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Two-dimensional views into four-dimensional physics

Soutenue le 10 décembre 2014 devant le jury composé de :

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Foreword

The aim of this dissertation is to provide a concise overview of my recent research in theoretical physics and to put it in a broader international context. It encompasses the time I spent as a post-doctoral fellow at the Racah Institute in Jerusalem, as *maître de conférences* at Institut d'astrophysique de Paris and finally at Laboratoire de physique théorique et hautes énergies.

I will summarize the work presented in most of my publications during this period, whose list is given in the appendix, and some unpublished results that nevertheless deserved to be couched on paper. Finally I will mention several works in progress as well as some other projects for the near future.

Given the size constraints, I will skip most of the technicalities and give the emphasis on the methods used and on the physical interpretation of the results; for more details I refer the reader to the published articles. Writing this manuscript gave me also the opportunity to forge stronger links between works in different areas, and to revisit old problems that I could not solve at that time.

The short overviews that are given at the beginning of each section are strongly biased by my personal implication in the field that I focused on in this review. The bibliography is by no means comprehensive and only meant to orient the reader; I apologize in advance for any important omission.

Acknowledgements

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Introduction

String theory as a field of research has evolved significantly during the last ten years. From a rather unified set of techniques and concepts, it has morphed into a cluster of research areas that originated from string theory or were inspired by it. For instance the Anti de Sitter/conformal field theory correspondence, especially in its recent developments in relation with condensed matter, does not necessarily invoke a string theory origin; flux compactifications and their phenomenological applications rely mostly on a large-volume effective supergravity approach for which many interesting techniques have been developed as generalized geometry.

The original approach of string theory, as two-dimensional superconformal field theories mapping the worldsheet of the string to some target space-time, remains nevertheless the backbone of the field. Many fundamental questions remain unanswered, as the general relation between the algebraic data of the two-dimensional theory and the geometry of space-time, if there is any notion of it. This is by no means an academic question since there isn't any compelling evidence that compactifications relevant to describe the real four-dimensional world are in the realm of large-volume supergravity models. In the context of heterotic strings, that I have focused on recently, it is clearly not the case.

Two-dimensional physics encodes also many non-trivial aspects of gauge theory dynamics in higher dimensions, at different levels. Non-trivial field theories and some of their quantum dualities can be obtained from boundary states and the associated open string sectors. Back-reacted brane geometries are in several interesting cases amenable to worldsheet methods and lead to holographic dualities that can be probed in the regime of strong curvature and strong gauge coupling. Finally, time-dependent decay processes in these non-gravitational theories can be studied in detail as open string tachyon condensation.

Most of my work revolved around this perspective of understanding non-trivial aspects of space-time physics from the abstract two-dimensional field theory perspective. I used mainly the methods of (super)conformal field theory, with or without boundaries, and recently started to explore the possibilities offered by gauged linear sigma models. In many cases though some detailed analysis in quantum field theory or supergravity were used to complete and extend the picture.

This manuscript is organized in four sections, corresponding to the main themes of my recent research. In the first section I will discuss aspects of non-BPS D-branes in string theory and open string tachyon decay. In the second section are gathered works about the interplay between branes constructions and quantum field theory dynamics in various dimensions and with various amounts of supersymmetry, both at the 'pre-holographic' level (*i.e.* without back-reaction) and in the context of gauge/gravity duality. The third section is devoted to several aspects of heterotic compactifications with torsion, using supergravity, conformal field theories and gauged-linear sigma-model approaches. Finally in the last section I will illustrate by some examples the ambiguities in associating a geometry to a string theory, and non-geometric vacua that can be obtained in relation with these symmetries.

Contents

1

Non-BPS branes and tachyon condensation

Time-dependent phenomena are hard to describe in string theory, despite their relevance in cosmology or in the holographic description of decay processes. The difficulties are both technical – one cannot use unitary conformal field theories on Riemann surfaces – and more physical. For instance one may not know what is the final state of the system; in other examples quantum production of heavy strings, enhanced by the Hagedorn density of states, may signal a breakdown of perturbation theory.

Open string tachyon condensation is associated with the decay of non-BPS D-branes or of brane/antibrane pairs. In the superstring case, the tachyon field rolls towards the absolute minimum of its potential, identified as the closed string vacuum; formulated first as a conjecture, this statement has been proven using string field theory (see [1] for a review and references therein). Following the seminal work of Sen [2], 'rolling tachyon' solutions have received lot of attention since they provide tractable examples of time-dependent backgrounds at tree-level, described by solvable boundary conformal field theories.

As for closed string time-dependent backgrounds, one may worry that, because of closed string radiation, the worldsheet description breaks down. It should actually be the case to some degree as the total radiated energy per unit area should be equal to the D-brane tension, which is infinite in the tree-level approximation; at non-zero string coupling a high-energy cut-off should appear. In order to overcome this difficulty Sen proposed an interesting *open/closed completeness conjecture* [3] which states that the quantum theory of open strings gives a complete description of the tachyon condensation process, and in particular correctly reproduces the properties of the tachyon vacuum, while the dual closed string description breaks down at late times. This conjecture followed from the study of tachyon condensation in flat space-time. Remarkably, it has been shown to be true in two-dimensional string theory [4], which has a dual description in terms of a matrix model.

Besides the worldsheet conformal field theory approach, an effective field theory has been developed, first by requiring that it reproduces at late times the expected behaviour of the system as a pressure-less gas [5] and second by comparing the one-shell effective Lagrangian with the disk partition function [6]. This Dirac–Born–Infeld-like effective action for the tachyon field, which is valid around the rolling tachyon solution in a flat space-time background, has been generalized in various directions. It has been covariantized in order to accommodate for arbitrary background fields and extended to multiple non-BPS branes and brane/antibrane pairs. These effective actions have been used in a variety of applications, stretching from the holographic description of chiral symmetry breaking [7] to cosmological models of brane inflation [8].

I had been interested in probing open string tachyon physics in situations where these descriptions could break down, in other words systems which would challenge the universality of the tachyon condensation process. An example that I have studied in detail with Flavien Kiefer, my Ph.D. student at the time, was a configuration of a D-brane and an anti-D-brane at finite distance [9], which is interesting since the tachyon can be made arbitrarily light. I have also considered several superficially inequivalent classes of unstable D-brane configurations near NS5-branes, whose decays are expected to be governed by similar effective actions [10]. Finally, the decay of a non-BPS D-particle in anti-de-Sitter space was studied with Eliezer Rabinovici [11], as an attempt to challenge the completeness conjecture of Sen, as one expects a field-theoretic rather than Hagedorn asymptotic density of states in this space-time. Before studying the actual rolling tachyon solutions in the last two cases, I had first to construct the boundary states corresponding to the unstable D-branes. This will be the object of the next two sections.

1. Unstable D-brane configurations near five-branes

The effective action for the open string tachyon on a non-BPS Dp-brane:

$$(1)\quad S = -T_p \int d^p x\, V(T) \sqrt{-\det(\eta_{\mu\nu} + \partial_\mu T\, \partial_\nu T + \cdots)}\,, \text{ with } \quad V(T) = 1/\cosh(T/\sqrt{\alpha'})\,,$$

is quite similar to the Dirac–Born–Infeld (DBI) action for a BPS D-brane if one views the tachyon field $T(x^\mu)$ as a warped extra dimension. This analogy prompted Kutasov to find a brane configuration without tachyons reproducing the same dynamics [12]. He considered a BPS D-brane in the background of Neveu–Schwarz five-branes that induce a warp factor for the D-brane/NS5 brane distance field strikingly similar to the tachyon potential $V(T)$.

Following this work, Sen proposed a set-up in which these two configurations were conjectured to be identical [13]. One considers a background of two NS5-branes placed at antipodal points around a compact transverse dimension $x^6 \sim x^6 + 2\pi R$. A first type of unstable D-brane (G-type) in this background consists in a BPS D-brane – for instance, a D0-brane in type IIA – sitting halfway between the two five-branes. The D-particle is unstable against a small perturbation of its position, which will make it fall onto one of the five-branes. The second type of D-brane (U-type) is very different as one considers a non-BPS D-brane ending on both NS5-branes, *e.g.* a stretched D1-brane in type IIA; being of the wrong dimensionality, it has a tachyon as in flat space-time. In the limit where the radius R of the transverse circle is very small, the mass of the 'geometric tachyon' on the BPS D0-brane and of the 'universal tachyon' on the non-BPS D1-brane agree, as well as the respective tensions of these objects. This suggests that both D-branes become identical in this regime where α' corrections become important.

I obtained a proof of this conjecture in [10] using a slightly different configuration. I have considered a set of k five-branes, distributed evenly on a circle of radius ρ in the plane (x^5, x^6). One can construct in this background two types of D-branes in close analogy with Sen's set-up, see fig 1; first BPS D-branes sitting at the origin of the circle, similar to the G-type D-branes, and second non-BPS D-branes stretched between antipodal NS5-branes, similar to the U-type D-branes.

