TD 2 : The Blume – Capel model

On each site of a complete graph ("fully connected" model) one sets a spin-1 variable $S_i = \pm 1, 0$.

Coupling strength $J_0 > 0$ (for $\forall S_i$).

$$H = -\frac{J_0}{2} \sum_{i \neq j} S_i S_j + \Delta \sum_i S_i^2 \qquad \Delta > 0,\ J_0 > 0.$$

Energetic analysis :

• $\Delta = 0$

1) The interaction energy of all pairs of spins is the same.

$\Rightarrow$ CURIE - WEISS MAGNET.

Their interaction depends on N: to get H extensive, $O(N)$,

2) we need to have $J_0 \to \frac{J}{N}$.

Infinite range interactions basically leads to the absence of any physical dimension.

Hamiltonian H is space - independent.

3) Nature of the phase transition.

# of solutions depend on the temperature T.

Without field there is only 1 solution ($m_0 = 0$) at $\beta < \frac{1}{J}$.

$F = U - TS$ dominated by the entropy (DISORDERED PHASE NO SPONTANEOUS MAGNET.)

As $T \downarrow$, 2 solutions appear $m \pm 1$.

At low T, F is dominated by the internal energy $\Rightarrow$ ORDERED PHASE

$\beta_c = \frac{1}{J}$.

• $\Delta \neq 0$

1) Role played by Δ.

Δ is a measure of the ENERGY DIFFERENCE between the state $S_i = +1\ \forall i$ and the state $S_i = 0\ \forall i$, let's say BETWEEN THE FM AND THE PM configurations

2) Two state to be expected as ground states?

$S_i = \pm 1 \quad \forall i \qquad$ or $\qquad S_i = 0 \quad \forall i$

$$E_{FM} = -\frac{J}{2N}\sum_{i\neq j} 1 + \Delta \cdot N = -\frac{J}{2N}(N^2 - N) + \Delta N =$$

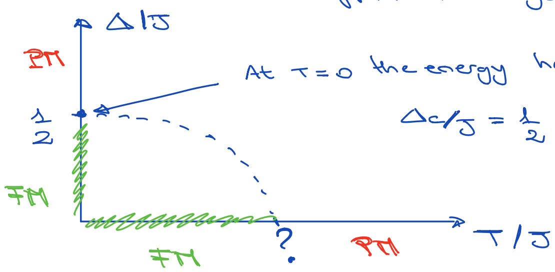
$$= \left(-\frac{J}{2} + \Delta\right)N + \frac{J}{2} \xrightarrow[N\gg 1]{\lim} \left(-\frac{J}{2} + \Delta\right)N$$

$$E_{PM} = H(\{s_i = 0\}) = 0 \Rightarrow \Delta E = E_{FM} - E_{PM} \simeq \left(\Delta - \frac{J}{2}\right)N$$

$\Delta > \frac{J}{2}$: $\Delta E > 0$ Paramagn. = GS

$\Delta < \frac{J}{2}$: $\Delta E < 0$ Ferromagn. = GS

We can then make a guess for the phase diagram, from an energetic analysis at T=0.



At T=0 the energy has to be minimized (entropic effects do not matter).

- CANONICAL ENSEMBLE

① $$Z = \sum_{\{s_i = \pm 1, 0\}} e^{-\beta H[\{s_i\}]}$$ function of (J, Δ, β)

First remark: it does not depend on all parameters. The phase diagram depend on $(\beta J, \Delta/J)$

Sometimes you can find $K = \beta J \quad \delta = \Delta/J$

② Introduce the auxiliary variable $x = \underbrace{\frac{1}{N}\sum_i s_i}_{}$

A GLOBAL VARIABLE!

$$-\beta H[\{s_i\}] = +\frac{\beta J}{2N}\sum_{i\neq j} s_i s_j - \beta\Delta\sum_i s_i^2$$

$$N\underbrace{\frac{1}{N}\sum_i s_i \frac{1}{N}\sum_j s_j}_{N\left(\frac{1}{N}\sum_i s_i\right)^2} - \underbrace{\frac{1}{N}\sum_i s_i^2}_{1 \text{ (if Ising spins)}}$$

negligible w.r.t Δ

$$-\beta H[\{s_i\}] = \frac{\beta J}{2}N\left(\frac{1}{N}\sum_i s_i\right)^2 - \left(\frac{\beta J}{2N} + \beta\Delta\right)\sum_i s_i^2$$

- If the spins were Ising variables $x = -N, -N+\frac{2}{N}, -N+\frac{4}{N}\dots$

One could replace $\sum_{\{s_i\}} \to \int dx$ and the ENTROPY would be easy.

