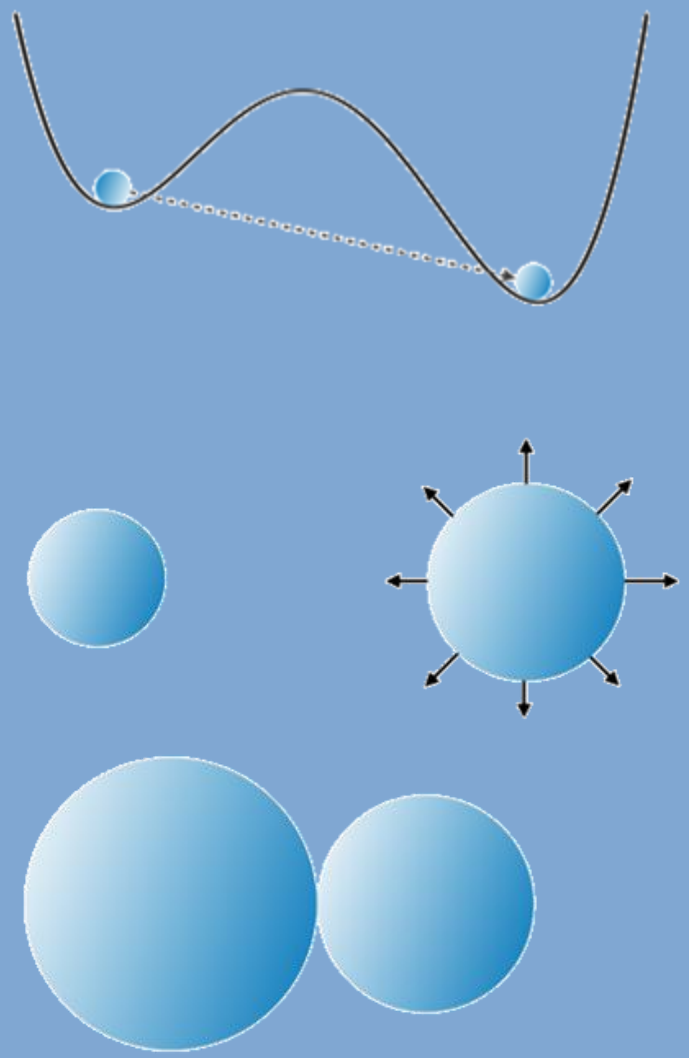


From: Vacuum Decay in our Universe | Alternator

Scattering and induced false vacuum decay in the two-dimensional quantum Ising model

Luka Pavešić, Marco Di Liberto and Simone Montangero (2025)

