Quantum disordered systems

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September 2014, Cargèse, France

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- Pasquale Calabrese (Università di Pisa, Italia)
- Laura Foini (Université de Genève, Suisse)
- Marco Schiró (IPhT, Saclay, France)
- Guilhem Semerjian (LPT-ENS, Paris, France)

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- Guilhem Semerjian (LPT-ENS, Paris, France) - quantum cavity
- Pasquale Calabrese (Università di Pisa, Italia) - quantum quenches
- Laura Foini (Université de Genève, Suisse) - quantum quenches
- Marco Schiró (IPhT, Saclay, France) - quantum quenches

Motivation

Why should one care about quantum fluctuations ?

Physical

– High-energy physics

– Condensed matter clear

– Atomic physics

$$\hbar\omega \gtrsim k_B T$$

Motivation

Why should one care about quantum fluctuations ?

Physical

– High-energy physics

– Condensed matter clear

– Atomic physics & cold atom experiments

revived fundamental questions concerning equilibration in classical and quantum closed systems

Motivation

Why should one care about quantum fluctuations ?

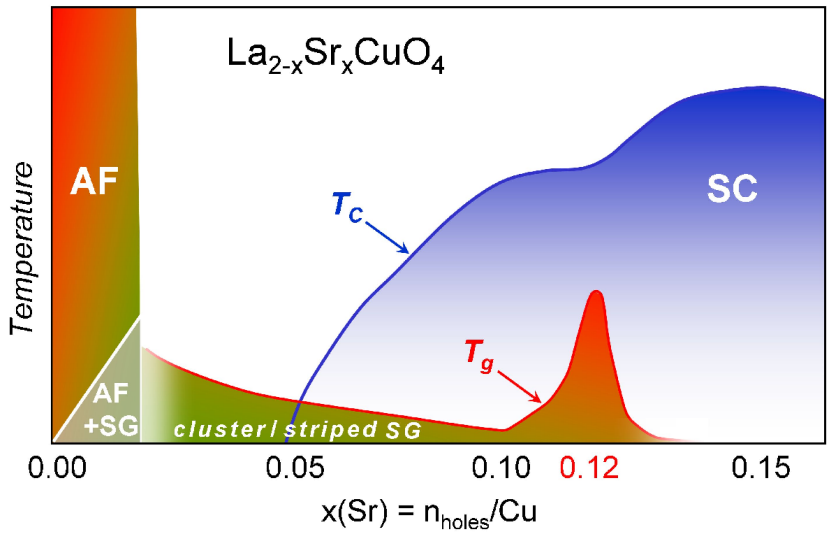
Physical

- High-energy physics
- Condensed matter clear
- Atomic physics
- Glassy oriented crowd ?

Motivation: physics

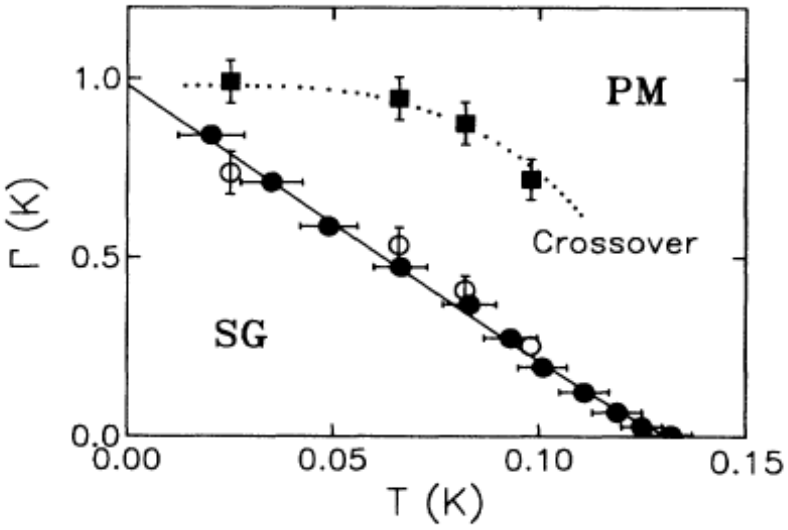
Some putative quantum spin-glass phases

High Tc SCs $La_{2-x}Sr_xCu_2O_4$



M-H Julien et al. 03

Dipolar systems $LiHo_xY_{1-x}F_4$

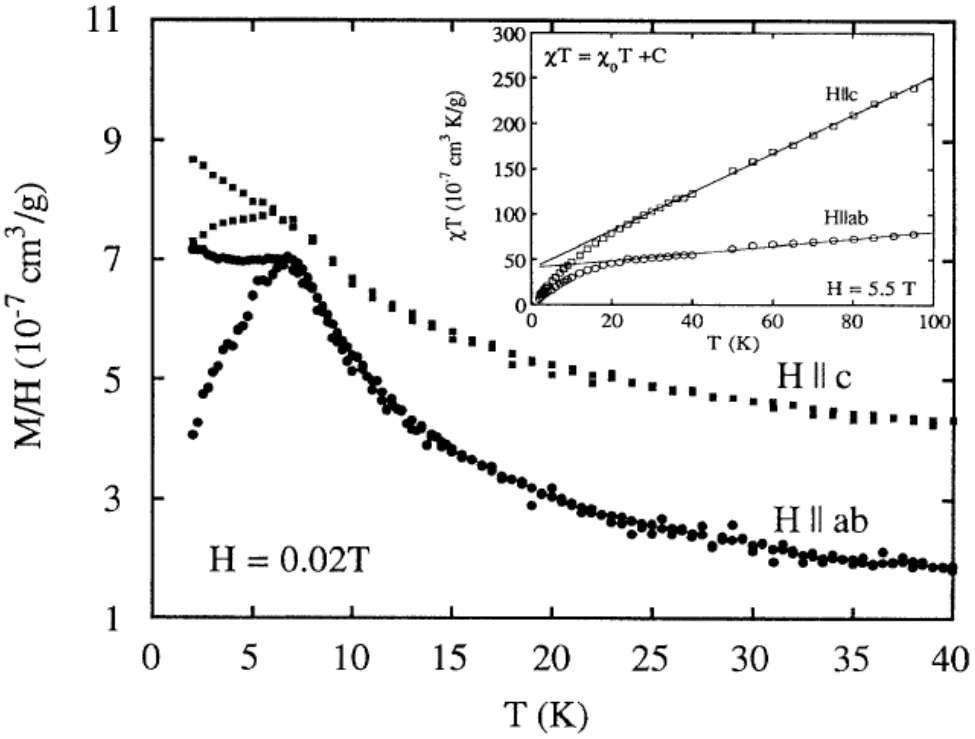


G. Aeppli et al. 90s

Motivation: physics

Field-cooled vs zero field-cooled magnetisation

$La_{1.96}Sr_{0.04}CuO_4$

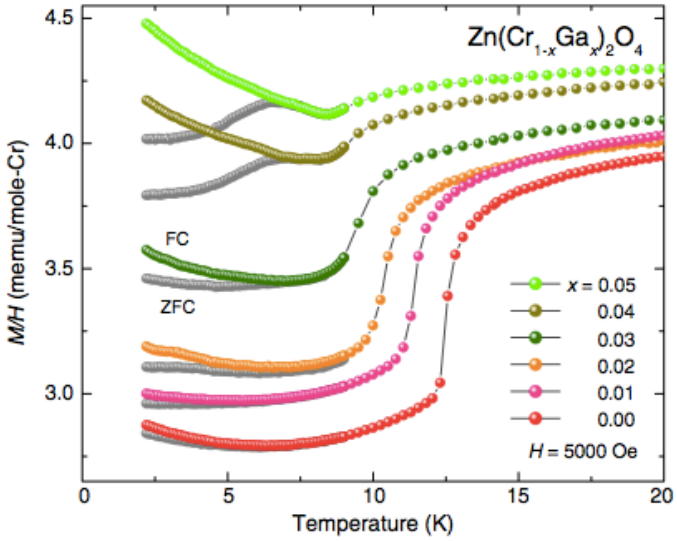


Chou et al. 95

Motivation: physics

Field-cooled vs. Zero field-cooled magnetisation

$ZnCr_{2(1-x)}\ Ga_{2x}\ O_4$



LaForge, Pulido, Cava, Chan, Ramírez 13

A geometrically frustrated magnet – no quenched disorder.

Proposals to realise quantum spin-glasses with atoms in optical cavities.

Motivation: physics

Methods from glassy physics

Statics

TAP Thouless-Anderson-Palmer	fully-connected (complete graph)
Replica theory	Gaussian approx. to field-theories
Cavity or Peierls approx.	dilute (random graph)
Bubbles & droplet arguments	finite dimensions
RG	finite dimensions

Dynamics

Generating functional for classical field theories (MSRJD).

Perturbation theory, renormalization group techniques, self-consistent approximations, droplet methods.

Extensions?

Motivation : computer science

Quantum annealing

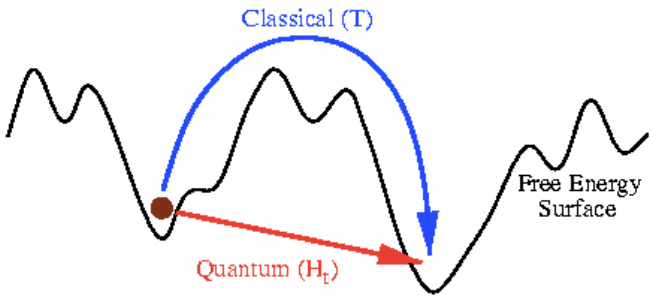
Goal : use quantum fluctuations to solve (hard) optimisation problems.

Idea : once mapped onto a classical physical Hamiltonian, find its ground state by following a well–chosen path in parameter space that takes the system into the quantum realm and then back to classical.

Quantum fluctuations are efficient to tunnel through tall (but not wide) barriers

while temperature fluctuations are efficient to jump over short (but possibly wide) barriers

Arrhenius



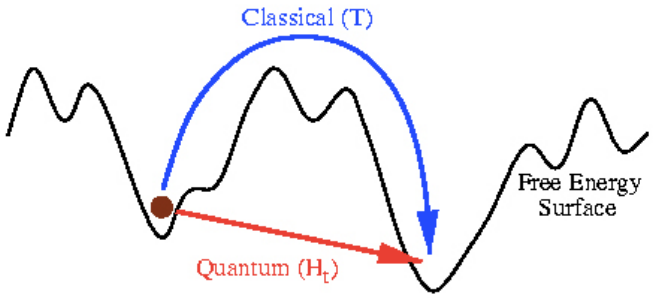
Tunneling

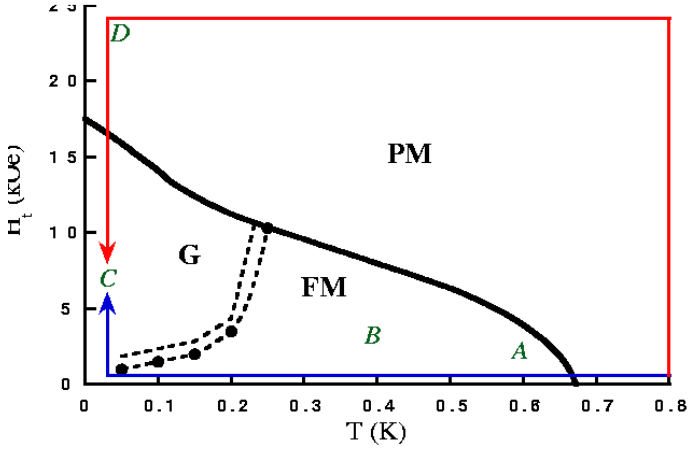
Motivation : computer science

Quantum annealing

Goal : use quantum fluctuations to solve (hard) optimisation problems.

Idea : similar to simulated annealing but in an 'enlarged' phase diagram.





Quantum tunneling & thermal activation

Motivation

Why should one care about quantum fluctuations ?

Physical

- High-energy physics
- Condensed matter clear
- Atomic physics
- Glassy oriented crowd ?

  Understand these materials.

  Use our toolbox to deal with these problems.

  Develop formalism.

Plan

Quantum fluctuations

- Canonical equilibrium.
  - Classical disordered models & optimisation problems.
  - Quantum disordered models & optimisation problems.
  - The bath. Effects on equilibrium phase diagrams.
- Dynamics.
  - Closed systems and questions on equilibration.
  - Open systems, Markov vs. non-Markov dynamics.
  - A single dissipative quantum particle.
  - Quantum macroscopic dissipative systems.

Plan

Quantum fluctuations

- **Canonical equilibrium.** Preliminaries.
  - Classical disordered models & optimisation problems.
  - Quantum disordered models & optimisation problems.
  - The bath. Effects on equilibrium phase diagrams.
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Statistical physics

No need to solve the classical dynamic equations!

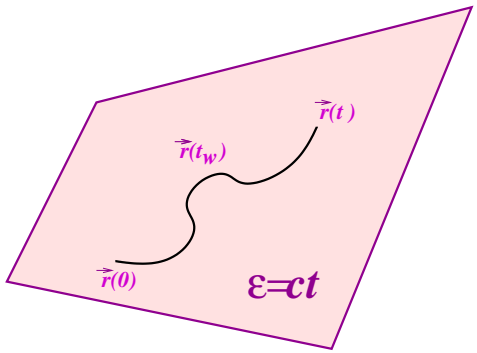
Under certain circumstances, *ergodic hypothesis*, after some equilibration time, t_{eq}, the macroscopic observables can be, on average, obtained with a *static* calculation, as an average over all configurations in phase space weighted with a probability distribution function $P(\{\vec{p}_i, \vec{x}_i\})$:

$$\langle A \rangle = \int \prod_i d\vec{p}_i d\vec{x}_i \; P(\{\vec{p}_i, \vec{x}_i\}) \; A(\{\vec{p}_i, \vec{x}_i\})$$

Recipes for $P(\{\vec{p}_i, \vec{x}_i\})$ are given and depend upon the conditions under which the system evolves, whether it is isolated or in contact with an environment.

L. Boltzmann, late XIX

Ensembles



Isolated system $\Rightarrow$ total energy is conserved

$$\mathcal{E} = H(\{\mathcal{C}\}) = H(\{\vec{p}_i, \vec{x}_i\})$$

Flat probability density

$$P(\{\mathcal{C}\}) \propto \delta(H(\{\vec{p}_i, \vec{x}_i\}) - \mathcal{E})$$

Microcanonical distribution

$S_\mathcal{E} = k_B \ln V(\mathcal{E})$ Entropy

$\beta \equiv \frac{1}{k_B T} = \left.\frac{\partial S_\mathcal{E}}{\partial \mathcal{E}}\right|_\mathcal{E}$ Temperature

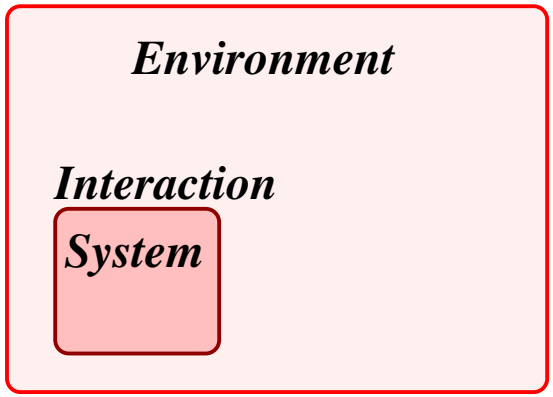
$\mathcal{E} = \mathcal{E}_{syst} + \mathcal{E}_{env} + \mathcal{E}_{int}$

Neglect $\mathcal{E}_{int}$ (short-range interact.)

$\mathcal{E}_{syst} \ll \mathcal{E}_{env}$

$P(\{\mathcal{C}\}) \propto e^{-\beta H(\{\vec{p}_i, \vec{x}_i\})}$

Canonical ensemble



Quantum mechanics

Notation & reminder

Each dynamical variable or observable (e.g. position, translational momentum, etc.) is associated with a Hermitian operator, say $\hat{A}$.

The state a quantum system is represented by a vector in a Hilbert space, say $|a\rangle$.

The eigenvalues of the operator, $\hat{A}|a\rangle = a|a\rangle$, correspond to the possible values of the dynamical variable.

If the system is in a general state $|\psi\rangle$ the value a is obtained with probability $p = |\langle a|\psi\rangle|^2$.

Observables associated to operators that do not commute are not simultaneously measurable, e.g. $\hat{p}$ and $\hat{x}$.

Quantum mechanics

Notation & reminder

Take a quantum particle's momentum, $\hat{p}$, and position, $\hat{x}$, operators satisfying the commutation relation $[\hat{p}, \hat{x}] = -i\hbar$.

