Dynamics of thermal first-order phase transitions

Leticia Cugliandolo, Marco Picco
& Francesco Chippari (PhD 2023)
& F. Corberi, M. Esposito, O. Mazzarisi, Salerno University

LPTHE LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES — SORBONNE UNIVERSITÉ — cnrs

1) Introduction & aims

The project is centred around the study of the **dynamics of the bi and tri-dimensional ferromagnetic and nearest-neighbours interacting *Potts* models** with **$q \gg 1$ states**, undergoing a thermal **first-order phase transition**, on different lattices topologies (square, cubic, honeycomb and triangular).

Aims:

- Characterization of the relaxational dynamics.
- Analysis of the dynamical regimes through which the models pass before reaching a stable or metastable equilibrium.
- **Description of metastability and freezing and, when the system is able to escape from these equilibria, the dynamical escaping process.**

2) The *Potts* Model

The interacting model we used is defined by the following energy function:

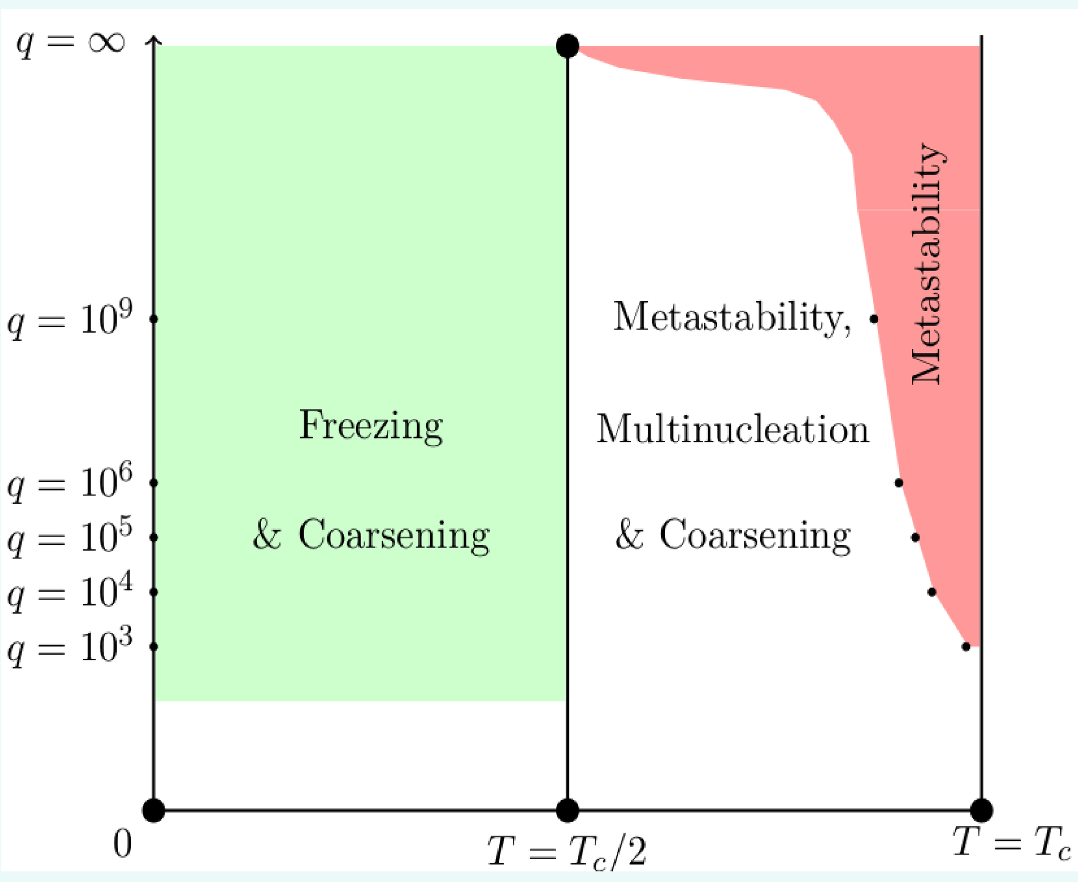
- $$\mathbf{H_J^{Potts}(\{s_i\}) = -\frac{J}{2}\sum_{\langle i,j\rangle} \delta_{s_i,s_j}}$$

with $\mathbf{s_i \in \{1, 2, \ldots, q\}}$ and $\mathbf{i \in \{1, \ldots, N\}}$, $\mathbf{N = L^2}$ or $\mathbf{N = L^3}$,

where the sum runs on the nearest neighbours spins ($\langle i,j\rangle$) on a lattice with **PBC** and side **L**.

