

PROGRAM FOR TODAY

0 - SLIDES WITH MOTIVATION 30-45 min

(THE NUMBERS FOLLOW THE # OF THE SECTIONS IN THE LECT. NOTES)

1-1 - BASIC NOTIONS 30 min

1-2 - THIS YEAR WE WILL NOT DISCUSS THE INEQUIVALENCE OF ENSEMBLES.

1-3 - BATHS & THEIR MODELING.

15 min

≈ 2h

REFERENCES

DETAILS FROM SLIDES

• BOOKS

- ALTLAND - SIMONS CMT
- GOLDENFELD PHASE TRANS RG
- PALISI STAT PHYS
- BERLINSKY - HARRIS INTRO STAT PHYS
- LESNE PHASE TRANS.
- CASTIGLIONE ET AL FORMALISM

THE COURSE WILL BE AN
INTRODUCTION TO THEORET. METHODS IN
STATISTICAL PHYSICS

CHAPTER 1

INTRODUCTION

1.1 BASIC NOTIONS

STATISTICAL PHYSICS IS ABOUT GOING
FROM PARTICLES / AGENTS / ELEMENTS
TO

MACROSCOPIC BEHAVIOUR VIA
OBSERVABLES / ORDER PARAM

$N \gg 1$ ELEMENTS w/ MICROSC. DYNAMICS



PROB TH & STAT. METH.

A FEW OBSERVABLES w/ NO (OR SIMPLE)
TIME DEPENDENCIES

Note that going from $N \gg 1$ to a few
may not need an "equilibrium" hypothesis
(to be made explicit below)

PROB THEORY & STATISTICAL METHODS
CAN BE APPLIED TO
DIFF MICROSC DYN. eg.

CLASSICAL OR QUANTUM
NON-RELAT OR RELAT.



COLLECTIVE - EMERGENT PHENOMENA

EMPHASIS ON PHASE TRANSITIONS

START FROM CLASSICAL NON-RELAT

HERE, ALL IN EQUILIBRIUM (RECALL STAT PHYS LECT.)

BUT Comments on out of equilibrium

"STANDARD" METHODS BUT

Comments on new aspects too

OUT OF EQUILIBRIUM

- (1) EXP. TIMES ARE TOO SHORT TO LET THE SYST EVOLVE TO EQUIL.
- (2) THERE ARE EXT. DRIVING FORCES
- (3) TOO MANY INT. OF MOTION (INTEGRABILITY)

MACROSC SYST OUT OF EXP.
A FINE FIELD OF RESEARCH

RECALL : IN NOTES RECAP ON PASSAGE
FROM MECHANICS $\rightarrow$ STAT PHYS

(SHOULD BE KNOWN -)

MORE IN CASTIGLIONE ET AL, FOR EX

GOAL. UNDERSTAND THE
MACROSCOPIC BEHAVIOUR AS A FUNCTION OF
A FEW QUANTITIES CHARACTERISE THE SYST "MACRO STATE"

- ENERGY, VOLUME, NUMBER PART CLOSED
 E V N $O(N)$
- TEMP., PRESSURE, CHEM POT OPEN
 T P μ $O(1)$

OBSERVABLES DERIVABLE FROM
THERMODYN POTENTIALS

eg. $F(\beta J, \beta h)$ ^{OPEN} FREE ENERGY

AS DERIVATIVES eg. $m = - \frac{\partial F}{\partial h}$

THE PROBLEM IS : HOW CAN ONE COMPUTE
THE THERMODYN. POTENTIALS

TO DO IT, ONE NEEDS TO IDENTIFY

- THE SYSTEM & ITS INTERNAL INTERACTIONS
- THE SITUATION : OPEN, CLOSED & EVENTUAL ROLE OF ENVIRONMENT.