The system's Hamiltonian is $\hat{H}_{syst} = \frac{\hat{p}^2}{2m} + V(\hat{x})$.

Take a quantum spin 1/2 such that $\hat{S}^z|\pm\rangle = \pm\frac{\hbar}{2}|\pm\rangle$.

The spin operator $\hat{S}^a$ with $a = x,\ y,\ z$ satisfies the commutation relations $[\hat{S}^a, \hat{S}^b] = \mathrm{i}\hbar\epsilon_{abc}\hat{S}^c$.

The time-dependent state of the system is represented by a vector in a Hilbert space $|\psi(t)\rangle$.

It evolves in time following Schrödinger's equation

$$\mathrm{i}\hbar\,\mathrm{d}_t|\psi(t)\rangle = \hat{H}_{syst}|\psi(t)\rangle$$

Quantum mechanics

Notation & reminder : density operator

Take a time-dependent state $|\psi(t)\rangle$ with expansion $|\psi\rangle = \sum_n a_n(t)|u_n\rangle$ in an orthonormal basis $|u_n\rangle$ and assume it is normalised.

The time-dependent density operator is defined as $\hat{\rho}(t) \equiv |\psi(t)\rangle\langle\psi(t)|$.

Since $|\psi(t)\rangle$ is normalised, $\mathrm{Tr}\hat{\rho}(t)$ is normalised as well.

The quantum average of an operator $\hat{\mathcal{O}}$ is given by $\langle\hat{\mathcal{O}}\rangle = \mathrm{Tr}\hat{\rho}(t)\hat{\mathcal{O}}$.

The density operator evolves according to

$$\mathrm{i}\hbar\,\mathrm{d}_t\hat{\rho} = [\hat{H}_{syst}, \hat{\rho}]$$

The density matrix elements are given by $\rho_{mn}(t) = \langle u_m|\hat{\rho}(t)|u_n\rangle = a_m(t)a_n^*(t)$.

Quantum mechanics

Notation & reminder : statistical ensembles

The system may be in state $|\psi_n\rangle$ with probability p_n.

When ? if we prepare a system (an atom, say) many times.

The density operator is then $\hat{\rho} \equiv \sum_n p_n |\psi_n\rangle\langle\psi_n|$ with $|\psi_n\rangle$.

The density matrix $\langle\psi_n|\hat{\rho}|\psi_m\rangle$ is the quantum-mechanical analogue to a classical phase-space probability measure, $P(\mathcal{C})$ of statistical physics.

In canonical equilibrium the density operator is $\hat{\rho} \equiv \mathcal{Z}^{-1}\, e^{-\beta \hat{H}}$ with $\mathcal{Z} = \mathrm{Tr} e^{-\beta \hat{H}}$.

One studies $\hat{\rho}$ to infer phase diagrams of open quantum systems.

Plan

Quantum fluctuations

- **Canonical equilibrium.**
  - **– Classical disordered models & optimisation problems.**
  - – Quantum disordered models & optimisation problems.
  - – The bath. Effects on equilibrium phase diagrams.
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Disordered spin systems

Classical p-spin model

$$H_{syst} = \sum_{i_1<\cdots<i_p}^{N} J_{i_1 i_2 \ldots i_p} s_{i_1} s_{i_2} \ldots s_{i_p}$$

Ising, $s_i = \pm 1$, or spherical, $\sum_{i=1}^{N} s_i^2 = N$, spins. Drawing

Sum over all p-uplets on a complete graph: fully-connected model.

Random exchanges $P(J_{i_1 i_2 \ldots i_p}) \propto e^{-p!\, J_{i_1 i_2 \ldots i_p}^2 / (2N^{p-1} J^2)}$

Extensions to random graphs possible: dilute models.

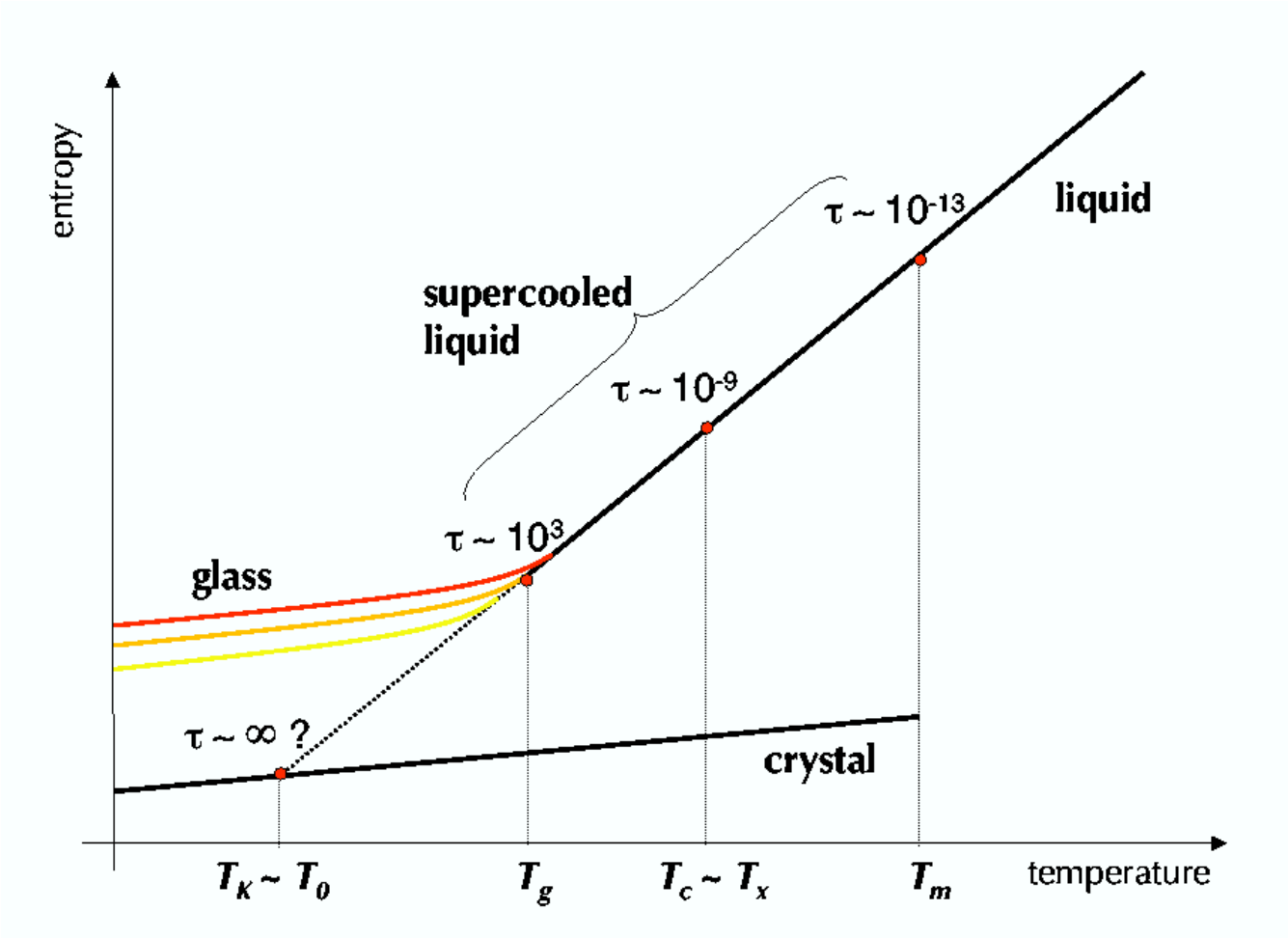
$p = 2$ Ising: Sherrington-Kirkpatrick (SK) model for spin-glasses

$p = 2$ spherical $\approx$ mean-field ferromagnet.

$p \geq 3$ Ising or spherical: models for fragile glasses.

Disordered spin systems

Classical p-spin model & fragile glasses



T_s T_d

T. Kirkpatrick, Thirumalai & Wolynes 80s

Disordered spin systems

Random K-sat problem

A clause is the 'logical or' between K requirements imposed on Boolean variables x_i chosen randomly from a pool of N of them.

A formula is the 'logical and' between M such clauses, $F = \bigwedge_{\ell=1}^{M} \bigvee_{i=1}^{K} x_i^{(\ell)}$. It is satisfied when all M clauses are.

The search for a solution can be set as the search for the spin configuration(s) with vanishing energy

$$H_{syst} = \alpha 2^{-K} N + \sum_{R=1}^{K} (-1)^R \sum_{i_1 < \cdots < i_R}^{N} J_{i_1 i_2 \ldots i_R} \, s_{i_1} s_{i_2} \ldots s_{i_R}$$

with $\alpha = M/N$, Ising classical spins, $s_i = \pm 1$, and interactions $J_{i_1 \ldots i_R} = 2^{-K} \sum_{\ell=1}^{M} C_{\ell,i_1} \ldots C_{\ell,i_R}$ with $C_{\ell,i_k} = +, -$ for the condition $x_{i_k}^{(\ell)} =$ T,F and $C_{\ell,i_k} = 0$ otherwise.

Sum of classical dilute $p \leq K$-spin models

Optimisation problems

Status

Consensus: there exist families of cost functions of N discrete variables such that no algorithm can find their global minimum by executing a number of operations smaller than some polynomial of N.

P $\neq$ NP conjecture

Consequence: classical algorithms need an exponentially large (in the system size) number of operations to solve hard instances in the NP class,

$$t \simeq e^{aN}$$

Such hard instances exist in Random $K \geq 3$-sat for special values of the parameter α (close to the threshold between satisfiable and unsatisfiable phases).

Challenges

Classical disordered systems & computer science

Glasses. Go beyond mean-field models (fully-connected and dilute) disordered spin systems and understand the behaviour of particle systems with short-range interactions.

Fully understand the glassy arrest.

Optimisation. Dilute spin models are the focus of study. Understand all their possible dynamics, physical and unphysical.

Find algorithms that solve hard instances in polynomial time,

and disprove P $\neq$ NP,

or prove that this is not possible and then establish P $\neq$ NP.

Plan

Quantum fluctuations

- **Canonical equilibrium.**
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  - **Quantum disordered models & optimisation problems.**
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Disordered spin systems

Quantum p-spin model

$$\hat{H}_{syst} = \sum_{i_1<\cdots<i_p}^{N} J_{i_1\ldots i_p}\hat{\sigma}^z_{i_1}\ldots\hat{\sigma}^z_{i_p} + \Gamma\sum_{i=1}^{N}\hat{\sigma}^x_i$$

$\hat{\sigma}^a_i$ with $a = 1, 2, 3$ the Pauli matrices, $[\hat{\sigma}^a_i, \hat{\sigma}^b_i] = 2\mathrm{i}\epsilon_{abc}\hat{\sigma}^c_i$.

Γ transverse field. It induces quantum fluctuations.

In the limit $\Gamma \to 0$ the classical limit should be recovered.

Sum over all p-uplets on a complete graph (extensions to random graphs)

$P(J_{i_1 i_2\ldots i_p}) \propto e^{-p!\, J^2_{i_1 i_2\ldots i_p}/(2N^{p-1}J^2)}$

$p \geq 2$ Ising: quantum Sherrington-Kirkpatrick and p-spin models.

$p \geq 2$ continuous variables : quantisation achieved by adding a kinetic energy.

Quantum systems

Quantum fluctuations

- Take an isolated quantum system with Hamiltonian $\hat{H}_i$
- Initialize it in, say, $|\psi_0\rangle$ the ground-state of $\hat{H}_i$.
- Evolve it with a different, possibly time-dependent, Hamiltonian $\hat{H}(t)$

$$\mathrm{i}\hbar \mathrm{d}_t |\psi(t)\rangle = \hat{H}(t)|\psi(t)\rangle$$

Can these dynamics help reach the ground state of a cost function ?

- For example, choose $\hat{H}(t)$ such that $\hat{H}(t_0) = \hat{H}_i$ and $\hat{H}(t_f) = H_f$ the (classical) cost function in question, and try to find in this way its ground state.

Disordered spin systems

Quantum p-spin model and random K-sat problem

$$\hat{H}_{syst}(t) = \frac{\alpha N}{2^K} + \sum_{R=1}^{K} (-1)^R \sum_{i_1<\cdots<i_R}^{N} J_{i_1 i_2 \ldots i_R}\, \hat{\sigma}^z_{i_1} \hat{\sigma}^z_{i_2} \ldots \hat{\sigma}^z_{i_R} + \Gamma(t) \sum_i \hat{\sigma}^x_i$$

with $\alpha = M/N$,

the Pauli matrices, $[\hat{\sigma}^a_i, \hat{\sigma}^b_j] = 2\mathrm{i}\delta_{ij}\epsilon_{abc}\hat{\sigma}^c_i$,

and the interactions

$J_{i_1 \ldots i_R} = 2^{-K} \sum_{\ell=1}^{M} C_{\ell,i_1} \ldots C_{\ell,i_R}$

with $C_{\ell,i_k} = +, -$ for the condition $x^{(\ell)}_{i_k} =$ T,F and $C_{\ell,i_k} = 0$ otherwise.

Sum of quantum dilute $p \leq K$-spin models

Interpolate between $\Gamma(0) \gg 1$ and $\Gamma(t_f) = 0$ (easy to hard)

Optimisation problems

Adiabatic theorem and quantum annealing

If a quantum system is prepared in the ground state of a simple Hamiltonian, $\hat{H}_i$, and one gives a *slow enough* evolution to the Hamiltonian, $\hat{H}(t)$, the *adiabatic theorem* **M. Born & V. Fock 31** ensures that the system remains, with high probability, in the *instantaneous ground state* of $\hat{H}(t)$ at all subsequent times.

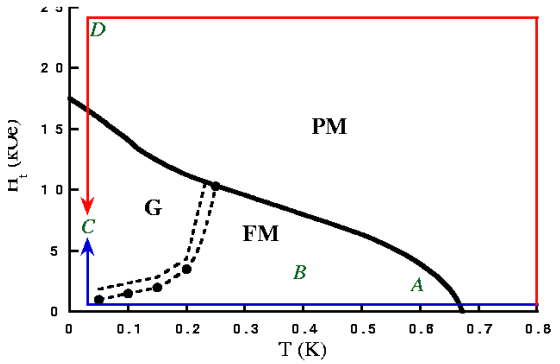
Purpose : use this property to take the system to the ground state of a desired (classical) Hamiltonian $H_f = \hat{H}(t_f)$.

Quantum annealing

Kadowaki & Nishimori 98

Dipolar spin-glass

G. Aeppli et al. 90s



Optimisation problems

Quantum annealing

Take, slowly, the system from the ground state of a simple Hamiltonian, $\hat{H}_i$, to the ground state of a desired (classical) Hamiltonian $H_f = \hat{H}(t_f)$.

But, how slow is slow ?

The running time should be $t_f > \Delta_{\min}^{-2}$

with $\Delta_{\min} = E_1 - E_0$ the minimal gap between the energy of the first excited state, E_1, and the energy of the ground state, E_0, encountered along the evolution.

M. Born & V. Fock 31

Optimisation problems

Adiabatic theorem and quantum annealing

Interesting optimisation problems have first order phase transitions when rendered quantum. Technical details below and in **Semerjian's** talk.

At the first order phase transition the gap closes exponentially in the system size

$$\Delta_{\min} \simeq N e^{-aN}$$

Jörg, Krzakala, Kurchan & Maggs 08

Bapst, Foini, Krzakala, Semerjian & Zamponi 13

Therefore an exponentially long running time is also needed to follow the ground state.

$$t_f \simeq \Delta_{\min}^{-2} \simeq e^{2aN}$$

No gain...

Motivation: physics

Methods from glassy physics

Statics

TAP Thouless-Anderson-Palmer	fully-connected (complete graph)
Replica theory	Gaussian approx. to field-theories
Cavity or Peierls approx.	dilute (random graph) Semerjian
Bubbles & droplet arguments	finite dimensions Friday
RG	

Dynamics

Generating functional for classical field theories (MSRJD).

Perturbation theory, renormalization group techniques, self-consistent approximations, droplet methods.

Matsubara replica calculation

A sketch

$$-\beta f = \lim_{N\to\infty} \frac{\ln \mathcal{Z}}{N} = \lim_{N\to\infty} \lim_{n\to 0} \frac{[\mathcal{Z}^n] - 1}{Nn}$$

$\mathcal{Z}^n$ partition function of n independent copies of the system: **replicas**.

