Advanced Statistical Physics TD Correlation functions

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Correlation functions

We now analyse the distribution of the spatial correlation of Ising chains with random interactions [1]. We saw in the lectures that

$$\langle s_i s_{i+R} \rangle = \prod_{j=i}^{i+R-1} \tanh(\beta J_j) , \qquad (1)$$

a product of *i.i.d.* random variables.

- 1. Similarly to what we have done in the analysis of the free-energy density, take the logarithm, and use the resulting form to relate the disorder average to the most probable or typical value.
- 2. Compute the disorder averaged correlation $[\langle s_i s_{i+R} \rangle]$. Is the result the same as the one derived in the previous item?
- 3. Take a Gaussian probability distribution of the interaction strengths J_i with mean J_0 and variance J^2 and give some concrete values to these parameters. Compute the two correlation functions numerically as a function of the spin distance R. Conclude.

Depending on the physical quantities one wants to study, the correlation function has to be calculated following one or the other procedure. For example, if the spin s_i is fixed to be +1 and one asks how the magnetisation $\langle s_{i+R} \rangle$ decreases as a function of the distance R, one must use the first definition. On the other hand, if one wants to calculate the average magnetic susceptibility by summing the correlation functions, one has to use the second one. The difference between the two can be a huge factor.

Note the implication of the observation made in this example for numerical simulations: in Monte Carlo simulations it is impossible to measure $[\langle s_i s_{i+R} \rangle^2]$, almost all the data being concentrated around $\langle s_i s_{i+R} \rangle^2_{\text{typ}}$.

References

 B. Derrida and H. Hilhorst, On correlation functions in random magnets, J. Phys. C: Solid State Phys. 14, L539 (1981).