À quoi sert-elle la physique statistique ?

Leticia F. Cugliandolo

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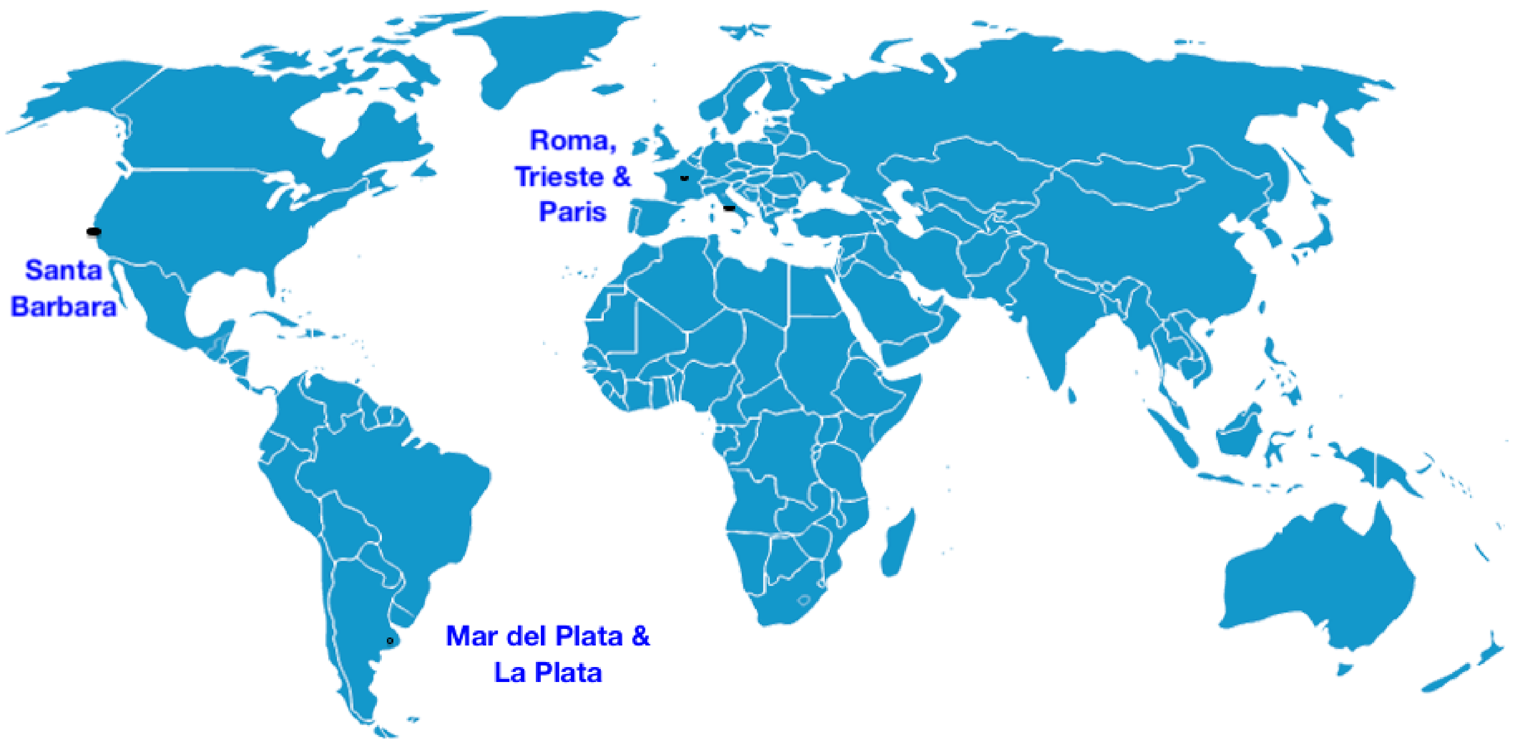
Sorbonne, 2023

- Career
    - Studies
    - Research
    - Mentoring
    - Responsibilities
- Physics

World

International career



Argentina

Universidad Nacional de Mar del Plata

1st & 2nd year

Electronic Engineering

3er año

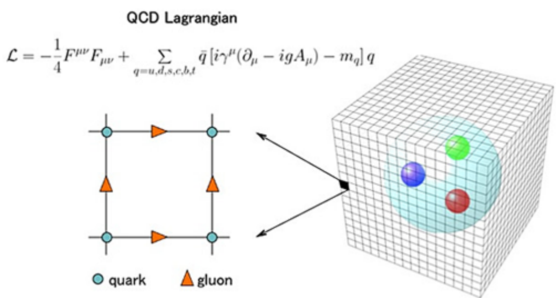
Physics

Argentina

Universidad Nacional de La Plata, Physics Department

Argentina

Theoretical physics



4th & 5ft year

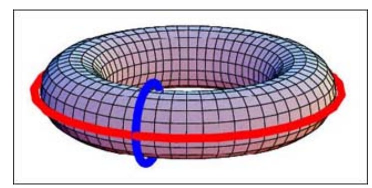
Licenciatura (Master) in Physics

Zero-modes on the lattice : the vortex fermion system

Doctorado (PhD) in Physics

UNLP 1988-1991

Topological quantum field theories



Supervisors Fidel A. Schaposnik (UNLP) & Eduardo Fradkin (University of Urbana-Champaign, USA)

Subjects related to the 2016 Nobel Prize (Thouless, Kosterlitz & Haldane) & Fields Medal 1990 (Witten)

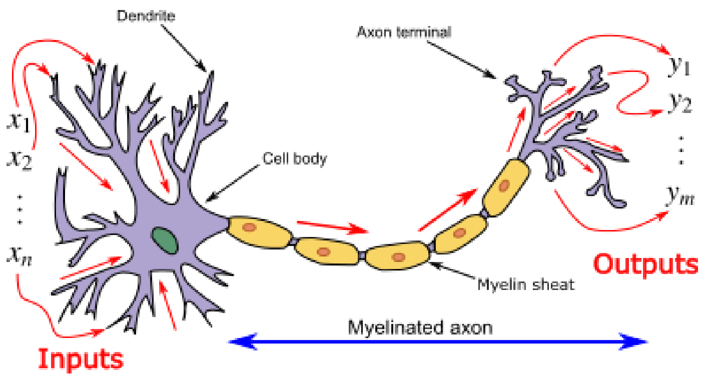
Italia

Università di Roma I ‘La Sapienza’

SAPIENZA
UNIVERSITÀ DI ROMA

Roma

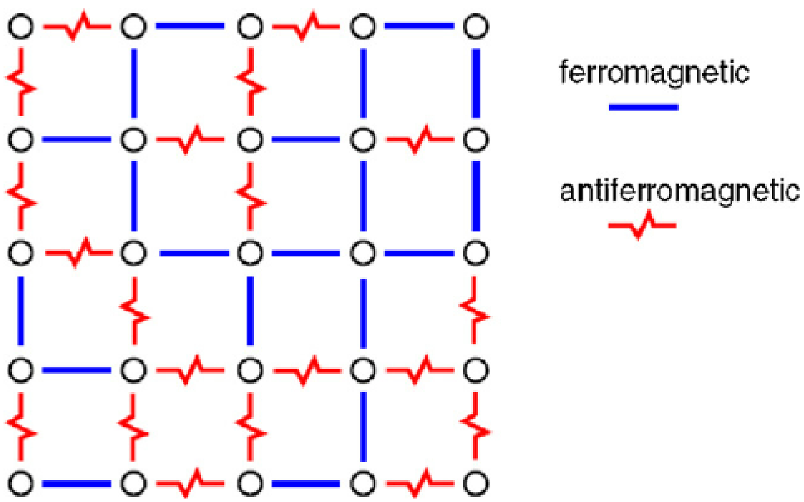
Post-doc : from Field Theory to Statistical Physics



Neural networks (91-92)

Spin glasses (92-94)

$$H = -\sum_{i \neq j} J_{ij} s_i s_j$$



s_i up-down spins ↑↓ or

neuron firing/quiescent

J_{ij} magnetic coupling or

Hebb memory rule

More later

Subjects related to the 2021 Nobel Prize (Giorgio Parisi)

Paris

2nd post-doc and positions

1994-1996 CEA Saclay

Glass theory

1997-2003 ENS Paris

Quantum disordered systems

2003-present Sorbonne Université

Active matter

Frustrated magnetism

Quantum out of equilibrium systems

Formalism etc. etc. etc.

Long term visits to University of California at Santa Barbara, Harvard, ICTP Trieste, The University of Cambridge, Universidad de Buenos Aires

Students/post-docs

Still working with

G. Semerjian	Assist. Prof. ENS
C. DaSilva	Private sector PT
D. Loi	Informatics IT
A. Sicilia	Blogger-periodista
C. Aron	CNRS at ENS
A. Jelić	Associate ICTP
E. Katzav	Prof. Jerusalem
D. Levis	Prof. Barcelona
L. Foini	CNRS Saclay
J. Bonart	Finance UK
H. Ricateau	Informatics FR
A. Tartaglia	Informatics IT
M. Casiulis	post-doc Israel
D. Barbier	post-doc Suisse