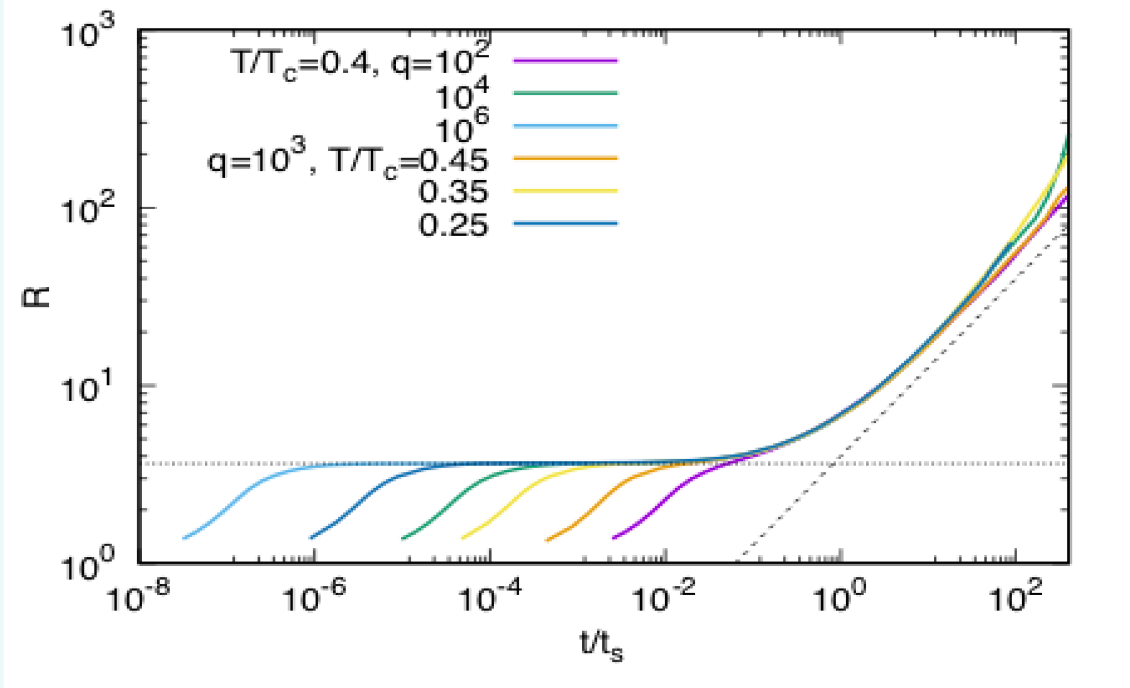
3) Results

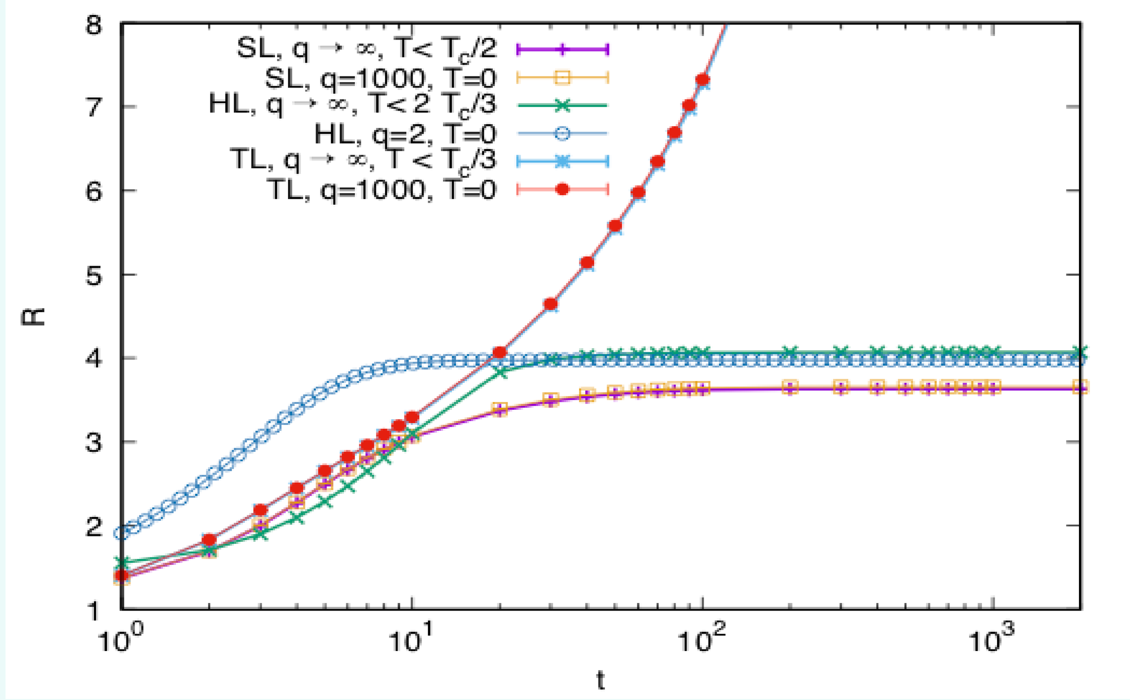
- A **q-independent spinodal temperature**, $\mathbf{2T_c/z}$ with z the coordination number of the lattices.
- For $\mathbf{T \leq 2T_c/z}$, **a universal low temperature dynamical behaviour** with blocked states, escaped in universal way after a proper **time-scale of the Arrhenius form** $\mathbf{e^{J/T}}$ [2].
- For $\mathbf{T \geq 2T_c/z}$, metastability, multinucleation then coarsening [3].
- Close to T_c, only metastability [1].



4) Low T