<u>However</u>, we have spin-1 variables $\Rightarrow$ Use ANOTHER METHOD.

③ We introduce then a HUBBARD-STRATONOVICH TRANSFORMATION useful to LINEARIZE A QUADRATIC EXPRESSION

$$e^{bm^2} = \sqrt{\frac{b}{\pi}}\int_{-\infty}^{+\infty} dx\, e^{-bx^2 + 2bmx}$$

This identity can be applied to "open" $\left(\sum_i s_i\right)^2$

$$Z = \sum_{\{s_i = \pm 1, 0\}} e^{-\beta\Delta\sum_i^N s_i^2}\, \underbrace{e^{+\frac{\beta J}{N}\left(\sum_i s_i\right)^2}}_{\text{Let's focus first on this piece}}$$

$$\Rightarrow \sqrt{\frac{\beta J N}{2\pi}} \int_{-\infty}^{+\infty} dx\; e^{-\frac{\beta J N}{2} x^2 + \frac{2\beta J N}{2}\left(\frac{1}{N}\sum_i s_i\right) x}$$

where we can use: $m = \frac{1}{N}\sum_i s_i$ $\quad b = \frac{\beta J N}{2}$

- Note that the "DIAGONAL TERM", generated by having transformed $\sum_{i \neq j}$ in $\sum_{ij}$ is $\ll$ than the $\sum_{ij}$ one $O(1)$ versus $O(N)$ $\Rightarrow$ We neglect it.

Thanks to Hubbard - Stratonovich, we decoupled the spins at the exponential.

$$-\beta\Delta \sum_i s_i^2 + \beta J \cancel{N} \frac{1}{\cancel{N}} \sum_i s_i\, x = \sum_i \left[-\beta\Delta s_i^2 + \beta J s_i x\right]$$

$$= -\beta \sum_i \left(\Delta s_i^2 - J x s_i\right)$$

Now we can compute the sum over $\sum_i x_i$ (before integrating over x) because:

$$e^{-\sum_i g_i} = \prod_i e^{-g_i}$$

$$\sum_{\{s_i = \pm 1, 0\}} e^{-\sum_i g_i} = \prod_i \left(\sum_{s_i = \pm 1, 0} e^{-g_i}\right)$$

factorization property (factorizing the sum over all individual spins).

$$\Downarrow$$

$$\underbrace{e^{-\beta(\Delta - Jx)}}_{s_i = +1} + \underbrace{e^{-\beta(\Delta + Jx)}}_{s_i = -1} + \underbrace{1}_{s_i = 0}$$

$$Z = \sum_{\{s_i = \pm 1, 0\}} e^{-\beta\Delta \sum_i^N s_i^2} \times \sqrt{\frac{\beta J N}{2\pi}} \int_{-\infty}^{+\infty} dx\, e^{-\frac{\beta J N}{2} x^2 + \frac{2 \beta J N}{2} \left(\frac{1}{N} \sum_i s_i\right) x}$$

$$= \sqrt{\frac{\beta J N}{2\pi}} \int_{-\infty}^{+\infty} dx\, e^{-\frac{\beta J N}{2} x^2} \left[1 + e^{-\beta(\Delta - Jx)} + e^{-\beta(\Delta + Jx)}\right]^N$$

$$= \sqrt{\frac{\beta J N}{2\pi}} \int_{-\infty}^{+\infty} dx\, e^{-N\beta \tilde{f}(x)}$$

Where

$$\boxed{\tilde{f}(x) = \frac{J x^2}{2} - \frac{1}{\beta} \ln\left[1 + e^{-\beta\Delta}\, 2\cosh(\beta J x)\right]}$$

This is the Ginzburg-Landau free energy density

as a function of x ("magnetization")

↓

ORDER PARAMETER of THE

FM - PM phases.

(5) Show that - in the $N \to \infty$ limit, the saddle point of $x = m$.

