Homework
The One-Dimensional Elephant Random Walk

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In this homework we study a non-Markovian stochastic process known as the *Elephant Random Walk* (ERW), introduced to model random motion with long-term memory. You are assumed to be familiar with the standard one-dimensional random walk, its mean displacement, variance, and diffusive scaling.

1 Definition of the Elephant Random Walk

Consider a walker on the one-dimensional lattice. The position after N steps is

$$X_N = \sum_{k=1}^{N} \sigma_k,$$

where each increment $\sigma_k = \pm 1$.

- The first step σ_1 is chosen randomly:

$$\mathbb{P}(\sigma_1 = \pm 1) = \frac{1}{2}.$$

- For each step $n \geq 2$:
    1. A previous time $k \in \{1, \ldots, n-1\}$ is chosen uniformly at random.
    2. With probability p, the walker repeats the chosen step: $\sigma_n = \sigma_k$.
    3. With probability $1-p$, the walker takes the opposite step: $\sigma_n = -\sigma_k$.

The parameter $p \in [0, 1]$ controls the strength of memory.

2 Comparison with the Standard Random Walk

1. Recall the mean displacement $\langle X_N \rangle$ and variance $\langle X_N^2 \rangle$ of the standard unbiased random walk.

2. Explain why the Elephant Random Walk is *non-Markovian*.

3 Mean Displacement

1. Show that the expected value of the increment satisfies

$$\langle \sigma_n \rangle = \frac{2p-1}{n-1} \sum_{k=1}^{n-1} \langle \sigma_k \rangle.$$

2. Deduce the scaling behavior of the mean displacement $\langle X_N \rangle$ for large N.

4 Mean Square Displacement

1. Using $X_N = \sum_{k=1}^{N} \sigma_k$, express $\langle X_N^2 \rangle$ in terms of correlations $\langle \sigma_i \sigma_j \rangle$.

2. The long-time behavior is known to be

$$\langle X_N^2 \rangle \sim \begin{cases} N, & p < \frac{3}{4}, \\ N \log N, & p = \frac{3}{4}, \\ N^{4p-2}, & p > \frac{3}{4}. \end{cases}$$

Interpret these regimes physically.

5 Physical Interpretation

1. Why is the model called the *Elephant* Random Walk?

2. Give one physical or interdisciplinary system where such memory effects could be relevant.

References

G. M. Schütz and S. Trimper, *Elephants can always remember: Exact long-range memory effects in a non-Markovian random walk*, Phys. Rev. E **70**, 045101 (2004).