Probing the fractal phase of a random matrix using replicas

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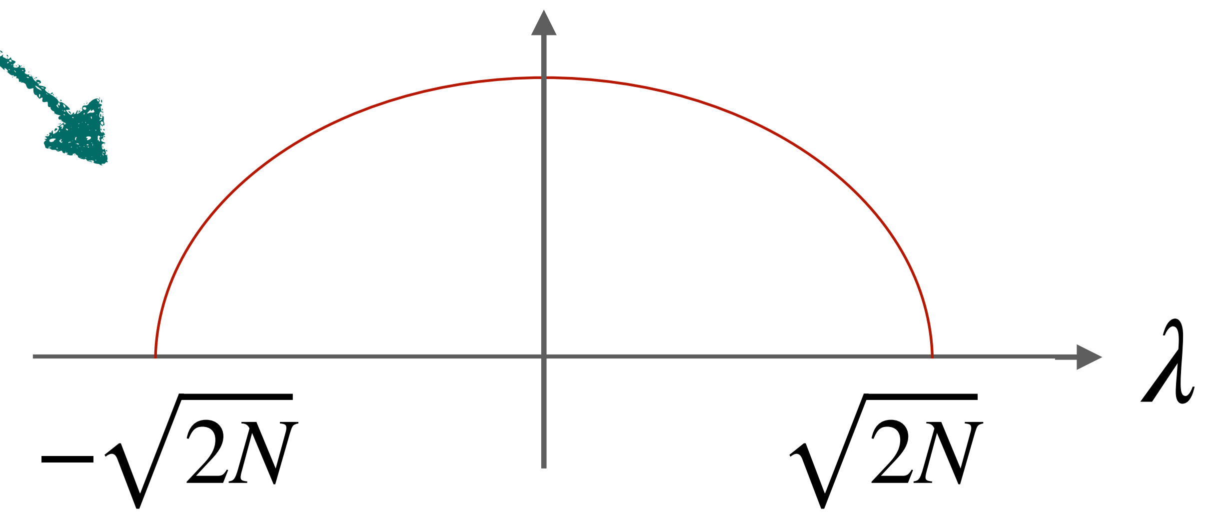
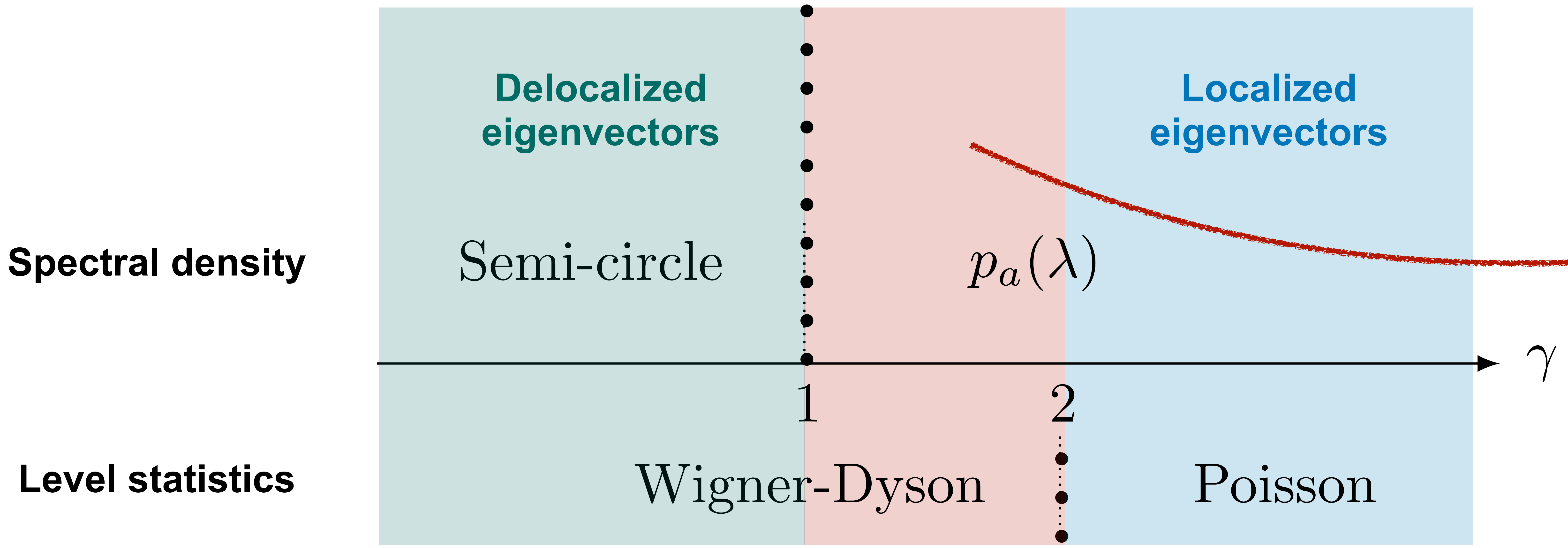
The generalized Rosenzweig-Porter model