Quantum mechanically, $\mathcal{Z} = \mathrm{Tr}\, e^{-\beta \hat{H}}$

and $\mathcal{Z} = \int_{\{s_i(0)\}}^{\{s_i(\beta\hbar)\}} \mathcal{D}s_i(\tau)\, e^{-\frac{1}{\hbar} S^e_{syst}[\{s_i(\tau)\}]}$ continuous

or $\mathcal{Z} = \sum_{s_i(\tau_k)=\pm 1} e^{-\frac{1}{\hbar} S^e_{syst}[\{s_i(\tau_k)\}]}$ discrete

the form of the Euclidean action, S^e_{syst}, depends on whether we use

trully SU(2) quantum spins or the 'spherical' version of the model.

Feynman-Matsubara construction of functional integral over imaginary time.

Matsubara replica calculation

A sketch

$$\boxed{-\beta f = \lim_{N\to\infty} \frac{\ln \mathcal{Z}}{N} = \lim_{N\to\infty} \lim_{n\to 0} \frac{[\mathcal{Z}^n] - 1}{Nn}}$$

Self-averageness

average over disorder

Quantum mechanically, $\mathcal{Z} = \mathrm{Tr}\, e^{-\beta \hat{H}} \Rightarrow \{\int, \sum\}\, e^{-\boxed{\frac{1}{\hbar}} S^e_{syst}[\{s_i\}]}$

No i contrary to the dynamic path-integral (that will appear later).

Mapping to $d+1$ classical statistical physics problem with anisotropic (imaginary-time $\neq$ spatial) interactions.

Feynman-Matsubara construction of functional integral over imaginary time.

Matsubara replica calculation

A sketch

Average over disorder $\Rightarrow$ coupling between replicas

$$\sum_{i_1 \neq \cdots \neq i_p} J_{i_1 \ldots i_p} \int d\tau \sum_a s^a_{i_1}(\tau) \ldots s^a_{i_p}(\tau) \;\Rightarrow\; \int d\tau d\tau' \sum_{ab} \left(\sum_i s^a_i(\tau) s^b_i(\tau') \right)^p$$

One introduces the auxiliary two-time dependent replica matrix

$$\delta \left(Q_{ab}(\tau, \tau') - N^{-1} \textstyle\sum_i s^a_i(\tau) s^b_i(\tau') \right)$$

In terms of the replica indices $Q_{ab}(\tau, \tau')$ is still a 0×0 matrix.

Slightly intricate imaginary-time & replica index structure. Recipes to deal with them

Bray & Moore 80 and the **Parisi** Ansatz

Matsubara replica calculation

Spherical case

$Q_{ab}(\tau,\tau')$ can be evaluated by saddle-point if one exchanges the limits $N \to \infty$ $n \to 0$ with $n \to 0$ $N \to \infty$.

Stationary behaviour expected. The equation to solve is

$$\left(-\frac{1}{\Gamma}\frac{\partial^2}{\partial\tau^2} + z\right) Q_{ab}(\tau)$$

$$= \delta_{ab}\delta(\tau) + \frac{p}{2}\int_0^{\beta\hbar} d\tau' \sum_c Q_{ac}^{\bullet(p-1)}(\tau-\tau')Q_{cb}(\tau')$$

with periodic boundary conditions, $Q_{ab}(\beta\hbar) \overset{c}{=} Q_{ab}(0)$.

In terms of the replica indices $Q_{ab}(\tau)$ is still a 0×0 matrix.

Bray & Moore 80, just $q_d(\tau)$, and the **Parisi** Ansatz for $a \neq b$

Note the similarity with the MCT equations.

Quantum TAP & cavity method

Quantum TAP

Legendre transform of f with respect to $\{m_i(\tau)\}$ and $C(\tau - \tau')$ with $m_i(\tau) = \langle s_i(\tau) \rangle$ and $C(\tau - \tau') = N^{-1} \sum_i \langle s_i(\tau) s_i(\tau') \rangle$.

In fully-connected models one finds the exact free-energy functional $f(m_i(\tau), C(\tau - \tau'))$ and the saddle-point equations.

Derivation & analysis of this functional for quantum p-spin disordered models

Biroli & LFC 01 ; Andreanov & Müller 12 (SK)

Quantum cavity methods allow one to deal with **dilute quantum spin models** **Krzakala, Rosso, Semerjian & Zamponi 08**, **Laumann, Moessner, Scardicchio & Sondhi 09**

Semerjian

Quantum TAP

Legendre transform of f with respect to $\{m_i(\tau)\}$ and $C(\tau-\tau')$ with $m_i(\tau)=\langle s_i(\tau)\rangle$ and $C(\tau-\tau')=N^{-1}\sum_i\langle s_i(\tau)s_i(\tau')\rangle$.

$f(m_i(\tau),C(\tau-\tau'))$.

The TAP equations for the quantum p-spin disordered (spherical) models

$$\begin{aligned}
\Gamma^{-1}\partial_\tau^2 C(\tau) &= -\frac{p}{2}\int_0^{\beta\hbar} d\tau'\,[C^{p-1}(\tau-\tau')-q^{p-1}][C(\tau')-q] \\
&\quad +z[C(\tau)-q]-\delta(\tau) \\
zm_i &= \sum_{i_2<\cdots<i_p} J_{i,\ldots i_p} m_{i_2}\ldots m_{i_p} \\
&\quad +\frac{p}{2}m_i\int_0^{\beta\hbar} d\tau'[C^{-p-1}(\tau')+(p-2)q^{p-1}-(p-1)C(\tau')q^{p-2}]
\end{aligned}$$

$q=N^{-1}\sum_i m_i^2$.

Biroli & LFC 01 ; Andreanov & Müller 12 (SK)

Quantum p-spin models

Some results

$$\hat{H}_{syst} = \sum_{i_1<\cdots<i_p}^{N} J_{i_1\ldots i_p}\hat{\sigma}^z_{i_1}\ldots\hat{\sigma}^z_{i_p} + \Gamma\sum_{i=1}^{N}\hat{\sigma}^x_i$$

$\hat{\sigma}^a_i$ with $a = 1, 2, 3$ the Pauli matrices, $[\hat{\sigma}^a_i, \hat{\sigma}^b_i] = 2\mathrm{i}\epsilon_{abc}\hat{\sigma}^c_i$.

Γ transverse field. It induces quantum fluctuations.

In the limit $\Gamma \to 0$ the classical limit should be recovered.

Sum over all p-uplets on a complete graph (extensions to random graphs)

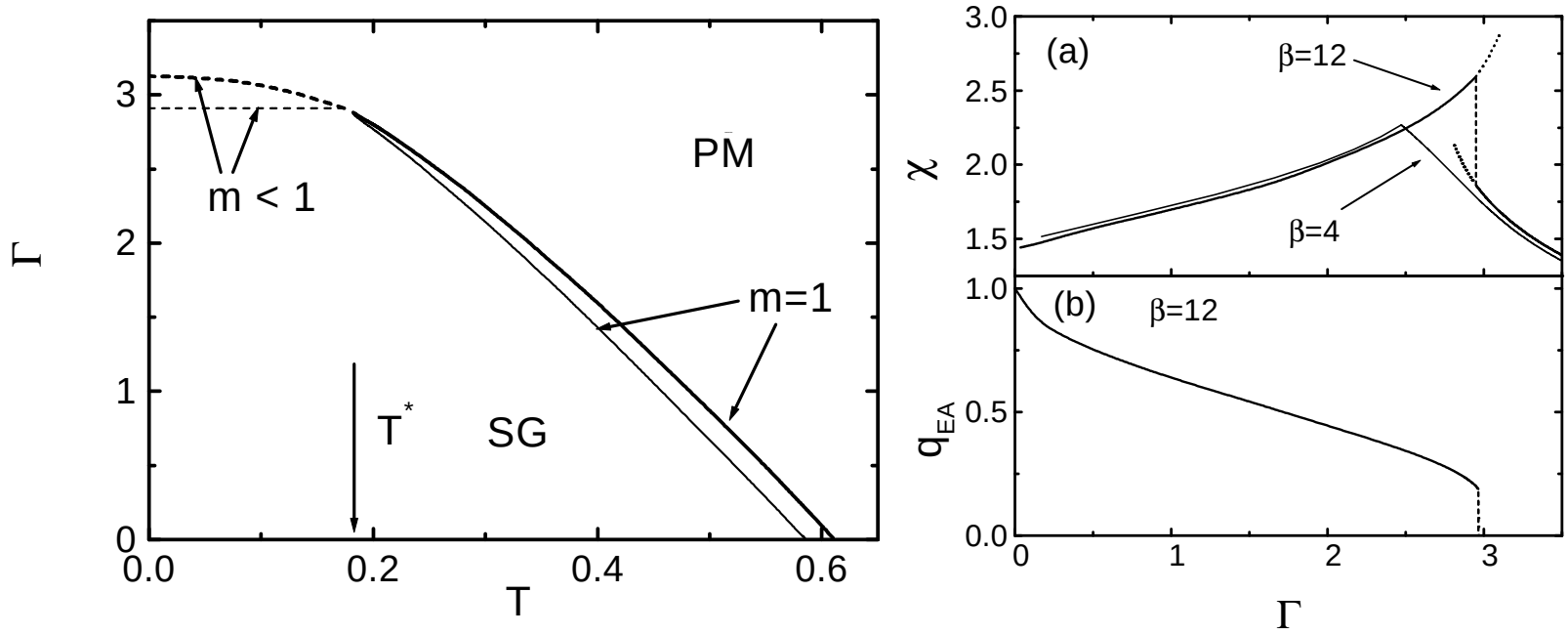
$P(J_{i_1 i_2\ldots i_p}) \propto e^{-p!\, J^2_{i_1 i_2\ldots i_p}/(2N^{p-1}J^2)}$

$p \geq 2$ Ising: quantum Sherrington-Kirkpatrick and p-spin models.

$p \geq 2$ continuous variables : quantisation achieved by adding a kinetic energy.

1st order phase transition

Quantum fully-connected $p \geq 3$ spin model



Focus on the thin dashed and solid inner lines: static phase transition.

Jump in the susceptibility across the dashed part of the critical line.

LFC, Grempel & da Silva Santos 00

In dilute disordered $p \geq 3$ models, review:

Bapst, Foini, Krzakala, Semerjian & Zamponi 13

Combinatorial optimisation

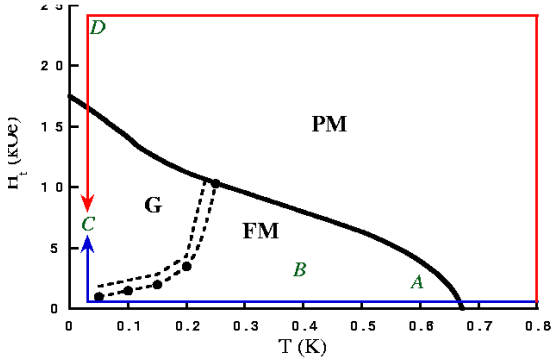
K**-satisfiability** is written in terms of $p(\leq K)$**- spin models on a random (hyper-)graph**.

Quantum annealing

Kadowaki & Nishimori 98

Dipolar spin-glass

G. Aeppli et al. 90s



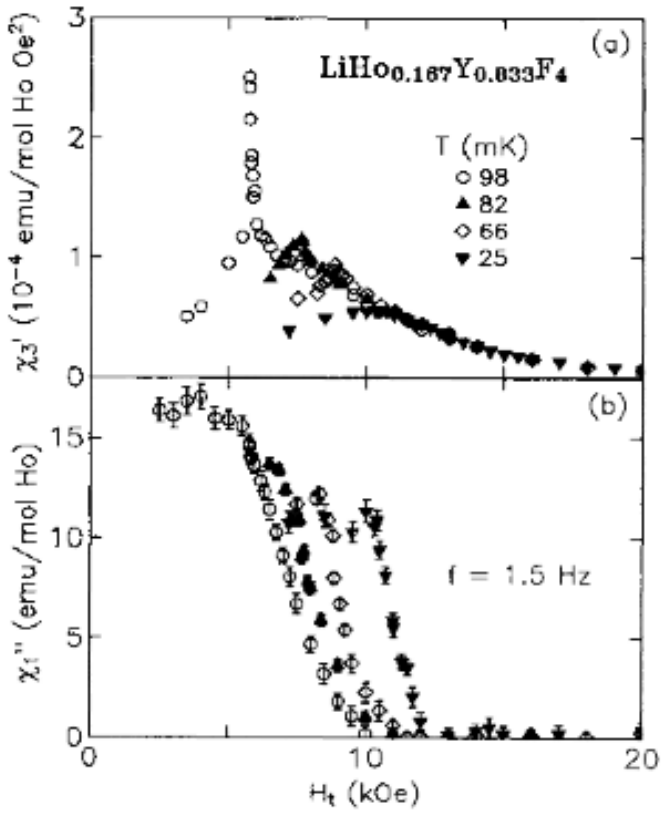
1st order transitions : trouble for **quantum annealing** techniques as

$$t_f \simeq e^{aN}$$

Jörg, Krzakala, Kurchan, Maggs, Pujos 08-09

1st order phase transition?

Dipolar glasses



Γ

Non-linear susceptibility

χ_3

The divergence disappears at low T

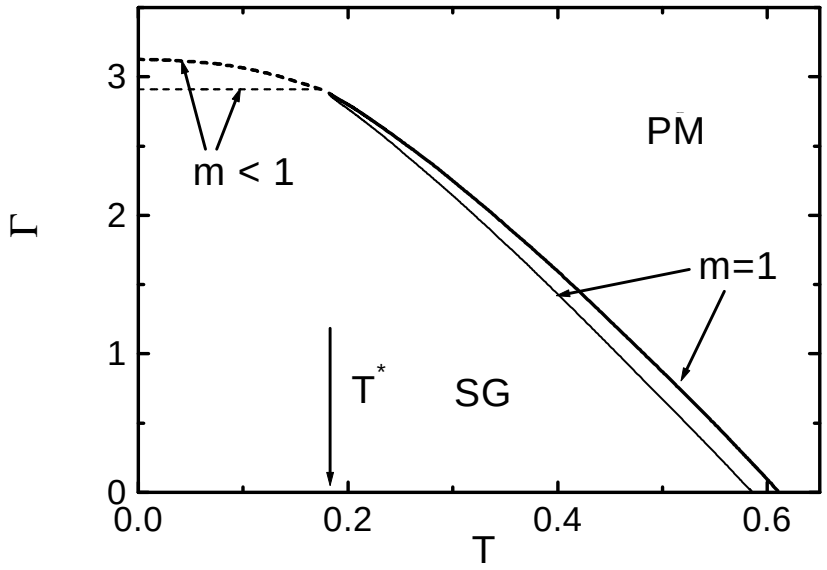
Out of phase linear susceptibility

χ_1''

Wu et al 93

1st order phase transition

Quantum fully-connected $p \geq 3$ spin model



Focus on the thick dashed and solid inner lines: dynamic phase transition.

Found with marginality condition (replicon vanishing)

LFC, Grempel & da Silva Santos 00

In dilute disordered $p \geq 3$ models, review:

Bapst, Foini, Krzakala, Semerjian & Zamponi 13

Plan

Quantum fluctuations

- **Canonical equilibrium.**
  - Classical disordered models & optimisation problems.
  - Quantum disordered models & optimisation problems.
  - **The bath. Effects on equilibrium phase diagrams.**
- Dynamics.
  - Closed systems and questions on equilibration.
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  - A single dissipative quantum particle.
  - Quantum macroscopic dissipative systems.

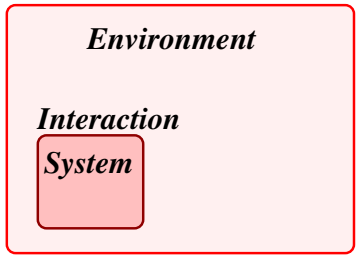
Dissipative systems

Aim

Interest in describing the **statics** and **dynamics** of a **classical or quantum physical system** coupled to a **classical or quantum environment**.

The Hamiltonian of the ensemble is

$$H = H_{syst} + H_{env} + H_{int}$$



The dynamics of all variables are given by Newton or Heisenberg rules, depending on the variables being classical or quantum.

The total energy is conserved, $E = \text{ct}$, but each contribution is not, in particular, $E_{syst} \neq \text{ct}$, and we'll take $E_{syst} \ll E_{env}$.