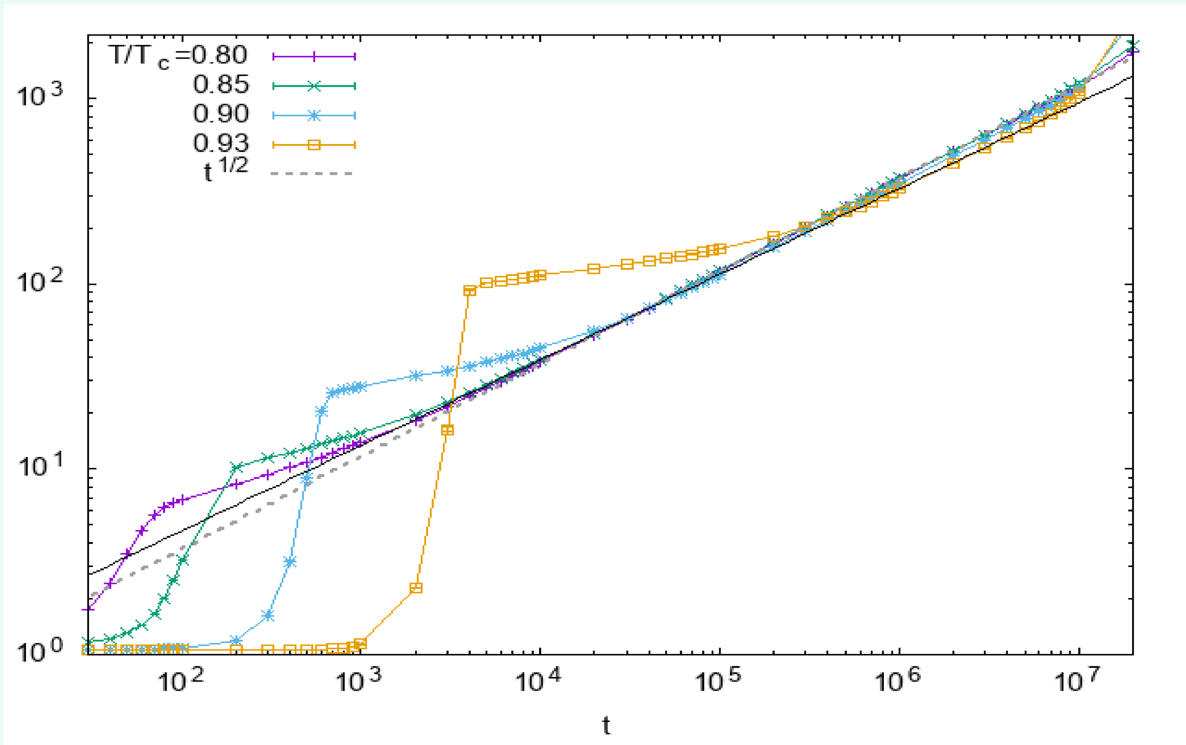
We computed the growing length, $R(t; q, T/T_c)$, which quantifies the typical linear extent of the ordered geometric spin clusters. It is associated to the total length of clusters interfaces and to the energy of the system and allows us to understand in which dynamical regime is the system at each timestep. The rescaled $R(t/t_s)$, vs. t/t_s, with $t_s = e^{J/T}$, shows a **universal behaviour** in the low temperature region, corresponding to the green part of the "dynamics phase diagram"