AND FIGURE OUT HOW TO DEAL WITH THE MANY BODY SYST IN (STRONG) INTERACTION

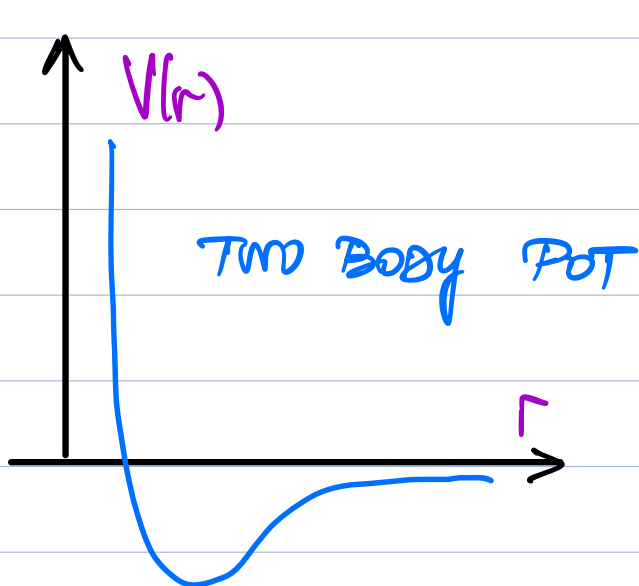
METHODS PERTURBS THEORY, MEAN-FIELD, RENORM GROUP.

⇒ ALLOW ONE TO
COMPUTE THERMODYN
POTENTIAL
& THEN ANALYZE IT

● MICROSCOPIC MODELS

PHYSICAL PARTICLE SYSTEMS FOLLOWING
NEWTON CLASSICAL DYNAMICS

SOLID - LIQUID - GAS



MICROSCOPIC
INTERACTIONS

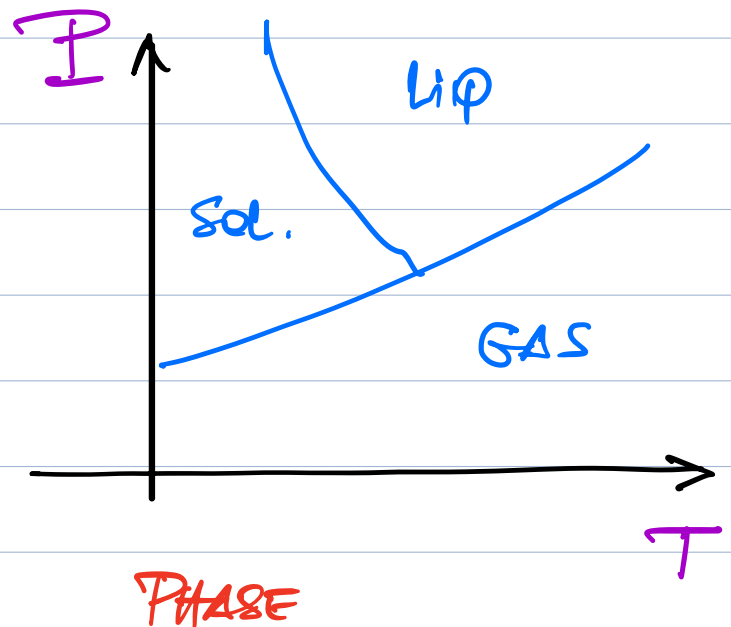


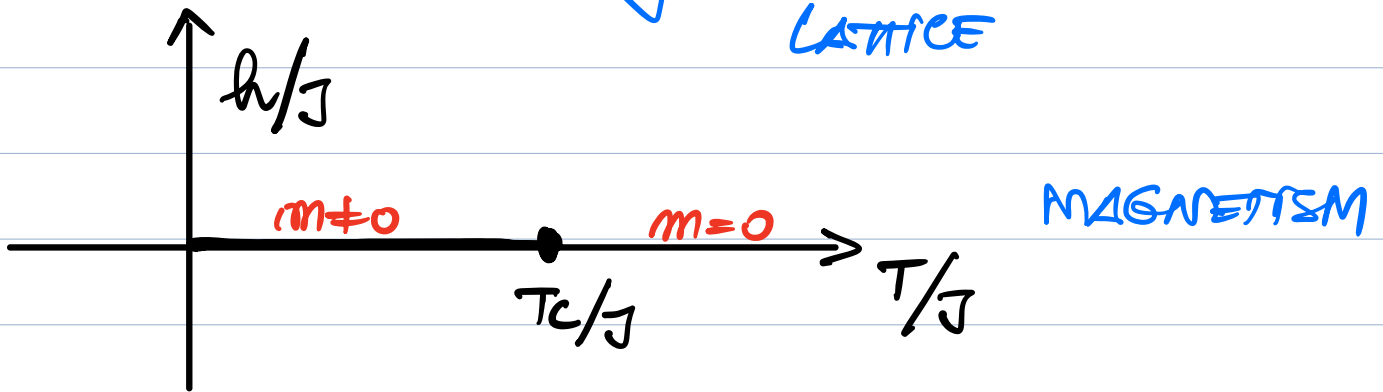
DIAGRAM
(CONST V, N)