X. Turkeshi
G. Alfaro-Miranda
R. Agrawal

A. Kolton — Academia
S. Bustingorry
P. Guruciaga
JL Iguain
H. Lozza
N. Nessi

MP Loureiro
Z. G.-Arenas

M. Kennett ↦
P. Charbonneau
A. Velenich

A. Suma
P. Digregorio
I. Petrelli
O. Mazzarisi
C. Caporusso

Many in academia & some in private sector: mainly journalism, finance & info

Students/post-docs

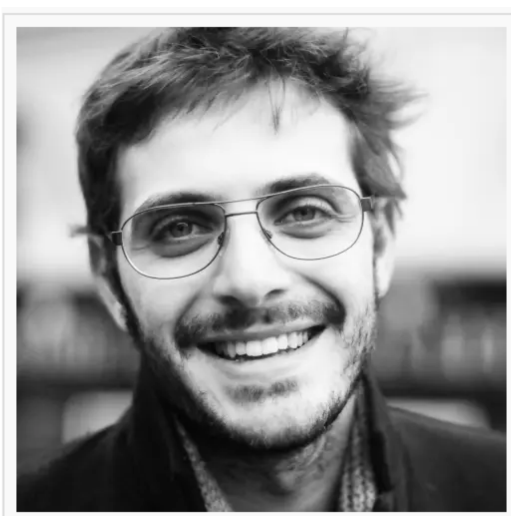
Parcours

A. Sicilia Adventurous journalist

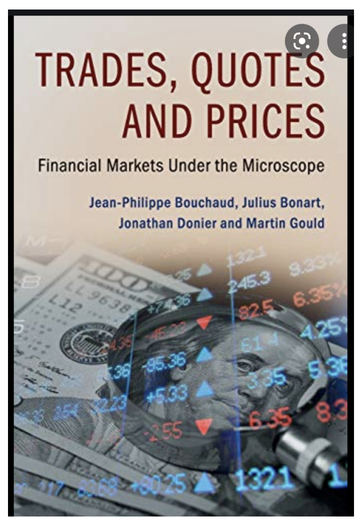
C. Aron CR CNRS at ENS

M. Kasiulis Lutetian Project NYU

D. Levis Prof. at Barcelona



J. Bonart Citadel



L. Foini CR CNRS at IPhT CEA Saclay

Les Houches

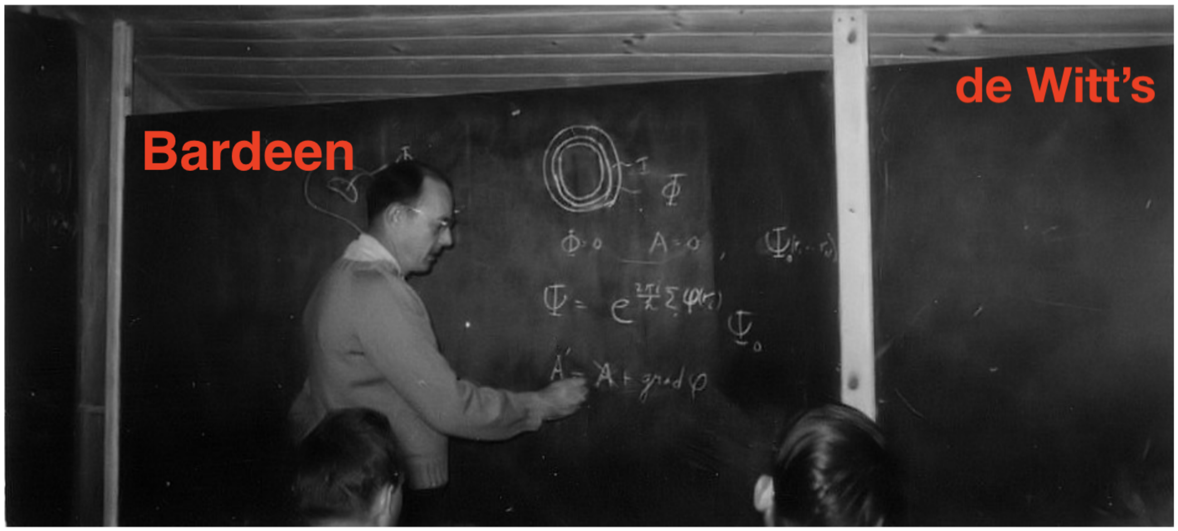
ÉCOLE DE PHYSIQUE
des HOUCHES

UNIVERSITÉ
Grenoble
Alpes



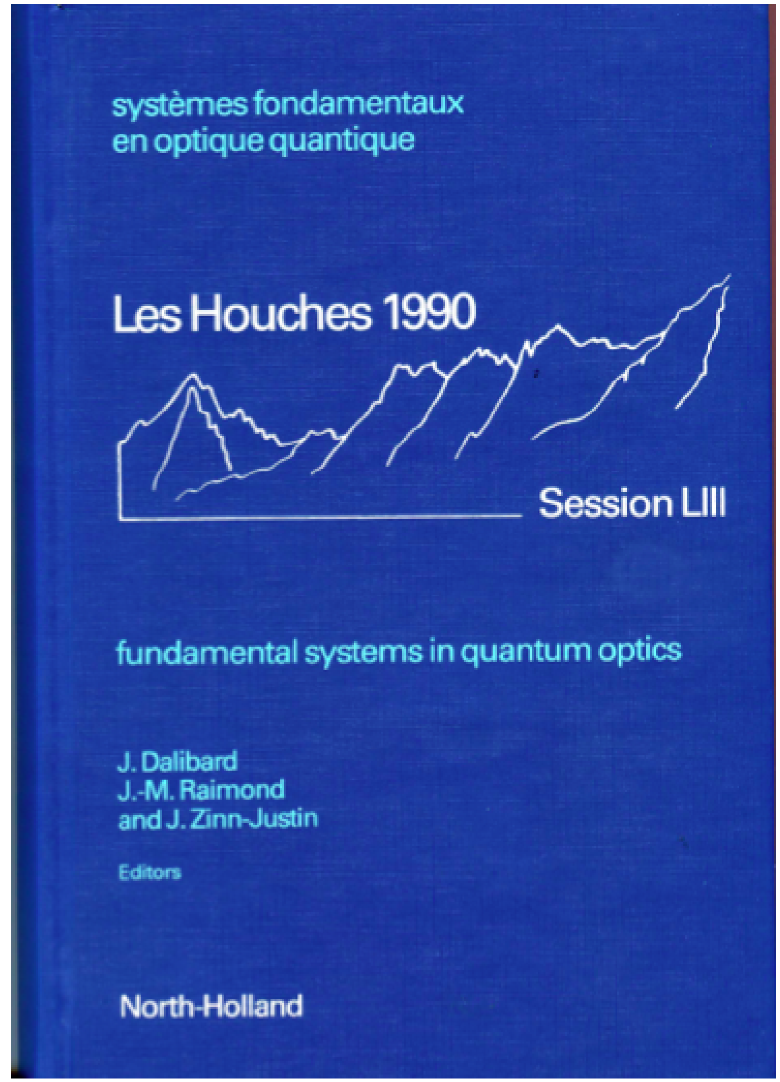

Les Houches

Ecole de Physique des Houches



ÉCOLE DE PHYSIQUE
des HOUCHES

UNIVERSITÉ
Grenoble
Alpes



Statistical Physics

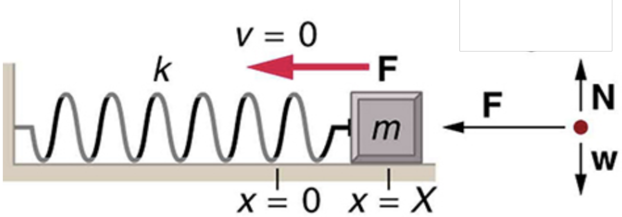
and disordered systems

Classical mechanics

Newton - Hamilton - Lagrange

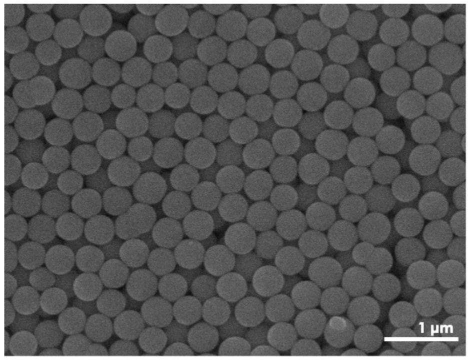
Newton (Physics 101) $m\vec{a} = \vec{F}$

- Solve simple problems especially for gradient forces $\vec{F}(\vec{x}) = -\vec{\nabla}V(\vec{x})$ *e.g.*



- What happens if instead of one single particle there are many in interaction ?

$$\dot{\vec{p}}_i \equiv m\vec{a}_i = \vec{F}_i(\{\vec{x}_j\}) \qquad i, j = 1, \dots, N \gg 1$$



Very hard to solve.

Approximations & numerics

Collective phenomena

Interest in **macroscopic**

Statistical physics

Advantage

No need to solve the Newton dynamic equations!

Under the *ergodic hypothesis*, after some *equilibration time* $t_{\rm eq}$, *macroscopic observables* can be, on average, obtained with a *static* calculation, as an average over all configurations in phase space weighted with a probability distribution function $\boldsymbol{P(\{\vec{p}_i, \vec{x}_i\})}$

$$\langle A \rangle = \int \prod_i d\vec{p}_i d\vec{x}_i \, \boldsymbol{P(\{\vec{p}_i, \vec{x}_i\})} \, A(\{\vec{p}_i, \vec{x}_i\})$$

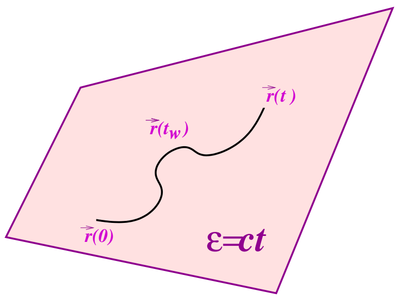
$\langle A \rangle$ should coincide with $\overline{A} \equiv \lim_{\tau \to \infty} \frac{1}{\tau} \int_{t_{\rm eq}}^{t_{\rm eq}+\tau} dt' \, A(\{\vec{p}_i(t'), \vec{x}_i(t')\})$

the *time average* typically measured experimentally

Boltzmann, late XIX

Statistical physics

Ensembles : recipes for $P(\{\vec{p}_i, \vec{x}_i\})$ according to circumstances



Microcanonical distribution

$$\boxed{P(\{\vec{p}_i, \vec{x}_i\}) \propto \delta(\mathcal{H}(\{\vec{p}_i, \vec{x}_i\}) - \mathcal{E})}$$

Flat probability density

Isolated system

$$\mathcal{E} = \mathcal{H}(\{\vec{p}_i, \vec{x}_i\}) = ct$$

$$S_{\mathcal{E}} = k_B \ln g(\mathcal{E})$$

Entropy

$$\beta \equiv \frac{1}{k_B T} = \left.\frac{\partial S_{\mathcal{E}}}{\partial \mathcal{E}}\right|_{\mathcal{E}}$$

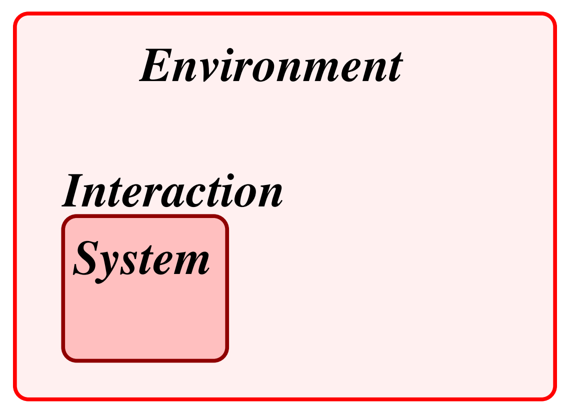
Temperature

$$\mathcal{E} = \mathcal{E}_{syst} + \mathcal{E}_{env} + \mathcal{E}_{int}$$

Neglect $\mathcal{E}_{int}$ (short-range interact.)

$$\mathcal{E}_{syst} \ll \mathcal{E}_{env} \qquad \beta = \frac{\partial S_{\mathcal{E}_{env}}}{\partial \mathcal{E}_{env}}$$

$$\boxed{P(\{\vec{p}_i, \vec{x}_i\}) \propto e^{-\beta \mathcal{H}(\{\vec{p}_i, \vec{x}_i\})}}$$