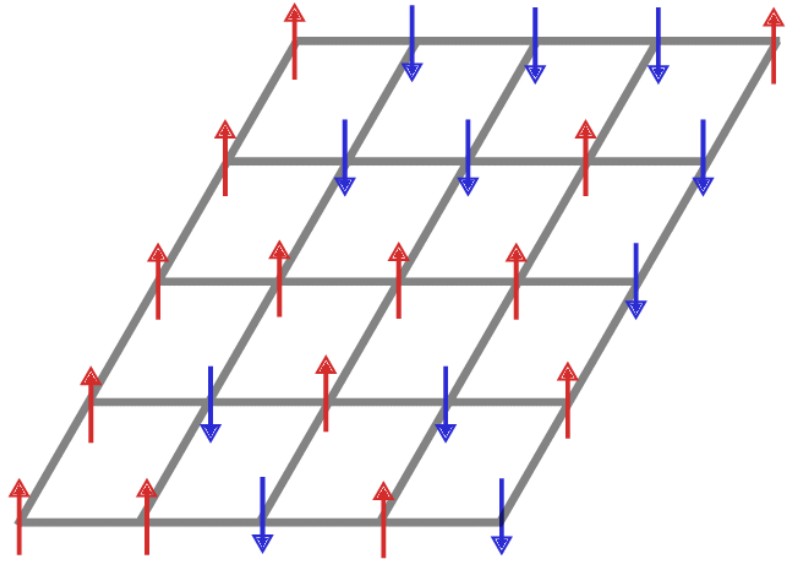
Daan Uijlenhoet
Quantum Dynamics

Contents

Quantum Ising model

- Classical Ising model in 2D

$$H = -J \sum_{\langle i,j \rangle} Z_i Z_j - h \sum_i Z_i$$



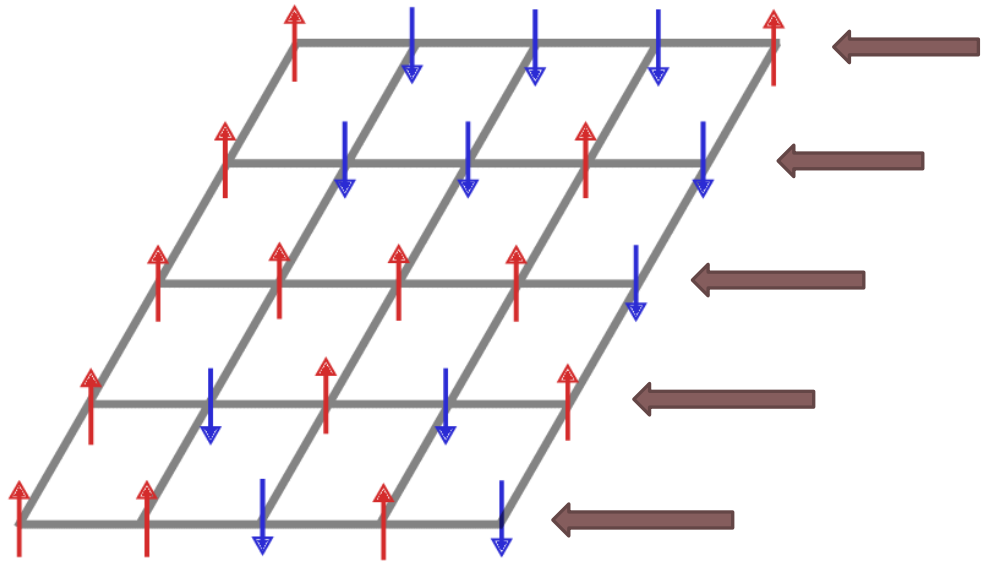
From: Thermalisation and Relaxation of Quantum Systems - Scientific Figure on ResearchGate.

Quantum Ising model

- Quantum Ising model in 2D

$$H = -J \sum_{\langle i,j \rangle} Z_i Z_j - g \sum_i X_i - h \sum_i Z_i$$

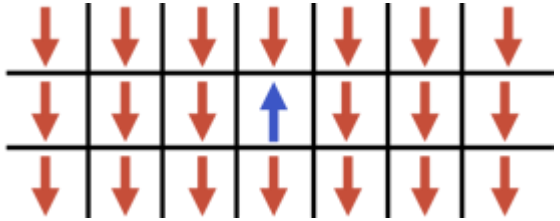
- Quantum phenomena
- *Lectures*: Quantum Ising chain (1D)



From: Thermalisation and Relaxation of Quantum Systems - Scientific Figure on ResearchGate.

Quantum Ising model

- Consider: ferromagnetic phase with $g < g_c \sim 3.04J$, $h = 0$
- *Magnons*: elementary excitations



$$H = -J \sum_{\langle i,j \rangle} Z_i Z_j - g \sum_i X_i$$

Perturbation theory
g<<J

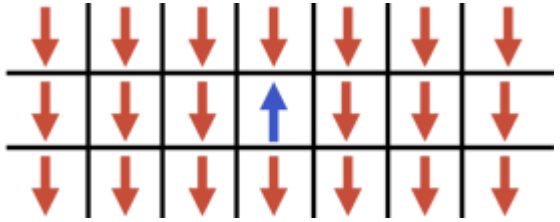
$$H = H_0 + \lambda \hat{V}$$

$$\epsilon_1(\vec{k}) = E_0 + 8J - \frac{g^2}{4J} [1 + (\cos(k_x) + \cos(k_y))]$$