Reduced system

Model the environment and the interaction

E.g., an ensemble of harmonic oscillators and a linear in q_a and non-linear in x, via the function $\mathcal{V}(x)$, coupling: using the single particle notation

$$H_{env} + H_{int} = \sum_{\alpha=1}^{\mathcal{N}} \left[\frac{\pi_\alpha^2}{2m_\alpha} + \frac{m_\alpha \omega_\alpha^2}{2} q_\alpha^2 \right] + \sum_{\alpha=1}^{\mathcal{N}} c_\alpha q_\alpha \mathcal{V}(x)$$

Equilibrium. Imagine the whole system in contact with a megabath at inverse temperature β. Compute the reduced classical partition function or quantum density matrix by tracing away the bath degrees of freedom.

Dynamics. Classically (coupled Newton equations) and quantum (easier in a path-integral formalism) elimination of the bath variables.

In all cases one can integrate out the oscillator variables as they appear only quadratically, for this choice of $H_{env} + H_{int}$

Reduced system

Statistics of a classical system

Imagine the coupled system in canonical equilibrium with a megabath

$$\mathcal{Z}_{syst+env} = \sum_{env,\, syst} e^{-\beta H}$$

Integrating out the environmental (oscillator) variables

$$\mathcal{Z}^{red}_{syst} = \sum_{syst} e^{-\beta\left(H_{syst} - \frac{1}{2}\sum_a \frac{c_a^2}{m_a\omega_a^2}\,[\mathcal{V}(x)]^2\right)} \neq \mathcal{Z}_{syst} = \sum_{syst} e^{-\beta H_{syst}}$$

One possibility: assume weak interactions and drop the new term.

Trick: add $H_{counter}$ to the initial coupled Hamiltonian, and choose it in such a way to cancel the quadratic term in $\mathcal{V}(x)$ to recover

$$\boxed{\mathcal{Z}^{red}_{syst} = \mathcal{Z}_{syst}}$$

i.e., the partition function of the system of interest.

Reduced system

Model the quantum environment and the interaction

An ensemble of quantum harmonic oscillators and a bi-linear coupling, again using the single particle notation

$$\hat{H}_{env} + \hat{H}_{int} = \sum_{\alpha=1}^{\mathcal{N}} \left[\frac{\hat{\pi}_\alpha^2}{2m_\alpha} + \frac{m_\alpha \omega_\alpha^2}{2} \hat{q}_\alpha^2 \right] + \sum_{\alpha=1}^{\mathcal{N}} c_\alpha \hat{q}_\alpha \hat{x}$$

Quantum mechanically (easier in a Matsubara path-integral formalism) one can also integrate out the oscillator variables.

One obtains a reduced density operator, $\hat{\rho}_{syst}^{red}$.

Reduced system

Statics of a (dissipative) quantum system

One integrates the oscillator's degrees of freedom to get the reduced density matrix

$$\rho^{red}_{syst}(x'',x') = \mathcal{Z}^{-1}_{red}\int_{x'}^{x''}\mathcal{D}x(\tau)\, e^{-\frac{1}{\hbar}\left[S^{e}_{syst}-\int_0^{\beta\hbar}d\tau\int_0^{\tau}d\tau'\; x(\tau)K(\tau-\tau')x(\tau')\right]}$$

Even choosing the counter-term to cancel a quadratic term in $x^2(\tau)$ a non-local (possibly long-range) interaction with kernel

$$K(\tau) = \frac{2}{\pi\hbar\beta}\sum_{n=-\infty}^{\infty}\int_0^{\infty}d\omega\;\frac{I(\omega)}{\omega}\;\frac{\nu_n^2}{\nu_n^2+\omega^2}\,e^{i\nu_n\tau}$$ remains.

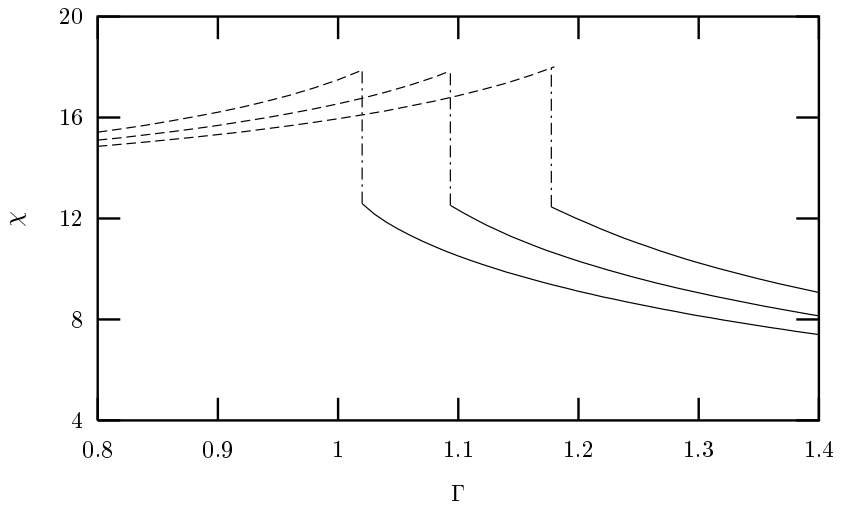
No obvious 'weak-coupling' argument can be used to drop it.

What are the effects of this term ?

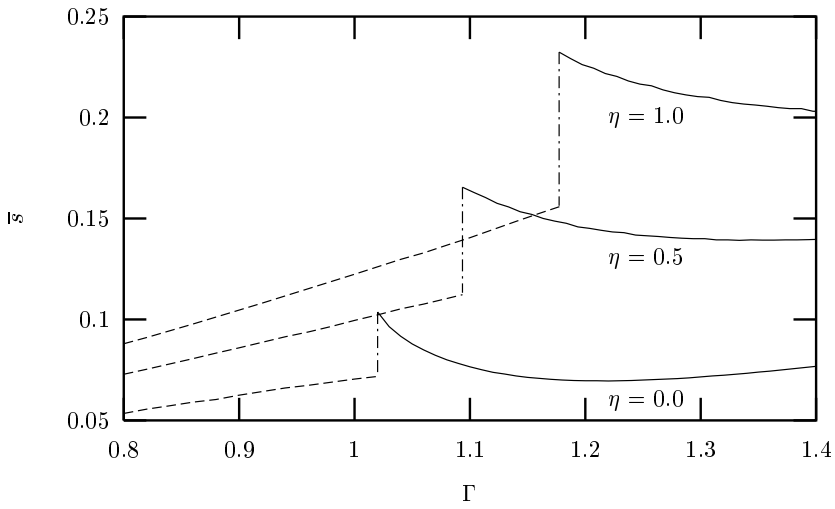
Noise-dependent transitions

Quantum $p = 3$-spin model with $I(\omega) = \eta\omega$

Magnetic susceptibility



Averaged entropy density



$$\eta = 0,\ 0.5,\ 1$$

η is the parameter measuring the strength of the coupling to the bath

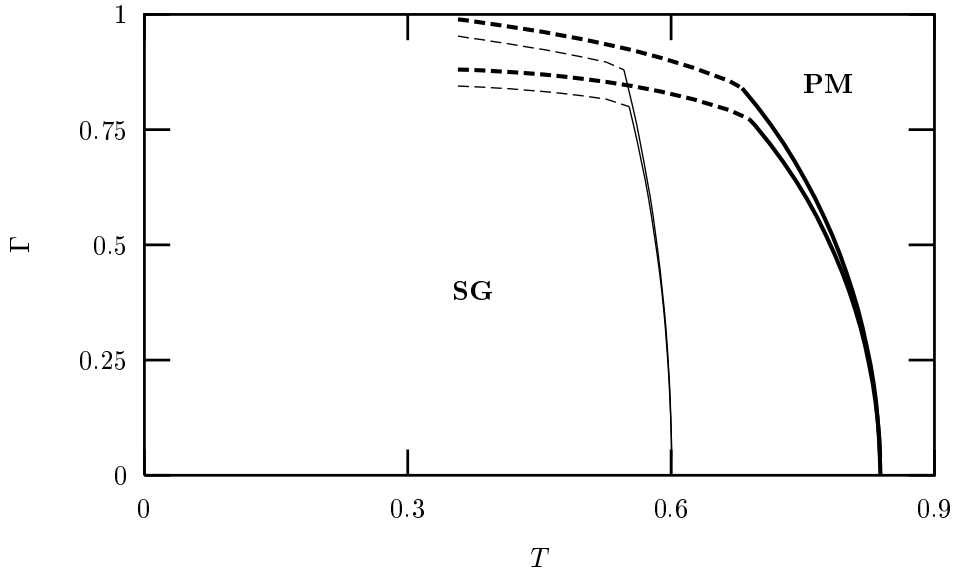
LFC, Grempel, Lozano, Lozza & da Silva Santos 02

Same kind of phenomena for $p = 2$, SU(2) spins, rotors, fermion bath, etc.

Static & dynamic phase diagram

Quantum $p = 3$-spin model with $I(\omega) = \eta\omega$

dashed = 1st order, solid = 2nd order thin = static, bold = dynamic



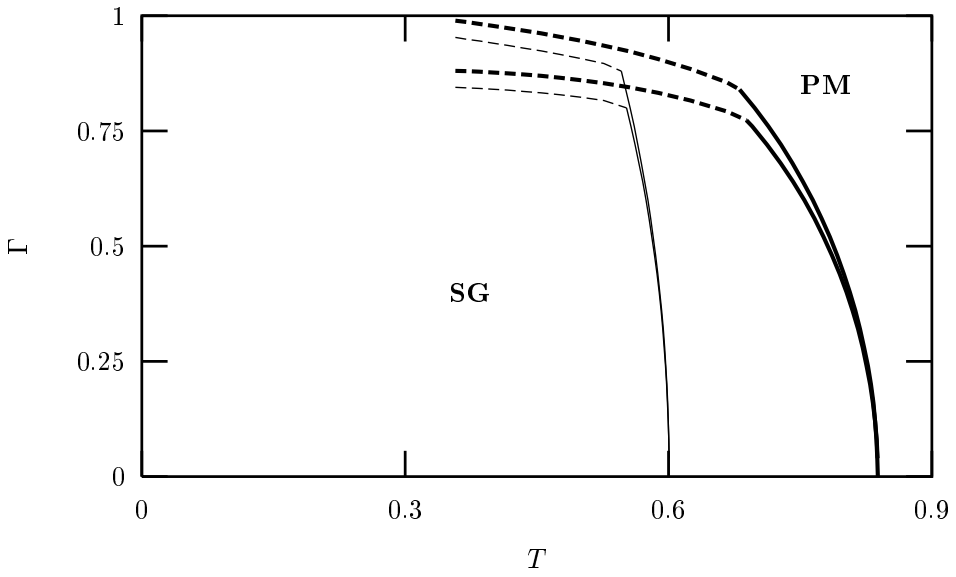
$\eta = 0,\ 0.5$ **LFC, Grempel, Lozano, Lozza & da Silva Santos 02**

The ordered phase is stabilized by the environment

Static & dynamic phase diagram

Quantum $p = 3$-spin model with $I(\omega) = \eta\omega$

dashed = 1st order, solid = 2nd order thin = static, bold = dynamic



Recall RFOT

for fragile glasses

$T_s \neq T_d$

No η-dependence at $\Gamma \to 0$

$\eta = 0,\ 0.5$ **LFC, Grempel, Lozano, Lozza & da Silva Santos 02**

The ordered phase is stabilized by the environment

Engineering environments

Statics of quantum disordered systems

Goal : use the coupling to an engineered bath to take the system to a desired, glassy or ordered, phase and then switch-off the bath.

Summary

Statics of quantum disordered systems

- We introduced quantum p-spin disordered models.
- We very briefly mentioned that the TAP and replica approaches as well as the cavity method **Semerjian** can be applied to them.
- We showed that these models have first order phase transitions in the low temperature limit.

  Problems for quantum annealing methods.
- A quantum environment induces long-range interactions in the imaginary-time direction and can have a highly non-trivial effect quantum mechanically.

Similar results for quantum Ising chains. For dilute models ?

SK model & connection to electron glasses : talk to **Müller**

Plan

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Isolated systems

Dynamics of classical systems

A few particles: dynamical systems

Many-body: foundations of statistical physics

Questions:

Does the dynamics of a particular system reach a flat distribution over the constant energy surface in phase space ?

Ergodic theory ($\in$ mathematical physics at present).

Can some part of the system, say modes, be taken as a bath with respect to others ?

Etc.

Isolated quantum systems

Quantum quenches

- Take an isolated quantum system with Hamiltonian $\hat{H}_i$
- Initialize it in, say, $|\psi_0\rangle$ the ground-state of $\hat{H}_i$.
- Unitary time-evolution with $\hat{U} = e^{-\frac{i}{\hbar}\hat{H}t}$ with a Hamiltonian $\hat{H}$.

Does the system reach some steady state ?

Note that it is the ergodic theory question posed in the quantum context.

Motivated by cold-atom experiments & exact solutions of $1d$ quantum models.

After a quantum quench, i.e. a rapid variation of a parameter in the system, **are at least some observables described by thermal ones? When, how, which?**

Calabrese, Foini & Schiró

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Classical dynamics (coupled Newton equations)

Assuming the environment is coupled to the sample at the initial time, t_0, and that its variables are characterized by a Gibbs-Boltzmann distribution or density function at inverse temperature β

One finds a **colored Langevin equation with multiplicative noise**

Reduced system

Dynamics of a classical system: general Langevin equations

The system, p, x, coupled to an **equilibrium environment** evolves according to the multiplicative noise non-Markov **Langevin equation**

Inertia **friction**

$$\overbrace{m\ddot{x}(t)}+\mathcal{V}'(x(t))\overbrace{\int_{t_0}^{\infty}dt'\,\gamma(t-t')\dot{x}(t')}\,\mathcal{V}'(x(t'))=$$

$$\underbrace{-\frac{\delta V(x)}{\delta x(t)}}+\mathcal{V}'(x(t))\,\underbrace{\xi(t)}$$

deterministic force **noise**

The friction kernel is $\gamma(t-t')=\Gamma(t-t')\theta(t-t')$

The **noise** has zero mean and correlation $\langle\,\xi(t)\xi(t')\,\rangle = k_BT\,\Gamma(t-t')$ with T the temperature of the bath and k_B the Boltzmann constant.

Reduced system

Dynamics of a classical system : general Langevin equations

The system, p, x, coupled to an **equilibrium environment** evolves according to the multiplicative noise non-Markov **Langevin equation**

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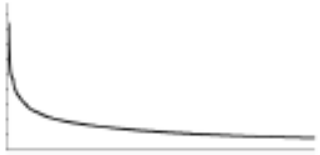
$$\overbrace{m\ddot{x}(t)}+\mathcal{V}'(x(t))\overbrace{\int_{t_0}^{\infty}dt'\,\gamma(t-t')\dot{x}(t')}\,\mathcal{V}'(x(t'))=$$

$$\underbrace{-\frac{\delta V(x)}{\delta x(t)}}+\mathcal{V}'(x(t))\,\underbrace{\xi(t)}$$

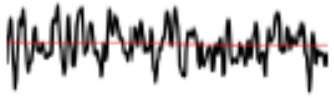
deterministic force **noise**

$\mathcal{E}_{syst}(t) \neq$ ct



Friction

Zoom



Noise

Separation of time-scales

Additive classical white noise

In classical systems one usually takes a bath kernel with the smallest relaxation time, $t_{env} \ll$ all other time scales.

The bath is approximated by the white form $\boxed{\Gamma(t-t') = 2\gamma\delta(t-t')}$

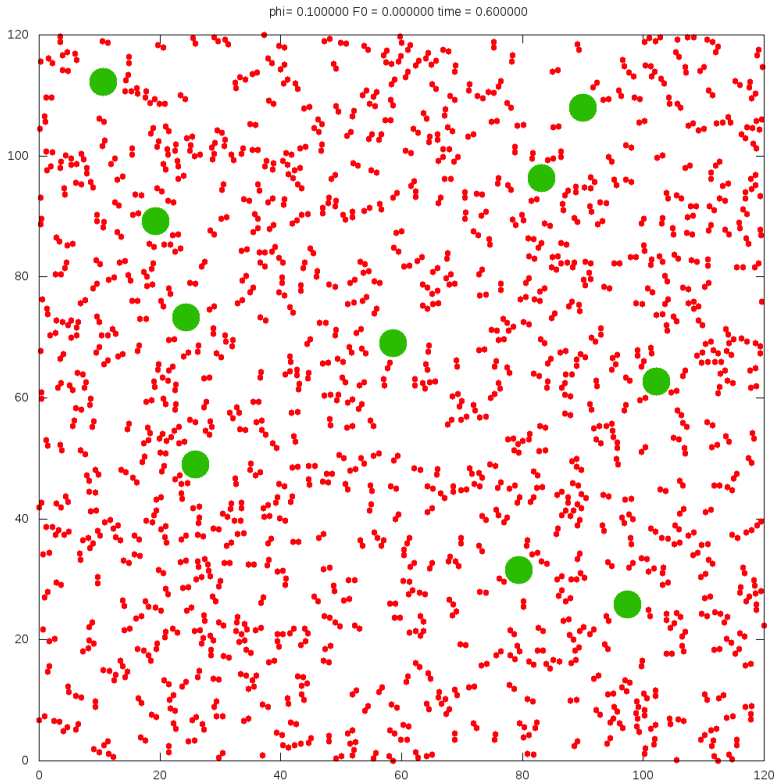
Moreover, one assumes the coupling is bi-linear, $H_{int} = \sum_a c_a q_a x$.

