polyakov (2005); Basko, Aleiner, Altshuler (2006)]

$$C(t) = \frac{4}{L}\sum_{i=1}^{L}\left\langle\left(\hat{n}_i(t)-\tfrac{1}{2}\right)\left(\hat{n}_i(0)-\tfrac{1}{2}\right)\right\rangle$$



Ergodicity breaking transition:

In the MBL phase the system retains memory of the initial condition after infinite time

Many-body localization (MBL)

The XXZ spin-1/2 chain with a random field

$$\hat{\mathcal{H}} = \frac{1}{2}\sum_{i=1}^{L}\Big(\underbrace{\hat{c}_i^\dagger \hat{c}_{i+1} + \mathrm{h.c.}}_{\substack{\text{nearest-neighbor}\\ \text{hopping}}} + \underbrace{2\Delta \hat{n}_i \hat{n}_{i+1}}_{\text{interactions}} + \underbrace{2h_i \hat{n}_i}_{\substack{\text{disorder}\\ h_i \in [-W, W]}}\Big)$$

[Oganesyan, Huse (2007)]

Many-body localization (MBL)

The XXZ spin-1/2 chain with a random field

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$h_i \in [-W, W]$

[Oganesyan, Huse (2007)]

For any generic quantum state

$$|\Psi\rangle = \sum_{I=1}^{\mathcal{N}} \psi_I |I\rangle$$

where $\mathcal{N}$ is the dimension of the Hilbert space, and $|I\rangle$ is a basis state (i.e. particle configuration)

Many-body localization (MBL)

The XXZ spin-1/2 chain with a random field

$$\hat{\mathcal{H}} = \frac{1}{2}\sum_{i=1}^{L}\Big(\underbrace{\hat{c}_i^\dagger \hat{c}_{i+1} + \text{h.c.}}_{\text{nearest-neighbor hopping}} + \underbrace{2\Delta \hat{n}_i \hat{n}_{i+1}}_{\text{interactions}} + \underbrace{2h_i \hat{n}_i}_{\text{disorder}}\Big)$$

$h_i \in [-W, W]$

[Oganesyan, Huse (2007)]

For any generic quantum state

$$|\Psi\rangle = \sum_{I=1}^{\mathcal{N}} \psi_I |I\rangle$$

where $\mathcal{N}$ is the dimension of the Hilbert space, and $|I\rangle$ is a basis state (i.e. particle configuration)

Hilbert space picture:

Recast the many-body quantum dynamics into the diffusion of a single fictitious particle on the Hilbert (or Fock) space graph

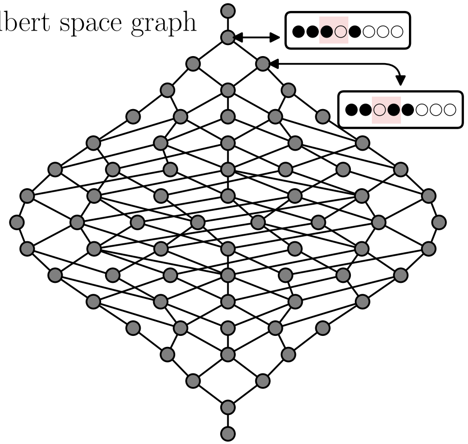
[Altshuler, Gefen, Kamenev, Levitov (1997)]

$$\hat{\mathcal{H}} = \sum_I E_I |I\rangle\langle I| + \sum_{\langle I,J\rangle} T_{IJ}\left(|I\rangle\langle J| + |J\rangle\langle I|\right)$$

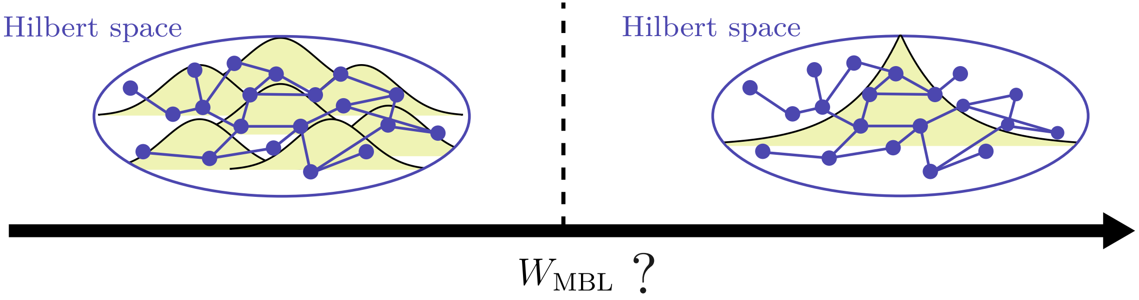
$E_I = \langle I|\sum_{i=1}^{L}(\Delta \hat{n}_i \hat{n}_{i+1} + h_i \hat{n}_i)|I\rangle$ correlated energies

$T_{IJ} = \langle I|\frac{1}{2}\sum_{i=1}^{L}(\hat{c}_i^\dagger \hat{c}_{i+1} + \hat{c}_{i+1}^\dagger \hat{c}_i)|J\rangle$ nearest-neighbor hopping

XXZ Hilbert space graph



Is it a bona fide phase transition?