$$H = \begin{pmatrix} a_{11} & & & \\ & a_{22} & & \\ & & \ddots & \\ & & & a_{NN} \end{pmatrix} + \frac{\nu}{N^{\gamma/2}} \begin{pmatrix} & & \\ & \textbf{GOE} & \\ & & \end{pmatrix}$$

$p_a(a_{ii})$



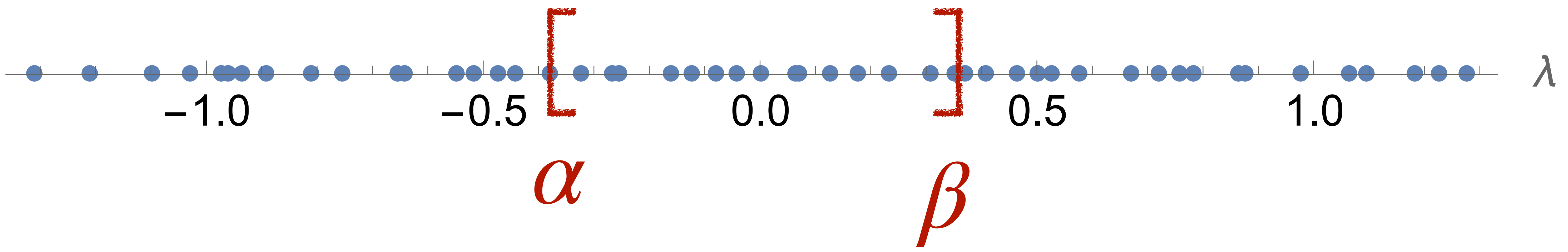
$$\rho_N(\lambda) = \frac{1}{N}\sum_{i=1}^{N}\delta(\lambda - \lambda_i)$$



$1 < \gamma < 2$

Fractal phase: partially delocalized eigenvectors

Local level statistics: # of eigenvalues in an interval



$$I_N[-\alpha, \beta] \equiv N \int_{\alpha}^{\beta} \mathrm{d}\lambda \, \rho_N(\lambda)$$