BUT DIFFICULT TO DEAL WITH ANALYTICALLY
BETTER FOCUS ON OTHER CASES WHICH
ARE GOOD ENOUGH TO DISCUSS & UNDERST.
PHASE TRANS.

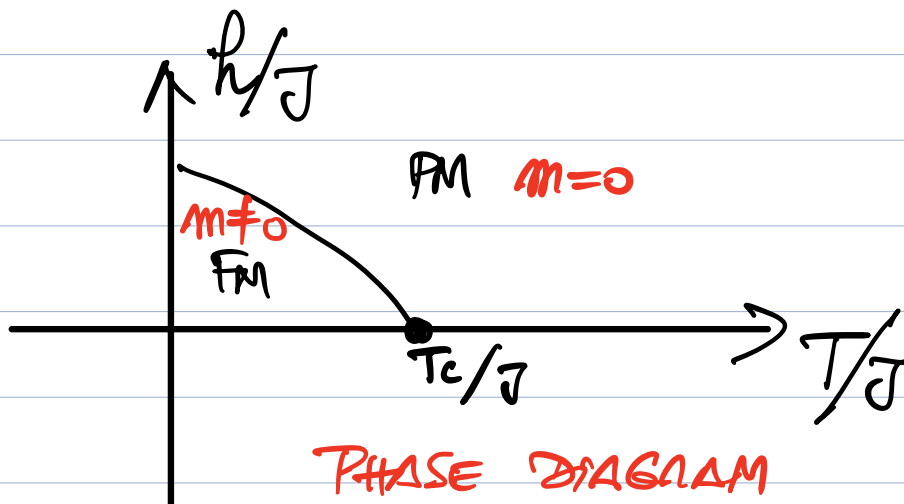
SPIN MODELS

EASIER TO DEAL WITH ANALYTICALLY & NUMERICALLY

eg. ISING MODEL ON A LATTICE



OR IN MEAN FIELD



MF: CURIE-WEISS

$$H[\{S_i\}] = -\frac{J}{N} \sum_{i \neq j} S_i S_j$$

$$S_i = \pm 1$$

QUANTUM CASES

eg. QUANTUM GASES / LIQUIDS / SOLIDS

NEW PHASES, SUCH AS SUPERFLUID ONES
(FLOW WITH NO DISSIPATION)

ALSO QUANTUM SPIN SYSTEMS

$$\hat{H} [\{ \hat{s}_i^a \}] = -\frac{J}{N} \sum_{\langle ij \rangle} \hat{s}_i^z \hat{s}_j^z + \Gamma \sum_i \hat{s}_i^x$$

↘ OPERATORS

END OF CHOICE OF MODEL.

$$1h + 20 \text{ min} = 1h \frac{1}{2}$$

SOME CONCEPTS OF EQUILIBRIUM
PROPS

LIUVILLE & ERGODIC HYPOTHESIS

RECAP. FORMALISM — SEE LECTURES NOTES
CLASSICAL MECHANICS $\rightarrow$ STAT. PHYS.

N PART $\{ \vec{Q}, \vec{P} \}$

PHASE
SPACE

$\vec{Q} = \{ \vec{q}_1, \dots, \vec{q}_N \}$

$\vec{P} = \{ \vec{p}_1, \dots, \vec{p}_N \}$ $\mathcal{X} = 2dN$ -dim.

NEWTON DYN $\Rightarrow$ TRAJECTORY
FROM $\{ \vec{Q}(0), \vec{P}(0) \}$
TO $\{ \vec{Q}(t), \vec{P}(t) \}$

TAKE ENSEMBLE OF $\{ \vec{Q}(0), \vec{P}(0) \}$
WITH

$S_0(\vec{Q}_0, \vec{P}_0)$

Σ ENVELOPE
THEM.