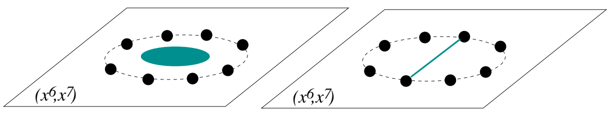


FIGURE 1. *G-type (left) and U-type (right) D-branes in the background of a ring of five-branes.*

In the *double-scaling limit*, where the radius of the circle goes to zero while keeping $\sqrt{\alpha'}g_s/\rho$ constant, the worldsheet theory corresponding to the back-reacted NS5-branes becomes exactly solvable, being T-dual to a diagonal $\mathbb{Z}_k$ orbifold of $\mathcal{N} = (2,2)$ Liouville theory of background charge $Q = \sqrt{2/k}$ and of an $\mathcal{N} = (2,2)$ minimal model of central charge $c = 6 - 3/k$. In a previous work with Ari Pakman and Jan Troost [14] we had constructed some of the D-branes in this set-up, by combining A-branes and B-branes of each of the two superconformal field theories. By extending these results I was able to construct the exact boundary states corresponding to the G-type D-brane, made of a B-brane of the minimal model combined with an A-brane of Liouville theory, and to the U-type D-brane, from A-type boundary states in each factor. The masses squared of the tachyon on these branes are $M^2 = -\frac{1}{k\alpha'}$ and $M^2 = -\frac{1}{2\alpha'}$ respectively. For generic k these boundary states are different, being constructed out of a different basis of Ishibashi states.

These G-type and U-type boundary states become non-trivially identical for $k = 2$, which corresponds to a pair of five-branes. It proves the exact equivalence between the geometric tachyon on the G-type brane and of the universal tachyon on the U-type brane.[1] This is very interesting since these two configurations are superficially very different before the α' corrections are taken into account. The rolling tachyon solution corresponding to the decay of these D-brane configurations has interesting properties that will be discussed in section 3. A similar tachyon condensation has a holographic interpretation as the decay of an unstable dyon in four-dimensional $\mathcal{N} = 1$ quiver theories near Argyres–Douglas points, see chapter 2.

2. D-branes in Lorentzian AdS_3

Anti-de-Sitter in three dimensions with NS-NS flux is an important string theory solution, being a rare example of curved space-time described by a solvable worldsheet theory, the Wess–Zumino–Witten (WZW) model at level k associated with the group manifold $SL(2,\mathbb{R})$, and holographically dual to a two-dimensional conformal field theory through the AdS_3/CFT_2 correspondence.

As was mentioned before conformal field theories associated with Lorentzian space-times are difficult to deal with. The problem is easy to circumvent in flat space-time, as the time coordinate is a free theory associated with the chiral currents $i\partial X^0$ and $i\bar{\partial} X^0$, and not solved in time-dependent backgrounds. AdS_3 is halfway between these two cases; while there exists a Killing vector associated with translations in the global time coordinate, it is not mapped in the CFT to affine currents but rather to a global $U(1)$ symmetry $J_0^3 + \bar{J}_0^3$, where J^3 is the generator of the (compact, time-like) elliptic $\hat{\mathfrak{u}}(1)$ subgroup of affine $\widehat{\mathfrak{sl}}(2,\mathbb{R})$. As a consequence there is a non-trivial relation between the worldsheet theory for the geometric coset $H_+^3 = SL(2,\mathbb{C})/SU(2)$, the analytic continuation of AdS_3, and the original model. The H_+^3 theory – which is not unitary – has been solved by Teschner on the sphere [15] and with Ponsot and Schomerus on the disk [16]. The relation with the Lorentzian model is not straightforward, as the H_+^3 model contains neither the discrete states nor the long string states[2] of AdS_3, which appear after Wick rotation in a subtle way [17].

I have found an alternate method of analytic continuation in order to deal with this issues. Rather than performing a Wick rotation on a coordinate one considers an *analytic continuation in the moduli space* of the theory. I have defined Euclidean AdS_3 as a family of sigma-models related to smooth Euclidean-signature metrics depending on R, a free parameter:

$$ds^2 = \alpha' k \left[dr^2 + \frac{R^2 \tanh^2 r \, d\varphi^2 \; + \; dx^2}{\tanh^2 r + R^2} \right] , \tag{2}$$

[1]There is no significant difference between this configuration and Sen's set-up, since none of the two D-branes under scrutiny warp the compact circle in the latter case.

[2]Long strings are finite energy states corresponding to macroscopic strings which grow linearly in global time and winds w times around the angular direction, w being the sector of *spectral flow* of the affine algebra.

with non-trivial H-flux and dilaton profile. The underlying conformal field theory is unitary for any real R; a T-dual description is given the orbifold $SL(2,\mathbb{R})/U(1)|_k \times U(1)_{kR^2}/\mathbb{Z}_{Nk}$, where N indicates the periodicity of the Euclidean time $x \sim x + 2\pi N$. This space has only two commuting Killing vectors, hence break most of the $SO(2,2)$ symmetry of AdS_3; however its spectrum shares most relevant features with the latter, including discrete states and long strings. Global Lorentzian AdS_3 is obtained by the analytic continuation $R^2 \to -1$, and a non-compact time coordinate (*i.e.* the universal cover of the group manifold) corresponds to the limit $N \to \infty$. We shall use the following prescription: all the computations are done in the Euclidean theory, for generic R, then the analytic continuation $R \to i$ is taken at the end; this may be subtle for non-analytic expressions like the bulk three-point function but of no concern here.

I have used this method to obtain the boundary states for D-branes in Lorentzian AdS_3 [18], which were not known previously. I have combinined the boundary states of the Euclidean black hole CFT $SL(2,\mathbb{R})/U(1)|_k$ [19] with Dirichlet or Neumann branes of a free boson in the diagonal orbifold theory, before performing the analytic continuation as described above. The relevant boundary states in the Euclidean black hole correspond to D0-branes localized at the tip, space-filling D2 branes and D1-branes with Dirichlet boundary conditions along the compact Euclidean time direction. Depending on the relative boundary conditions chosen in each factor, one obtains either boundary states preserving one copy of the chiral algebra – corresponding to (twined) conjugacy classes [20], for instance AdS_2 sub-manifolds – or symmetry-breaking branes [21]. The geometry of these branes has been shown to match the semi-classical results. I have also computed the corresponding annulus amplitudes and boundary two-points functions. Of particular interest is the symmetry-breaking D0-brane whose decay will be discussed below.

3. Tachyon condensation on non-BPS D0-branes

Universality of open string tachyon decay – in particular the open/closed conjecture of Sen – could be challenged in space-times with a different high-energy density of closed string states compared to Minkowski space. A different behaviour (UV-finite decay) was indeed suggested in [22] for non-critical strings, and could also be expected in anti-de-Sitter space-time, given the field-theoretical density of states. We have examined carefully these two systems with Eliezer Rabinovici [11] and found that, contrary to these expectations, the endpoint of the tachyon decay is still the closed string vacuum, with a 'tachyon dust' of closed strings with interesting properties.

The 'half S-brane' rolling tachyon solution in flat space-time corresponds to the following boundary deformation on the disk:

$$\delta S = \lambda \oint_{\partial D} d\ell \, \psi^0 e^{X^0/\sqrt{\alpha'}} \sigma^1 \,, \tag{3}$$

where σ^1 is a Chan-Patton factor associated with the fermionic boundary zero-modes, and describes the rolling of the tachyon, set at the maximum of its potential at minus infinity, towards its minimum.[3] This boundary interaction acts as a time-dependent source for the closed string field, leading to a coherent state that is associated with the 'tachyon dust', *i.e.* the remnant of the D-brane at the end of the decay. By standard field theory arguments the probability amplitude for a given closed string mode is given by its one-point function on the disk. By tracing over the whole spectrum one obtains the total average number of emitted closed strings as the imaginary part of the annulus

[3]It can be obtained as a $c \to 3/2$ limit of time-like super-Liouville theory, keeping the boundary cosmological constant λ finite but sending the bulk cosmological constant to zero.

amplitude; in flat space-time, this quantity is UV-divergent as massive string states are massively produced.

We have considered type IIB superstrings in an $AdS_3 \times S^3 \times T^4$ space-time with string coupling $g_6 = \sqrt{k/Q_1}$, corresponding to the back-reaction of Q_1 fundamental strings and k NS5-branes. The rolling tachyon boundary state for a non-BPS D0-brane in this background was obtained using the methods of section 2, taking into account that the diagonal orbifold discussed there acts on the boundary cosmological constant in eq. (3) as $\lambda \to \exp(\pi\sqrt{2/k})\lambda$.