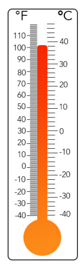
Canonical ensemble

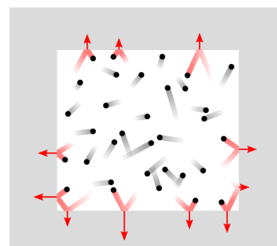
Statistical physics

Accomplishments

- Microscopic definition & derivation of **thermodynamic** concepts

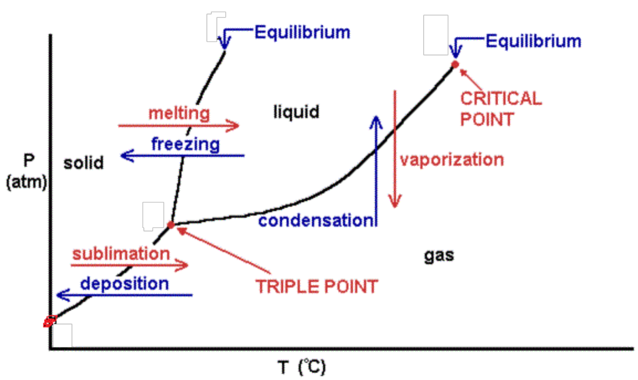
  (temperature, pressure, *etc.*) and laws (equations of state, *etc.*)





$$PV = nRT$$

- Theoretical understanding of **collective effects** $\Rightarrow$ **phase diagrams**

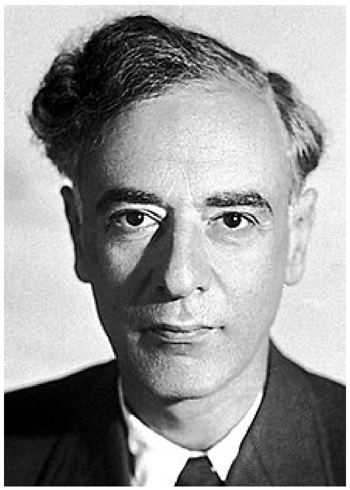
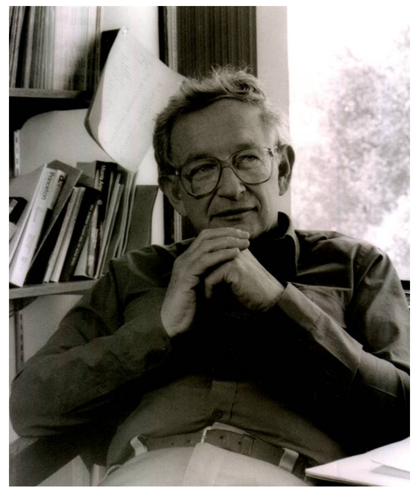


Phase transitions : sharp changes in the macroscopic behavior when an external (*e.g.* the temperature of the environment) or an internal (*e.g.* the interaction potential) parameter is changed

- Calculations can be difficult but the theoretical frame is set beyond doubt

Statistical physics

Four very important players & concepts

L. D. Landau	K. Wilson	P. W. Anderson	D. J. Thouless
			
Phase transitions Symmetry breaking	Renormalization Universality	Higgs Mechanism Disorder, Localization	Topology Disorder, Localization
1962	1982	1977	2016 Nobel Prizes

Theoretical description of phase transitions

Importance of randomness – More is different

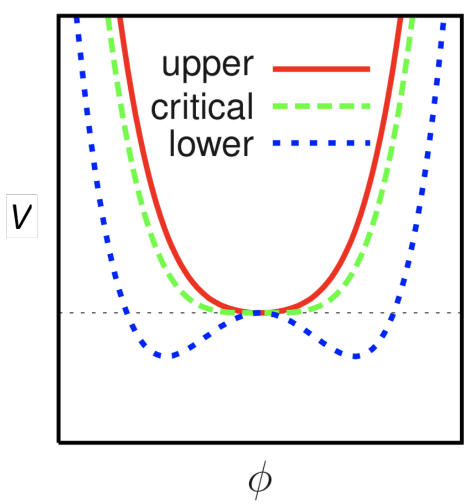
Landau Theory

A phase-transition : change of state

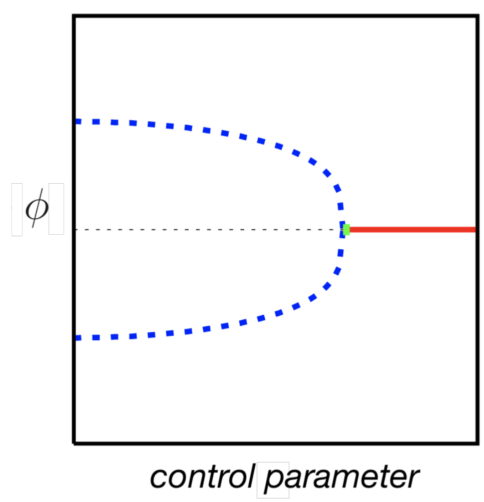
A point representing the global state (a macroscopic observable) of the system

In the "upper" phase, the *effective potential* in which it moves has only one minimum, $\phi = 0$.

In the "lower" phase, the effective potential has two minima $\phi = \pm\phi_0 \neq 0$.



Landau free-energy



Order parameter

Disorder

Geometric randomness

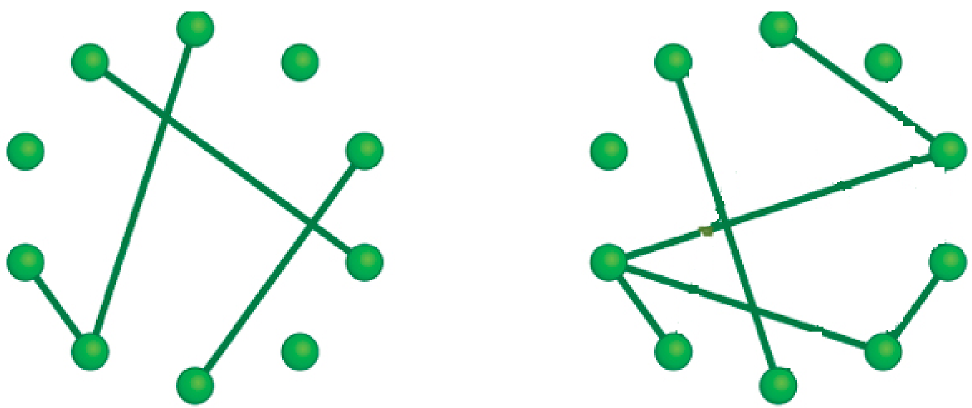
Random graphs

Fixed random – quenched/frozen – objects

Different realisations, heterogeneities

Simplest example, random graphs

Take N vertices and draw a link joining each pair with probability p



Two realisations

Geometric randomness

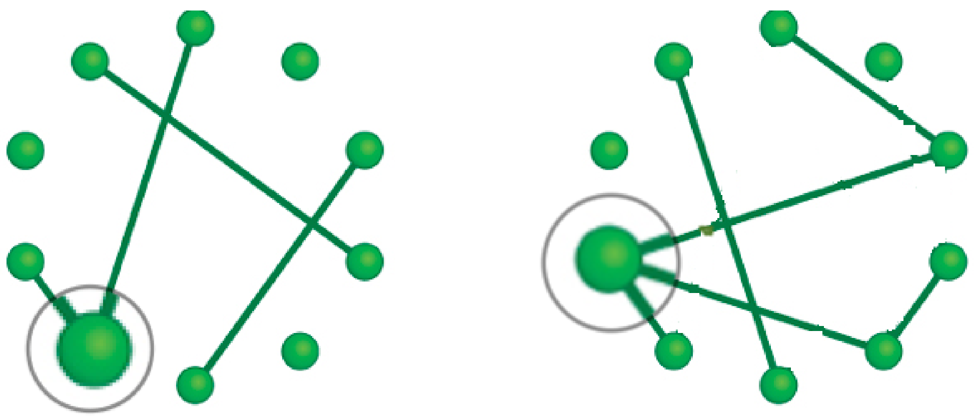
Random graphs

Fixed random – quenched/frozen – objects

Different realisations, heterogeneities

Simplest example, random graphs

Take N vertices and draw a link joining each pair with probability p

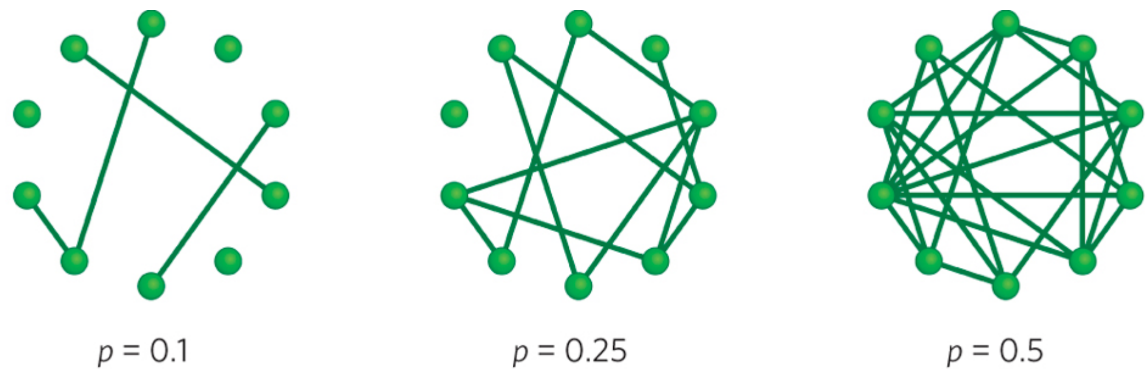


Heterogeneity fluctuations

Geometric randomness

Mathematics & applications

Erdös-Rényi (1959)



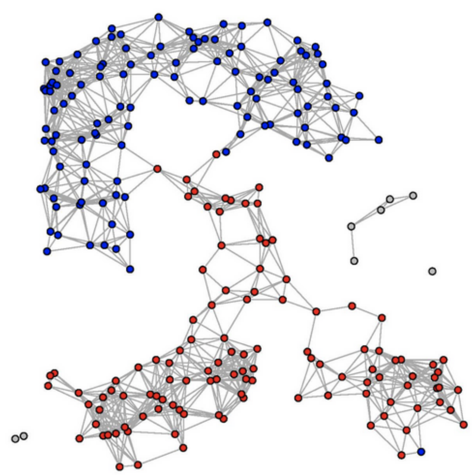
Questions :

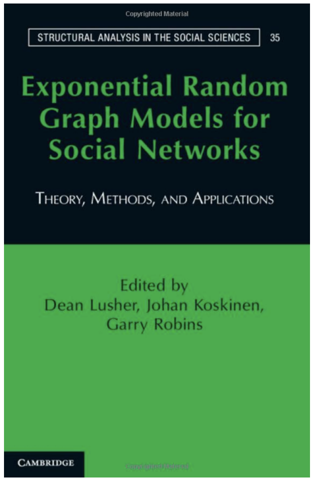
complete subgraphs ?

is the graph connected ?

etc.