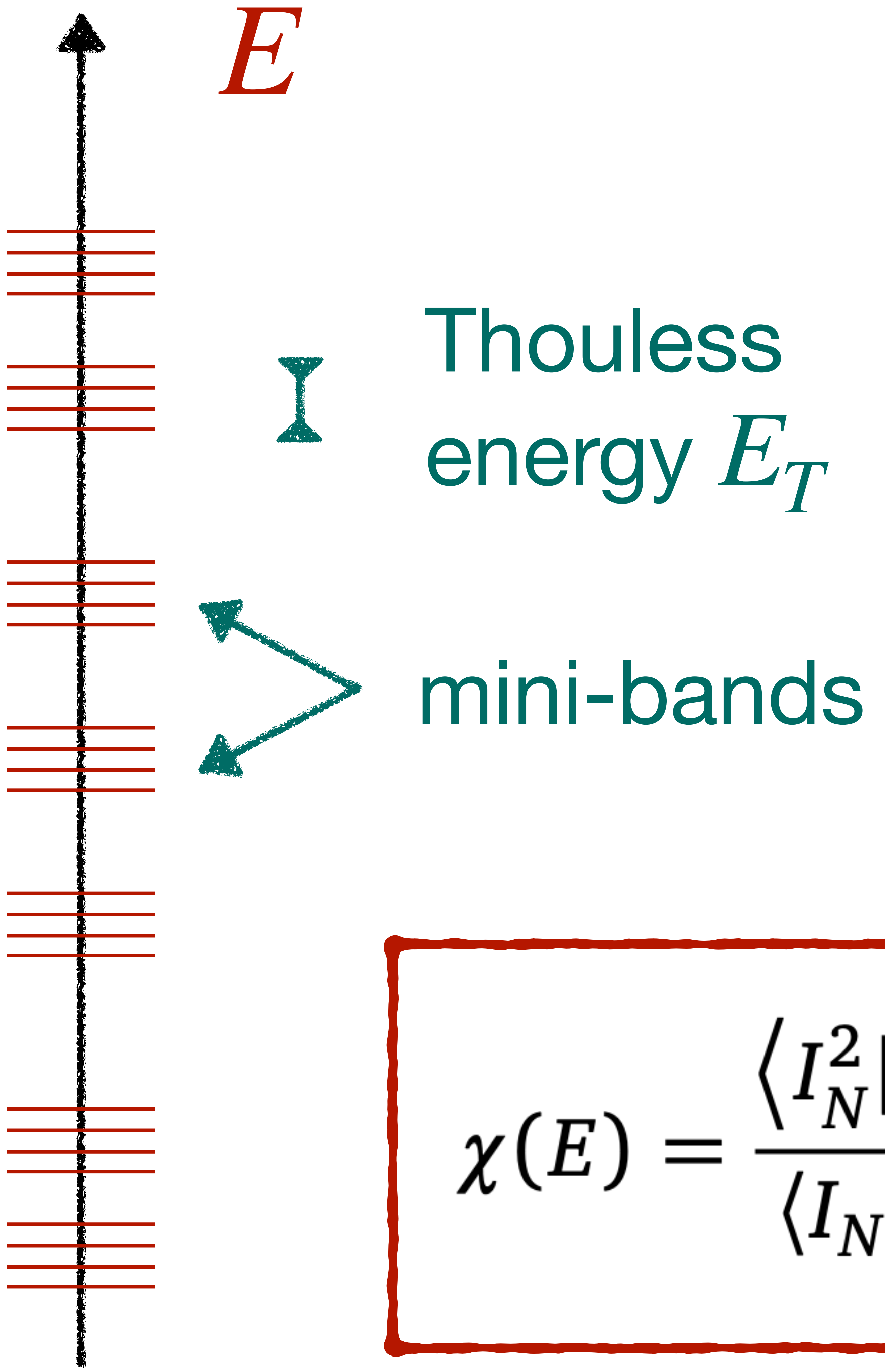
Large deviation function:

$$\mathcal{F}_{[-E,E]}(s) \equiv \lim_{N \to \infty} \frac{1}{N} \ln \left\langle e^{-s I_N[-E,E]} \right\rangle$$