WHAT HAPPENS w/ $S(\vec{Q}(t), \vec{P}(t), t)$?

Liouville's eq. $\frac{dS}{dt} = \frac{\partial S}{\partial t} + \{ H, S \}$

REMO

CASTIGLIONE,
FALCIONI, LESANE
VULCANI

Phys Stat:
Chaos et approx.
multiechelles

ERGODIC HYPOTHESIS

AFTER SOME TIME

$$S \rightarrow S(t)$$

eg. NO EXPLICIT TIME DEP
only THROUGH $H(\vec{Q}, \vec{P})$

- OBSERVABLE CONSEQUENCES

THE TIME-AVER. OF NON-PATHOLOGICAL
OBSERV Θ WHICH DEP ON TIME ONLY
THROUGH $(\vec{Q}(t), \vec{P}(t))$ REACHES A
CONST.

$$\bar{\Theta} = \lim_{\tau \gg t_0} \frac{1}{\tau} \int_{t_0}^{t_0 + \tau} dt' \Theta(\vec{Q}(t'), \vec{P}(t'))$$

INDEP. OF t

AND COINCIDES WITH

$$\langle \Theta \rangle = \int d\vec{P} d\vec{Q} \mathcal{S}(H(\vec{P}, \vec{Q})) \Theta(\vec{P}, \vec{Q})$$

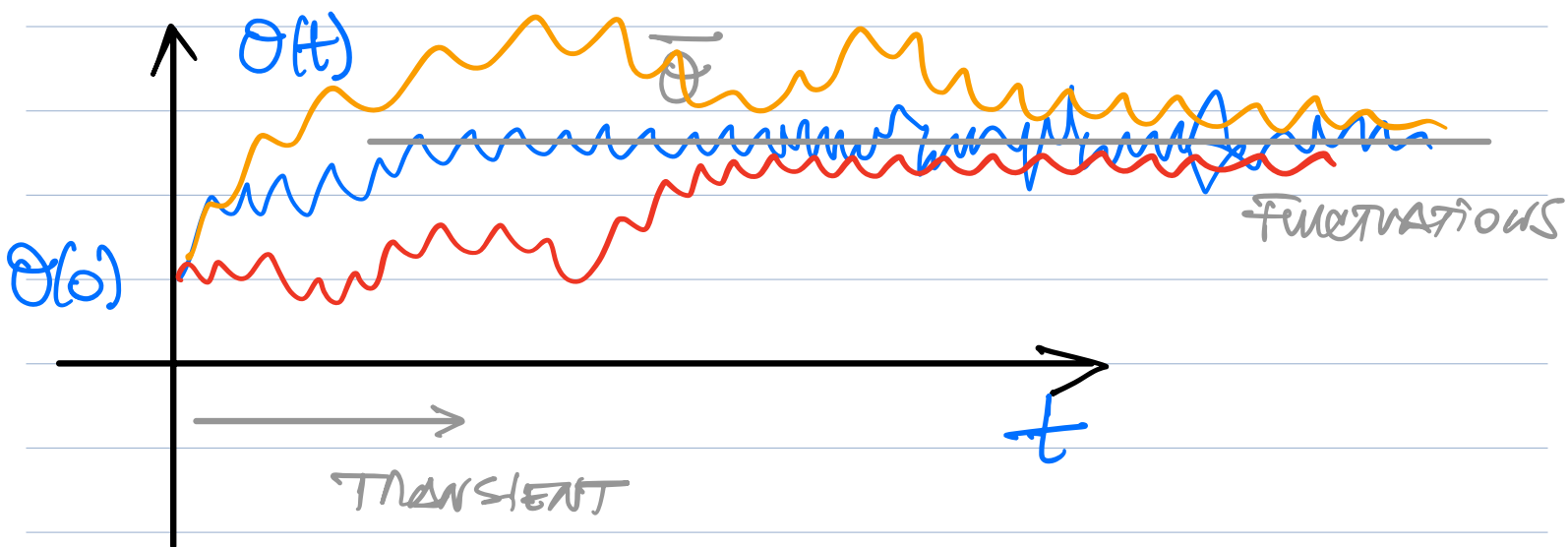
AN ENSEMBLE AVERAGE :

$$\bar{\Theta} = \langle \Theta \rangle$$

VERY IMPORTANT

ASSUME FOR SIMPLICITY E IS THE ONLY CONST MOTION

MANY EX. IN PREVIOUS EXAMS & LECTURE NOTES.