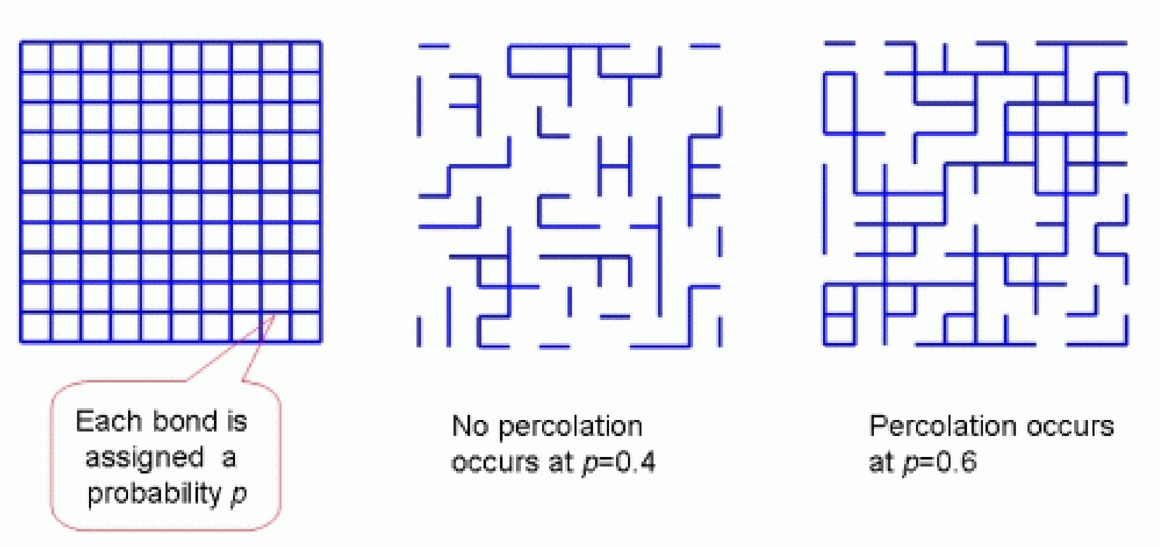
Networks





Geometric randomness

Percolation



Probability Π

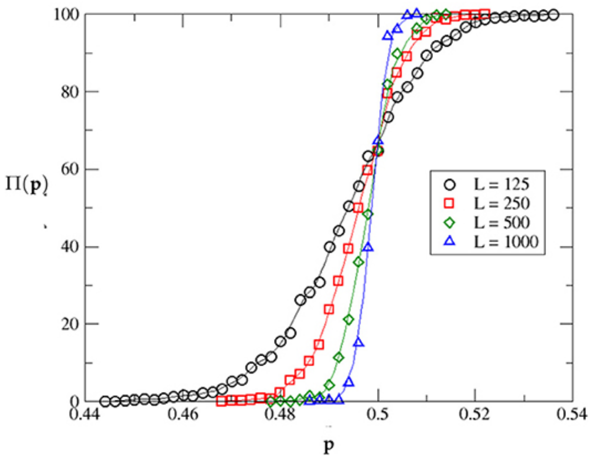
of there being a path

taking from one end to the other

as a function of p

for different system sizes L

Phase transition



Physics : spin-glasses

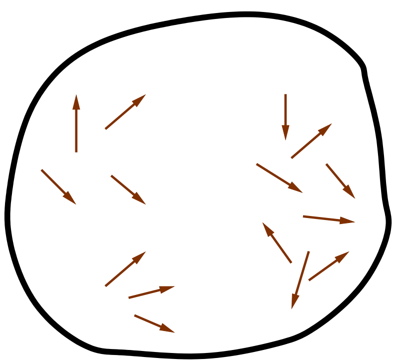
Magnetic impurities (spins) randomly placed in an inert host

$\vec{r}_i$ are random and time-independent since

the impurities do not move during experimental time-scales $\Rightarrow$

quenched randomness

Magnetic impurities in a metal host



spins can flip but not move

RKKY interaction potential

$$V(r_{ij}) \propto \frac{\cos 2k_F r_{ij}}{r_{ij}^3} s_i s_j$$

very rapid oscillations about 0

positive & negative

slow power law decay.

Physics : spin-glasses

Models on a lattice with random couplings

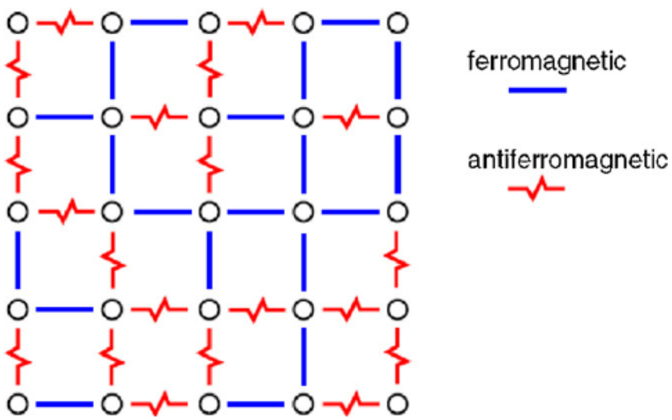
Ising spins $s_i = \pm 1$ sitting on a lattice

J_{ij} are random and time-independent since

the impurities do not move during experimental time-scales $\Rightarrow$

quenched randomness

Magnetic impurities in a metal host



spins can flip but not move

Edwards-Anderson model

$$H_J[\{s_i\}] = -\sum_{\langle ij \rangle} J_{ij} s_i s_j$$

J_{ij} drawn from a pdf with

zero mean & finite variance

Rugged landscapes

Beyond the Landau potential

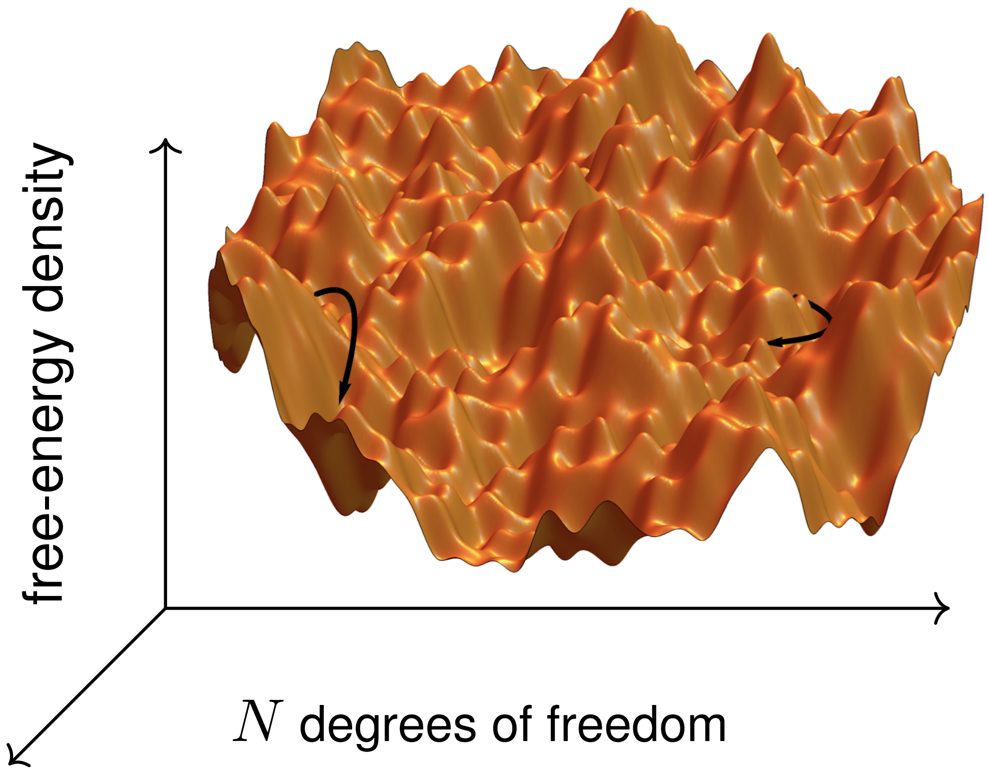


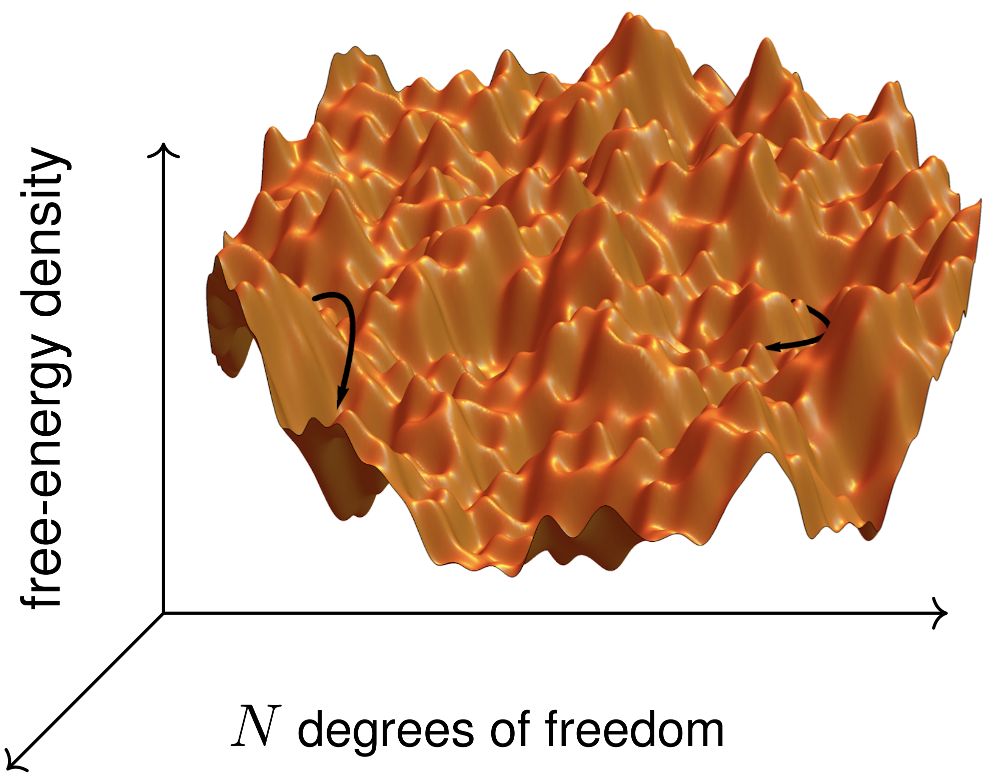
Figure adapted from a picture by **C. Cammarota**

Topography of the landscape on the N-dimensional substrate made by the N order parameters ?