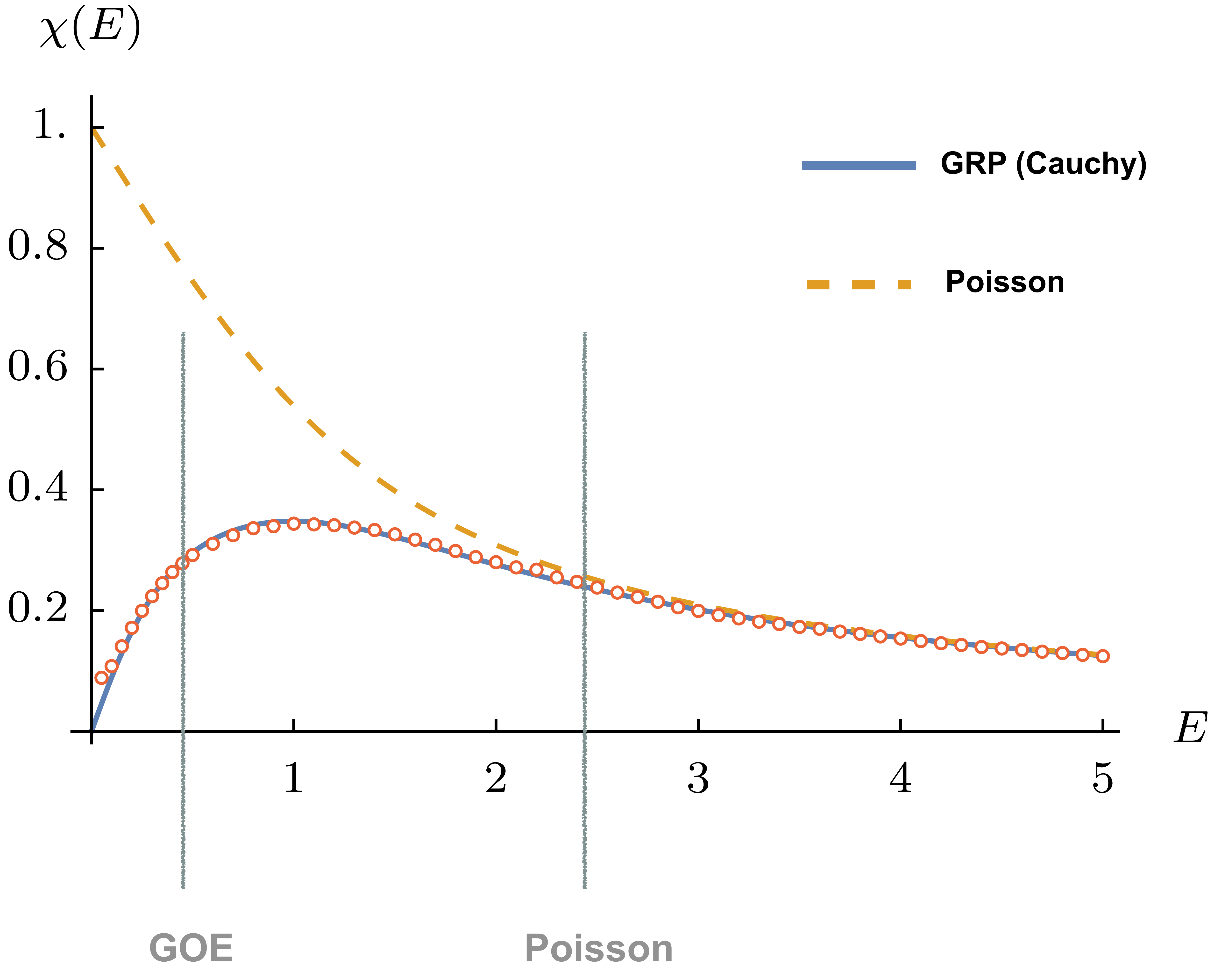
can be accessed using the **replica** method

