Advanced Statistical Physics

Exam - 2nd Session

January 2019

Surname :
Name :
Master :

Take a particle with mass m in a one dimensional harmonic potential

$$V(x) = \frac{1}{2} m\omega^2 x^2 \tag{1}$$

with real frequency ω taken from a probability distribution $q(\omega)$. The position of the particle is given by the real variable x.

1. Compute the free-energy at fixed ω and inverse temperature $\beta = 1/(k_B T)$, with k_B Boltzmann's constant.

2. Compute the average over disorder of F_ω without the use of the replica trick. Pose the calculation for generic $q(\omega)$ and then take the particular case $q(\omega) = (2\pi\sigma^2)^{-1/2}\ e^{-\omega^2/(2\sigma^2)}$. First establish the parameter dependence of the result and then go on and find the explicit form of it.

3. Explain the replica method.

4. Find an expression for the disorder averaged free-energy following the usual steps of the replica method.

5. Take now a Gaussian probability with zero mean and variance σ^2 for the frequency ω. What is the averaged free-energy?

Note that the volume of a sphere of unit radius in an n-dimensional space is $\Omega_n = 2\frac{\pi^{n/2}}{\Gamma\left(\frac{n}{2}\right)}$. This is the form to exploit in the calculations above.

We also know $\int_0^\infty dr \frac{r^{n-1}}{\sqrt{1+r^2}} = \frac{1}{2}\Gamma\left(\frac{n}{2}\right)\Gamma\left(1-\frac{n}{2}\right)$

A useful integral will be $\int_0^\infty dy\ e^{-ay^2}\ln y = -\frac{1}{4}\left(C+\ln 4a\right)\sqrt{\frac{\pi}{a}}$ where C is a constant given by $-C = \int_0^1 dx \ln\ln 1/x$.

The Gamma function admits a series expansion
$\Gamma(1+z) = \sum_{k=0}^\infty c_k z^k$ with $c_0 = 1$, $c_1 = -C$, and C the same constant as in the integral above. Therefore, $\Gamma\left(1-\frac{n}{2}\right) = 1 + C\,\frac{n}{2} + \mathcal{O}(n^2)$.

Take the **random field Ising model**

$$H = -J\sum_{\langle ij\rangle} s_i s_j - \sum_{i=1}^{N} h_i s_i \tag{2}$$

with $J > 0$ and h_i taken from a probability distribution $P(h_i)$ with zero mean and variance σ_h^2. The first sum runs over nearest neighbours on a cubic lattice in dimension d.
Is its free-energy density self-averaging? Justify your answer.

Explain the Imry-Ma droplet argument that allows one to estimate in which dimension this model can sustain an ordered phase.

Which kind of phases do you expect?

What is(are) the order parameter(s) in this problem?