There are several differences between the decay in flat-space and anti-de-Sitter, which conspire to make the physics rather different. The analysis of the annulus amplitude gives two types of contributions to the 'tachyon dust'. The first one comes from the localized states, or short strings, corresponding to world-sheets trapped by the confining potential of anti-de-Sitter space. The number of produced short strings at tree-level is finite; this is made possible by the field-theory-like entropy of high-energy states $S(E) \sim \sqrt{E}$, compared to the usual Hagedorn entropy of string theory in flat-space $S(E) \sim E$.

The contribution from long strings is associated with macroscopic string states of finite energy. Using the exact expression for the boundary state and the asymptotic high-energy density of string states one finds that the average number of long strings with winding number w behaves like:

$$\overline{\mathcal{N}(w)} \sim \frac{1}{\sqrt{w}}\, e^{-\pi\sqrt{\frac{k}{2}}\left(1-\frac{1}{2k}\right)w} \sum_N N^{-2} e^{2\pi\sqrt{\left(2-\frac{1}{k}\right)N}} \times e^{-2\pi\sqrt{\frac{2}{k}}\frac{N}{w}} \tag{4}$$

Unlike in flat space-time the sum over the oscillator number N is exponentially convergent, as the exponential falloff from the one-point function (last term) wins over the asymptotic density of states growth (last but one term); however the remaining sum over the spectral flow w is logarithmically divergent. One should impose a cut-off at $w \sim \sqrt{Q_1}$, such that the typical energy of long strings is of order $1/g_6$, and that the back-reaction of the long strings is small.

The physical picture that we have obtained is quite interesting. The decay of a localized D-particle in AdS_3 produces a gaz of macroscopic heavy long strings, having in the perturbative regime very large mean winding numbers, radial momenta and oscillator numbers, and growing linearly with global time, together a negligible proportion of short strings, see fig 2. We have computed the rate of open string production, which is also finite hence negligible compared to long closed strings production. The non-local behaviour of the system, which is very different from what happens in flat space-time, is due to the peculiar nature of quantum gravity in three-dimensions with a negative cosmological constant.

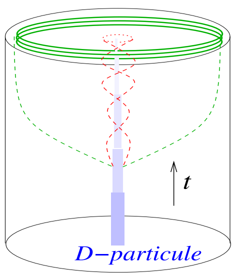


FIGURE 2. *Closed strings emission from D0-brane decay in global AdS_3.*

Anti-de-Sitter gravity in three-dimensions is expected to be holographically dual to a conformal field theory in two-dimensions. This should allow in principle to find a dual description of the brane decay. In the S-dual description the holography theory is identified in the UV with the gauge theory of the D1-D5 system, flowing to an $\mathcal{N} = (4,4)$ sigma-model on the moduli space of instantons on T^4. With only NS-NS fluxes turned on, as in our study, this theory is singular as a fundamental string can leave the system at no cost of energy, signalling the opening of a Coulomb branch [23].[4] A D-instanton in the bulk is associated with a BPS instanton in this theory, while the non-BPS D-particle that we have considered is expected, as in AdS_5 [24], to be the sphaleron associated with the maximum of the instanton barrier. We have observed that, oddly, the point-like D-instanton behave like a crosscap state in the dual conformal field theory. We did not pursue this study any further.

The second system that we were interested in is the decay of a non-BPS D1-brane in eight-dimensional type IIA non-critical strings, or equivalently in the background of two NS5-branes. As we have seen in section 1 it has another description as a BPS D0-brane falling onto one of the five-branes. The analysis is similar to the previous case, but simpler as the time direction is not curved. The boundary state is obtained from the tensor product of an A-type boundary state in $\mathcal{N} = 2$ Liouville theory and of the ordinary rolling tachyon. While the asymptotic density of states is lower than in flat ten-dimensional space-time, behaving like $\log D(N) \sim 2\pi\sqrt{3N/2}$ rather than like $\log D(N) \sim 2\pi\sqrt{2N}$, the coefficient of the boundary state in Liouville shifts the saddle point of the integral over the radial momentum in the such a way that the tree-level closed string production is divergent as in flat space. This corrects the claim made in [22], where only the effect of the density of states was taken into account.

4. Tachyon condensation in the brane-antibrane system

A pair of BPS Dp-branes with opposite Ramond–Ramond charges breaks space-time supersymmetry and feels an attractive force mediated both by the exchange of gravitons and RR fields. In the open string channel, the open string sectors of each D-brane get coupled by a non-supersymmetric sector of open strings ending on both D-branes, leading to a non-zero one-loop vacuum amplitude on the cylinder. If the D-brane and the anti-D-brane are close enough to each other (explicitly for distances $d < d_c = \pi\sqrt{2\alpha'}$), the lightest complex scalar mode becomes tachyonic. This system was the main topic of research of Flavien Kiefer during his Ph.D. under my supervision.

In the tree-level approximation, the system of a D-brane and an anti-D-brane at fixed finite distance is well-defined and, for separations below the critical distance d_c, one may in principle study the decay of the open string tachyon while holding fixed the separation between them. This system is interesting as, approaching d_c from below, the tachyon can be made arbitrarily 'light', or, equivalently, the tachyon potential very shallow. Starting at a fixed sub-critical distance and zero relative velocity, one can always choose the string coupling constant small enough such that the time-scale associated with the tachyon decay is much smaller than the time-scale associated with the collision between the branes.

These expectations from the qualitative analysis are in blatant contradiction with the known effective action for the system. Starting from the non-Abelian DBI-like $U(2)$ action for two coincident non-BPS D-branes [25], Garousi obtained this action after an orbifold by the left-moving fermion number [26]. In flat space-time, the dynamics of

[4]These are the macroscopic 'long string' solutions in AdS_3 that were discussed in this section, which are copiously emitted by the D0-brane decay.

the D0/anti-D0 system is claimed to be governed by the action (considering topologically trivial tachyon configurations):

$$S = -T_0 \int dt\, V(|T|) \sqrt{1 + \frac{|T|^2 d^2}{4\pi^2 \alpha'} - \frac{\dot{d}^2}{4} + \alpha' |\dot{T}|^2}\,. \tag{5}$$

It can be checked that when the tachyon modulus $|T|$ is displaced from the origin, the equations of motion force the distance d to evolve also with time. We have investigated numerically whether, with generic initial conditions, the system evolves towards an attractor solution that could be the starting point of a worldsheet analysis. For instance, one could have brane coalescence before the onset of tachyon condensation. We have observed that, on the opposite, the system is unstable as energy can be constantly exchanged between the distance and tachyon fields.

These conclusions should be taken with a grain of salt, as the range of validity of this effective action has not been given. The non-BPS brane tachyonic effective action, see eq. 1, is valid for small and slowly varying perturbations around the rolling tachyon solution, while the ordinary non-Abelian DBI action for a stack of BPS branes is valid for small and slowly varying perturbations around constant field configurations, and at separations small compared to the string length. This ensures a hierarchy of scales between the fields that are kept in the effective action and string oscillators. In the effective action (5), one keeps both light fields and fields with a mass squared scale of order $1/\alpha'$, the tachyon when the distance is small or the distance field close to the critical value.

These considerations motivated a detailed study of the brane-antibrane system from the worldsheet point of view with Flavien Kiefer. This work was inspired by an article of Bagchi and Sen [27], who showed that the rolling tachyon in a D-brane/anti-D-brane system at finite and fixed distance is an exactly conformal boundary CFT, but only for brane separations in the range $0 \leqslant |d| < d_c/\sqrt{2}$. This was a surprise as we expected that the physics was the same up to the critical distance. We have found in [9] convincing evidence that it is indeed the case.

The starting point for this study was the super-space action for the T-dual system, a pair of a D1-brane and an anti-D1-brane wrapped on a circle with Wilson lines. The boundary action on the disk reads in components

$$S_b = \oint_{\partial D} d\ell\, \left(\eta\dot{\bar{\eta}} - \eta\psi T - \bar{\eta}\bar{\psi}\bar{T} - F\bar{F} - FT - \bar{F}\bar{T} \right)\,, \tag{6}$$

with a tachyon of the form $T = \frac{\lambda}{2\pi} \exp\left(\omega X^0 + irX\right)$, where $0 \leqslant r < r_c = {}^{1}/\sqrt{2}$ is the T-dual of the brane separation and $\omega^2 + r^2 = {}^{1}/{}_{2}$. Canonical quantization of the bottom component η of the boundary Fermi super-field yields the Chan-Patton algebra, while integrating out the auxiliary field F gives a *contact term*. If one introduces a short-distance cut-off ε it leads to a quadratic interaction in the tachyon $\frac{\lambda^2}{4\pi^2}\, \varepsilon^{1-4r^2} \oint d\ell\, :e^{2\omega X^0}:$ which plays an important role.