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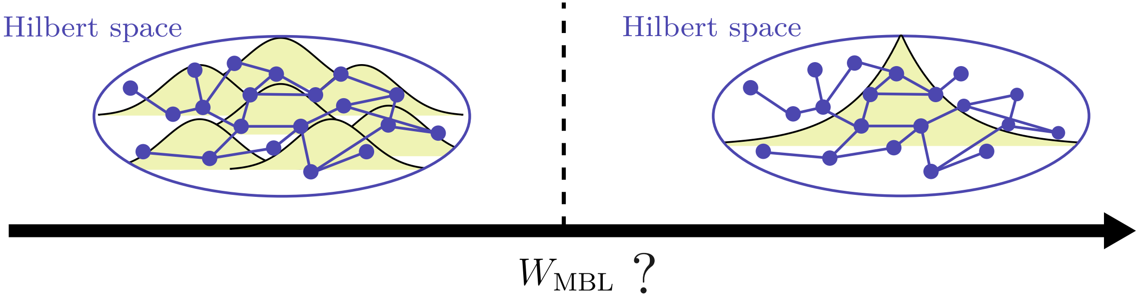
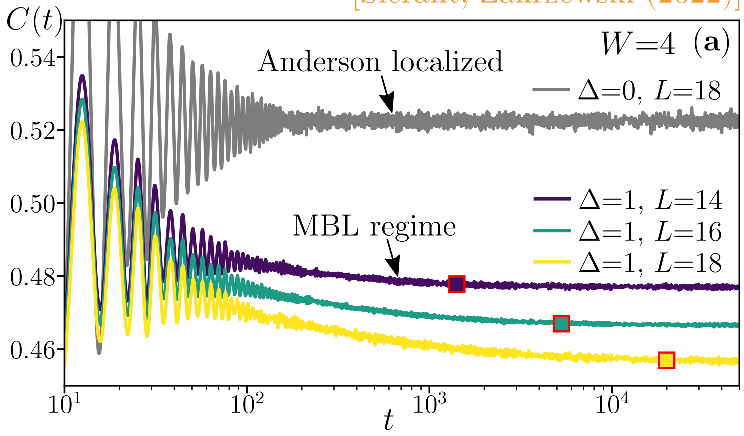


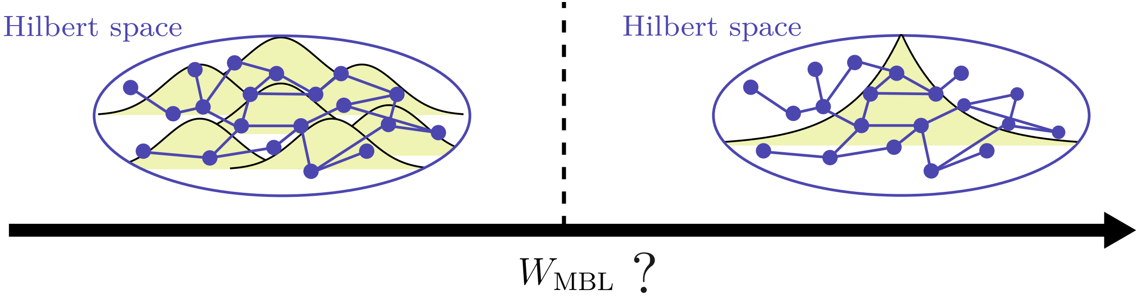
It is hard to draw conclusions!

- Numerics are constrained to small systems $L \sim 20$
  [Luitz, Laflorencie, Alet (2015)]
- Avalanche instability in the thermodynamic limit?
  [De Roeck, Huveneers (2017); Szołdra et al. (2024)]
- **Rare many-body resonances**
  [Morningstar et al. (2022); Ha et al. (2023)]
  [Colbois et al. (2024)]

[Sierant, Zakrzewski (2022)]



Is it a bona fide phase transition?