From: The Ising model

Quantum Ising model

- Consider: ferromagnetic phase with $g < g_c \sim 3.04J$, $h = 0$
- *Magnons*: elementary excitations



$$H = -J \sum_{\langle i,j \rangle} Z_i Z_j - g \sum_i X_i$$

Perturbation theory
g<<J

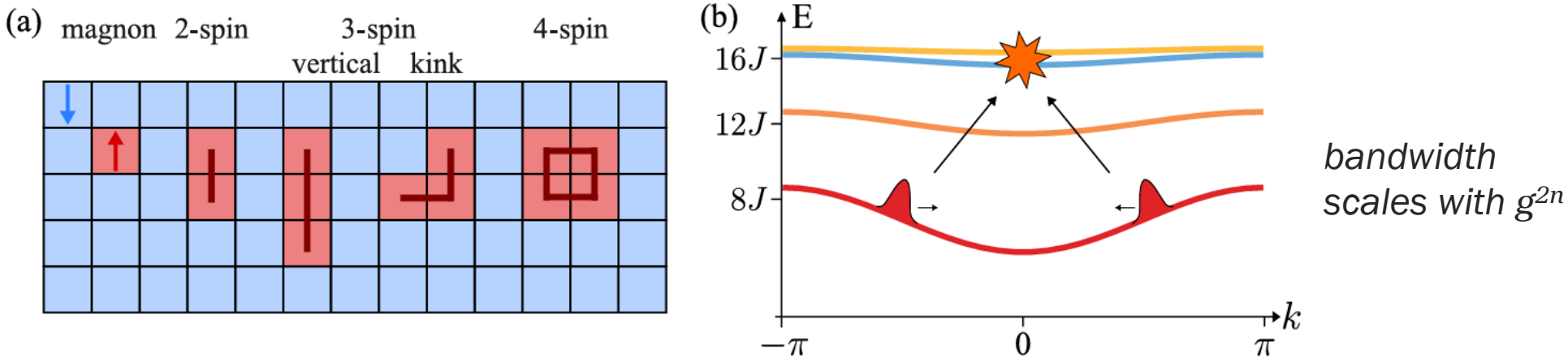
$$\sum_i X_i \, |\uparrow \uparrow \cdots\rangle = |\downarrow \uparrow \cdots\rangle + |\uparrow \downarrow \cdots\rangle + \cdots$$

$$\epsilon_1(\vec{k}) = E_0 + 8J - \frac{g^2}{4J}[1 + (\cos(k_x) + \cos(k_y))]$$

From: The Ising model

Scattering

- Two magnons with equal but opposite momenta $\mathbf{k} = \pm(k,k)$
- Different values of g/J
- At $g = 0$: resonance between 2-magnon and 3- and 4-spin excitations

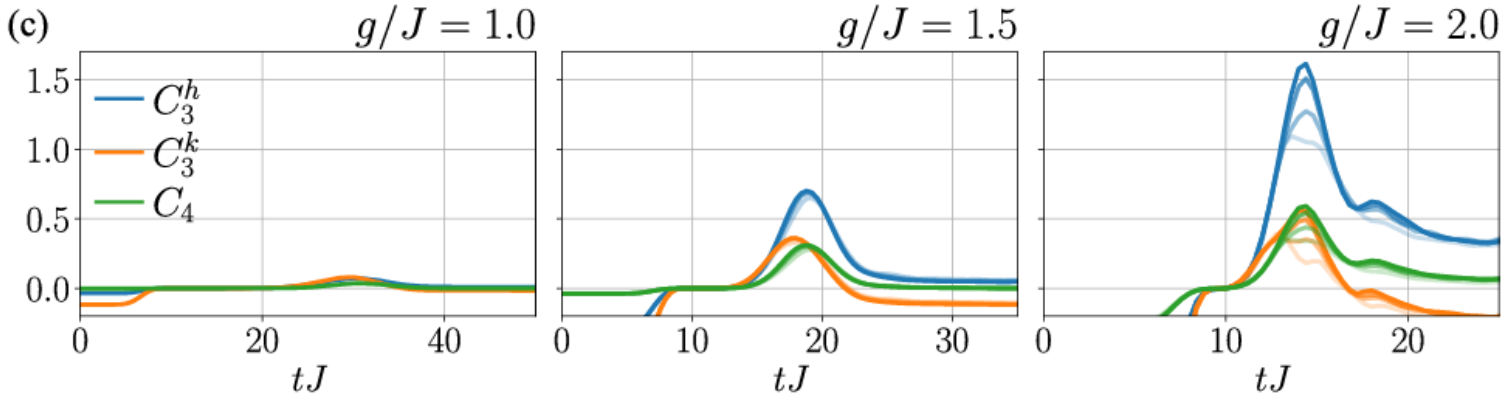


bandwidth scales with g^{2n}

From: Pavešić et al. (2025), Fig. 1

Scattering

- At small g/J: elastic scattering
- At larger g/J: resonant reactions
  - Intermediate resonance
  - Three-particle process