The Langevin equation becomes

$$\boxed{m\ddot{x}(t) \; + \; \gamma\dot{x}(t) \; = \; -\frac{\delta V(x)}{\delta x(t)} \; + \; \xi(t)}$$

with $\langle \xi(t) \rangle = 0$ and $\langle \xi(t)\xi(t') \rangle = 2k_B T\gamma \, \delta(t-t')$.

Brownian motion



First example of dynamics of an *open system*

The system : the Brownian particle

The bath : the liquid

Interaction : collisional or potential

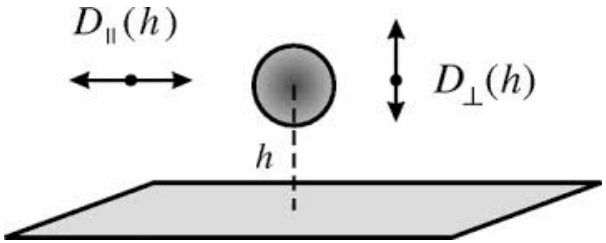
'Canonical setting'

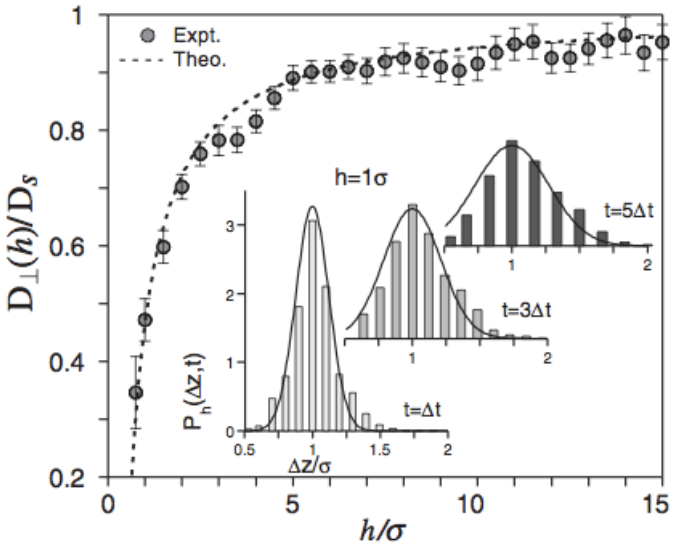
A few Brownian particles or tracers ● imbedded in, say, a molecular liquid.

Late XIX, early XX (Brown, Einstein, Langevin)

Interesting effects

Multiplicative noise

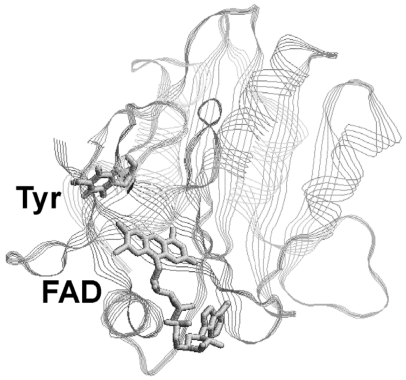


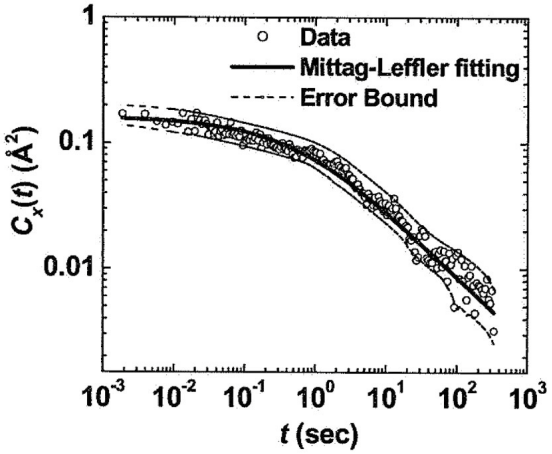


Varying diffusion constant

Carbajal-Tinoco et al. 07

Colored noise





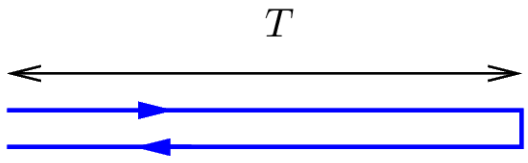
Non-exponential relaxation

Yang et al. 03

Formulation

Dissipative quantum dynamics

- **Path-integral Schwinger-Keldysh** formalism.



- Choose the system+reservoir initial density matrix at $t = 0$.

  Could be a factorized density operator

$$\boxed{\hat{\rho}(0) = \hat{\rho}_{syst}(0) \otimes \hat{\rho}_{env}(0)}$$

  or not.

- Integrate out the bath degrees of freedom

- Obtain an effective action $\boxed{S = S_{syst} + S_{influence}}$

- $\boxed{S_{influence}}$ is non-local in time.

Markov limit

in dissipative quantum physics ?

A very delicate question of time-scales and coupling constants.

t_{syst}, t_{env} and η.

Spohn 80, Gardiner 90s, Girvin - Les Houches 11

Search for a local differential equation, a master equation, for the reduced density operator

$$\mathrm{i}\hbar\, \mathrm{d}_t \hat{\rho}^{red}_{syst} = \underbrace{[\hat{H}_{syst}, \hat{\rho}^{red}_{syst}]}_{\text{Unitary}} + \underbrace{\hat{\mathcal{L}}(\hat{\rho}^{red}_{syst})}_{\text{Non-unitary evolution}}$$

Lindblatt operators

OK in quantum optics, quantum machines not in atomic physics, cond-mat

NB no closed Fokker-Planck eq. for a Langevin process with coloured noise.

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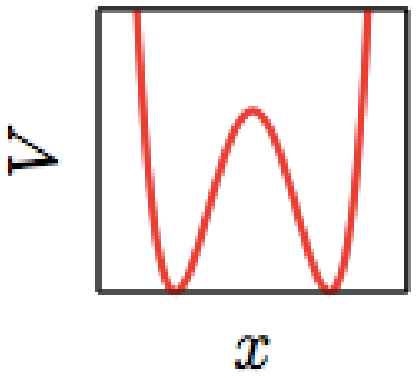
Quantum dynamics

Non-trivial effects under Ohmic dissipation $I(\omega) = \eta\omega$



$P_{tunn} \to 0$

Suppression of tunnelling or

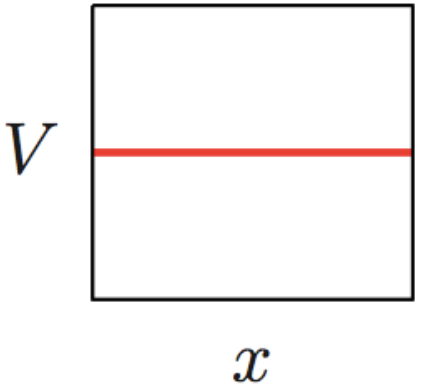
Localisation in a double well potential

at $k_B T = 0$ for $\eta > 1$

Bray & Moore 82, Leggett et al 87

Slowed-down diffusion

$$\langle \hat{x}^2(t) \rangle \to \begin{cases} \dfrac{2k_B T}{\eta}\, t & \text{Classical } k_B T \neq 0 \\ \dfrac{\hbar}{\pi\eta}\, \ln t & \text{Quantum } k_B T = 0 \end{cases}$$

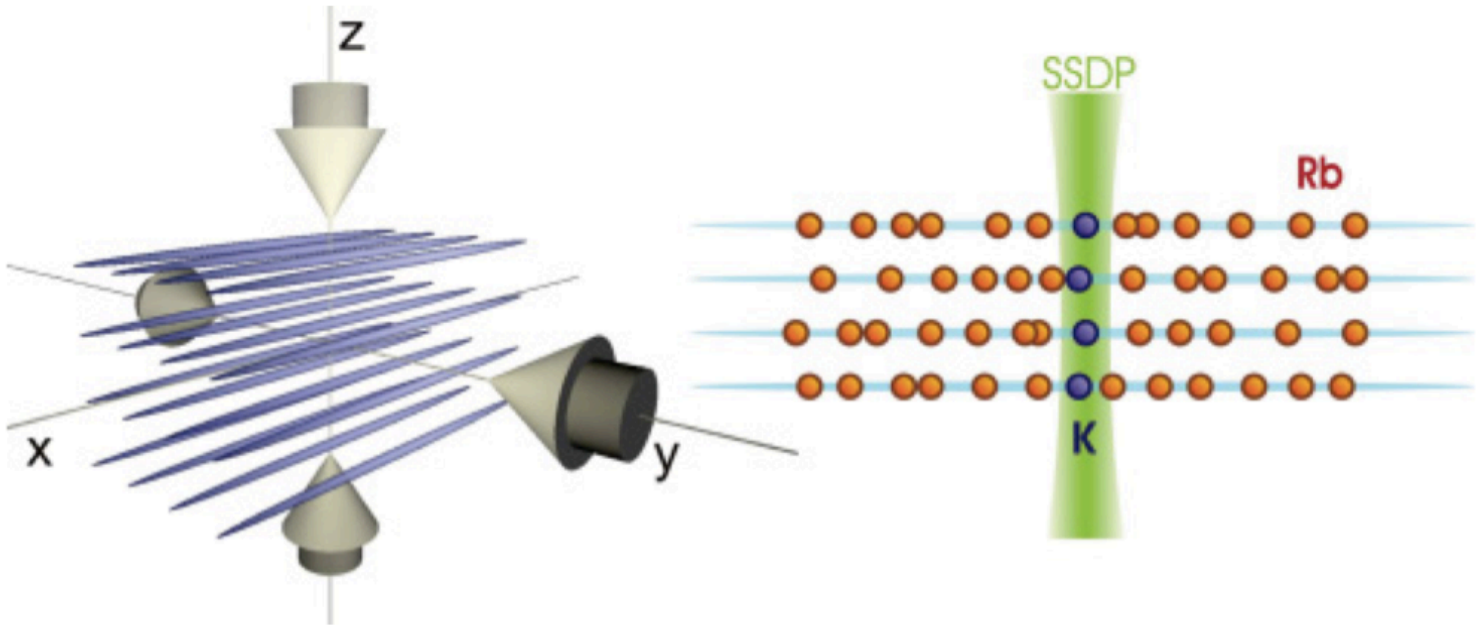


Schramm-Grabert 87

Other non-trivlal effects at $T \simeq 0$ or non-Ohmic, $I(\omega) \simeq \omega^{\alpha}$ baths.

A quantum impurity

in a one dimensional harmonic trap



K atom : the impurity (1.4 on average per tube) $T \simeq 350$ nK

Rb atoms : the bath (180 on average per tube) $\hbar\beta\sqrt{\kappa_0/m} \simeq 0.1$

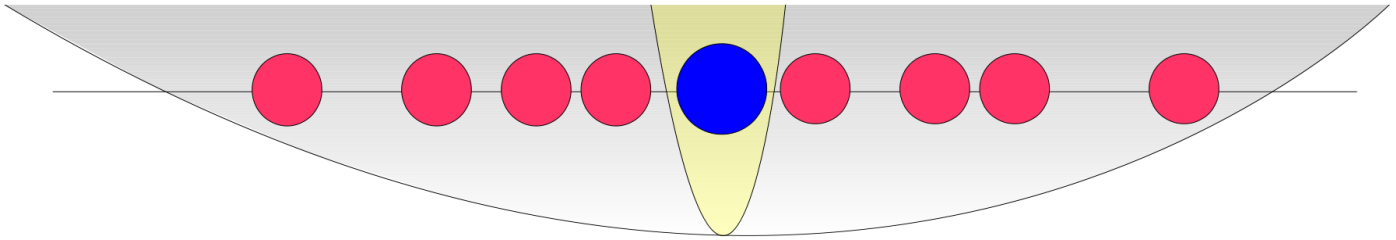
all confined in one dimensional tubes

Catani et al. 12

Experiment

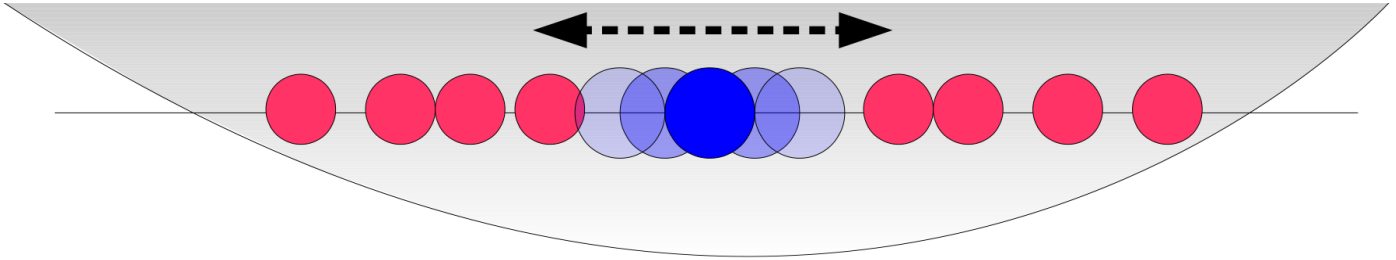
Sketch

Initially, the impurity is localized at the centre of the harmonic potential.



At $t = 0$, the impurity is released.

It subsequently undergoes quantum Brownian motion in the quasi $1d$ harmonic potential.



Protocol

A quench of the system

Initial equilibrium of the coupled system : $\boxed{\hat{\rho}(t_0) \propto e^{-\beta \hat{H}_i}}$

with $$\hat{H}_i = \hat{H}^i_{syst} + \hat{H}_{env} + \hat{H}_{int}$$

and $$\hat{H}^i_{syst} = \frac{1}{2m}\,\hat{p}^2 + \frac{1}{2}\kappa_0\,\hat{x}^2$$

At time $t_0 = 0$ the impurity is released, the laser blade is switched-off and the atom only feels the *wide* confining harmonic potential $\boxed{\kappa_0 \longrightarrow \kappa}$ as well as the bath made by the other species.

What are the subsequent dynamics of the particle ?

Use it to characterise the environment

Functional formalism

Influence functional

Feynman-Vernon 63, Caldeira-Leggett 84

Obtain the generating functional $\mathcal{Z}_{red}[\zeta] = \int \mathcal{D}\text{variables}\, e^{\frac{i}{\hbar}S[\zeta]}$

with the action given by $S = S_{\rm det} + S_{\rm init} + S_{\rm diss} + S_{\rm sour}[\zeta]$

where $S_{\rm det}$ characterises the deterministic evolution, $S_{\rm init}$ the initial density matrix, $S_{\rm diss}$ the dissipative and fluctuating effects due to the bath, and $S_{\rm sour}$ the terms containing the sources ζ.

Correlations between the particle and the bath at the initial time $t_0 = 0$ are taken into account *via* $\hat{\rho}(t_0)$ and then $S_{\rm init}$.

Once written in this way, the usual field-theoretical tools can be used. In particular, the minimal action path contains all information on the dynamics of quadratic theories.

The model

The bath in the experiment

The environment is made of interacting bosons in one dimension that we model as a Luttinger liquid.

The local density operator is $\hat{\varrho}(x) = \varrho_0 - \frac{1}{\pi}\frac{d}{dx}\hat{\phi}(x)$.
A canonical conjugate momentum-like operator $\hat{\Pi}(x)$ is identified.

One argues

$$\hat{H}_{env} = \frac{\hbar}{2\pi}\int dx \left[\frac{u}{K}\left(\frac{d\hat{\phi}(x)}{dx}\right)^2 + \frac{uK\pi^2}{\hbar^2}\hat{\Pi}^2(x)\right]$$

The sound velocity u and LL parameter K are determined by the microscopic parameters in the theory. For, e.g., the Lieb-Liniger model of bosons with contact potential $\hbar\omega_L \sum_{i<j}\delta(\hat{x}_i - \hat{x}_j)$, one finds $u(\gamma)K(\gamma) = h\pi\varrho_0/m_b$ and an expression for $K(\gamma)$ with $\gamma = m_b\omega_L/(\hbar\varrho_0)$.