We define $I(\beta) = \lim_{N\to\infty} \int dx\, e^{-N\tilde{f}(x,\beta)} = e^{-N \operatorname{Inf}_x \tilde{f}(x,\beta)}$

We need to find the ABSOLUTE minimum x^* and evaluate $\tilde{f}(x,\beta)$ on it.

$$-\beta f(\beta J, \beta\Delta) = \frac{\ln Z}{N} = \frac{1}{N} \ln e^{-N \operatorname{Inf}_x \beta \tilde{f}(x; \beta J, \beta\Delta)}$$

$$\boxed{f(\beta J, \beta\Delta) = \operatorname{Inf}_x \tilde{f}(x; \beta J, \beta\Delta)}$$

Note also that $\tilde{f}(x; \beta J, \beta\Delta) = \tilde{f}(-x; \beta J, \beta\Delta)$, symmetric function w.r.t $x = 0$.

- What is the $\operatorname{Inf}_x \tilde{f}$?

It's equal to $\langle \frac{1}{N}\sum_i s_i \rangle = x^* \Rightarrow$ magnetization density

Proof

$$\langle \frac{1}{N}\sum_i s_i \rangle = \frac{1}{Z}\sum_{\{s_i\}} e^{-\beta H} \left(\frac{1}{N}\sum_i s_i\right) =$$

$$= \frac{1}{\beta}\frac{\partial}{\partial h} \ln Z(h)\Big|_{h=0} \quad \text{generating fnct. method}$$

where $Z(h) = \sum_{\{s_i\}} e^{-\beta H + \beta h \frac{1}{N}\sum_i s_i}$

This corresponds to $\mathcal{H} \to \mathcal{H} - h \frac{1}{N} \sum_i s_i$

uniform field coupled to magnet. density

As before:

$$\beta \tilde{f}_h (x; \beta J, \beta \Delta) = \beta \tilde{f} (x; \beta J, \beta \Delta) + \beta x h$$

$$\Rightarrow \frac{1}{N} \ln Z(h) \xrightarrow[N \to \infty]{} + \frac{\cancel{N}}{\cancel{N}} \operatorname{Inf}_x \left[\beta \tilde{f} (x; \beta J, \beta \Delta) + \beta x h \right]$$

$$\frac{1}{\beta N} \cdot \frac{\partial \ln Z(h)}{\partial h} \xrightarrow[N \to \infty]{} + \frac{\cancel{N}}{\cancel{N}} \frac{\cancel{\beta} x^*}{\cancel{\beta}} = x^*$$

$$\boxed{\frac{1}{N} \sum_i \langle s_i \rangle = x^*}$$

⑥ Identify the extrema of $\tilde{f}(x; \beta J, \beta \Delta)$.

$$\tilde{f}(x; \beta J, \beta \Delta) = \frac{J x^2}{2} - \frac{1}{\beta} \ln \left[1 + e^{-\beta \Delta} 2 \cosh (\beta J x) \right]$$

$$\frac{\partial \tilde{f}}{\partial x} = 0 = \cancel{J} x - \frac{1}{\cancel{\beta}} \frac{e^{-\beta \Delta} 2 \sinh (\beta J x) \cdot \cancel{\beta J}}{\left[1 + e^{-\beta \Delta} 2 \cosh (\beta J x) \right]}$$

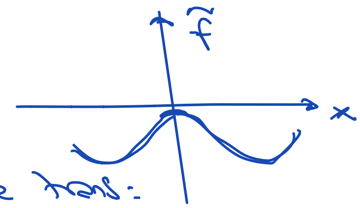
However, IMPLICIT EQUATION for x^*!

CANNOT be solved analytically! $x = \frac{e^{-\beta \Delta} 2 \sinh (\beta J x)}{\left[1 + e^{-\beta \Delta} 2 \cosh (\beta J x) \right]}$

Note however that $x = 0$ is always solution.

⑦ Taylor expansion around $x=0$

Look for the parameters such that the quadratic coefficient < 0



Signature of a 2nd order phase trans:

$\tilde{f}(0)=0$

$\tilde{f}'(x=0)=0$

$$\tilde{f}''(x\simeq 0) = J - \frac{2e^{-\beta\Delta}\left[\beta J \cosh(\beta J x)\left(1+e^{-\beta\Delta}2\cosh(..)\right)\right.}{\left[1+e^{-\beta\Delta}2\cosh(\beta J x)\right]^2} - \left.\frac{\sinh(\beta J x)\, e^{-\beta\Delta}\, 2\sinh(\beta J x)\cdot \beta J}{\cdots}\right|_{x=0}$$

Not a very good strategy.