The boundary CFT defined by the boundary action (6) cannot be solved exactly hence one needs to resort to conformal perturbation theory. When the inter-brane distance is in the range $0 \leqslant r < {}^{r_c}/\sqrt{2}$ the perturbative integrals are finite and the theory is conformal. Divergences appear whenever the fusion of tachyon vertex operators produce marginal or relevant operators. The first divergence of this kind occurs for $r = r_c/\sqrt{2}$, where $:e^{2\omega X^0}:$ becomes marginal; this was noticed by Bagchi and Sen in [27] who concluded that the theory is not conformal any more past this value. We have shown that the contact term already discussed, dictated by worldsheet supersymmetry, precisely cancels this divergence and leads to UV-finite perturbative integrals.

In order to study in detail whether this model is conformal for larger separations we have computed the beta-functions for the boundary couplings, including the tachyon, the distance-changing operator and the operators of the form $: e^{n\omega X^0} :$ that can appear by fusion of tachyon vertices. It is convenient to work in the minimal subtraction scheme, for which only logarithmic divergences contribute to the beta-functions [28]. Following the discussion above the theory is conformal to all orders up to the next 'resonance' at $r = \sqrt{7}/4$ where $: e^{4\omega X^0} :$ becomes marginal. The perturbative integrals are actually finite at least at fourth order for $r < \sqrt{7}/4$ thanks to the contact term. The explicit computation of the regularized perturbative integrals at fourth order, which was complicated by the path ordering of operators, indicated that no logarithmic divergence is present when $r \to \sqrt{7}/4^-$. A power-law divergence remains for $r > \sqrt{7}/4$, corresponding to four tachyon colliding, which cannot be removed by the contact term, but does not contribute to the beta-functions. This shows that the theory is conformal to all orders at least up to the next resonance at $r = \sqrt{17}/6$. From this value and up to the critical distance $r_c = 1/\sqrt{2}$ one would need to study sixth-order and higher perturbative integrals that are quite intractable. We shall safely assume that the conclusion would be the same as no change in behaviour is expected.

This study has important consequences as far as the effective action of the system is concerned. It implies first that the effective action of Garousi, eq. (5), is not valid around the rolling tachyon solution that we have discussed, even when the two branes are very close to each other. At quadratic order in the tachyon modulus τ, the effective Lagrangian should indeed be of the form $\mathcal{L} \sim \sqrt{1-2r^2}\,(\frac{\tau^2}{2} + \frac{\dot{\tau}^2}{1-2r^2}) + \mathcal{O}(\tau^4)$ in order to admit this solution. Writing the full non-linear action is not possible without further assumptions. One can also compare the effective Lagrangian evaluated on-shell on the rolling tachyon solution with the disk partition function; at quadratic order is gives a non-trivial relation between the space-time and worldsheet perturbations

$$\tau(t) = \left(\frac{1}{\sqrt{1-2r^2}} \frac{\Gamma(2-4r^2)}{\Gamma^2(1-2r^2)} \right)^{1/2} \lambda\, e^{\omega t} \,. \tag{7}$$

The left-hand-side vanishes when r approaches the critical distance $1/\sqrt{2}$. The worldsheet theory is also peculiar in this limit as an infinite number of operators become marginal; it would be interesting to compare with the theory obtained by taking the limit from above.

Finding the full non-linear action for the tachyon and distance fields remains an interesting open problem. It is an important question, both for our understanding of brane dynamics and for applications in cosmology or holography. Further constraints could be obtained by finding other exact solutions of the world-sheet theory[5] and by computing explicitely the disk partition function at any brane/antibrane separation,which seems out of reach at the moment.

[5]For the rolling tachyon on non-BPS D-branes there exist both the 'half S-brane' solution, eq. (3) and the 'full S-brane' perturbation in $\cosh(X^0/\sqrt{\alpha'})$. We have shown that there is no similar marginal perturbation at non-zero distance between the branes.

2

Field theory dynamics from brane configurations

Brane configurations provide invaluable insights into the quantum dynamics of gauge theories. The low-energy dynamics on a stack of D-branes is given by $U(N)$ super-Yang–Mills in ten dimensions and dimensional reductions thereof, including $\mathcal{N} = 4$ SYM in four dimensions. UV completions of gauge theories with fewer supersymmetries can be obtained by wrapping branes around supersymmetric cycles of (compact or non-compact) Calabi-Yau manifolds and flavoured theories can be considered by adding higher-dimensional supersymmetric D-branes to the geometry. Supersymmetric orientifolds give rise to orthogonal or symplectic gauge groups.

Many aspects of these constructions – as geometric transitions and associated field theory dualities – are simpler to understand in equivalent, T-dual set-ups which consist in D-branes ending on NS5-branes in an ambient flat space-time. The fivebranes, being much heavier than the D-branes in the perturbative regime (their tension scales like $1/g_s^2$ compared to $1/g_s$), can be considered as static objects that impose boundary conditions in the world-volume theory of the D-branes ending on them, preserving some supersymmetry; gauge theory degrees of freedom can be then decoupled from gravity by sending the string coupling to zero. These models are usually referred as Hanany–Witten constructions [29]; see [30] for a review. An example of such brane engineering, which was used to derive an infrared Seiberg-like duality in an $\mathcal{N} = 1$ Chern–Simons theory with matter in three dimensions, will be given in section 1; this work was done in collaboration with Adi Armoni, Amit Giveon and Vasilis Niarchos [31].

Neglecting the back-reaction of the fivebranes in the field theory limit, even if gravity naively decouples, is questionable. The four-dimensional Yang–Mills coupling is related to the string coupling and the typical length scale L of the separation between the fivebranes in string units as $1/g_{YM}^2 \sim L/g_s$. The field theory limit corresponds to $g_s \to 0$ with g_{YM} held fixed. As the fivebranes become closer to each other, the fivebrane world-volume theory remains interacting; this limit corresponds to a *double scaling limit* of the NS5-brane theory [30]. While the world-volume theory on the NS5-branes and D-branes decouple from gravity, the gravitational background of the fivebranes is taken to its near-horizon limit, giving a fivebrane holographic duality akin to AdS/CFT. Engineering field theories from D-branes in the near-horizon background of fivebranes is not only more correct from the conceptual point of view, but allows also to derive important rules of branes dynamics as the brane creation effect. In an article with Adi Armoni, Gregory Moraitis and Vasilis Niarchos [32], these techniques were used to propose a Seiberg-type duality in a non-supersymmetric four-dimensional gauge theory; in this case the exact construction of D-brane boundary states in the near-horizon fivebranes background was even more necessary as the 'brane cartoons' did not make sense. This will be explained in section 3.

The D-brane set-up that realises this non-supersymmetric gauge theory is embedded in an unusual string theory that, despite having no space-time fermions, has no tachyons, which is very interesting on its own right. It is a non-critical analogue of the type 0′B orientifold of Sagnotti [33,34], with neither RR nor NSNS tadpoles, in contrast with the critical case. This work was done with Vasilis Niarchos [35] and will be discussed in section 2; to our knowledge it was the first string theory without fermions stable at tree-level. A key ingredient in this model was a particular crosscap state of $\mathcal{N} = 2$ Liouville theory. In a preceding article, we have analysed several types of orientifolds of this superconformal field theory, including the relevant ones for this construction [36], and considered also supersymmetric Hanany–Witten set-ups associated with orthogonal and symplectic gauge groups in the back-reacted fivebrane geometry.

The fivebrane world-volume theory is decoupled from gravity in the $g_s \to 0$ limit, as mentioned already, giving a *little string theory*. In type IIB the low-energy excitations of the theory (which has features alien to field theories as T-duality symmetry) contain a super-Yang–Mills multiplet, giving a holographic correspondence between non-gravitational gauge theories and near-horizon backgrounds of five-branes [37]. In this context I have obtained in [38] a holographic duality between $\mathcal{N} = 1$ quiver gauge theories in four dimensions, near Argyres–Douglas infrared fixed points, and perturbative string theory backgrounds with a known worldsheet description. This seems to be the first case of $\mathcal{N} = 1$ holographic duality solvable on the string theory side; this work will be discussed in section 4. I have studied also the D1-branes in this holographic background which correspond to the dyons of the $\mathcal{N} = 1$ gauge theory. Finally in section 5 I will present a case of heterotic five-brane holographic solution, given by a smooth non-Kähler resolved conifold with torsion preserving $\mathcal{N} = 1$ supersymmetry, and which is expected to be dual to a confining gauge theory. This example is borrowed from the analysis of heterotic flux vacua developed in chapter 3.