$N_{\text{meas.}}$

WHILE

$$\langle \theta \rangle = \frac{1}{N_{\text{meas.}}} \sum_{a=1} \theta_a$$

AVERAGE OVER $\neq$ MEASUREMENTS, ALL AT THE SAME TIME AFTER PREPARATION (IN EXP. OR IN NUM. SIMULATION)

THIS SHOULD HOLD IF REASONABLE OBSERV.
(SUFFICIENTLY "GLOBAL")

NB ONE CAN HAVE OUT OF EQ CASES IN WHICH,
eg E_{kin} IS OK BUT E_{pot} IS NOT

MICRO-STATE VS. MACRO-STATE

eg. A PRECISE POINT IN PHASE SPACE

$$(\vec{Q}(t), \vec{P}(t))$$

OR A PRECISE SPIN CONF WILL BE
CALLED A MICRO-STATE

INSTEAD AN ENSEMBLE OF MICRO STATES
WITH THE SAME MACRO PROPS WILL
BE CALLED A MACRO-STATE

1.2 THE CLASSICAL GIBBS ENSEMBLES

- SITUATION OPEN-CLOSED

MICROCANONICAL ENSEMBLE

CLOSED SYSTEM $E = \text{ct.}$ $V = \text{ct.}$ $N = \text{ct.}$

HYPOTHESES $S(H) = \begin{cases} S_0 & \forall \text{ conf } w/ E \\ 0 & \text{other confs.} \end{cases}$

ALL CONFS ON THE CONST ENERGY $N-1$ DIM SURFACE IN PHASE SPACE ARE EQUALLY PROBABLE

DENSITY OF STATES

$$g(E) \equiv \int d\vec{p} d\vec{q} \delta(H(\vec{p}, \vec{q}) - E)$$

VOLUME OF PHASE SPACE WITH $H(\vec{p}, \vec{q}) = E$

MICROCANON.

ENTROPY

$$S(E) = k_B \ln g(E)$$

MICROCANONICAL TEMPERATURE

$$\frac{1}{T(E)} = \frac{\partial S(E)}{\partial E}$$

NB.

if THERE ARE OTHER CONSERVED QUANTITIES,
(eg. MOMENTUM $\vec{P}$ OR ANGULAR MOM. $\vec{L}$)
RESTRICT FURTHER THE AVAILABLE PHASE
SPACE, BUT STILL ASSUME $S(H, \vec{P}, \text{etc.}) = S_0$

FROM MICROCANONICAL TO CANONICAL

TRACE OUT A PART OF TOTAL SYST

ASSUME IT'S SHORT-RANGED COUPLED TO
PART OF INTEREST

ASSUME "BATH" $\gg$ "SYSTEM" IN SIZE

FOLLOW NOTES

$$S(\{\vec{q}, \vec{p}\}) = \frac{e^{-\beta H(\{\vec{q}, \vec{p}\})}}{\mathcal{Z}}$$

$\mathcal{Z}$ PARTITION FUNCTION

β $1/k_B T$ FIXED BY BATH PROP.

$-pF = \ln \mathcal{Z}$ it's a funct of β , AND PARAM
IN H

EQUILIBRIUM : MINIMIZE F

ALSO HOMEWORK

FLUCTUATION-DISSIPATION THEOREM

VARIATIONS WRT PARAM $\equiv$

FLUCTUATIONS

$$\underbrace{\frac{\partial \langle E \rangle}{\partial T}}_{\text{A SUSCEPTIBILITY}} = k_B \underbrace{\beta^2 \langle (E - \langle E \rangle)^2 \rangle}_{\text{A FLUCTUATION}}$$

A SUSCEPTIBILITY

A FLUCTUATION

- IT'S MODEL INDEPENDENT
- HOLDS IN THE CANONICAL ENSEMBLE
- IT HOLDS IN TOTAL GENERALITY, BEYOND ANY EXPANSION

PROOF

$$\langle E \rangle = - \frac{\partial \ln Z(\beta)}{\partial \beta}$$

TAKE THE DERIVATIVE
EXPLICITLY $\Rightarrow$
DONE.