Go back to $\tilde{f}'(x)$ and expand around $x\simeq 0$

$$\tilde{f}'(x) = Jx - \frac{2J\, e^{-\beta\Delta}\sinh(\beta J x)}{1+e^{-\beta\Delta}2\cosh(\beta J x)} = 0$$

$$\simeq \cancel{J}\cancel{x} - \frac{2J e^{-\beta\Delta}\,\cancel{J}\beta\cancel{x}}{1+e^{-\beta\Delta}\cdot 2} = 0$$

$$\Rightarrow 1 - \frac{2e^{-\beta\Delta}\cdot\beta J}{1+2e^{-\beta\Delta}} = 0$$

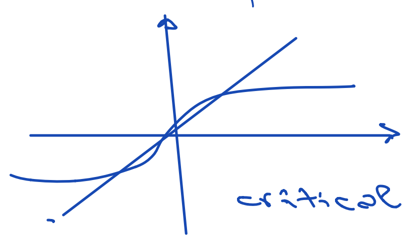
$$1 + 2e^{-\beta\Delta} - 2\beta J e^{-\beta\Delta} = 0$$

$$\boxed{\frac{1}{2} e^{\beta\Delta} + 1 - \beta J = 0}$$

SECOND ORDER CRITICAL LINE

By imposing $\Delta = 0 \Rightarrow \left(\frac{T}{J}\right)_c = \frac{2}{3}$ 2nd-order phase transition at this T_c.

This is somehow similar to $x = \tanh(\beta x)$ for the usual Ising case. Our functional dependence is more complex, but same idea.



critical β:

$x \simeq \beta x \Rightarrow \beta_c = 1$

$$\frac{2e^{-\beta\Delta} \sinh(\beta x)}{1 + e^{\beta\Delta} 2\cosh(\beta x)} \xleftrightarrow[\text{with } J=1]{\text{same role}} \tanh(\beta x)$$

We can increase Δ (with $\beta\Delta \ll 1$)

$$0 = \frac{1}{2}\beta\Delta + 1 - \beta J \Rightarrow \beta J = 1 + \frac{1}{2}\beta\Delta$$

We want to look at the phase diagram for $(\Delta|J)$:

$$\beta J = 1 + \frac{1}{2}\beta J\left(\frac{\Delta}{J}\right) \longrightarrow \beta J = \frac{1}{1 - \frac{1}{2}\Delta|J}$$

Is this wrong? I expanded $e^{\beta\Delta}$ assuming $\beta\Delta \ll 1$

OK as long as β is finite.

$$\boxed{\frac{T}{J} = 1 - \frac{1}{2}\Delta/J}$$

⑦ 2nd order phase transition ending at a TRICRITICAL POINT $(\Delta/J, T/J)_c$

where the quadratic and quartic coefficients of the expansion both vanish.

We should observe phase transitions of different orders in T, Δ according to their values.

$$\tilde{f}(x; \beta J, \beta\Delta) = \frac{Jx^2}{2} - \frac{1}{\beta}\ln\left[1 + e^{-\beta\Delta}\, 2\cosh(\beta J x)\right]$$

- this form is symmetric under reversal $x \to -x$.
- The two $x_{sp}^* \neq 0$ must correspond to <u>minima</u> of $\tilde{f}$, separated by a maximum at $x_{sp} = 0$.

Let's expand $f(x)$ around $x \simeq 0$:

$$f'(x) \sim f'(0) + f''(0)\,x + O(x^2)$$

$$f'(x) \simeq 0 + J\left[1 - \frac{e^{-\beta\Delta}\, 2\beta J}{1+2e^{-\beta\Delta}}\right]x + O(x^2)$$

$$\simeq 0 + J\underbrace{\left[1 - \frac{2\beta J}{2+e^{\beta\Delta}}\right]}x + \dots$$

This coefficient is zero by construction at THE CRITICAL POINT

$2 + e^{\beta\Delta} - 2\beta J = 0$ fixes the CRITICAL LINE.