1. $\mathcal{N} = 1$ Chern–Simons dualities

$\mathcal{N} = 1$ Chern–Simons (CS) theories in three dimensions with matter are very interesting field theories, as several examples have strongly coupled infrared dynamics. Unlike theories with more supersymmetry there are no non-renormalization theorems – in particular because the scalar multiplets contain real, rather than complex, scalar fields – that could help to understand their dynamics beyond perturbation theory. Brane constructions of such theories are very helpful as they allow to postulate some Seiberg-like magnetic duals that probe their strong coupling regime.[1] An important application would be to obtain superconformal theories holographically dual to $\mathcal{N} = 1$ AdS_4 solutions, which are the generic vacua of type II flux compactifications.

In [31] we were interested in a theory for which a Seiberg duality was expected from a different perspective. In four-dimensional $\mathcal{N} = 1$ $SU(N)$ super-Yang–Mills the $U(1)_R$ symmetry is broken to $\mathbb{Z}_{2N}$ by instanton effects and the gaugino condensate breaks it further to $\mathbb{Z}_2$. There exist BPS domain walls interpolating between the N different vacua; as confining strings can end on them they are analogous to D-branes. A set of k domain walls forms a bound state which is expected to support on its world-volume $U(k)$ gauge dynamics. An elementary consequence of charge conjugation symmetry in the four-dimensional SYM theory is that a bound state of k walls interpolating between k vacua 'clockwise' is equivalent to a bound state of $(N - k)$ anti-walls interpolating 'anti-clockwise' between $N - k$ vacua.

A concrete proposal for the effective theory on the domain walls bound state world-volume was made by Acharya and Vafa [39]. $\mathcal{N} = 1$ $U(N)$ SYM is obtained from D6 branes wrapping the three-cycle of the deformed conifold, giving after a conifold transition a resolved conifold with RR flux; D4-branes wrapping the S^2 gives the domain wall theory, consisting in an $\mathcal{N} = 2$ $U(k)$ SYM theory with an $\mathcal{N} = 1$ Chern–Simons

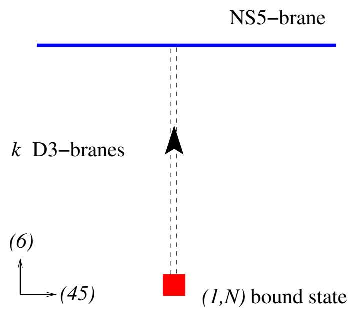
[1]A check of 't Hooft anomaly matching between the conjectured dual field theories is not available as the theory is defined in odd dimension.

term at level N induced by the RR flux:

$$(8)\quad S=\frac{1}{4g_{YM}^2}\int d^3x\,\mathrm{Tr}\left((D\phi)^2-F_{\mu\nu}F^{\mu\nu}+i\bar{\chi}\not{D}\chi+i\bar{\psi}\not{D}\psi+2i\bar{\chi}[\phi,\psi]\right)$$
$$+\frac{N}{4\pi}\int\mathrm{Tr}\,\left(A\wedge dA+\tfrac{2}{3}A\wedge A\wedge A\right)-\frac{N}{4\pi}\int d^3x\,\mathrm{Tr}\,\bar{\chi}\chi\,.$$

The flat direction parametrized by the vacuum expectation value of the adjoint scalar ϕ is lifted by quantum effects, except its $u(1)$ part; the two-loop attractive Coleman-Weinberg potential is compatible with the expected formation of bound states in the domain wall interpretation [40].

In order to engineer this Chern–Simons-SYM theory we started with a stack of k D3-branes ending on two separated NS5-branes, giving a $U(k)$ theory whose amount of supersymmetry depends on the angles between the fivebranes. Replacing one of the NS5-branes by an $(N,1)$-fivebrane – *i.e.* a bound state of N D5-branes and one NS5-brane – adds a CS term at level N, as can be seen by applying an $SL(2,\mathbb{Z})$ transformation to the boundary conditions set by NS5-branes [41]. We chose specific angles such that the configuration preserves $\mathcal{N}=1$ supersymmetry and that there exists an 'accidental' common world-volume direction x^3 to the fivebranes, corresponding to the classically massless scalar ϕ in eq. (8), see fig. 1. In this classical brane configuration the D3-branes seem free to separate from each other in the x^3 direction, which would contradict the field theory results quoted above.[2] This puzzle can be solved by a combination of two important principles of brane dynamics, that will also provide the dual magnetic description.



k D3-branes	:	012				|6|	
NS5-brane	:	012	3	4	5		
(N, 1) fivebrane	:	012	3		$\begin{bmatrix}5\\9\end{bmatrix}_{-\frac{\pi}{2}-\theta}$		8

FIGURE 1. *Brane set-up for the Acharya–Vafa theory. Supersymmetry sets* $\tan\theta=g_sN$.

A first important principle of brane dynamics is the *s-rule*, related by U-duality to the Pauli exclusion principle [42], which states that if several D3-branes stretched between an NS5-brane and a D5-brane coincide they break supersymmetry; it is still valid in the Acharya–Vafa configuration, i.e. with five-brane bound states. A second important ingredient is the *brane creation effect* [29]: when a D5-brane crosses an NS5-branes, a D3-brane is created. In the present case, the $(N,1)$ bound state crossing the NS5-brane creates N D3-branes, k of which annihilating with the D3-branes already present, that flipped their orientation during the transition. One obtains the *magnetic* theory, which is analogous to the original Acharya–Vafa theory but with a $U(N-k)$ gauge group. If we had started with $k>N$ D3-branes in the original 'electric' theory, we would have ended with $k-N$ anti-D3-branes, breaking all supersymmetry. Such supersymmetry-breaking configuration would have been discarded by the s-rule from the start in the 'electric' theory only if the D3-brane were forced to be on top of each other, because of an attractive force between them. In this case the s-rule indicates indeed that only the configurations with $k\leqslant N$ are supersymmetric, which is compatible

[2]For this set-up we could not easily use the near-horizon geometry of the fivebranes – that we advertised as the appropriate framework – given that the background has Ramond-Ramond flux.

with the domain wall interpretation and is confirmed by an explicit computation of the Witten index.

A quantum effect becoming classical in the magnetic theory is a familiar situation with Seiberg duality. Let us discuss in more detail the quantum dynamics of the 'electric' Acharya–Vafa theory defined by the action (8) and of its 'magnetic' dual. At energies well below the mass scale $m_{CS} = g^2_{YM}N$, one can ignore the kinetic terms of the gauge multiplet. The theory becomes $\mathcal{N} = 1$ Chern–Simons at level N coupled to an adjoint scalar multiplet. In the large N limit its dynamics is governed by the 't Hooft coupling $\lambda = k/N$. As we have observed there is a two-loop effective potential for the scalar giving a mass term of order $m_{CW} = m_{CS}/N$, which is parametrically smaller than m_{CS} at large N. Below this scale, the theory becomes topological, being pure CS with a decoupled free scalar multiplet corresponding to the $U(1)$ part of the gauge group.

At energies below the Chern–Simons scale m_{CS} the magnetic theory is strongly coupled, its 't Hooft coupling being $\tilde{\lambda} = 1 - \lambda$. In the energy range $m_{LOOP} \ll E \ll m_{CS}$ one expects that the relation between the electric and magnetic theory (which is justified from the brane construction as the brane motion does not go through any singularity) amounts to a non-trivial Seiberg-like duality. At energies well below m_{LOOP} the duality is nothing but the familiar level-rank duality of CS theory, namely the equivalence between $SU(k)$ CS at level $N - k$ and $SU(N-k)$ CS at level k.[3] The existence of a 'dynamical' duality above the scale m_{LOOP} can be justified by noticing that the wall tension formula

$$T_k = \frac{N^2\Lambda^3}{4\pi^2} \sin\frac{\pi k}{N}\,, \tag{9}$$

is invariant under Seiberg duality. This quantity, which is known exactly from the four-dimensional SYM perspective, is a non-perturbative statement in three dimensions about the dynamics of the Acharya–Vafa theory, giving the biding energy whose leading term in $1/N$ is the Coleman-Weinberg potential that we have mentioned.

We have studied also related three-dimensional $\mathcal{N} = 1$ theories having non-trivial dynamics in the deep infrared. For instance one can consider, in the set-up of figure 1, n NS5-branes instead of a single one. In this case the scalar multiplet corresponding to the x^8 direction, instead of a mass term, has a superpotential $W \sim \mathrm{Tr}\, X^{n+1}$. As before the quantum dynamics of the theory can be probed using a combination of the S-rule and of the brane creation effect. In this case we have postulated the existence of a conformal window as a function of the 't Hooft coupling $\lambda = k/N$ giving interesting interacting superconformal field theories; they are expected to be holographically dual to $\mathcal{N} = 1$ AdS_4 solutions, that could be found if one knew how to take into consideration the branes back-reaction.