$\gamma_{\rm exp} \simeq 1$ **Catani et al. 12**

t-DMRG of Bose-Hubbard model confirmation for $\hbar w$ small and $\hbar\omega_L$ large

Peotta et al. 13

The model

The interaction in the experiment

- The interaction is $\hat{H}_{\rm int} = \int dr dr'\, U(|r-r'|)\, \delta(\hat{x}-r')\, \hat{\rho}(r)$ with $\tilde{U}(p) = \hbar w e^{-p/p_c}$, quantized wave-vectors $p \to p_n = \pi\hbar n/L$, and L the 'length' of the tube.
- After a transformation to ladder operators $\hat{b}_n^\dagger,\ \hat{b}_n$ for the bath, the coupling $\hat{H}_{\rm int}$ becomes $\hat{H}_{\rm int} \propto \sum_{p_n} {\rm i}p_n \tilde{U}(p_n) e^{-\frac{{\rm i}p_n\hat{x}}{\hbar}} \hat{b}_{p_n} + {\rm h.c.}$
- One constructs the Schwinger-Keldysh path-integral for this problem.
- Low-energy expansion : $e^{\frac{{\rm i}p_n x_\pm}{\hbar}}$ to quadratic order, the action becomes the one of a particle coupled to a bath of harmonic oscillators with coupling constants determined by p_n. The spectral density $\mathbf{S}(\nu)/\nu$ is fixed. *A further approximation, $L \to \infty$, is to be lifted later.*

Impurity motion

Schwinger-Keldysh generating functional

The effective action has delayed quadratic interactions (both dissipative and noise effects) mediated by

$$\mathbf{\Sigma_B^K}(t-t') = 2\int_0^\infty d\nu\, \frac{\mathbf{S}(\nu)}{\nu}\, \cos[\nu(t-t')]$$

with the (Abraham-Lorentz) spectral density ($\hbar = 1$)

$$\mathbf{S}(\nu) \;=\; \frac{\pi}{2L}\sum_{p_n}\frac{K}{2\pi}\,|p_n|^3\,|\tilde{U}(p_n)|^2\,\delta(\nu - u|p_n|)$$

$$\rightarrow \quad \boxed{\eta\left(\frac{\nu}{\omega_c}\right)^3 e^{-\nu/\omega_c}} \qquad \text{continuum limit for} \quad L\to\infty$$

$\eta = Kw^2\omega_c^3/u^4$ with $\omega_c = up_c$ **Super-Ohmic diss.** $\boxed{\alpha = 3}$

K LL parameter, u LL sound velocity, $\hbar w$ strength of coupling to bath, ω_c high-freq. cut-off

The model

Schwinger-Keldysh generating functional

The action is quadratic in all the impurity variables.

The generating functional of all expectation values and correlation functions can be computed by the stationary phase method (exact in this case) as explained in, e.g.,

Grabert & Ingold's review

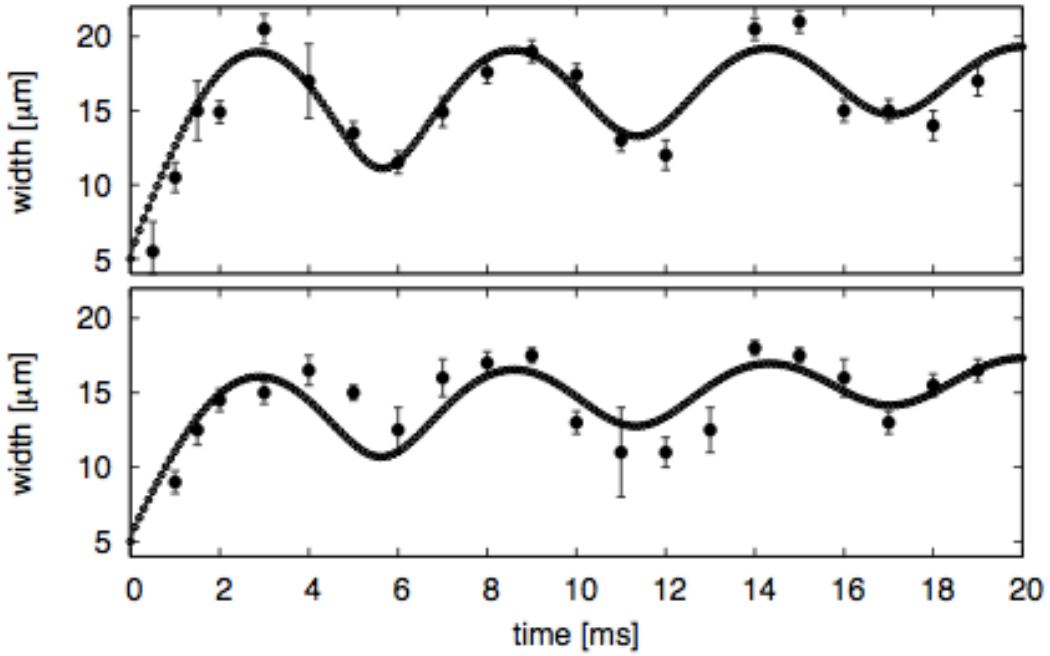
with some extra features : rôle of initial condition, quench in harmonic trap, non-Ohmic spectral density, possible interest in many-time correlation functions.

A polaron effect (mass renormalisation) and the potential renormalisation due to the fact that the bath itself is confined are also taken into account.

The equal-times correlation $\boxed{C_x(t,t) = \langle \hat{x}^2(t) \rangle}$ is thus calculated.

Breathing mode

Theory vs. experiment



$$\eta = w/\omega_L = 1$$

$$\eta = w/\omega_L = 4$$

Dynamics with m^* and κ^*, interpolation to $\lim_{t\to\infty}\langle\hat{x}^2(t)\rangle \to k_BT/\kappa$:

$$\langle\hat{x}^2(t)\rangle = \frac{\hbar^2\kappa_0}{4k_BT}\mathcal{R}(t) - \frac{\kappa^*}{k_BT}\mathcal{C}_{eq}^2(t) + \frac{k_BT}{\kappa^*} + \left(1-e^{-\Gamma t}\right)\left(\frac{k_BT}{\kappa} - \frac{k_BT}{\kappa^*}\right)$$

Bonart & LFC EPL 13

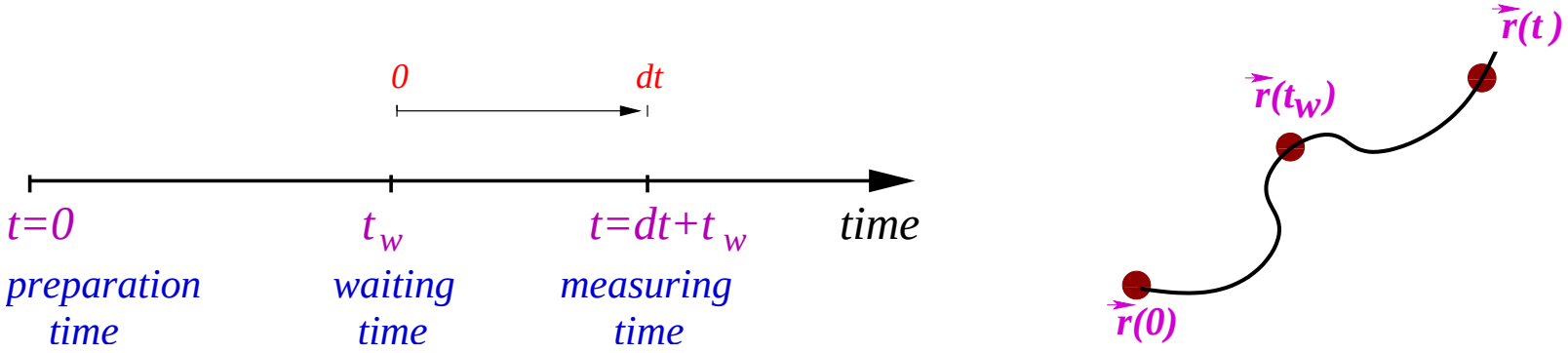
Plan

Quantum fluctuations

- **Canonical equilibrium.**
  - Classical disordered models & optimisation problems.
  - Quantum disordered models & optimisation problems.
  - The bath. Effects on equilibrium phase diagrams.
- **Dynamics.**
  - Closed systems and questions on equilibration.
  - Open systems, Markov vs. non-Markov dynamics.
  - A single dissipative quantum particle.
  - **Quantum macroscopic dissipative systems.**

Classical dynamics

Two-time correlation



t_w not necessarily longer than t_{eq}.

Correlations

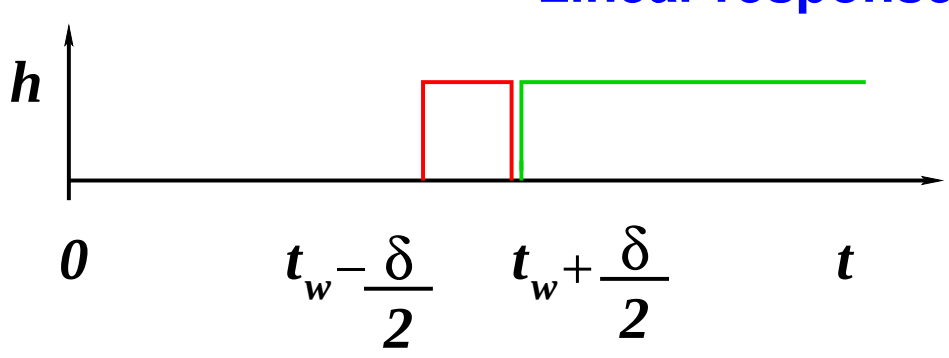
The two-time correlation between $A(\vec{r}(t))$ and $B(\vec{r}(t_w))$ is

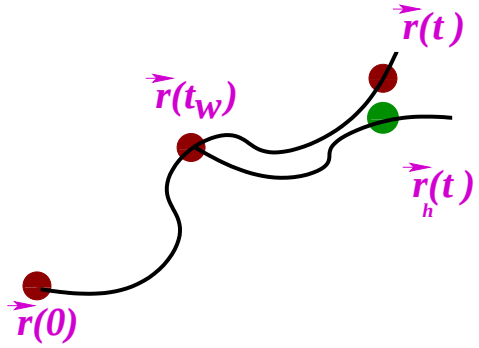
$$C_{AB}(t, t_w) \equiv \langle\, A(\vec{r}(t))B(\vec{r}(t_w))\,\rangle$$

the average is over realizations of the stochastic dynamics (random numbers in a MC simulation, thermal noise in Langevin dynamics, etc.)

Classical dynamics

Linear response





The perturbation couples linearly to the observable $E \rightarrow E - hB(\{\vec{r}_i\})$

The linear instantaneous response of another observable $A(\{\vec{r}_i\})$ is

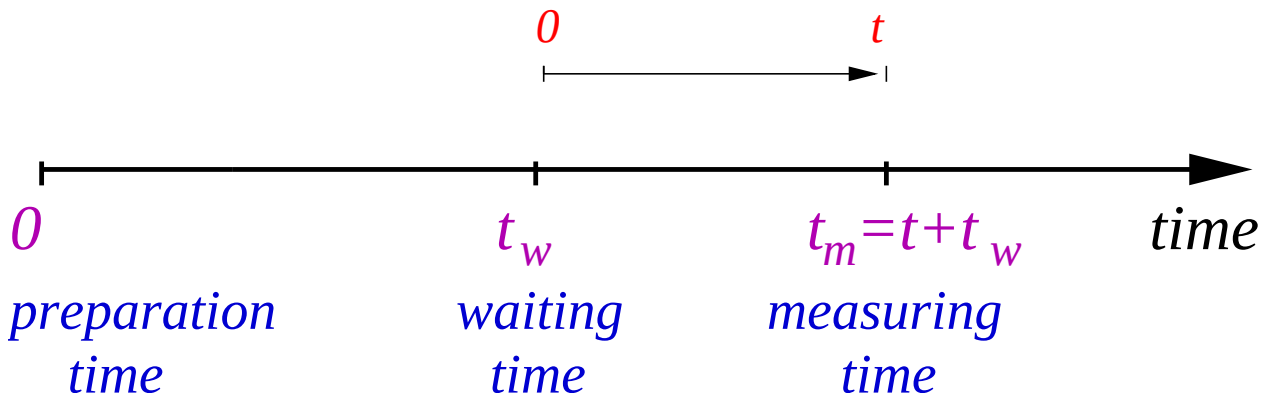
$$\mathbf{R_{AB}(t, t_w)} \equiv \left\langle \left. \frac{\delta A(\{\vec{r}_i\})(t)}{\delta h(t_w)} \right|_{h=0} \right\rangle$$

The linear integrated response or dc susceptibility is

$$\chi_{\mathbf{AB}}(\mathbf{t}, \mathbf{t_w}) \equiv \int_{t_w}^{t} dt' \, R_{AB}(t, t')$$

Real-time quantum dynamics

Two-time observables



Correlation

$$C(t+t_w,t_w) \equiv \langle [\hat{\mathcal{O}}(t+t_w),\hat{\mathcal{O}}(t_w)]_+ \rangle$$

Linear response

$$R(t+t_w,t_w) \equiv \left. \frac{\delta \langle \hat{\mathcal{O}}(t+t_w) \rangle}{\delta h(t_w)} \right|_{h=0} = \langle [\hat{\mathcal{O}}(t+t_w),\hat{\mathcal{O}}(t_w)]_- \rangle$$

Real-time dynamics

in equilibrium

If after τ_{eq} the system is in equilibrium with its environment :

- One-time quantities reach their equilibrium values,

$$\langle \hat{A}(t) \rangle \;\rightarrow\; \langle \hat{A} \rangle$$

- All time-dependent correlations are stationary,

$$\langle \hat{A}(t_1)\hat{A}(t_2)\cdots\hat{A}(t_n)\,\rangle = \langle\, \hat{A}(t_1+\Delta)\hat{A}(t_2+\Delta)\cdots\hat{A}(t_n+\Delta)\,\rangle$$

for any number of observables, n, and time-delay, Δ.

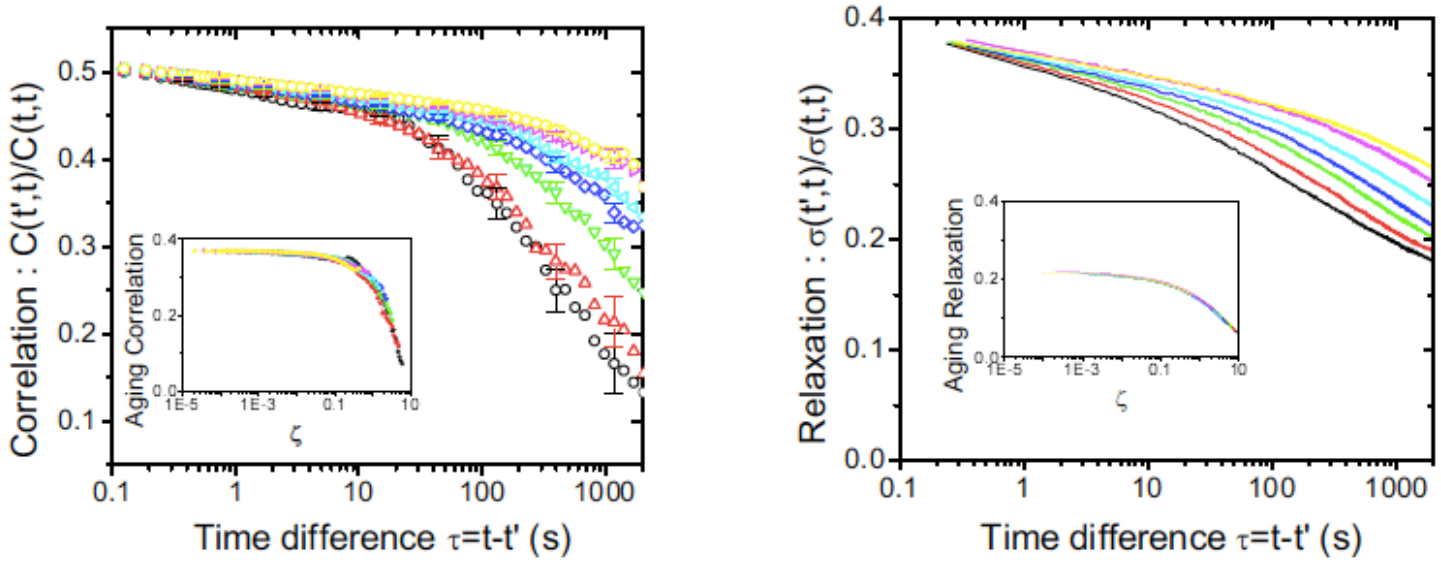
In particular, $C(t+t_w, t_w) = C(t)$.