Going on with the expansion, we must keep only even powers:

$$f'''(0) = 0$$

$$f^{IV}(x) = \frac{1}{A(x)}\Big\{-2J^4\beta^3\big[-32 + 4e^{2\beta\Delta} + e^{\beta\Delta}(-13 + e^{2\beta\Delta})\cosh(\beta J x)$$
$$- 4(-4 + e^{2\beta\Delta})\cosh(2\beta J x) + e^{\beta\Delta}\cosh(3\beta J x)\big]\Big\}$$

where $A(x) = [e^{\beta\Delta} + 2\cosh(\beta J x)]^4$

from which we get: $f^{IV}(0) = -\dfrac{2J^4\beta^3(-4 + e^{\beta\Delta})}{(2+e^{\beta\Delta})^2}$

TRICRITICAL POINT OBTAINED BY $f^{IV}(0) = 0$.

$$\boxed{4 - e^{(\beta\Delta)_c} = 0}$$

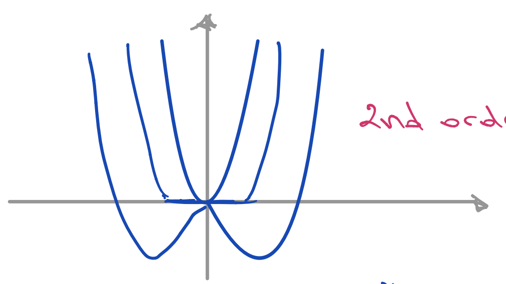
numerical solution :

$$\begin{cases} \left(\dfrac{\Delta}{J}\right)_c = \dfrac{\ln(4)}{3} \simeq 0.4621 \\ (\beta J)_c = 3 \end{cases}$$

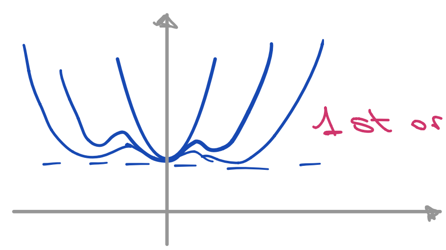
⑧ The 1st order phase transition corresponds to $\left(\frac{\Delta}{J}, \frac{T}{J}\right)$ on which $\tilde{f}(\beta J, \beta\Delta, x \neq 0) = \tilde{f}(\beta J, \beta\Delta, x = 0)$.

⑨ One need then to compare NUMERICALLY

$$\tilde{f}(x^* \neq 0; \beta J, \Delta/J) = \tilde{f}(0; \beta J, \Delta/J)$$

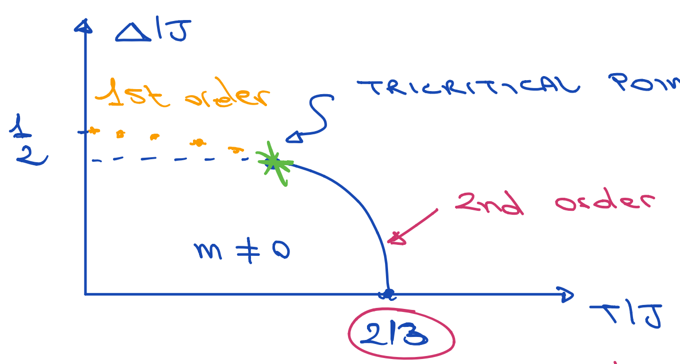


the minimum in x^* becomes a MAXIMUM



x^* remains a Minimum until a spinodal.

⑩



$\neq$ from the Ising model

MICROCANONICAL ENSEMBLE

N_+ spins taking value $+1$

N_- spins taking value -1

N_0 with value 0

① Constraint over these quantities.

$$N_+ + N_- + N_0 = N$$

② Write the total magnetization $M = \sum_i s_i$

- $M = N_+(1) + N_-(-1) = N_+ - N_-$

- Quadrupole moment $Q = \sum_i s_i^2 = N_+ + N_-$

From these 3 conditions, one can extract N_+, N_-, N_0 in terms of (N, M, Q).