2. Non-fermionic string theories

String theory is intimately tied with supersymmetry, which guarantees the absence of tachyons hence perturbative stability of the theory. Whether there exist non-supersymmetric string theories, or even more radically theories without closed string spacetime fermions, is an interesting fundamental question. Closed string theories of this sort would also be very important in view of gauge theory applications. Firstly, by adding appropriate D-branes one can engineer interesting non-supersymmetric field theories and study their dynamics, as we will see in section 3. Secondly, one expects that the holographic dual of pure Yang-Mills (or other gauge theories without fermionic gauge-invariant operators) is a non-critical string theory without fermions.

[3]The level of the CS terms are shifted by k (electric) and $N - k$ (magnetic) units respectively when one integrates out the massive gravitino χ.

An immediate objection to this project is a strong constraint coming from modular invariance. It has been argued by Kutasov and Seiberg [43] that the infrared divergence of the one-loop amplitude when a closed string tachyon is present is related by a $\tau \mapsto -1/\tau$ modular transformation to an exponential mismatch in the high-energy asymptotic densities of space-time bosons and fermions; hence a tachyon-free theory should exhibit *asymptotic supersymmetry* [44]. As usual this no-go theorem tells us where to look for in order to avoid its conclusion: a theory of *unoriented strings*, since the Klein bottle amplitude evades the modular-invariance constraint.

An example of such unoriented theory without tachyons, type 0′B, was indeed obtained by Sagnotti [33,34], by modding out the type 0B theory by the parity $\Omega' = (-)^{\bar{F}}\Omega$ where Ω is the usual worldsheet parity and $\bar{F}$ the right-moving worldsheet fermion number. The associated Klein-bottle amplitude gives a Ramond-Ramond tadpole that should be cancelled by space-filling D-branes. It leaves however an NS-NS tadpole uncancelled, indicating that the vacuum of the theory should be modified. In order to evade this issue of vacuum redefinition, and to have more freedom in the open string completion of the theory, we have considered with Vasilis Niarchos non-critical analogues of the type 0′B theory, given that their spectrum of delta-function normalizable states has a mass gap [35]. It was necessary to construct first the crosscap states of $\mathcal{N} = 2$ Liouville theory; I will briefly summarize our work [36] below.

2.1. Orientifolds in $\mathcal{N} = 2$ Liouville theory. Perturbative non-critical superstrings are built upon $\mathcal{N} = 2$ Liouville theory, which has an $\mathcal{N} = (2, 2)$ superpotential

$$W = \mu\, e^{-\frac{\Phi}{Q}} \,, \quad \Phi = \rho + i\varphi + \sqrt{2}\,\theta^{\alpha}\psi_{\alpha} + \cdots \,, \tag{10}$$

that regularizes the strong coupling region $\rho \to -\infty$ of the linear dilaton of background charge Q.[4] The basic parities compatible with $\mathcal{N} = 2$ supersymmetry are the A-type and B-type parities, which are exchanged by mirror symmetry [45]. The A-parity is a symmetry of the action with superpotential (10) provided that the cosmological constant μ is real; under the B-parity the superpotential is odd. Combining the B-parity with involutive symmetries we have defined B-type orientifolds leaving the action invariant, such as $s\Omega_B$, where s is the half-shift $\varphi \to \varphi + \pi/Q$, or $(-)^{\bar{F}}\Omega_B$. Other A-type and B-type parities were obtained with more general dressings by discrete symmetries.

A mirror description of $\mathcal{N} = 2$ Liouville is the axial super-coset $SL(2,\mathbb{R})/U(1)$ at level $k = 2/Q^2$ [46]; both are useful in understanding the orientifolds of the model. The $\mathbb{Z}_2$ symmetries of the WZW model that can be combined with worldsheet parity-reversal are easily seen with the embedding equation $X_0^2 + X_3^2 - X_1^2 - X_2^2 = k$ of AdS_3 in $\mathbb{R}^{2,2}$; they should reverse the orientation to preserve the Wess-Zumino term. For instance $\tau_2 : X^3 \mapsto -X^3$, which keeps invariant a space-like slice of AdS_3, descends in the coset to a space-filling B-type orientifold, identified in the mirror with the 'basic' A-parity of Liouville. This parity can be dressed by a half-winding shift $\tilde{s}$ along compact global time, giving another B-type orientifold. The crosscap states associated with each of the consistent parities have been found using different methods. In each case, the geometrical interpretation allowed us to postulate the form of the 'asymptotic' Klein bottle amplitude, *i.e.* associated with the delta-function normalizable states (insensitive to the strong coupling region $\rho \to -\infty$) which was used as a consistency check for the exact crosscap states.

The B-type orientifolds of $SL(2,\mathbb{R})/U(1)$ are the easiest to determine as there exists a localised B-type D-particle whose spectrum is made only of *identity* representations of the superconformal algebra. For both orientifolds $\tau_2\Omega_B$ and $\tilde{s}\tau_2\Omega_B$ we have determined the crosscap state by the *modular bootstrap* method. We started with the open

[4] For non-critical strings in $d = 2n$ dimensions with $SO(d-3,1)$ symmetry, $Q = \sqrt{5-n}$.

channel Möbius amplitude for the B-type D0-brane, that we postulate in view of the orientifold geometrical action. Having determined the $\mathcal{P}$ modular matrix, we obtained the closed string channel amplitude which is linear in the crosscap state. A cross-check was made by comparing the Klein bottle amplitude with the asymptotic expression determined by the orientifold action in the closed string sector. The wave-functions associated with these two orientifolds indicate that they consist in a localized source of odd (resp. even) winding modes together with an extended source of even (resp. odd) winding modes. This combination of localized and extended orientifolds, tied together

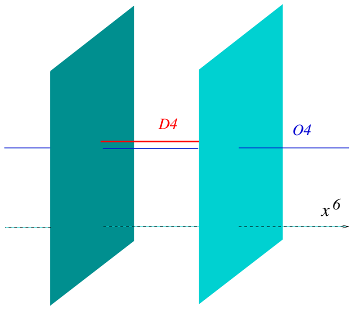


FIGURE 2. *Brane set-ups with O4-planes*

by the modular properties of the $\mathcal{N} = 2$ characters, fits nicely with the physics of type IIA Hanany–Witten set-ups with O4-planes, see fig. 2. The near-horizon limit of two parallel fivebranes gives non-critical superstrings in eight dimensions; then the B-type orientifold $\tau_2\Omega$ gives the O4-plane on the figure while the B-type localized boundary states give the 'suspended' D4-branes. The localized piece of the orientifold wave-function corresponds to the segment between the fivebranes, while the extended piece corresponds to the semi-infinite parts to the left and to the right; as expected these two parts have opposite Ramond-Ramond charges. We have computed the Möbius amplitude for this brane/orientifold configuration and showed that it vanishes thanks to supersymmetry.

As no localized A-type branes exist in the axial coset, we had to use a different technique for the A-type orientifolds . We first 'lifted' the result for the B-orientifolds of the axial coset to the H_3^+ theory, using the equivalence $SL(2,\mathbb{R})/U(1) \sim H_3^+/\mathbb{R}$. In this 'parent' theory the O2- and O1-plane wave-functions are related by and $SL(2,\mathbb{C})$ rotation, from which we obtained the O1-plane in the coset. We have checked that our result is compatible with partial conformal bootstrap results in H_3^+ [47] which couldn't fully determine the crosscap state.

2.2. Non-critical type 0′B theories. Some of the orientifolds of $\mathcal{N} = 2$ Liouville theory are particularly interesting as they allow to obtain non-fermionic string theories with neither tachyon nor tadpoles; they are unique examples with these features.

Type 0B non-critical strings in four dimensions is defined as the diagonal GSO projection of the tensor product $\mathbb{R}^{3,1} \times \mathcal{N} = 2$ Liouville theory with $Q = \sqrt{2}$. The physical spectrum contains only delta-function normalizable states w.r.t. the linear dilaton ρ, in particular a tachyon field T with arbitrary momentum and winding (n, w) along φ; the ground state $(n = w = 0)$ has negative mass squared. The B-type orientifold $s\,(-)^{\bar{F}}\Omega$ discussed in subsec. 2.1 projects out this ground state, while leaving invariant combinations of tachyon zero-modes which are massless or massive thanks to the universal mass shift $\alpha'\Delta m^2 = Q^2/2$ in the delta-function normalizable spectrum. Thus the one-loop amplitude, given by the sum of the torus and Klein bottle amplitude, is tachyon-free. This theory has a space-filling O5-plane, nevertheless it doesn't have any RR tadpole since its wave-function sources only odd winding modes. The theory is consistent and stable at tree-level, without needing an open string sector.