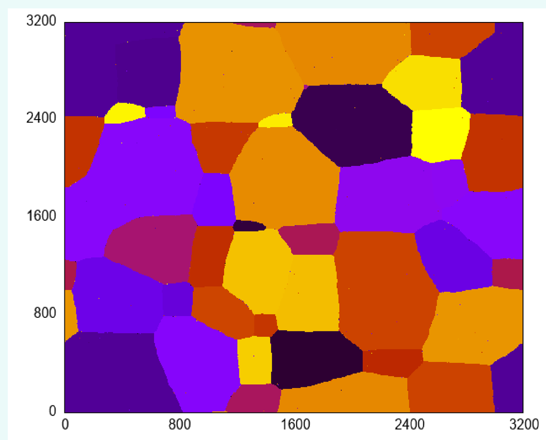
On the right, $R(t)$ vs. t for the square, honeycomb and triangular lattices to see similarities and differences (triangular) with respect to the square topology. Similar results in $3d$ for the cubic lattice [4].

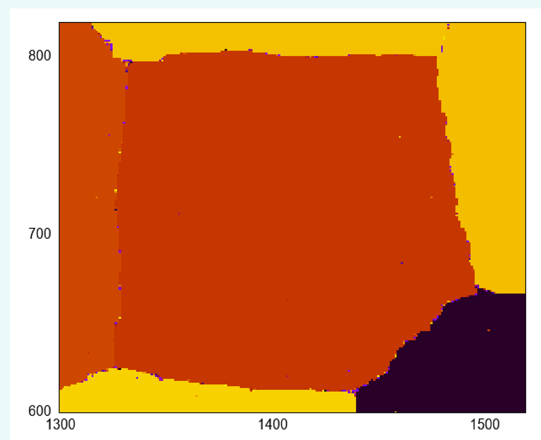
5) Metastability, multinucleation, coarsening



$R(t)$ vs t, $q = 10^3$, $L = 3200$.
snapshot for $t = 10^6$, $\frac{T}{T_c} = 0.93 \rightarrow$