Numerous studies by **theoretical physicists (TAP 1977)** and **probabilists**

Rugged landscapes

Beyond the Landau potential



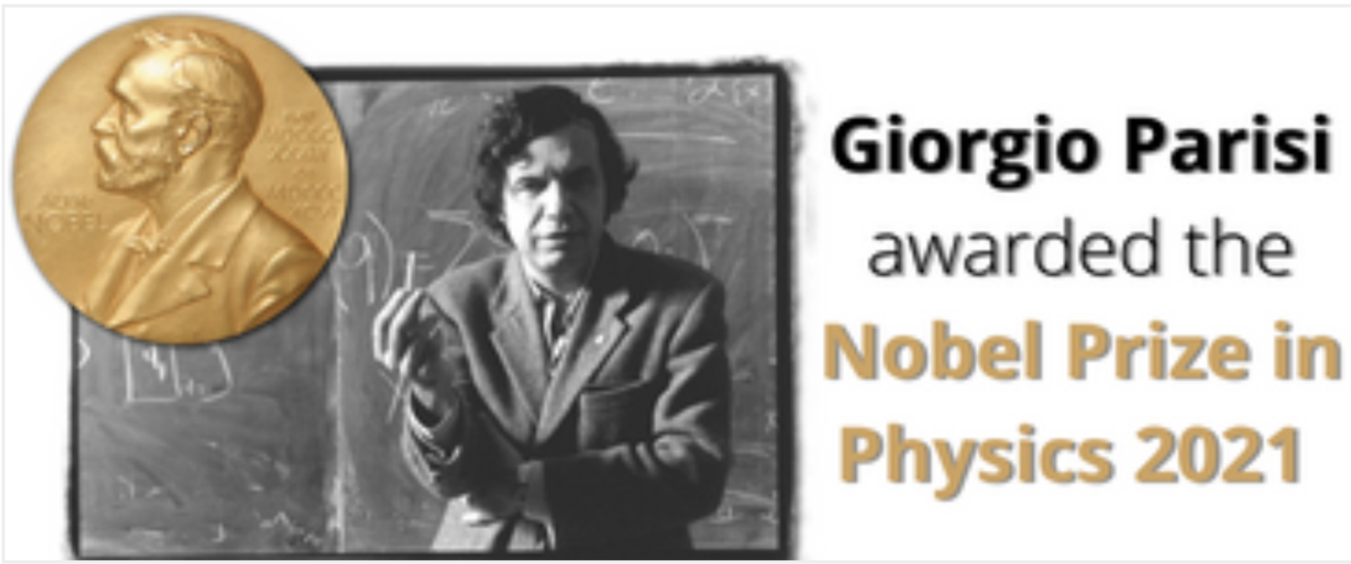
How to reach the absolute minimum ?

Thermal activation, surfing over tilted regions, quantum tunneling ?

Optimisation problem Smart algorithms ? Computer sc - applied math

Replica Theory

Giorgio Parisi



Replica method

A sketch

$$-\beta[f_J] = \lim_{N\to\infty} \frac{[\ln Z_N(\beta,J)]}{N} = \lim_{N\to\infty}\lim_{n\to 0} \frac{[Z_N^n(\beta,J)]-1}{Nn}$$

Z_N^n partition function of n independent copies of the system : **replicas**.

Gaussian average over disorder : coupling between replicas

$$\sum_a \sum_{i\neq j} J_{ij} s_i^a s_j^a \;\Rightarrow\; \sum_{i\neq j} \left(\sum_a s_i^a s_j^a\right)^2$$

Quadratic decoupling with the Hubbard-Stratonovich trick

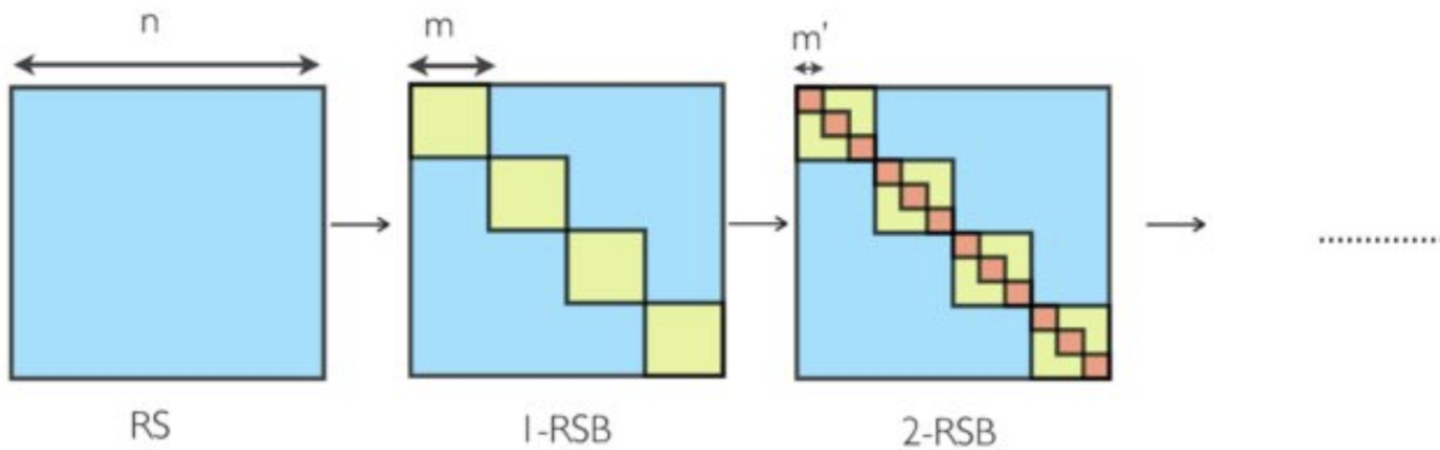
$$Q_{ab} \sum_i s_i^a s_i^b + \frac{1}{2} Q_{ab}^2$$

Q_{ab} is a 0×0 matrix but it admits an interpretation in terms of **overlaps**

The elements of Q_{ab} can evaluated by saddle-point if one exchanges the limits $N \to \infty$ $n \to 0$ with $n \to 0$ $N \to \infty$.

Replica Theory

The $n \times n$ matrix Q_{ab}

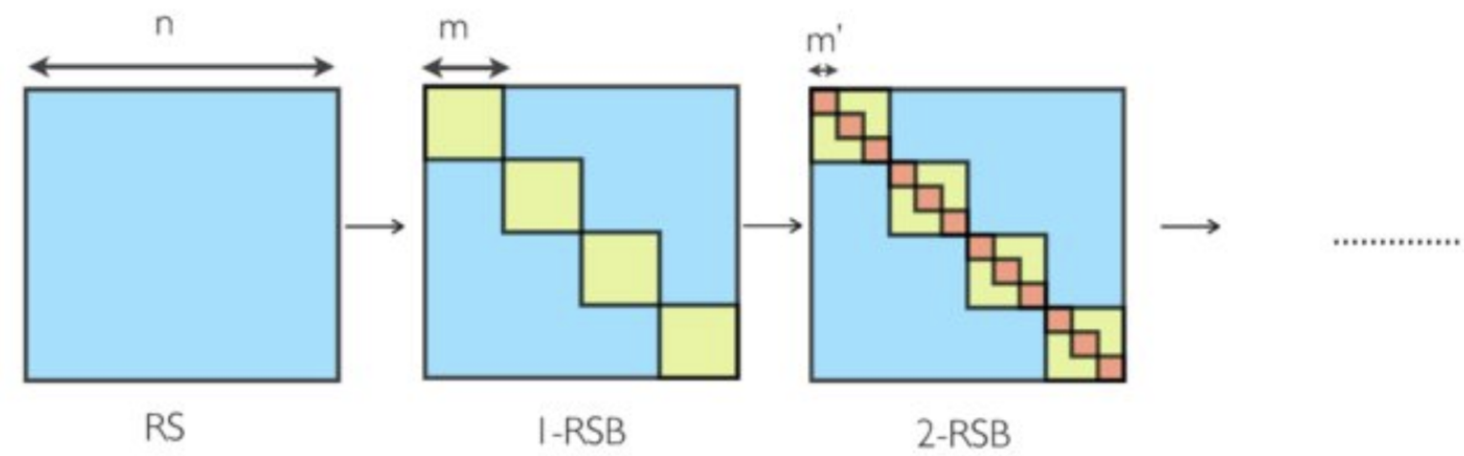


Replica symmetry breaking

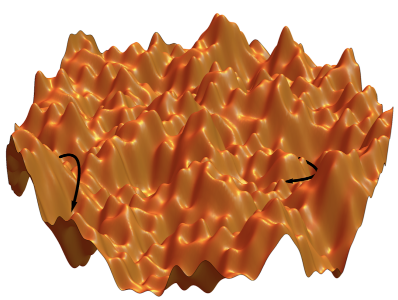
Parisi 1977-1979

Replica Theory

The $n \times n$ matrix Q_{ab}



Loosely speaking
the entries Q_{ab} tell us about
about the similarity between the
configurations in the different
valleys & the topology of the **landscape**