- Total energy $\mathcal{H} = -\frac{J}{2N} \sum_{i \neq j} s_i s_j + \Delta \sum_i s_i^2 =$

$$= -\frac{J}{2N}\left(\sum_i s_i\right)^2 + \frac{J}{2N} \cdot N + \Delta \sum_i s_i^2 =$$

$$= -\frac{J}{2N} M^2 + \frac{J}{2} + \Delta Q$$

Or in a similar way as a function of:

$$\mathcal{H} = -\frac{J}{2N}(N_+ - N_-)^2 + \cancel{\frac{J}{2}} + \Delta(N_+ + N_-) \simeq -\frac{J}{2N} M^2 + \Delta Q.$$

③ # of microscopic configurations Ω, compatible with the constraints.

$\ln \Omega \rightarrow$ "entropy"

$$\Omega = \frac{N!}{N_+!\, N_-!\, N_0!}$$

According to Stirling: $\boxed{\ln N! \simeq N \ln N - N}$

$$\ln \Omega = \ln N! - \ln N_+! - \ln N_-! - \ln N_0!$$

$$\begin{cases} N_0 = N - (N_- + N_+) = N - N_- - N_+ \\ M + Q = N_+ - \cancel{N_-} + N_+ + \cancel{N_-} = 2N_+ \\ M - Q = \cancel{N_+} - N_- - \cancel{N_+} - N_- = -2N_- \end{cases}$$

$$N_+ = \frac{M+Q}{2} = \frac{N(m+q)}{2}$$ with INTENSIVE ORDER PARAMETERS

$$N_- = \frac{Q-M}{2} = \frac{N(q-m)}{2}$$

$\Downarrow$

$$N_0 = N - N\left[\frac{q-m}{2} + \frac{m+q}{2}\right] = N - N\frac{\cancel{2}q}{\cancel{2}} = N(1-q).$$

$$\ln \Omega = \ln N! - \ln\left[\frac{N(m+q)}{2}\right]! - \ln\left[\frac{N(q-m)}{2}\right]! - \ln[N(1-q)]!,$$

$$\simeq N\ln N - N - \frac{N(m+q)}{2}\ln\left[\frac{N(m+q)}{2}\right] + \frac{N(m+q)}{2} +$$

$$- \frac{N(q-m)}{2}\ln\left[\frac{N(q-m)}{2}\right] + \frac{N(q-m)}{2} - N(1-q)\ln[N(1-q)] + N(1-q).$$

Simplifying, there are no linear terms.

$\rightarrow$ indep. of $\{m,q\}$

$$\ln\Omega \simeq N\ln N - \frac{N(m+q)}{2}\ln\left[\frac{N(m+q)}{2}\right] - \frac{N(q-m)}{2}\ln\left[\frac{N(q-m)}{2}\right] - N(1-q)\ln N(1-q).$$

Look at the terms $\propto N\ln N$:

$$N\ln N - \frac{N}{2}(m+q)\ln\frac{N}{2} - \frac{N}{2}(q-m)\ln\frac{N}{2} - N(1-q)\ln N =$$

$$= N\ln N - \frac{N}{2}(\cancel{m}+\cancel{q})\cancel{\ln} N - \frac{N}{2}(\cancel{q}-\cancel{m})\ln N - N(1-\cancel{q})\cancel{\ln} N$$

$$+ \frac{N}{2}(\cancel{m}+q)\ln(2) + \frac{N}{2}(q-\cancel{m})\cancel{\ln(2)} =$$

$$= \cancel{N\ln N} - \cancel{N\ln N} + Nq\ln(2) = Nq\ln(2)$$

$$\ln\Omega = -\frac{N}{2}(m+q)\ln(m+q) - \frac{N}{2}(q-m)\ln(q-m) - N(1-q)\ln(1-q) + Nq\ln(2).$$

4 $$S = k_B\ln\Omega \simeq -k_B N\left\{\frac{(m+q)}{2}\ln(m+q) + \frac{(q-m)}{2}\ln(q-m) + (1-q)\ln(1-q) - q\ln(2)\right\}.$$

• Now we want to write S as a function of $\{m, e\}$.