- **Rare many-body resonances**
  [Morningstar et al. (2022); Ha et al. (2023)] [Crowley, Chandran (2020)] [Garret, Roy, Chalker (2021)] [Colbois et al. (2024, 2025)] [Long et al. (2022)] [Villalonga, Clark (2020)] [De Tomasi et al. (2021)]

A resonance occurs when (broadly speaking)

$$|E_I - E_J| \lesssim |\langle I|\hat{\mathcal{H}}|J\rangle|$$

The eigenstate hybridizes i.e. has a finite support in both $|I\rangle$ and $|J\rangle$

Delocalization probability

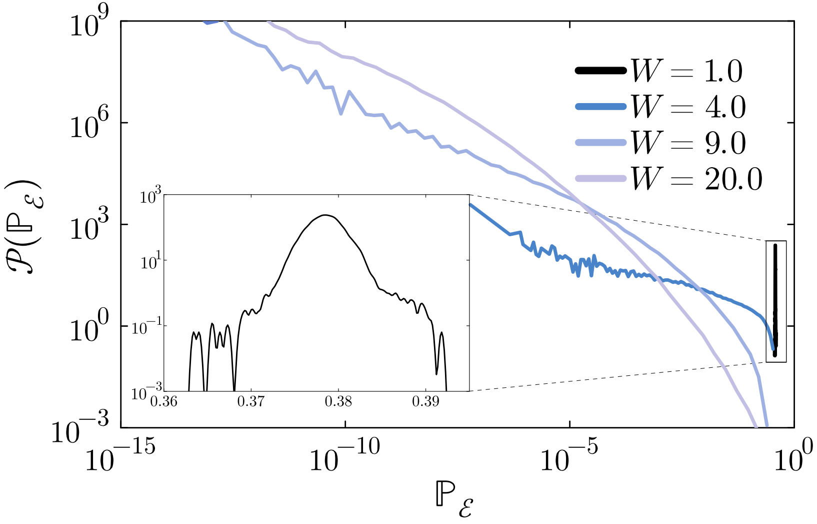
$|0\rangle = |●○○●○●○○○●○●●○●●\rangle$

$|f\rangle = |○●○○●●○○●○●○●○●●\rangle$

$$\mathbb{P}_{0\to f} = \lim_{t\to\infty} |\langle f| e^{-i\hat{\mathcal{H}}t} |0\rangle|^2$$

$$\mathcal{E} = \left\{ f \text{ s.t. } \sum_{i=1}^{L} n_i(0) n_i(f) = \frac{L}{4} \right\}$$

$$\mathbb{P}_{\mathcal{E}} = \sum_{f\in\mathcal{E}} \mathbb{P}_{0\to f}$$



[Alfaro Miranda et al. (2025)]