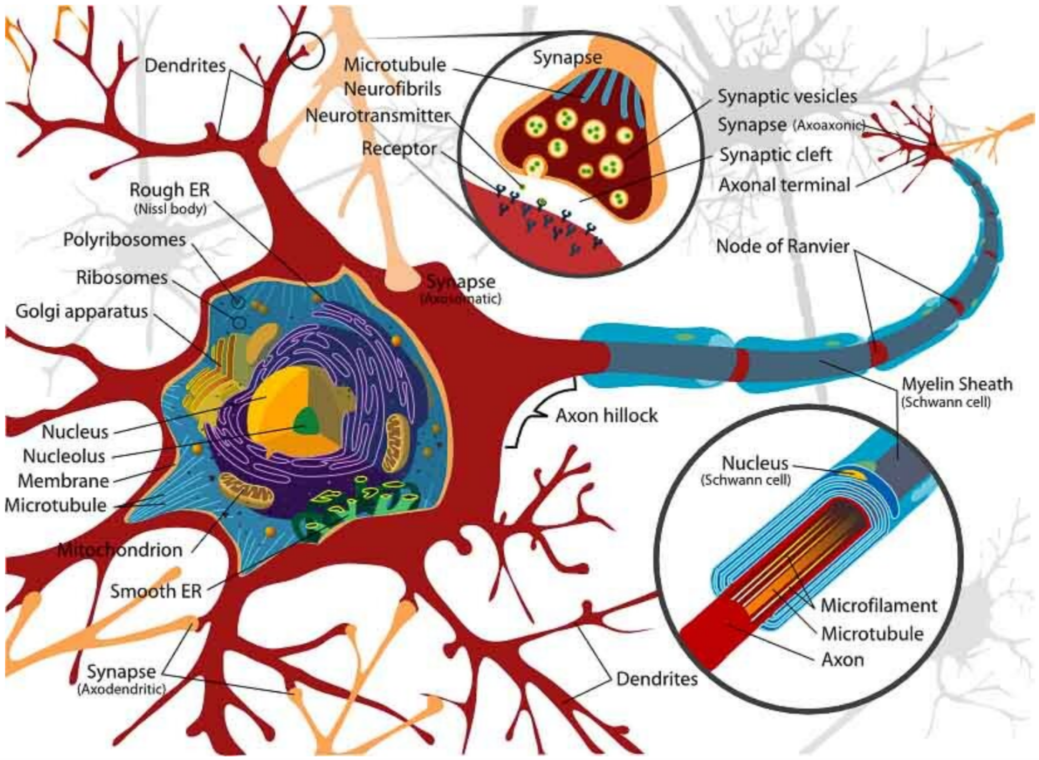


Parisi 1977-1979

Some applications

Neural Networks

Real neural network



Neurons connected by synapsis on a **random graph**

Figures from AI, Deep Learning, and Neural Networks explained, A. Castrounis

Neural networks

Models on graphs with random couplings

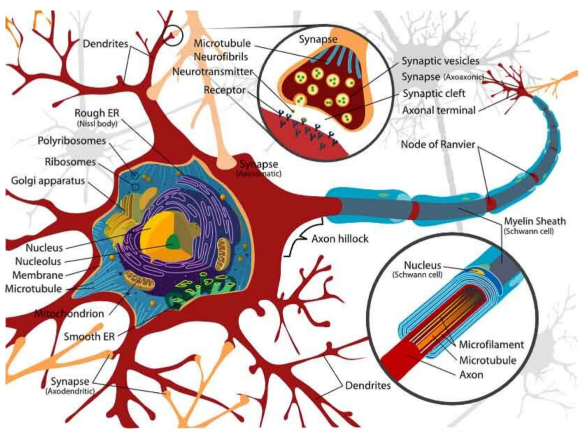
The neurons are Ising spins $s_i = \pm 1$ on a graph

J_{ij} are random and time-independent since

the synapsis do not change during experimental time-scales $\Rightarrow$

quenched randomness

The neural net



spins can flip but not move

Hopfield model

$H_J[\{s_i\}] = -\sum_{\langle ij \rangle} J_{ij} s_i s_j$

memory stored in the synapsis

$J_{ij} = 1/N_p \sum_{\mu=1}^{N_p} \xi_i^\mu \xi_j^\mu$

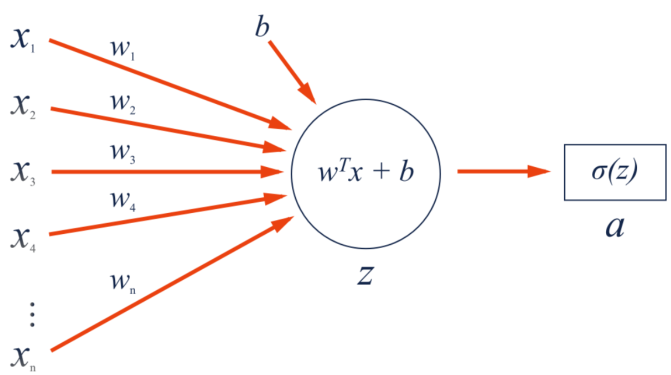
the patterns ξ_i^μ

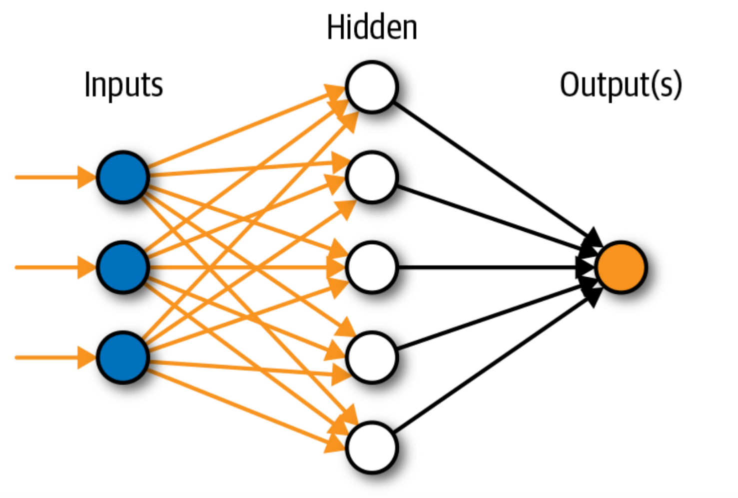
are drawn from a pdf with

zero mean & finite variance

Neural Networks

Sketch & artificial network





The connections in w^T may have a random component

The state of the neuron up (firing), down (quiescent) is a result of the calculation

In the artificial network on chooses the geometry (number of nodes in internal layer, number of hidden layers, connections between layers)

Figures from AI, Deep Learning, and Neural Networks explained, A. Castrounis

Optimisation problems

Constrained satisfaction problems

Problems involving variables which must satisfy some constraints

e.g. equalities, inequalities or both

studied in computer science to

compute their complexity or develop algorithms to most efficiently solve them

Typically, N variables, which have to satisfy M constraints.

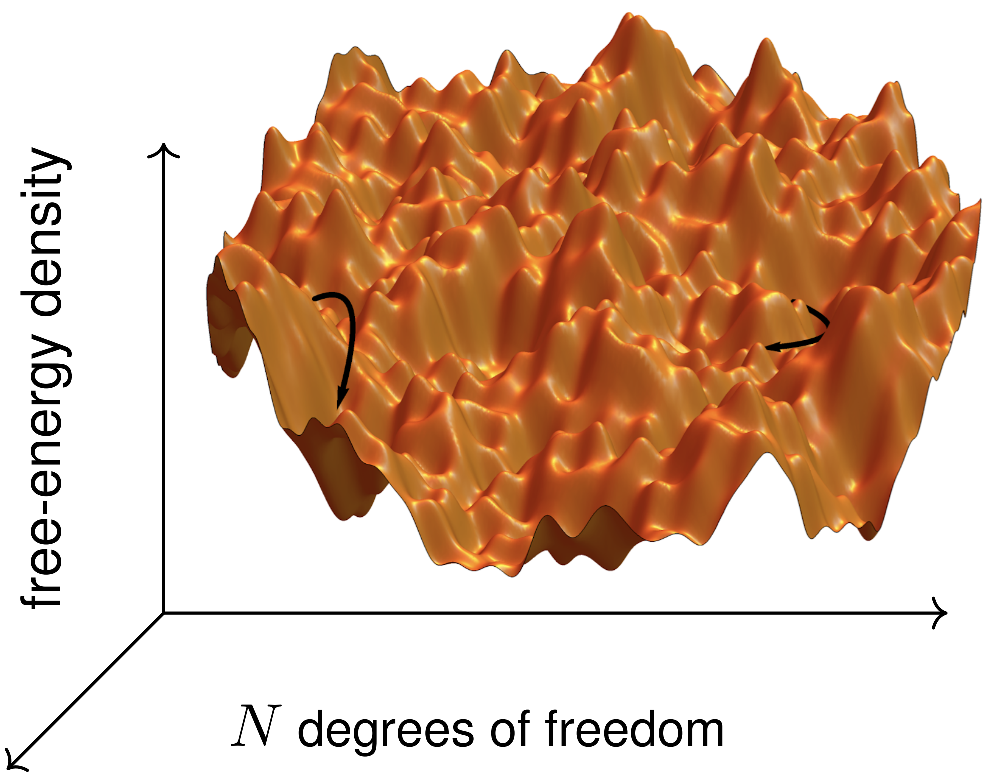
e.g. the variables could be the weights of a neural network, and each constraint imposes that the network satisfies the correct input-output relation on one of M training examples (e.g. distinguishing images of cats from dogs).

Statistical physics approach

thermodynamic limit $N \to \infty$ and $M \to \infty$ with $\alpha = M/N$ finite

Rugged landscapes

Beyond the Landau potential

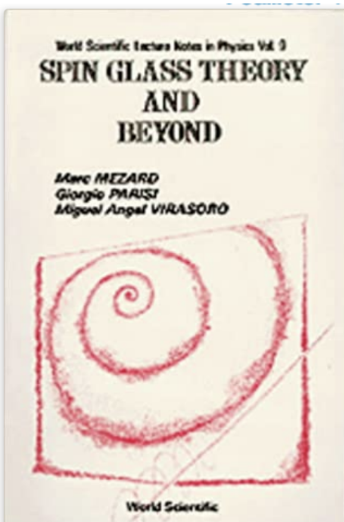


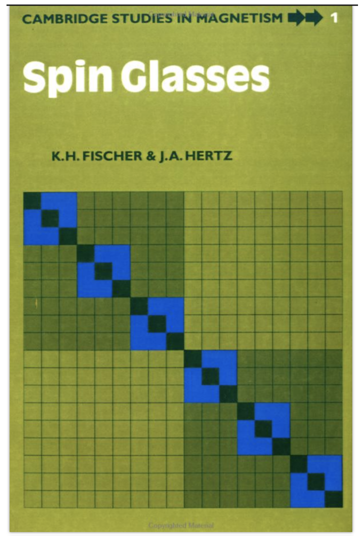
How to reach the absolute minimum ?

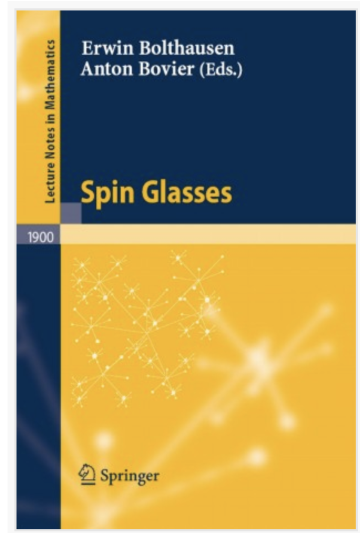
Thermal activation, surfing over tilted regions, quantum tunneling ?