We have to come back to the Hamiltonian:

$$\mathcal{H} = -\frac{J}{2N} M^2 + \Delta Q$$

$$\frac{\mathcal{H}}{N} = \varepsilon = \left\{ -\frac{J}{2N} (Nm)^2 + \Delta N q \right\} \frac{1}{N}$$

$$\varepsilon = -\frac{J}{2} m^2 + \Delta q \quad \Rightarrow \quad q = \left(\varepsilon + \frac{J}{2} m^2 \right) \frac{1}{\Delta}$$

$$S(m, e) = -K_B N \left\{ \frac{1}{2\Delta} \left(m\Delta + \varepsilon + \frac{J}{2} m^2 \right) \ln \left[\frac{1}{\Delta} \left(m\Delta + \varepsilon + J \frac{m^2}{2} \right) \right] \right.$$

$$+ \frac{1}{2\Delta} \left(\varepsilon + \frac{Jm^2}{2} - m\Delta \right) \ln \left[\frac{1}{\Delta} \left(\varepsilon + \frac{Jm^2}{2} - m\Delta \right) \right]$$

$$+ \left[1 - \frac{1}{\Delta} \left(\varepsilon + \frac{Jm^2}{2} \right) \right] \ln \left[1 - \frac{1}{\Delta} \left(\varepsilon + \frac{Jm^2}{2} \right) \right]$$

$$\left. - \left(\varepsilon + \frac{Jm^2}{2} \right) \frac{1}{\Delta} \ln(2) \right\} .$$

5) We fix the energy ε.

We look for equilibrium magnetization that maximize S.

$$\frac{\partial S(m, e)}{\partial m} = 0 \quad \Rightarrow \quad m^* \quad \text{EXTREMA}$$

$$\frac{\partial^2 S(m, e)}{\partial m^2} > 0 \quad \rightarrow \quad \text{minimum}$$

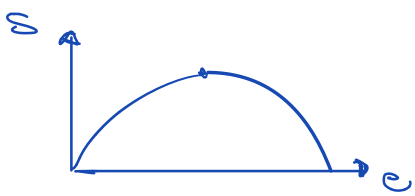
$$< 0 \quad \rightarrow \quad \text{maximum}$$

Replacing in S: $\quad S(e,m^*) = \sup_m S(e,m) = S(e)$.

NB: The energy is bounded from above.

One expect $S(\varepsilon)$ to be formed by 2 parts:

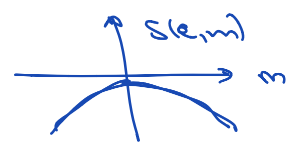
one with >0 slope and another with <0 slope



$$\frac{S(e,m)}{N} = S_0 + A_m m^2 + B_m m^4 + O(m^6) \quad \text{where } m^* = 0$$

$$S_0 = s(e, m=0) = -\left(1 - \frac{1}{\Delta}\varepsilon\right)\ln\left(1 - \frac{1}{\Delta}\varepsilon\right) - \frac{\varepsilon}{\Delta}\ln\left(\frac{\varepsilon}{2\Delta}\right)$$

In PM: $A_m < 0$, $B_m < 0$ $\Rightarrow$ $m^* = 0$ max. S



2nd order phase when:

$A_m = 0$, $B_m < 0$.

$\downarrow$

SIMILARLY TO GINZBURG-LANDAU

(the quadratic term disappears at the transition).

$$\frac{1}{T} = \frac{\partial S}{\partial E} \Rightarrow \frac{1}{K_B T} = \frac{1}{\Delta} \ln\left[\frac{1 - E/\Delta}{E/(2\Delta)}\right]$$

neglecting higher-order terms in ln'(...)...

Now we use $A_m = 0$ to obtain the

CRITICAL MICROCANONICAL TEMPERATURE

$$\begin{cases} \beta_c J = \frac{\exp(\beta/\Delta)}{2} + 1 \\ \beta_c J = \frac{\Delta}{E} \end{cases}$$

This determines the same 2nd order curve as in the CANONICAL APPROACH.

! To obtain the TRICRITICAL POINT: $A_m = 0$, $B_m = 0$.

One should find: $\frac{1}{2\Delta} = 1{,}0813$, $\beta J = \underbrace{3.0272}_{\neq \text{ CANONICAL!}}$

Small different but still a difference

ETWEEN CANONICAL $\leftrightarrow$ MICROCANONICAL ENSEMBLES.

* The microcanonical line extends beyond the canonical one (see also Ref.(5), by Campa, Dauxois & Ruffo (2009)).