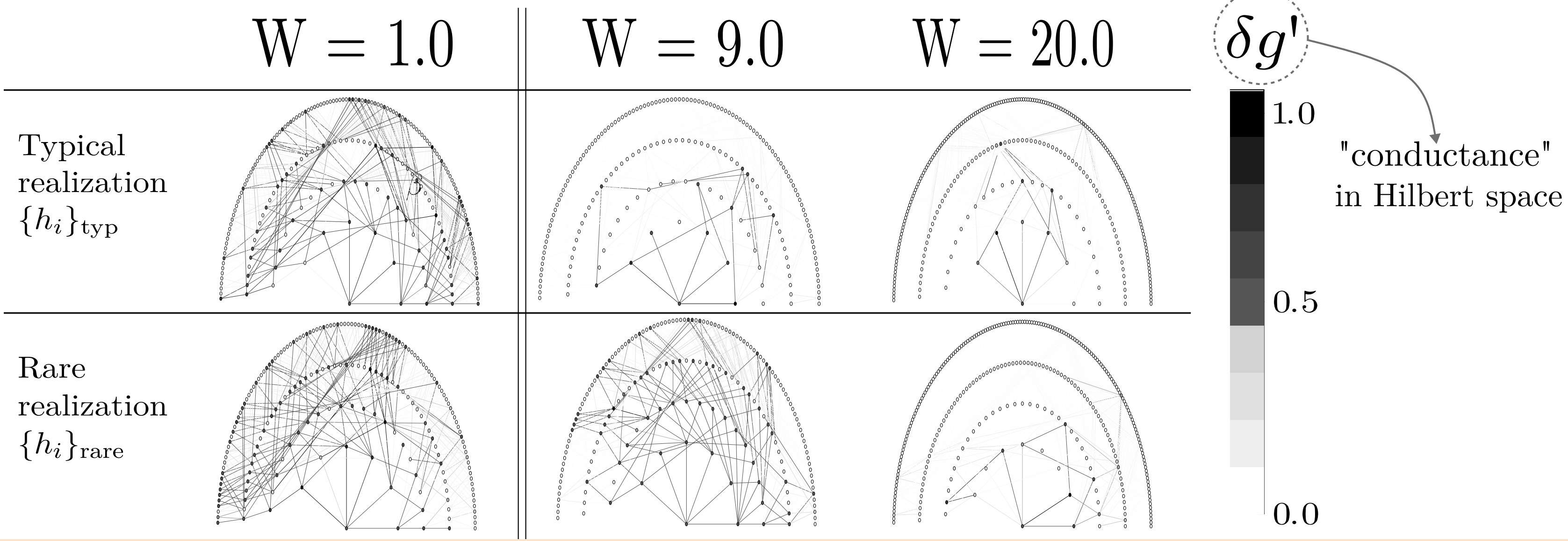
The large deviation method

[Biroli, Hartmann, Tarzia (2024)]
[Alfaro Miranda et al. (2025)]

$$\mathbb{P}_{\mathcal{E}} = \sum_{f \in \mathcal{E}} \mathbb{P}_{0 \to f}$$

$\mathbb{P}_{\mathcal{E}}$ receives contributions from exponentially many terms

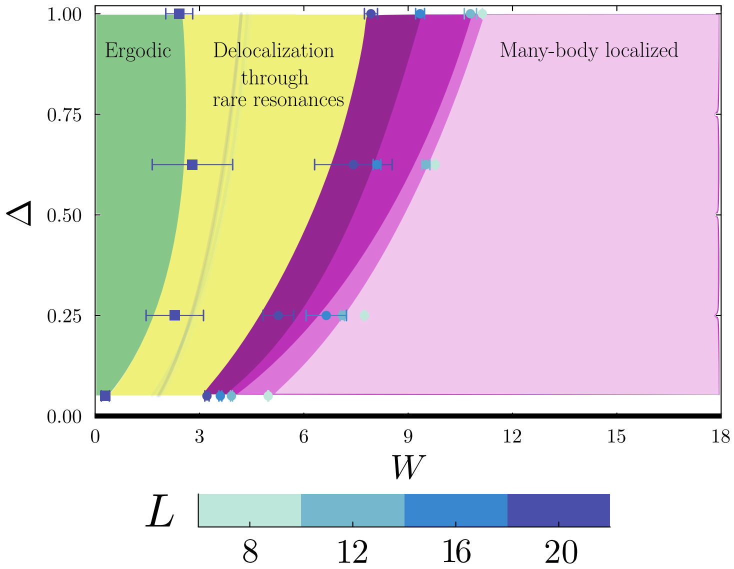
$\mathbb{P}_{\mathcal{E}}$ receives contributions from $O(1)$ large probabilities $\mathbb{P}_{0 \to f}$



The phase diagram

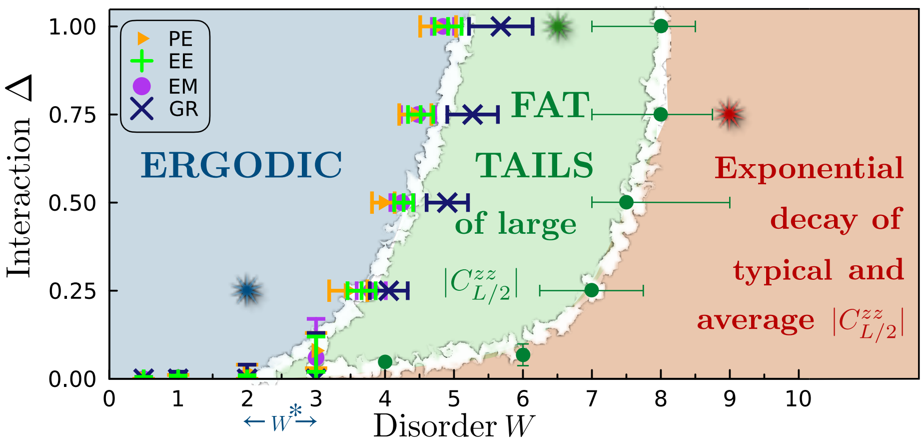
Large deviations in the many-body localization transition: The case of the random-field XXZ chain

Greivin Alfaro-Miranda,[1] Fabien Alet,[2] Giulio Biroli,[3]
Leticia F. Cugliandolo,[1] Nicolas Laflorencie,[2] and Marco Tarzia[4]