Optimisation problem Smart algorithms ? Computer sc - applied math

Some books


World Scientific Lecture Notes in Physics Vol. 9
SPIN GLASS THEORY AND BEYOND
Marc MEZARD
Giorgio PARISI
Miguel Angel VIRASORO
World Scientific


CAMBRIDGE STUDIES IN MAGNETISM 1
Spin Glasses
K.H. FISCHER & J.A. HERTZ


Lecture Notes in Mathematics
1900
Erwin Bolthausen
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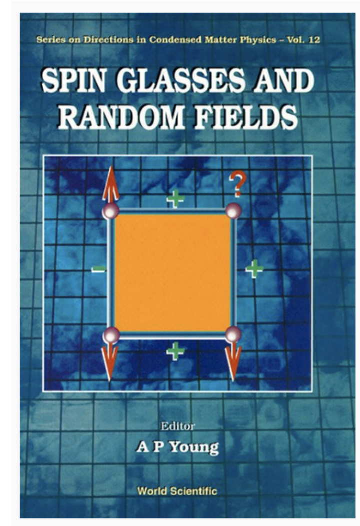
Series on Directions in Condensed Matter Physics – Vol. 12
SPIN GLASSES AND RANDOM FIELDS
Editor
A P Young
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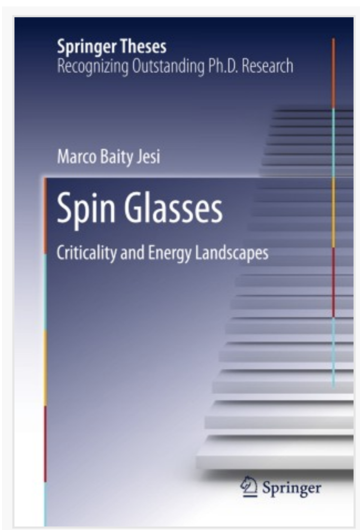
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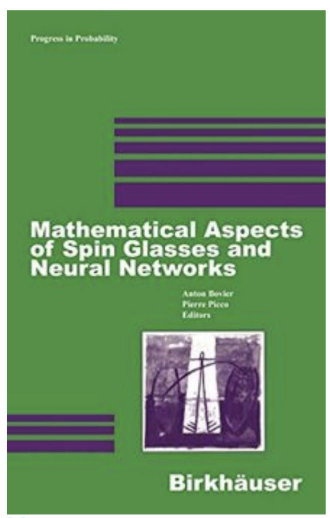
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Mathematical Aspects of Spin Glasses and Neural Networks
Anton Bovier
Pierre Picco
Editors
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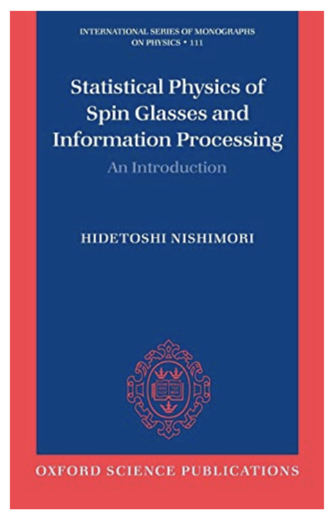
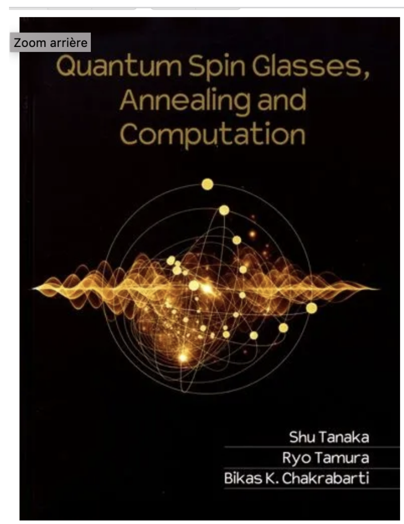

INTERNATIONAL SERIES OF MONOGRAPHS ON PHYSICS • 111
Statistical Physics of Spin Glasses and Information Processing
An Introduction
HIDETOSHI NISHIMORI
OXFORD SCIENCE PUBLICATIONS


Zoom arrière
Quantum Spin Glasses, Annealing and Computation
Shu Tanaka
Ryo Tamura
Bikas K. Chakrabarti

Out of equilibrium

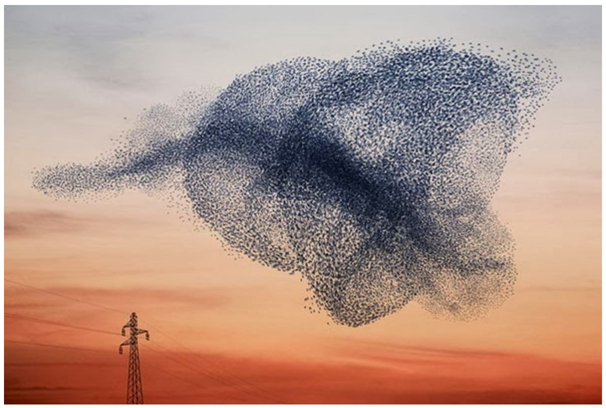
Driven systems

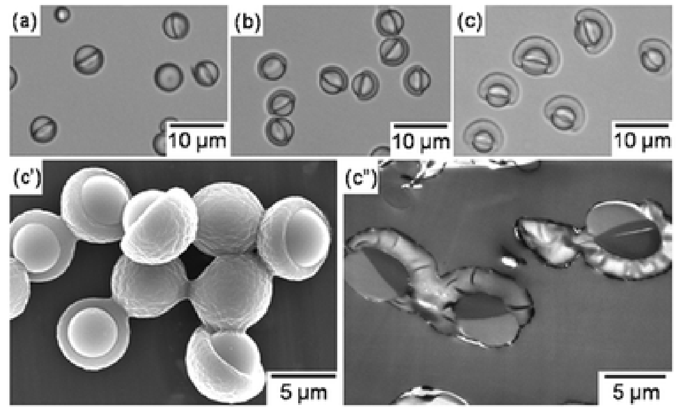
$$\boxed{\vec{F}_{\text{ext}} \neq -\vec{\nabla} V(\vec{x})}$$

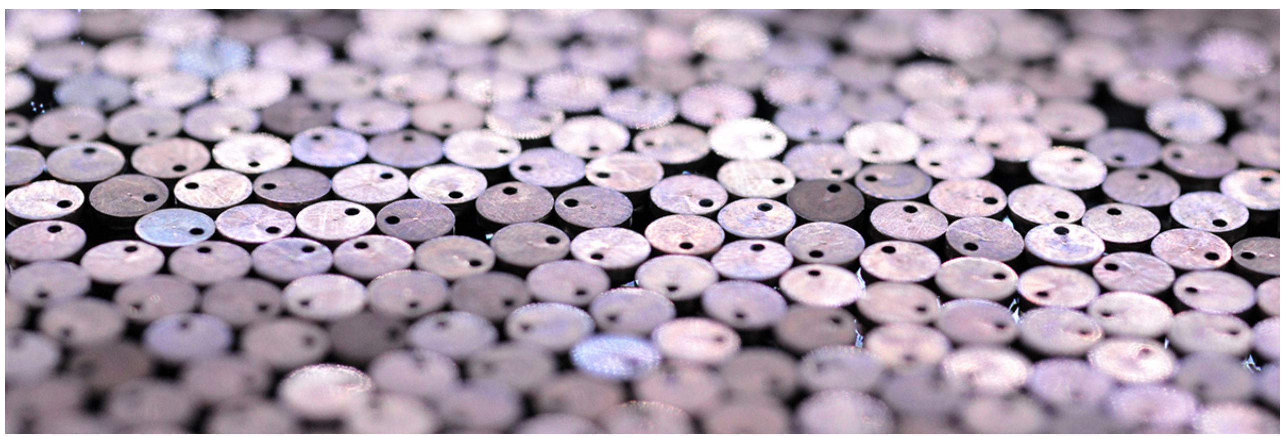
Energy injection $dE(t)/dt \neq 0$

Active matter

Natural & artificial : birds, bacteria, cells, grains, Janus particles





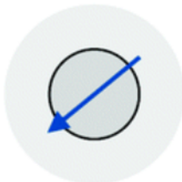


Experiments & observations **Bartolo** *et al.* Lyon, **Bocquet** *et al.* Paris, **Cavagna** *et al.* Roma, **di Leonardo** *et al.* Roma, **Dauchot** *et al.* Paris, just to mention some Europeans

Active Brownian particles

The standard model – ABPs

Spherical particles with diameter σ_d



Environment $\Longrightarrow$ Langevin dynamics

Scales $\Longrightarrow$ over-damped motion

Self-propulsion $\Longrightarrow$ active force $\vec{F}_{\rm act}$ along $\vec{n}_i = (\cos\theta_i(t), \sin\theta_i(t))$

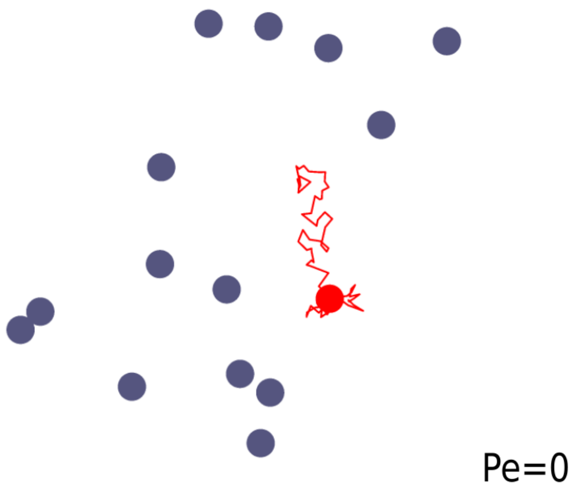
$$\underbrace{\gamma \dot{\vec{r}}_i}_{\text{friction}} = \underbrace{F_{\rm act}\vec{n}_i}_{\text{propulsion}} - \underbrace{\vec{\nabla}_i \sum_{j(\neq i)} V(r_{ij})}_{\text{inter-particle repulsion}} + \underbrace{\vec{\xi}_i}_{\text{translational white noise}} \qquad \underbrace{\dot{\theta}_i = \eta_i}_{\text{rotational white noise}}$$

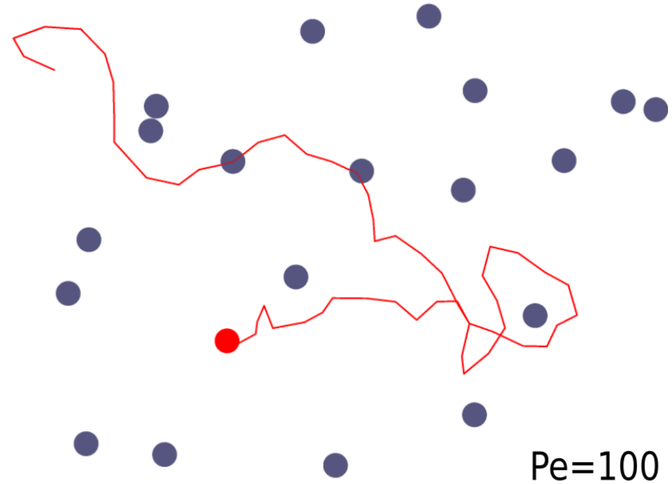
$2d$ packing fraction $\phi = \pi\sigma_d^2 N/(4S)$ Péclet number Pe = $F_{\rm act}\sigma_{\rm d}/(k_B T)$