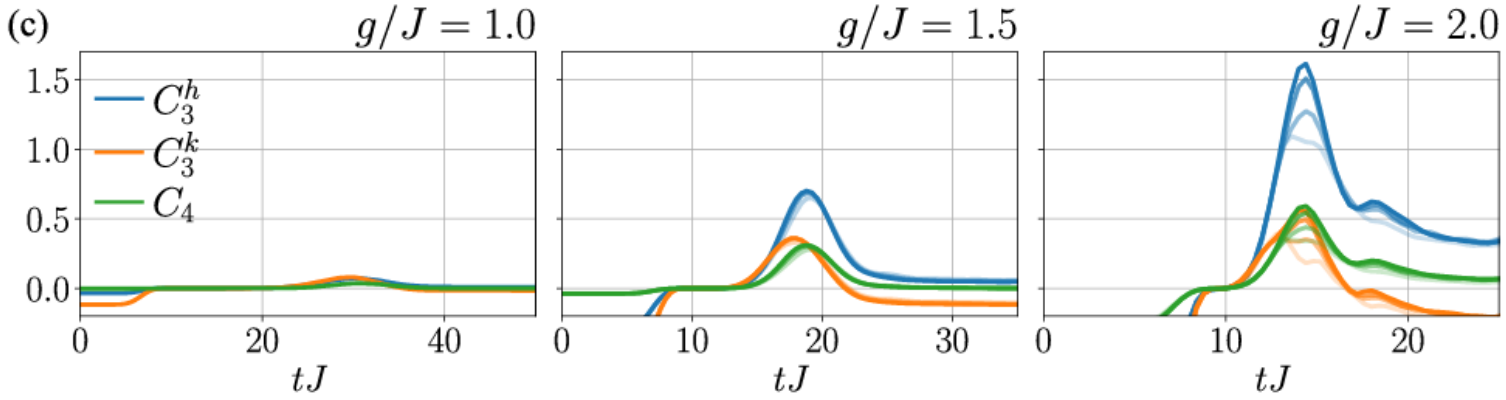


From: Pavešić et al. (2025), Fig. 2

Scattering

- At small g/J: elastic scattering
- At larger g/J: resonant reactions
  - Intermediate resonance
  - Three-particle process

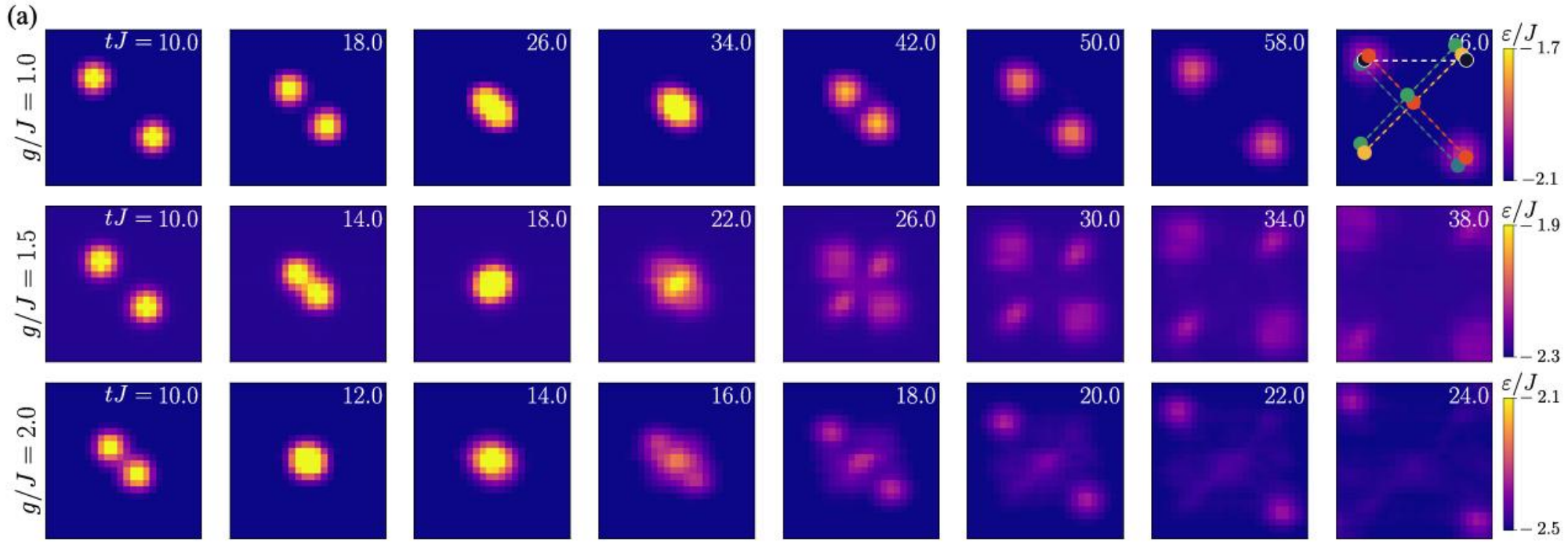
Elastic	**Inelastic**
$\|k_1, k_2\rangle \to \|\psi\rangle$	$\|k_1, k_2\rangle \to \|\Psi\rangle_{m+B}$
$\rho_m = \|\psi\rangle\langle\psi\|$	$\rho_m = \sum_\alpha p_\alpha \|\alpha\rangle\langle\alpha\|$