Classical glassy systems do not satisfy the second property and are out of equilibrium.

Real-time dynamics

out of equilibrium

In classical glassy systems $\tau_{eq} \gg \tau_{exp}$ and the system does not equilibrate with its environment ; it ages



Hérisson & Ocio 01

Quantum glassy systems ?

Spherical model

A particle in a random potential

$$\hat{H}_{syst} = \hat{H}_J(\{\hat{S}\}) + \sum_i \frac{\hat{\Pi}_i^2}{2M}$$

Potential energy — Kinetic energy

$$[\hat{\Pi}_i, \hat{S}_j] = -i\hbar\delta_{ij}$$ Canonical commutation rules

$$\sum_i \langle \hat{S}_i^2 \rangle = N$$ Spherical constraint

$$\Gamma \equiv \hbar^2/(JM)$$ Strength of quantum fluctuations

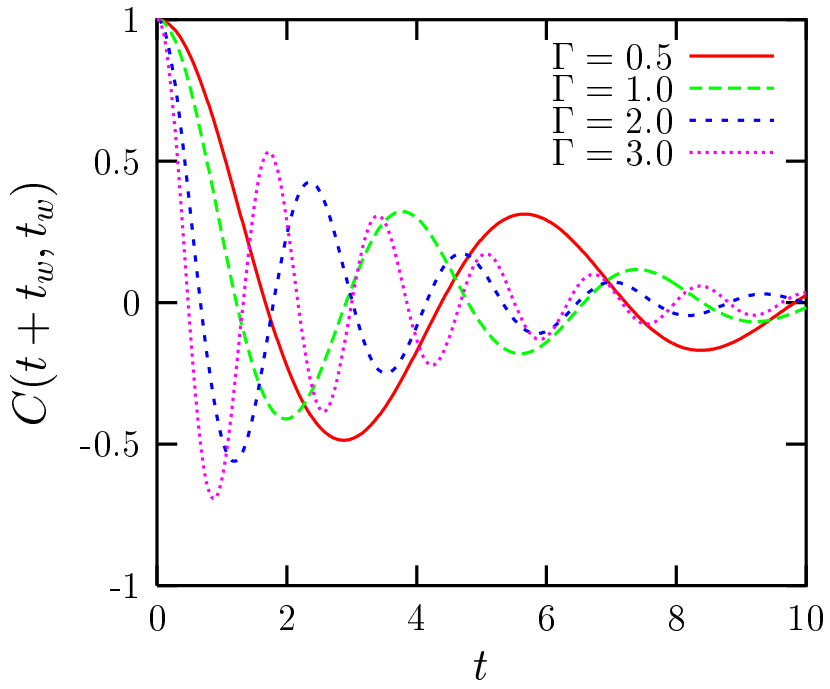
Coupled to a bath of quantum harmonic oscillators.

Results for the Ohmic case.

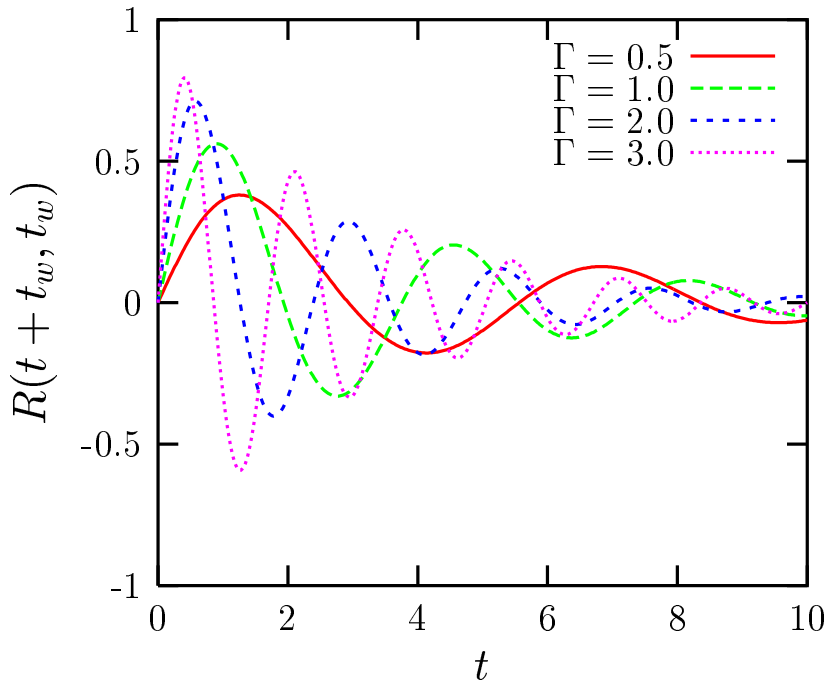
Real-time dynamics

Paramagnetic phase

Symmetric correlation



Linear response



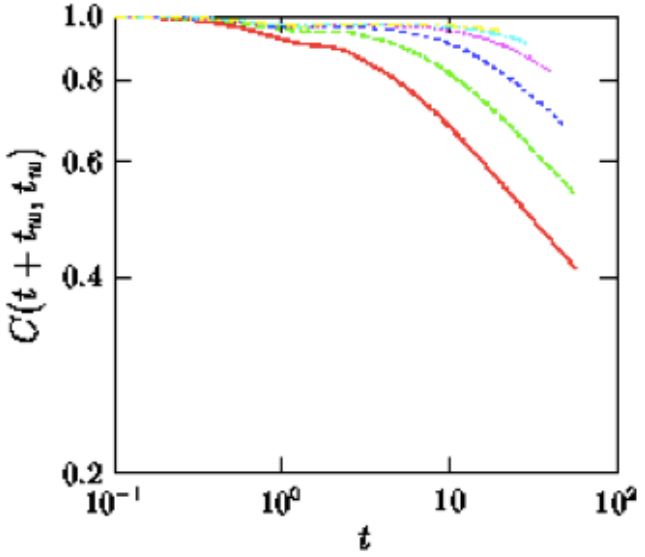
Dependence on the quantum parameter Γ

LFC & Lozano 98-99

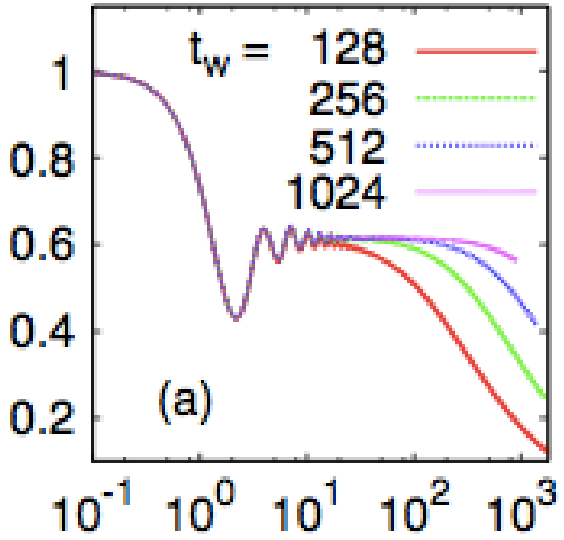
Real-time dynamics

Glassy or coarsening phases

Symmetric correlation



LFC & Lozano 98-99

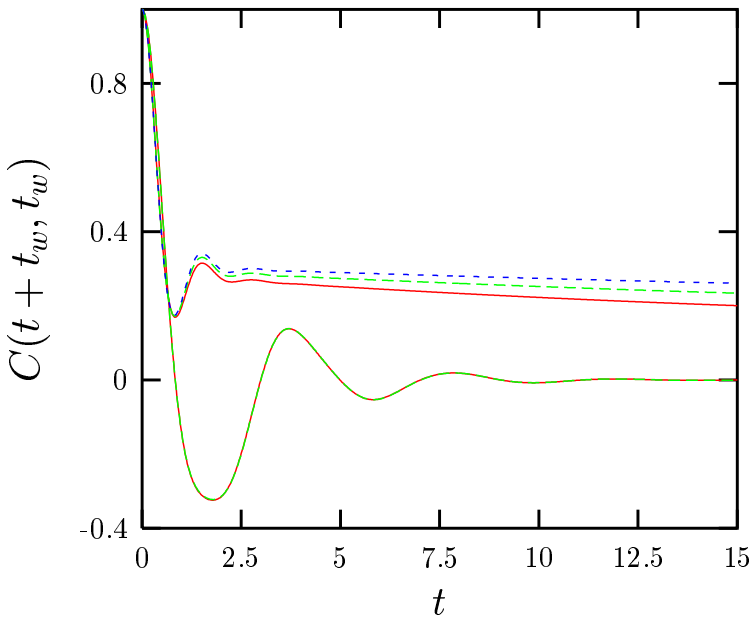


Aron, Biroli & LFC 09

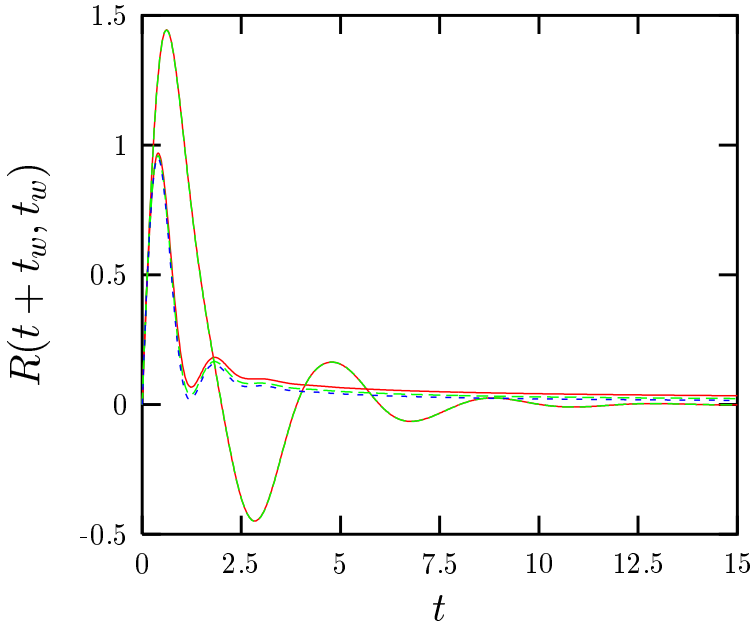
Real-time dynamics

Dependence on the coupling to the bath

Symmetric correlation



Linear response



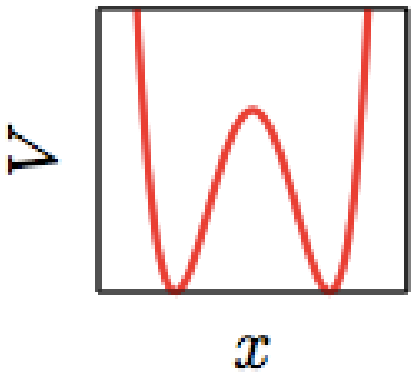
Comparison between $\eta = 0.2$ (PM) and $\eta = 1$ (SG)

LFC, Grempel, Lozano, Lozza & da Silva Santos 02

Localization

the Caldeira-Leggett problem

A quantum particle in a double-well potential coupled to a bath of quantum harmonic oscillators in equilibrium at $T = 0$.



Quantum tunneling for $0 < \eta < 1/2$

'Classical tunneling' for $1/2 < \eta < 1$

Localization in initial well for $1 < \eta$

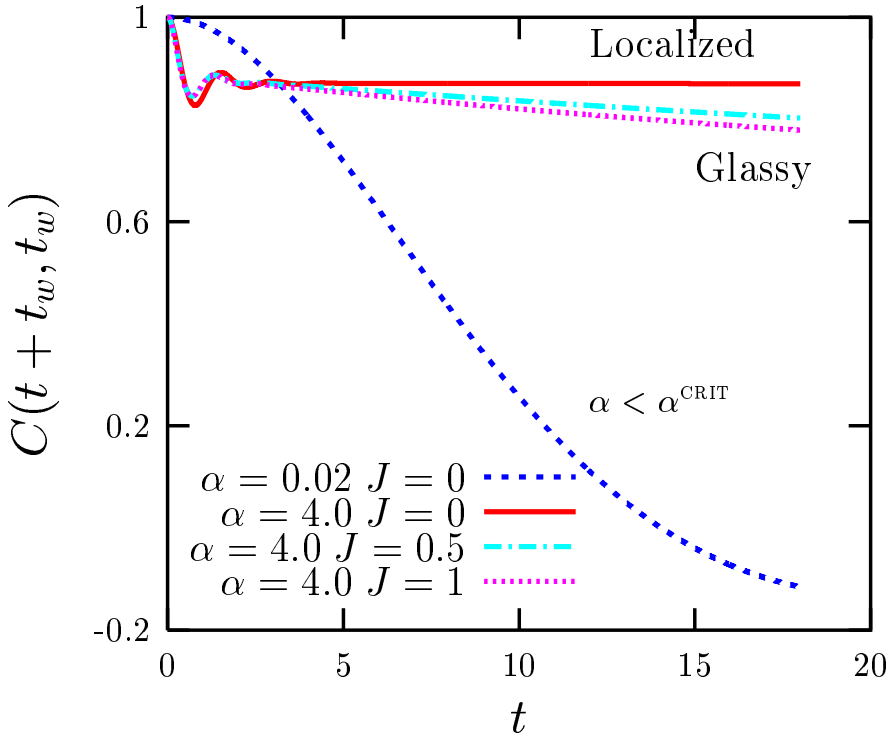
Bray & Moore 82

The same behaviour for a dissipative SU(2) spin in a transverse field

Leggett et al. 87

Real-time dynamics

Interactions against real-space localization



LFC, Grempel, Lozano, Lozza & da Silva Santos 02

Notation: α is the coupling to the bath here, that we called η in the rest of the talk

Real-time dynamics

Fluctuation-dissipation theorem in classical glassy systems

Focus on the time-integrated linear response

$$\chi(t+t_w,t_w) \equiv \int_{t_w}^{t+t_w} dt'\, R(t+t_w,t')$$

In equilibrium : $\chi(t+t_w,t_w) = \frac{1}{T}[C(t_w,t_w) - C(t+t_w,t_w)]$

In glasses : breakdown of the above FDT.

$$\chi(t+t_w,t_w) = \text{cst} - \frac{1}{T_{\texttt{eff}}} C(t+t_w,t_w)$$

in the long t_w and $t \gg t_w$ limit.

LFC & Kurchan 93

Real-time dynamics

Fluctuation-dissipation theorem in quantum glassy systems

The equilibrium FDT

$$R(t+t_w,t_w) = \frac{i}{\hbar}\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\, e^{-i\omega t}\, \tanh\left(\frac{\beta\hbar\omega}{2}\right) C(\omega,t_w)$$

becomes

$$\chi(t+t_w,t_w) \approx \text{cst} - \frac{1}{T_{\texttt{eff}}} C(t+t_w,t_w) \qquad t \gg t_w$$

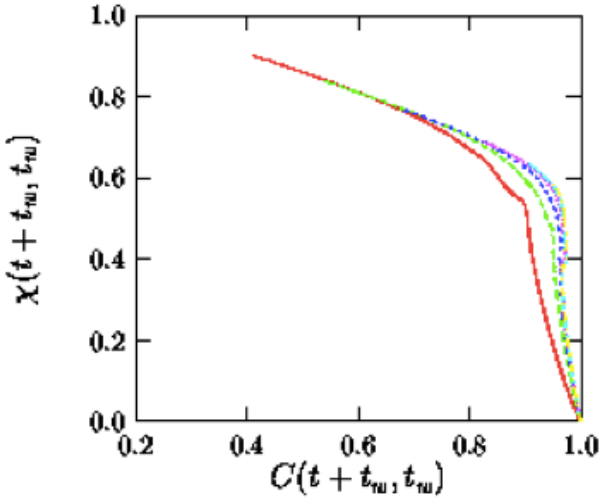
if the integral is dominated by $\omega t \ll 1$ and $T \to T_{\texttt{eff}}$ such that $\beta_{\texttt{eff}}\hbar\omega \to 0$.