Bialké, Speck & Löwen, Fily & Marchetti 12

Active Brownian particles

Typical motion of ABPs in interaction





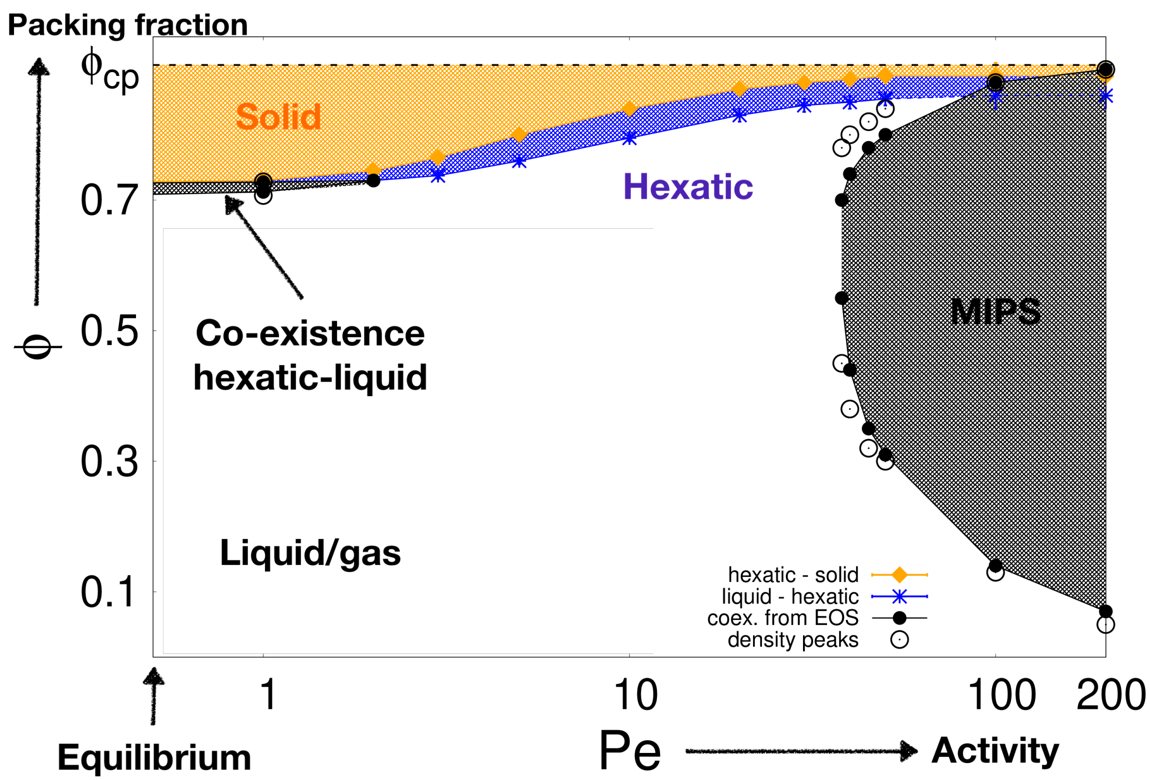
The activity induces a persistent random motion

Long running periods $\ell_p \propto \mathrm{Pe}\ \sigma_d$ and

sudden changes in direction

Active Brownian particles

Complex out of equilibrium phase diagram



Motility induced phase separation (MIPS)

gas & dense droplet

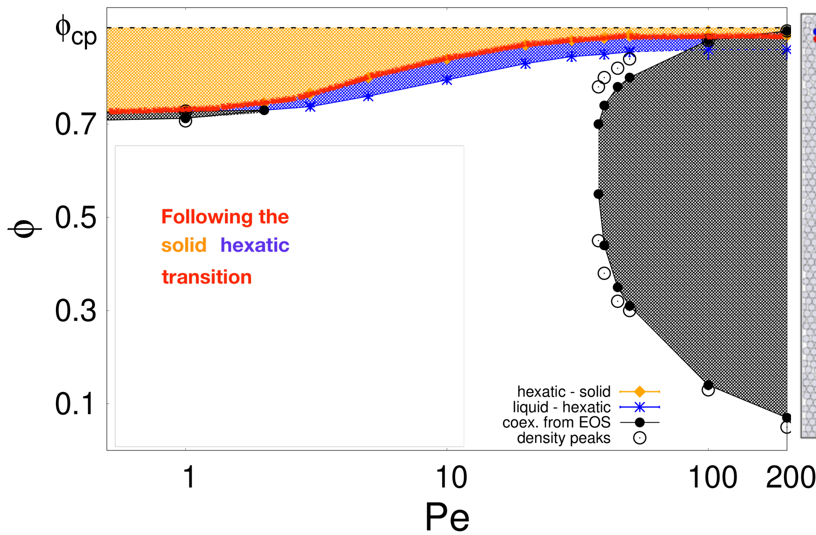
Cates & Tailleur 12

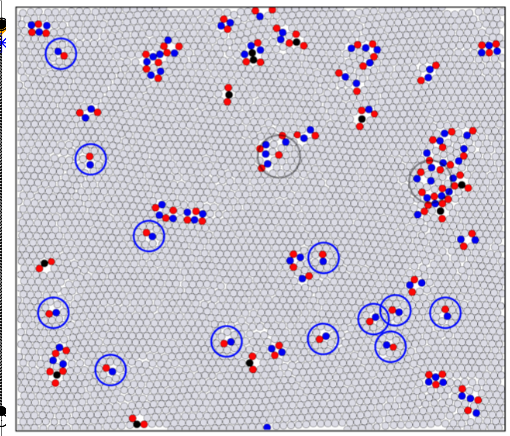
From virial pressure $P(\phi)$, translational and orientational correlations G_T and G_6, distributions of local density and hexatic order ϕ_i and ψ_{6i}, at fixed $k_B T = 0.05$

Digregorio, Levis, Suma, LFC, Gonnella & Pagonabarraga 18

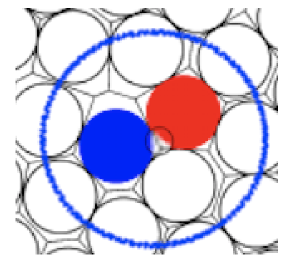
Active Brownian particles

Out of equilibrium phase diagram First question (out of many !)





Free dislocation: a 7-5 neighbor



$\neq$ from $\triangle$ lattice

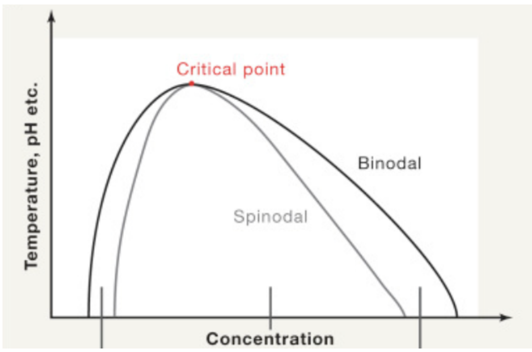
Solid - **Hexatic** transition at ϕ_{sh}, **driven by unbinding of dislocation pairs**

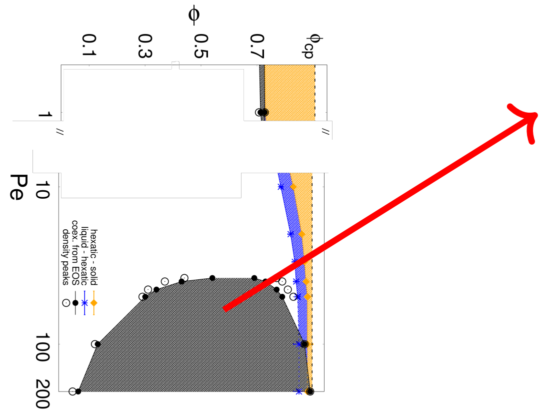
as in Berezinskii-Kosterlitz-Thouless-Halperin-Nelson-Young universality ?

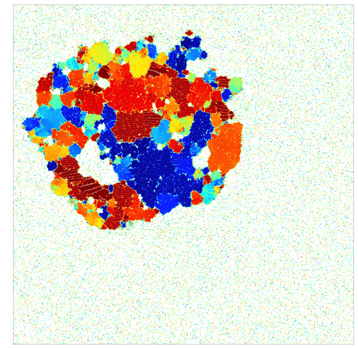
$$\rho_{disloc} \simeq a \, \exp\left[-b\left(\frac{\phi_{sh}}{\phi_{sh}-\phi}\right)^{\nu}\right] \qquad \nu \sim 0.37 \quad \forall \text{Pe ?}$$

Active Brownian particles

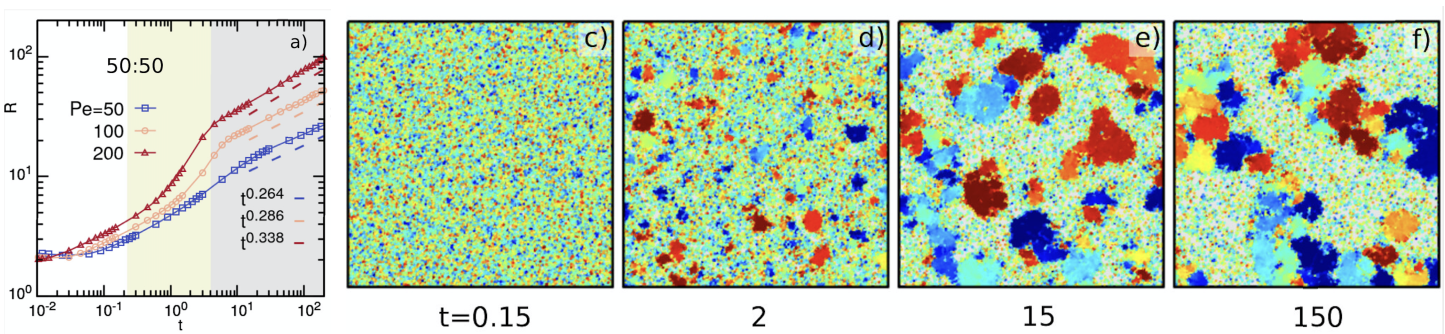
Out of equilibrium phase diagram **So many questions !**







Dynamics of formation of the dense phase ? but bubbles, hexatic order, ...



Universality with the Lifshitz-Slyozov law $\mathcal{R}(t) \simeq t^{1/3}$? Geometry ?

Redner ***et al*** **13, Stenhammar** ***et al*** **14, ... , Caporusso** ***et al*** **20, Caprini** ***et al*** **20, ...**

Thermodynamic notions ?

Conclusions

The talk showed some physics going from the general to the particular

statistical physics, disordered systems, out of equilibrium phenomena

Some basic statistical physics questions were discussed and concerned

phase diagrams, universality, effects of disorder, replicas...

Thermodynamic concepts out of equilibrium ?

Effective temperatures (heat flows, entropy production, partial equilibrations, fluctuations,...) importance of time-scales & observables. Also stochastic thermodynamics, fluctuation theorems, *etc.*

There is much more to be done and understood

Beyond

Econophysics

Social physics

Ecology

Biophysics

Computer science

X-physics