From: Pavešić et al. (2025), Fig. 2

Scattering

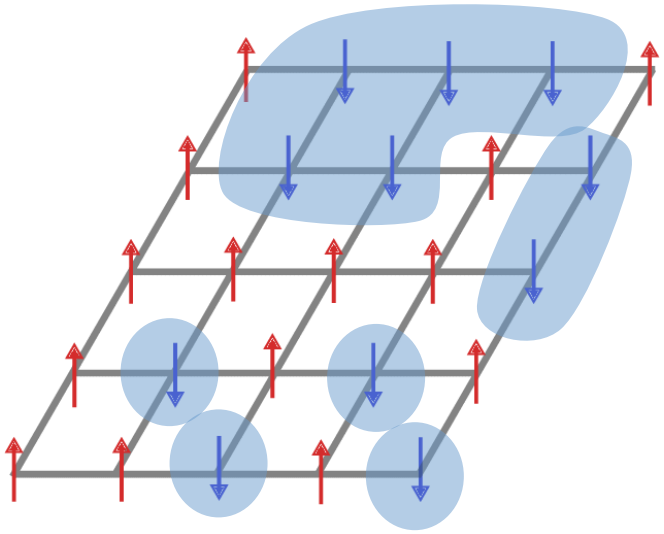
- Simulations:



From: Pavešić et al. (2025), Fig. 2

False vacuum decay

- Decay of *metastable* state to *truly stable* state
- Quantum Ising model: Set $h \neq 0$
- Bubbles of true vacuum