LFC & G. Lozano 98-99

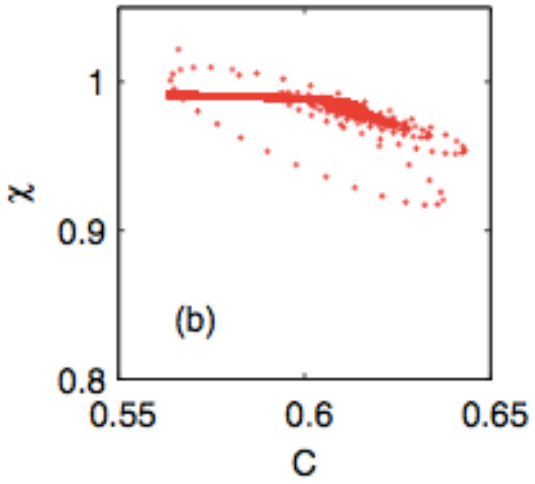
Real-time dynamics

Fluctuation-dissipation theorem in quantum glassy & coarsening systems

Parametric plot $\chi(C)$.



LFC & G. Lozano 98-99

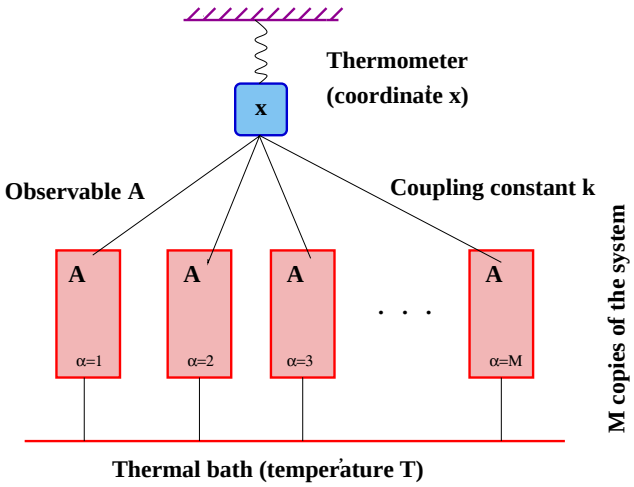


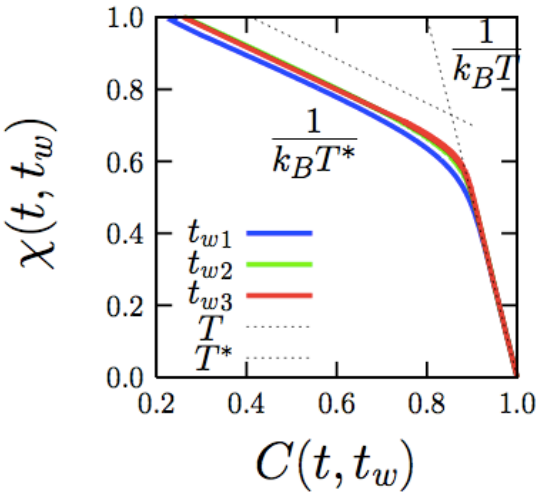
Aron, Biroli & LFC 09

FDT & effective temperature

Can one interpret the slope as a temperature ?

Yes, in classical glassy mean-field models **LFC, Kurchan, Peliti 97**





(1) Measurement with a **thermometer** with

- Short internal time scale τ_0, fast dynamics is tested and T is recorded.
- Long internal time scale τ_0, slow dynamics is tested and T^* is recorded.

(2) **Partial equilibration** (3) **Direction of heat-flow**

Quantum mechanically ?

Plan

Quantum fluctuations

- **Canonical equilibrium.**
  - Classical disordered models & optimisation problems.
  - Quantum disordered models & optimisation problems.
  - The bath. Effects on equilibrium phase diagrams.
- **Dynamics.**
  - Closed systems and questions on equilibration.
  - Open systems, Markov vs. non-Markov dynamics.
  - A single dissipative quantum particle.
  - Quantum macroscopic dissipative systems.

Quantum quench

Setting

- Take a quantum closed system and suddenly change a parameter.
- *E.g.*, the quantum Ising chain

$$H_{\Gamma_0} = -\sum \sigma_i^x \sigma_{i+1}^x + \Gamma_0 \sum \sigma_i^z$$

Transverse field $\Gamma_0 \to \Gamma$

Rieger & Iglói 90s

- **Questions :**

  Does the system reach a thermal equilibrium measure ?

  Under which conditions ?

  (*e.g.*, integrable vs. non-integrable systems ; sub *vs.* critical quenches)

  Calabrese & Cardy ; Rossini et al., etc.

  Is there some kind of emerging effective bath ?

Quantum quench

Previous studies

- Definition of T_e from time-independent observables :

  $\langle H_\Gamma \rangle_{\Gamma_0} = \langle H_\Gamma \rangle_{T_e}$

  $\langle M^x_\Gamma \rangle_{\Gamma_0} = \langle M^x_\Gamma \rangle_{T_e}$, *etc.*

  (We know these can be very misleading in glassy systems.)

- Definition of T_e from the functional form of correlation functions :

  $C(r) \equiv \langle \sigma^x_i(t)\sigma^x_j(t) \rangle_{\Gamma_0}$ *vs.* $C_{eq}(r) \equiv \langle \sigma^x_i(t)\sigma^x_j(t) \rangle_{T_e}$, *etc.*

  (Again, they can be misleading.)

- Proposal : put qFDTs to the test to check whether $T_{\rm eff}$ exists.

Fluctuation-dissipation theorem

Classical dynamics in equilibrium

The classical FDT for a stationary system with $\tau \equiv t - t_w$ reads

$$\boxed{\chi(\tau) = \int_0^\tau dt'\, R(t') = -\beta[C(\tau) - C(0)] = \beta[1 - C(\tau)]}$$

choosing $C(0) = 1$.

Linear relation between χ and C

Quantum dynamics in equilibrium

The quantum FDT reads

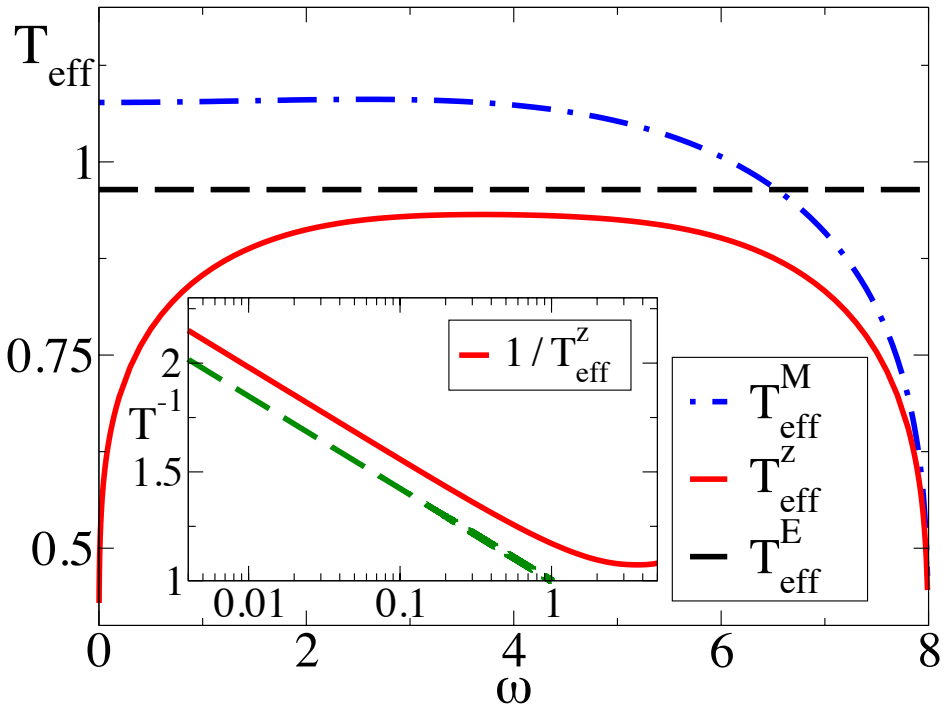
$$\boxed{\chi(\tau) = \int_0^\tau d\tau'\, R(\tau') = \int_0^\tau d\tau' \int_{-\infty}^{\infty} \frac{i d\omega}{\pi\hbar}\, e^{-i\omega\tau'}\, \tanh\left(\frac{\beta\hbar\omega}{2}\right) C(\omega)}$$

Complicated relation between χ and C

Quantum quench

$T_{\rm eff}$ **from transverse spin** $\hat{\sigma}_i^z$ **and** $\hat{M} = N^{-1}\sum_i \hat{\sigma}_i^z$ **qFDTs ?**

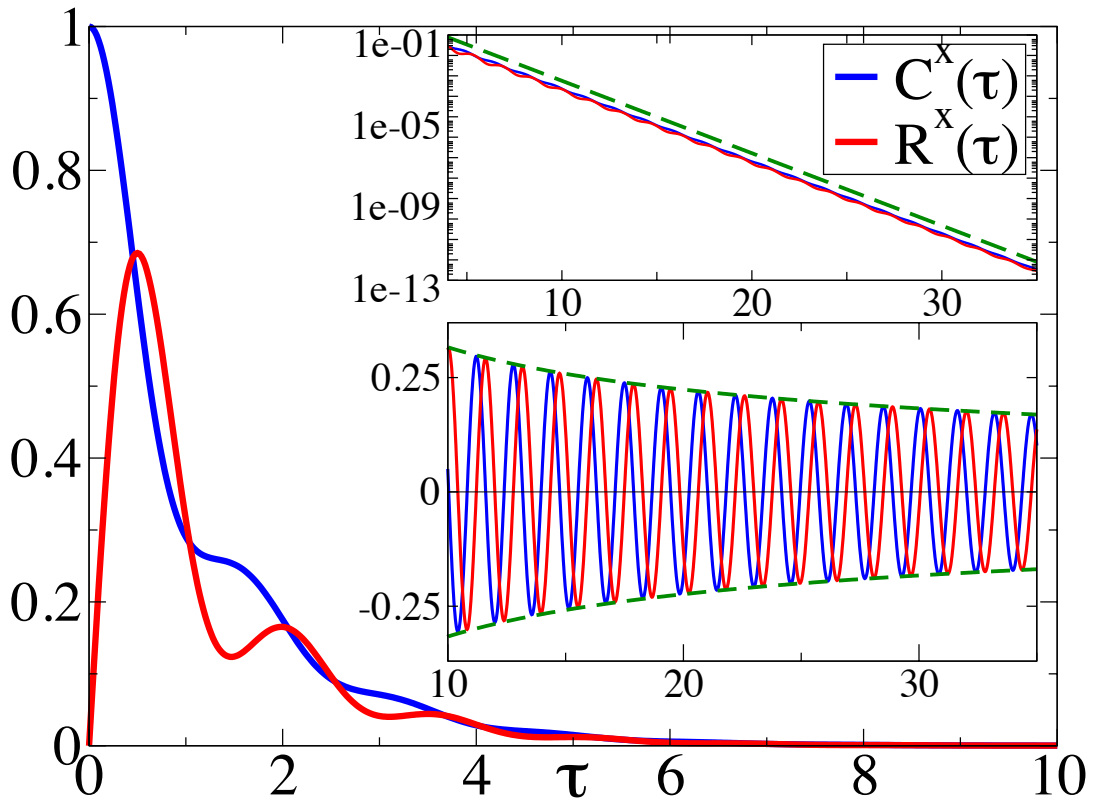
$$\mathrm{Im}R^z(\omega) = \tanh\left(\frac{\beta^z_{\rm eff}(\omega)\omega\hbar}{2}\right) C^z_+(\omega)$$



But $\beta^z_{\rm eff}(\omega) \neq \beta^M_{\rm eff}(\omega) \neq$ ct

Quantum quench

$T_{\rm eff}$ from longitudinal spin σ_i^x qFDT ?



$$C_+^x(\tau) \simeq A_C e^{-\tau/\tau_C}[1 - a_C \tau^{-2} \sin(4\tau + \phi_C)]$$
$$R^x(\tau) \simeq A_R e^{-\tau/\tau_R}[1 - a_R \tau^{-2} \sin(4\tau + \phi_R)]$$

Quantum quench

$T_{\rm eff}$ from longitudinal spin σ_i^x qFDT ?

For sufficiently long-times such that one drops the power-law correction

$$\boxed{-\beta^x_{\rm eff} \simeq \frac{R^x(\tau)}{d_\tau C^x_+(\tau)} \simeq -\frac{\tau_C A_R}{A_C}}$$

A constant consistent with a classical limit but

$$\boxed{T^x_{\rm eff}(\Gamma_0) \neq T_e(\Gamma_0)}$$

A complete study in the full time and frequency domains confirms that $T^x_{\rm eff}(\Gamma_0) \neq T^z_{\rm eff}(\Gamma_0) \neq T_e(\Gamma_0)$ (though the values are close).

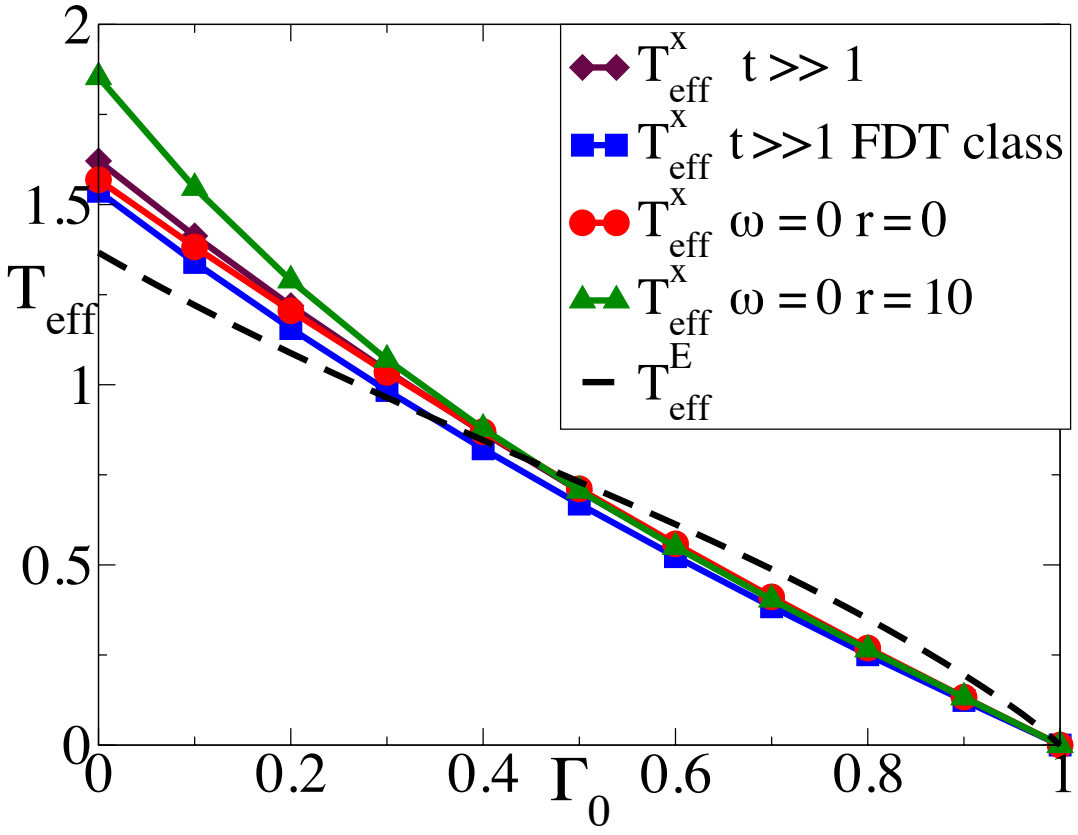
Fluctuation-dissipation relations as a probe to test thermal equilibration

No equilibration for generic Γ_0

Quantum quench

No $T_{\rm eff}$ from FDT

A quantum quench $\Gamma_0 \to \Gamma_c = 1$ of the **isolated Ising chain**



Foini, LFC & Gambassi 11, 13

More in the talk by **Foini**

Summary

Dynamics of quantum disordered systems

- We very briefly mentioned the Schwinger-Keldysh functional formalism & the delayed interactions induced by the coupling to a bath.
- the Markov limit & Lindblatt equation.
- An experimental realisation of quantum Brownian motion & its modelling.
- The real-time dynamics of dissipative quantum p-spin models.
- Quantum ageing and FDTs
- We used FDT ideas to check for equilibration in closed quantum systems.