CONDENSATION OF ENERGY FLUCTUATIONS

$$\langle E \rangle = \Theta(N) \quad \text{EXTENSIVITY}$$

$$\frac{\partial \langle E \rangle}{\partial T} = \Theta(N) \quad \text{AS WELL}$$
$$= \langle (E - \langle E \rangle)^2 \rangle$$

$$\Rightarrow \frac{\text{DISPERSION}}{\text{AVERAGE}} = \frac{\Theta(N^{1/2})}{\Theta(N)} = \Theta(N^{-1/2})$$

$$\begin{array}{c} \longrightarrow 0 \\ N \longrightarrow \infty \end{array}$$

NARROWER AND NARROWER $\mathcal{P}(E)$

$\Rightarrow$

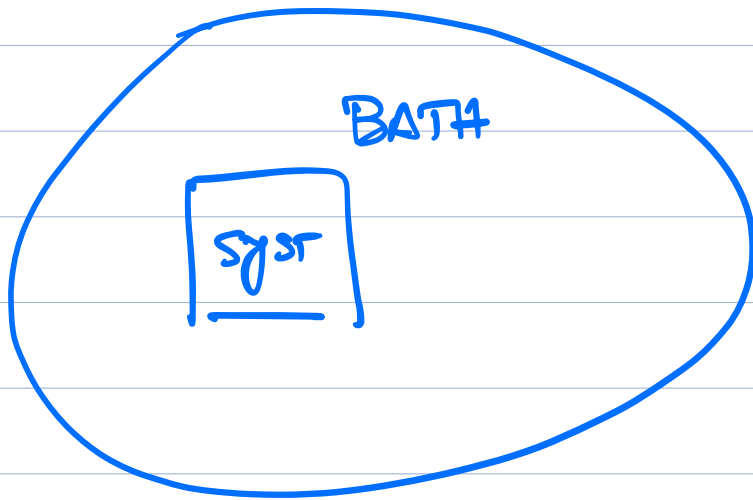
$$\langle E \rangle = E_{\text{microcanonical}}$$

JUSTIFICATION OF THE
EQUIVALENCE OF ENSEMBLES

4.3 REDUCED SYSTEMS

SYSTEMS AND BATHS

- HOW TO MODEL A BATH ?
- HOW TO "REDUCE" THE SYSTEM ?



MEGA BATH.

↓ FIXES β

$$\mathcal{Z} = \sum_{\text{conf bath}} \sum_{\text{conf syst}} e^{-\beta H}$$

$$H = H_{\text{syst}} + H_{\text{bath}} + H_{\text{int}}$$

SAY THAT THE BATH IS MADE OF
HARM OSC

$$H_{\text{bath}} = \sum_{a=1}^{N_{\text{osc}}} \left(\frac{\pi a^2}{2m_a} + \frac{m_a \omega_a^2}{2} q_a^2 \right)$$

AND SYST JUST ONE PART

$$H = \frac{P^2}{2M} + V(x)$$

BILINEAR INT (SIMPLEST CHOICE)

$$H_{\text{int}} = x \sum_a c_a q_a$$

H IS QUADRATIC IN ALL VARIABLES

$\Rightarrow$ EASY

INTEGRATE OUT $\{ \pi_a, q_a \}$:

$$\int_a \pi_a d p_a \int_a \pi_a d q_a$$

SUM OVER
CONFS.

$$Z_{\text{RED}} = \sum_{\text{conf syst}} e^{-\beta H_{\text{RED}}}$$

$$H_{\text{RED}} = H_{\text{syst}} - \frac{1}{2} \sum_{a=1}^{N_{\text{osc}}} \frac{c_a^2}{m_a \omega_a^2} x^2$$

$$H_{\text{RED}} \neq H_{\text{syst}}$$

- ARGUE $c_a^2 = \sigma (1/N_{\text{osc}})$

$\Rightarrow$ DLOF CORRECTION

• OR NOT

AT THE BASIS OF

- LANGEVIN EQ FOR STOCH DYN

- ENVIRON. DRIVEN EFFECTS.

SPECIALLY IMPORTANT QUANTUM
MECHANICALLY !

FEYNMAN - VERNON

LINDBLAD FORMALISM