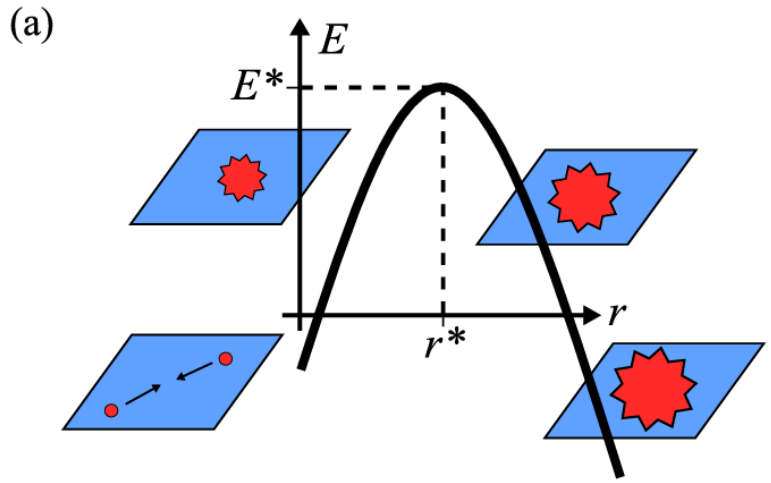
From: False vacuum - Wikipedia

False vacuum decay

- Bubble energy: positive interface tension versus negative bulk term

$$E(r) = A \cdot 2J \cdot r - B \cdot 2h \cdot r^2$$

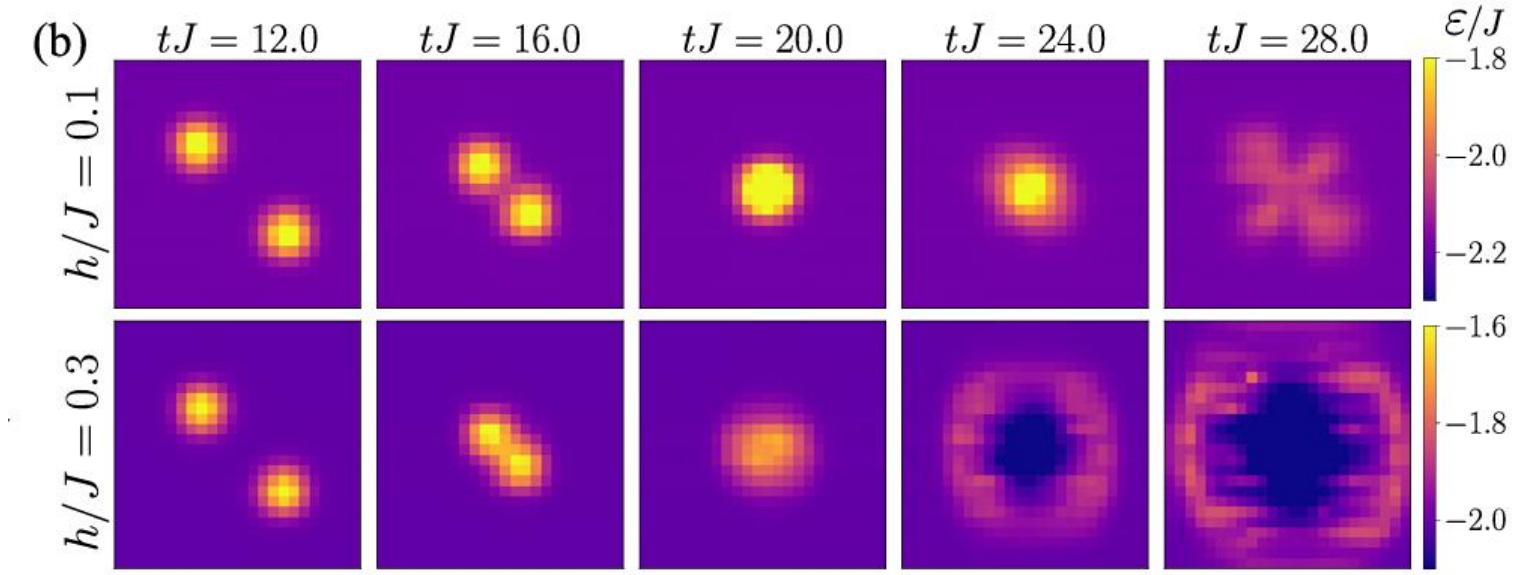
- Critical size r^*: Bubble growth
- Scattering: two true vacuum magnons



From: Pavešić et al. (2025), Fig. 3

False vacuum decay

- Fixed $g/J = 1.5$
- Increasing h: Critical size r^* decreases
- Different from theoretical predictions