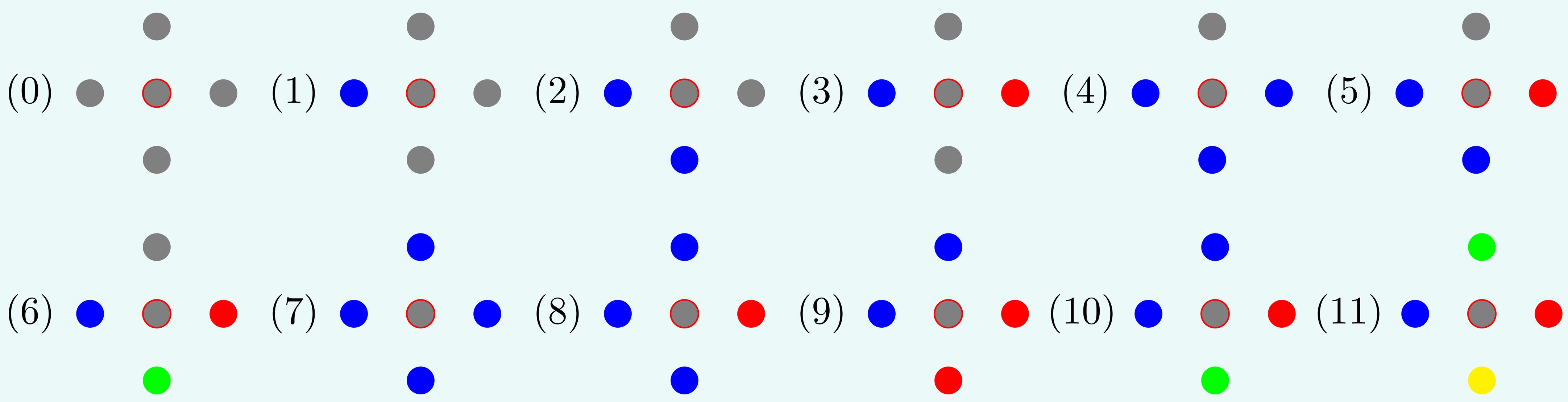
1. Metastability up to a finite time,
2. Multinucleation with a rapid increase of $R(t)$,
3. Coarsening $R(t) \simeq t^{\frac{1}{2}}$ (apart some "sand").





6) Metastability close to T_c

Enumeration : all the states can be described as



1. The evolution among these states is simple to describe. For example
$$P_{11\to 11} = \frac{q-4}{4e^{\beta}+q-4}, \quad P_{11\to 6} = \frac{4e^{\beta}}{4e^{\beta}+q-4}$$
2. Close to T_c, a master equation describes the evolution of the P_i's.
3. With a condition of stationarity, we solve the master equation and obtain the N_i's

T/T_c	p		N_{11}	N_6	$10^3 N_{3a}$	$10^3 N_{3b}$	$10^3 N_{3c}$	$10^3 N_{10a}$	$10^3 N_{10c}$
0.88	0.01017	numeric	0.9895816	0.0101646	0.0130	0.0260	0.1772	0.0020	0.0039
		analytic	0.9895916	0.0101674	0.0129	0.0259	0.1705	0.0020	0.0039
0.92	0.00725	numeric	0.9926679	0.0072481	0.0066	0.0132	0.0444	0.0020	0.0039
		analytic	0.9926690	0.0072485	0.0066	0.0131	0.0438	0.0020	0.0040
0.98	0.00459	numeric	0.9953845	0.0045892	0.0026	0.0053	0.0070	0.0020	0.0040
		analytic	0.9953847	0.0045892	0.0026	0.0053	0.0070	0.0020	0.0040
0.99	0.00428	numeric	0.9957020	0.0042752	0.0023	0.0046	0.0053	0.0020	0.0040
		analytic	0.9957023	0.0042751	0.0023	0.0046	0.0053	0.0020	0.0040

7) References

[1] O. Mazzarisi, F. Corberi, L. F. Cugliandolo, M. Picco: *Metastability in the Potts model: exact results in the large q limit*, J. Stat. Mech. (2020) 063214
[2] F. Chippari, L. F. Cugliandolo, M. Picco: *Low-temperature universal dynamics of the bidimensional Potts model in the large q limit*, J. Stat. Mech. (2021) 093201
[3] F. Corberi, L. F. Cugliandolo, M. Esposito, O. Mazzarisi, M. Picco: *How many phases nucleate in the bidimensional Potts model?*, J. Stat. Mech. (2022) 073204
[4] F. Chippari, M. Picco: *Freezing vs. equilibration dynamics in the Potts model*, J. Stat. Mech. (2023) 023201