Details of replica calculation

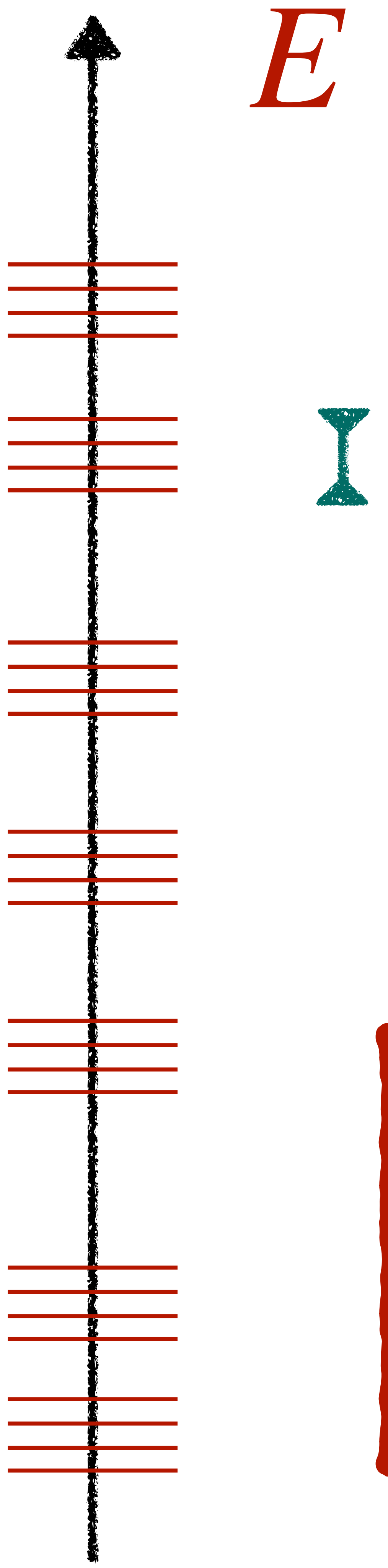
Level compressibility



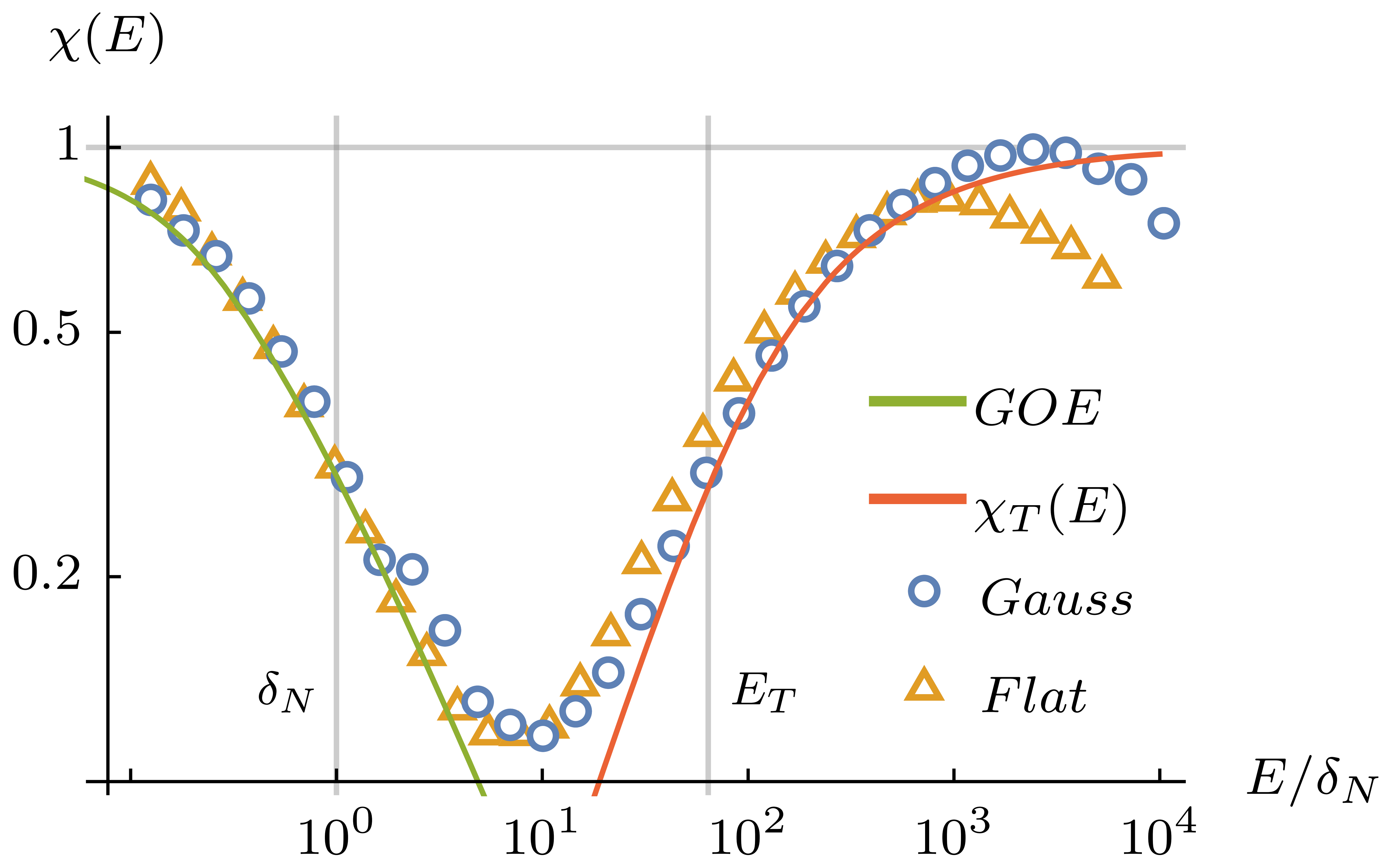
$$\chi(E) = \frac{\langle I_N^2[-E,E]\rangle_c}{\langle I_N[-E,E]\rangle}$$



Level compressibility



Thouless
energy E_T



$$\chi(E) \simeq \chi_T\left(y = E/E_T\right)$$
$$= \frac{1}{\pi y}\left[2y\arctan(y) - \ln(1+y^2)\right]$$

→ universal for $E \sim E_T$,
independent of $p_a(a_{ii})$

Venturelli, Cugliandolo, Schehr, Tarzia, SciPost Phys. 14, 110 (2023)