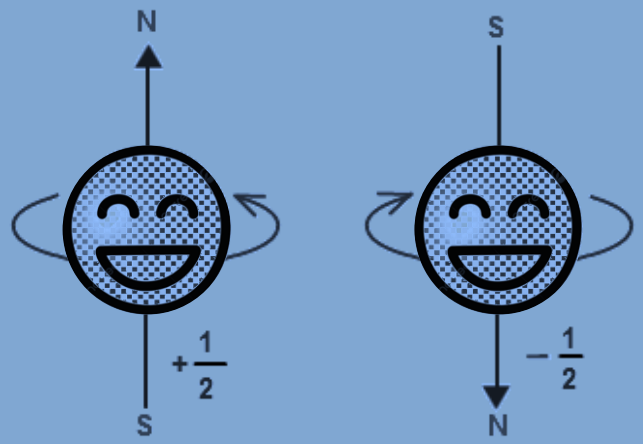
From: Pavešić et al. (2025), Fig. 3

Conclusion & Discussion

- Theoretical foundations
- Finite-size effects

Outlook

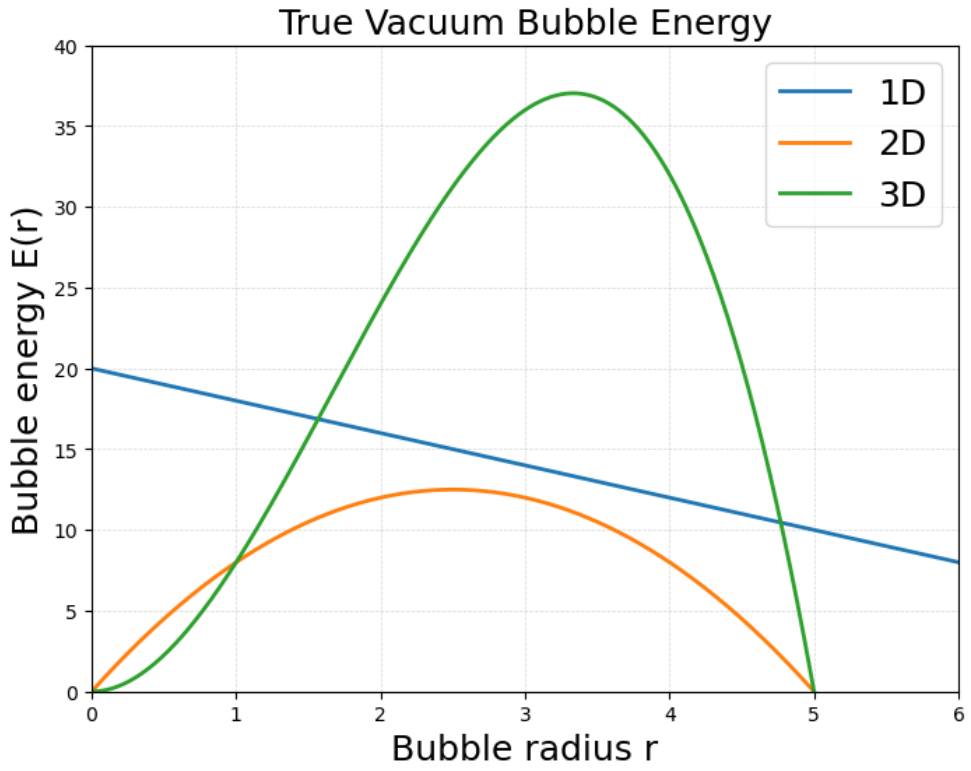
- From *1D to 2D*: Quench dynamics
- Systems in condensed matter and high-energy physics



Thank you for listening!

Additional Slide 1

- Dimensionality



Additional Slide 2

- Scattering states

$$|\Psi_{in}\rangle = |k_1, k_2\rangle \otimes |B\rangle$$

$$|\Psi_{out}\rangle = \frac{1}{\sqrt{2}}\Big(|k_1, k_2\rangle \otimes |B\rangle + |k_1', k_2'\rangle \otimes |B'\rangle\Big)$$

$$\begin{aligned}\rho_{\text{full}} = |\Psi\rangle\langle\Psi| = \frac{1}{2}\Big(&|k_1, k_2\rangle\langle k_1, k_2| \otimes |B\rangle\langle B| + |k_1, k_2\rangle\langle k_1', k_2'| \otimes |B\rangle\langle B'| \\ &+ |k_1', k_2'\rangle\langle k_1, k_2| \otimes |B'\rangle\langle B| + |k_1', k_2'\rangle\langle k_1', k_2'| \otimes |B'\rangle\langle B'|\Big)\end{aligned}$$

Additional Slide 3

- Perturbation theory

$$\langle \uparrow \uparrow \cdots | \, \hat{V} \, | \downarrow \uparrow \cdots \rangle = -g$$

$$E_n \sim E_n^{(0)} + \langle \psi_i^{(0)} | \, \hat{V} \, | \psi_j^{(0)} \rangle + \sum_{k \notin \mathrm{n}} \frac{\langle \psi_i^{(0)} | \, \hat{V} \, | \psi_k^{(0)} \rangle \, \langle \psi_k^{(0)} | \, \hat{V} \, | \psi_j^{(0)} \rangle}{E_i^{(0)} - E_k^{(0)}} + \cdots$$