Non-abelian gauge theories can be engineered by adding to this unoriented closed string theory D-branes localized in the Liouville direction, tensored with p-dimensional D-branes of $\mathbb{R}^{3,1}$. Only for p even one obtains boundary states without tachyon in the open string sector, similar to 'dyonic' D-branes of critical type 0B theory, because the localized D-branes in super-Liouville have A-type boundary conditions. It allows to obtain interesting non-supersymmetric gauge theories in three dimensions. Four-dimensional theories are obtained using six-dimensional non-critical strings.

The six-dimensional non-critical type 0′B theory, based on $\mathbb{R}^{5,1} \times \mathcal{N} = 2$ Liouville with $Q = 1$, was also considered. This model is obtained from the type 0B non-critical theory by quotienting with $s\,(-)^{\bar{F}}\Omega$; this choice is morally the same as in the critical case, as the compact boson is at the fermionic radius, justifying *a posteriori* the choice of B-type orientifold. The exact crosscap state was determined using the results of subsec. 2.1. As before the orientifold projects out the delta-function normalizable tachyon[5] and there are no RR or NSNS tadpoles from the Klein bottle amplitude. As $\mathcal{N} = 2$ Liouville theory with $Q = 1$ has an $\mathcal{N} = 4$ superconformal symmetry one can evade the constraints found in the previous case and get stable D-branes giving non-Abelian gauge theories in four dimensions without supersymmetry, related to $\mathcal{N} = 2$ SYM.

The near-horizon limit of two parallel fivebranes in type II gives six-dimensional non-critical type II theories. In this perspective, one can interpret the six-dimensional type 0′B theory as the mirror of a type 0A configuration of two parallel NS5-branes whose world-volume is spanned by $x^0, \ldots, x^5$ and an O′6-plane given by the quotient $\mathcal{I}_{789}(-)^{\bar{F}}\Omega$. This geometrical interpretation was confirmed by considering set-ups with an arbitrary number of fivebranes, and using the results of [48] for the orientifold geometry on the three-sphere. In this brane configuration there exists a 'bulk' closed string tachyon since the orientifold is not space-filling. However in the double-scaling limit leading to the non-critical string, only the 's-wave' mode in the directions transverse to the fivebranes is kept, therefore there is no tachyonic state at least at tree level.

Finally one can extend the Kutasov and Seiberg theorem [43] in light of our results. In the closed string sector, one can evade the requirement of asymptotic supersymmetry by considering unoriented strings. If the theory admits open string sectors the absence of asymptotic supersymmetry is related by channel duality to a coupling of the boundary state to a closed string tachyon. This leads us to postulate the following theorem: *a string theory admitting open strings descendants is tachyon-free in the closed string sector if and only if all the open string sectors associated with the allowed D-branes exhibit asymptotic supersymmetry in their spectrum.*

3. Seiberg duality in orientifold QCD

Engineering non-supersymmetric field theories from D-branes dynamics is only possible if there are no couplings between the D-branes and closed string tachyons, which would persist in the low-energy decoupling limit; hence a non-critical type 0′B theory is a good starting point. The map between these constructions and Hanany–Witten set-ups allows to get many insights into gauge theory dynamics. This led us to obtain a precise conjecture for Seiberg duality in a non-supersymmetric theory [32].

The field theory of interest, *orientifold QCD*, is known to be planar equivalent to $\mathcal{N} = 1$ SQCD at large N in their common sector [49], implying the existence of a non-supersymmetric Seiberg duality in this limit; the embedding in non-critical strings allowed us to extend these statements to *finite* N. The OQCD theory has a $U(N_c)$ gauge group, N_f scalar and fermion flavours and 'gauginos' in either the symmetric (OQCD-S) or antisymmetric (OQCD-AS) representations. This theory has classically a moduli

[5] In general there exist localized modes in super-Liouville with $Q \leqslant 1$ (discrete representations). In this model with $Q = 1$ the only localized mode is massless.

space as SQCD, which is lifted quantum mechanically at finite N_c. Embedding this theory in string theory is not straightforward. At large N_c one expects that the back-reaction of the D-branes will lead to holographic duality between OQCD and a string theory without fermions, given that there are no fermionic gauge-invariant operators, but without tachyons; this points towards a non-critical type 0′B theory.

3.1. Embedding in string theory. The orientifold of type 0B on $\mathbb{R}^{3,1} \times SL(2,\mathbb{R})_1/U(1)$ by $\Omega(-)^{Q_R}$, where Q_R is the left-moving R-charge, is the appropriate setting. It is B-type in $SL(2)/U(1)$ and acts on the NSNS ground states with momentum and winding (n, w) as $|n, w\rangle \mapsto (-)^{n+w+1}|-n, w\rangle$, projecting out the closed string tachyon; the corresponding crosscap state $|0'\rangle$ is a close analogue of the examples presented subsec. 2.2, but the geometrical interpretation is less clear here.

The 'colour' branes $|c\rangle$ are made of localized B-type boundary states in $SL(2)/U(1)$. The open string one-loop amplitude reveals that the spectrum contains a $U(N_c)$ gauge boson and a 'gaugino' in the symmetric or antisymmetric representation depending on the orientifold charge. The B-type 'flavour' branes $|f; s, \theta\rangle$, which extend along the linear dilaton direction ρ, are characterized by their Wilson line θ around the asymptotic circle φ and a parameter s indicating how close they get to the tip of the cigar geometry. The open string sectors between the colour and flavour branes give, for anti-flavour branes with $\theta = \pi$, a pair of quark 'multiplets' with mass $m = \sqrt{2}s$ in the fundamental and anti-fundamental of $U(N_c)$. With N_f such branes one gets the massless 'electric' OQCD theory for $s = 0$.

As in other brane set-ups the 'magnetic' theory is obtained when the two fivebranes cross each other. It corresponds to the transformation $\mu \mapsto -\mu$ on the cosmological constant in (10), which is equivalent to a half-period winding shift along φ. Inspired by similar considerations in supersymmetric theories [50] we have obtained the transformation of the boundary and crosscap states under this involution. The colour branes are mapped to themselves, the crosscap state $|0'B\rangle$ gets opposite RR charge and the flavour anti-branes pick a colour brane factor:

$$\overline{|f; 0, \pi\rangle} \overset{\mu \mapsto -\mu}{\longrightarrow} |f, 0, \pi\rangle + \overline{|c\rangle}\,. \tag{11}$$

This was obtained by examining how the boundary action for the 'flavour' branes behaves under $\mu \mapsto -\mu$. After tachyon condensation on the 'colour' D-$\bar{D}$ pairs one naively gets the magnetic theory. This cannot be the full story, as it violates RR-charge conservation. The latter requires the creation of four 'colour' branes or anti-branes (depending on the orientifold charge) during the transition, which is familiar in Hanany–Witten set-ups with orientifolds [30].[6] One gets that the magnetic theory is given by the massless open string states corresponding to the boundary and crosscap states

$$(N_f - N_c \mp 4)\overline{|c\rangle}\,, \quad N_f|f, 0, \pi\rangle\,, \quad \pm\overline{|0'\rangle}\,, \tag{12}$$

where $\pm$ refers to symmetric or antisymmetric fermions respectively. The magnetic theory is similar to the electric one, except that the gauge group is $U(N_f - N_c \mp 4)$. In both cases the global symmetry is $SU(N_f) \times SU(N_f) \times U(1)$.

Having determined the field content of the low-energy electric and magnetic theories, on can investigate whether they have some relevant couplings. Instead of computing the boundary N-point function, which is highly non-trivial, the interactions can be deduced from the following observation. In similar constructions of $\mathcal{N} = 1$ SQCD, it was argued [51] that the leading back -reaction of the D-branes, which gives the holographic beta-function, is proportional to $N_f - 2N_c$ and is consistent with a theory with a quartic superpotential. The same argument applies here, hence both the electric and

[6]As the orientifold projection enforces $\mu \in \mathbb{R}$ one cannot avoid the singularity at $\mu = 0$ where this phenomenon occurs.

magnetic theories have interactions in the fundamental and anti-fundamental quark 'multiplets' (ϕ, ψ) and $(\tilde{\phi}, \tilde{\psi})$, which reproduce a $\mathcal{N} = 1$ quartic superpotential in the large N_c limit, i.e. of the form $\mathcal{L}_{INT} = h\left(|\tilde{\phi}\phi\tilde{\phi}|^2 + 2\psi\tilde{\psi}\phi\tilde{\phi} + \cdots\right)$ up to subleading corrections in the $1/N_c$ expansion.