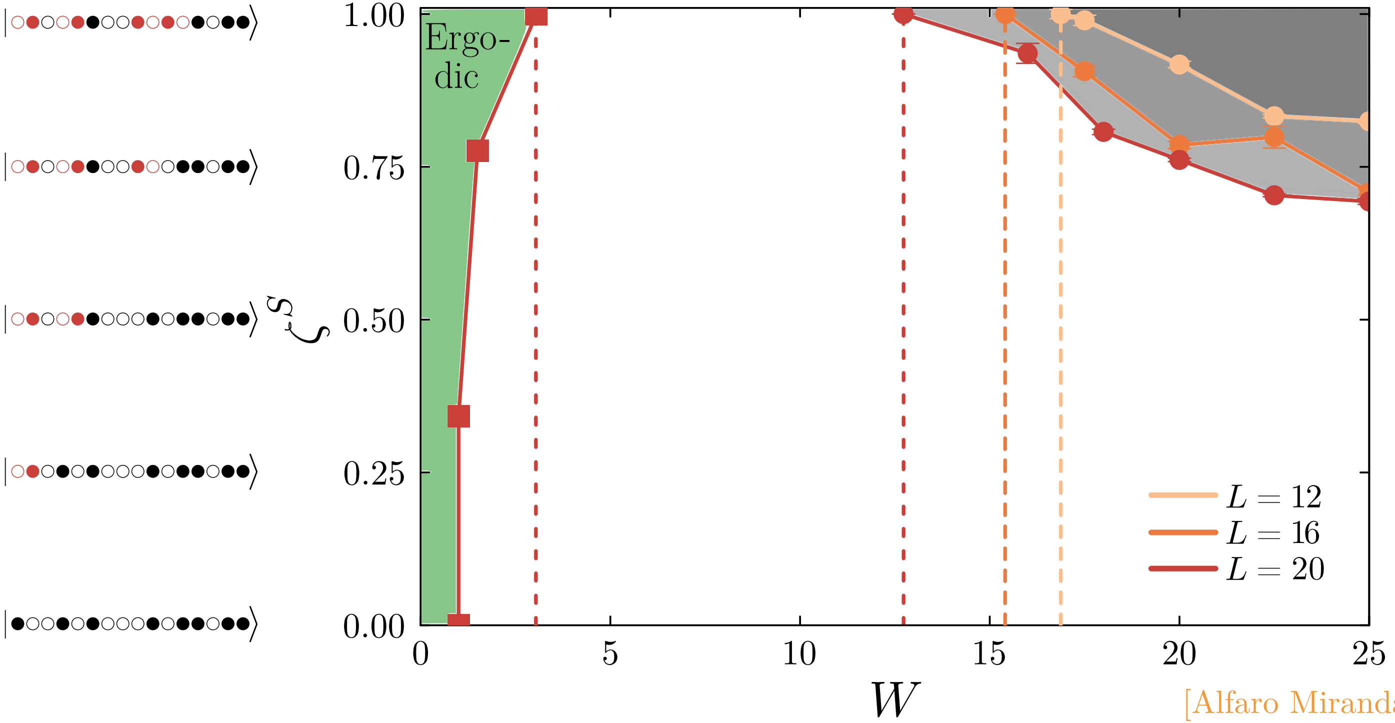


Statistics of systemwide correlations in the random-field XXZ chain: Importance of rare events in the many-body localized phase

Jeanne Colbois,[1,2,3,*] Fabien Alet,[1,†] and Nicolas Laflorencie[1,‡]



The 'size' of the many-body resonances



$\Delta = 1$

ζ^S : normalized distance in Hilbert space

Finite W_{MBL} at $L \to \infty$ accounting for the effect of long-range resonances

[Alfaro Miranda et al. (2025)]

Conclusions and Outlook

The Critical Clusters for Frustrated Spin Systems

- Is there another way of generating the clusters with $p_B < 0$?
- For the paramagnetic to spin glass transiton:
  Is the tuned I physically relevant?

The SWAP Method for Frustrated Spin Systems

- What happens within a spin glass phase with T_{SG} non-zero?
- Do a similar spatially-correlated pattern emerge in the structural case?

The Importance of Rare Events in Many-Body Localization

- Characterize the statistics of disorder realizations (importance sampling)
- Coupling to a thermal bath, can we see avalanches?
- Quasi-periodic potentials?

Percolation $\longrightarrow$ Frustrated spin systems

Glasses $\longrightarrow$ Spin glasses

Rare events $\left(\begin{array}{c}\text{classical}\\ \text{disordered}\\ \text{systems}\end{array}\right)$ $\longrightarrow$ Many-body localization

Thank you!

Percolation $\longrightarrow$ Frustrated spin systems

Glasses $\longrightarrow$ Spin glasses

Rare events $\left(\begin{array}{c}\text{classical}\\ \text{disordered}\\ \text{systems}\end{array}\right)$ $\longrightarrow$ Many-body localization