3.2. Seiberg duality and its consequences. In the large N_c limit,[7] where the non-supersymmetric orientifold QCD is planar equivalent to super-QCD, the duality we have obtained reduces to Seiberg self-duality for $\mathcal{N} = 1$ theories with a quartic potential [52]. At finite N_c the brane construction provides a precise prediction for an electric/magnetic duality beyond the planar limit:

- The electric theory is $U(N_c)$ Yang-Mills with a fermion in the symmetric (resp. antisymmetric) representation, N_f pairs of bosons and fermions in the fundamental and anti-fundamental representations of the gauge group and interactions analogous to a quartic superpotential in $\mathcal{N} = 1$ SQCD.
- The magnetic theory is $U(N_f - N_c - 4)$ (resp. $U(N_f - N_c + 4)$) YM with a fermion in the symmetric (resp. antisymmetric) representation, N_f pairs of bosons and fermions in the fundamental and anti-fundamental, a magnetic meson M and a pair of Weyl fermions $\chi, \tilde{\chi}$ (the 'mesinos') in the trivial representation,[8] which have both a mass term; after integrating out these massive fields one gets a 'quartic superpotential' similar to the electric theory, with inverse coupling.

We expect that the duality holds also in the absence of the 'quartic superpotential', as can be seen already in the planar limit.

In order to test the duality in orientifold QCD at the level of field theory, one cannot use the powerful tools of supersymmetry, such as holomorphy or exact beta-function formulae. A non-trivial check is provided by the 't Hooft anomaly matching, i.e. agreement between the global anomalies of the unbroken $SU(N_f) \times SU(N_f) \times U(1)_R$ global symmetry. In both OQCD theories we have obtained an exact agreement between the anomalies of the electric and magnetic descriptions, beyond the large N_c limit which was fixed by planar equivalence.

The duality in OQCD-S and OQCD-AS predicts the existence of a 'phase diagram' similar to SQCD. There exists a range of N_f/N_c giving a *conformal window* within which the theories have non-trivial infrared fixed points, see fig. 3. Its upper and lower

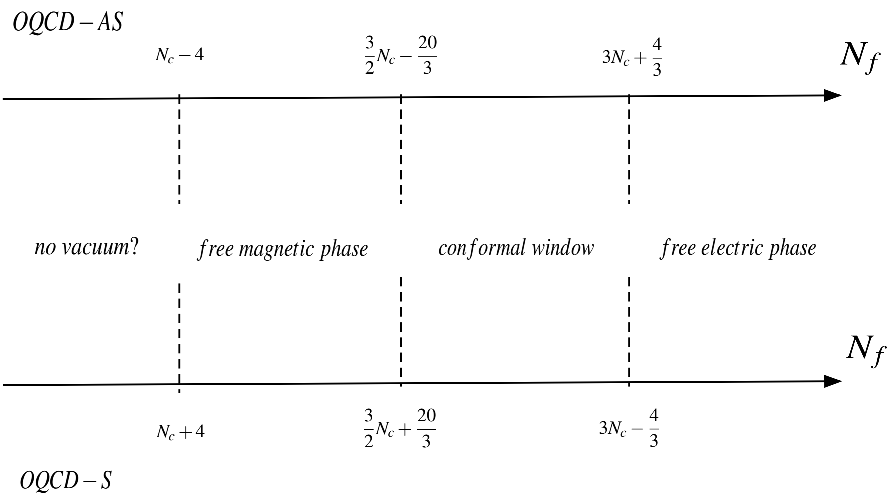


FIGURE 3. Phase diagram of orientifold QCD

[7]More precisely, in the Veneziano limit defined by $N_c \to \infty$ with N_f/N_c and $g^2_{YM}N$ held fixed.

[8]The 'mesinos' transform in the symmetric (resp. antisymmetric) representation of the (global) flavour group $SU(N_f)$.

bounds are given by the vanishing of the one-loop beta functions of the electric and magnetic theory respectively, since each of these descriptions is weakly coupled at one edge of the conformal window; this phase structure makes sense only for $N_c > 5$.

4. Holographic duals of supersymmetric quivers

In sections 1 and 3 we have studied field theory dynamics with D-branes suspended between NS5-branes. The gauge theories on the D-branes worldvolume are decoupled from gravity in the *double scaling limit*, where the fivebranes are brought close to each other while holding fixed the effective string coupling g_{EFF} in the back-reacted geometry.[9] The tree-level dynamics of light open string states reproduces the tree-level gauge theory, while quantum effects in the field theory appear at finite g_{EFF}. In the large N limit D-branes back-reaction becomes large and the resulting Ramond–Ramond background is expected to be holographically dual to the field theory. At finite N and large string coupling the strong coupling gauge dynamics can be probed in M-theory; for instance $\mathcal{N} = 2$ SYM is realized with an M5-brane wrapping a Seiberg–Witten curve. However in the eleven-dimensional supergravity regime the field theory degrees of freedom are not decoupled from the Kaluza-Klein states.

There exist some regions in the moduli space of $\mathcal{N} = 2$ $SU(n)$ super-Yang–Mills that can be probed with perturbative string theories. A singular *Argyres–Douglas* point [53] corresponds to a degeneration of the Seiberg–Witten curve to a 'small' torus of genus $\lfloor \frac{n-1}{2} \rfloor$:

$$y^2 \simeq \Lambda^n (x^n - \delta v) \ , \quad \delta v \to 0. \tag{13}$$

In the $\delta v \to 0$ limit, as $n(n-1)/2$ non-mutually local dyons (whose mass are given by the periods of the Seiberg–Witten differential) become simultaneously massless, one gets interacting superconformal field theories. A holographic description in the neighbourhood of these singularities is obtained in the *double scaling limit* $\delta v \to 0$ with fixed dyon masses; this universal limit can be reached from several Seiberg–Witten curves having the same singularity. In M-theory one can take the double-scaling limit of an M5-brane wrapping the curve (13); one can equally describe the theory with an NS5-brane wrapping the same curve in type IIA [54], by dimensionally reducing M-theory along another transverse circle, which is irrelevant in this limit. The back-reacted NS5-brane background is then given [55] by non-critical type IIA[10] on $\mathbb{R}^{3,1} \times SU(2)/U(1)|_n \times \mathcal{N} = 2$ Liouville with $Q = \sqrt{1/2 + 1/n}$, and a generalized GSO projection.

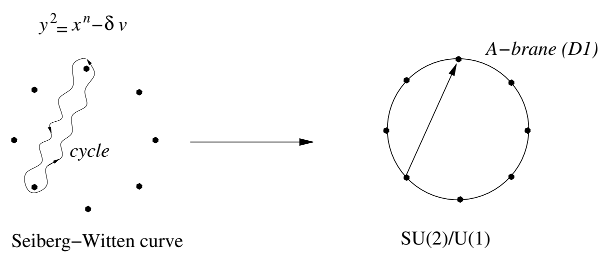


FIGURE 4. Cycles of the SW curve (13) and corresponding D1-branes in $SU(2)/U(1)|_n$.

I have shown in [38] that the dyons becoming massless at the Argyres–Douglas point correspond to localized D1-branes in the holographic dual, built out of a D1-brane of the super-coset $SU(2)/U(1)|_n$. There is a one-to-one correspondence between

[9]Coincident fivebranes, or equivalently orbifold singularities which are not supported by a B-field, lead to a strong coupling singularity in string theory, as stretched/wrapped D-branes become massless.
[10]It generalizes the six-dimensional non-critical string already discussed in the context of type 0'B.

the periods of the Seiberg–Witten differential around pair of branch points of the SW curve (13), distributed with a $\mathbb{Z}_n$ symmetry, and BPS D-branes of the $\mathbb{Z}_n$ super-parafermionic theory, see fig. 4.

I have constructed the boundary states for these holographic dyons and evaluated their mass from the one-point function on the disk, which is weighted by $1/g_{\text{EFF}}$. Mirror symmetry between the axial coset $SL(2,\mathbb{R})/U(1)$ and $\mathcal{N}=2$ Liouville relates g_{EFF} to the cosmological constant μ in (10), which is in turn holographically dual to the vacuum expectation value $(-)^n\langle \det \phi\rangle = 2\Lambda^n + \delta\nu$. It predicts that the dyon masses scale near the singularity like $m_D \sim (\delta\nu)^{1+2/n} \sin\frac{\pi}{n}(1+2\jmath)$, matching the field theory results.

The main goal of the work that I presented in [38] was to extend these models to holographic duals of $\mathcal{N}=1$ gauge theories. At the level of Hanany–Witten constructions, a configuration of MN D4-branes of world-volume $x^{0,1,2,3,6}$ ending along x^6 on two parallel NS5-branes filling $x^{0,1,2,3,4,5}$ gives $\mathcal{N}=2$ SYM with $U(MN)$ gauge group. A supersymmetric quotient $\mathbb{C}^2/\mathbb{Z}_N$ along (x^4+ix^5, x^8+ix^9) breaks supersymmetry to $\mathcal{N}=1$ and gives an $\hat{A}_{N-1}$ affine quiver gauge theory, with $SU(M)$ gauge groups at the nodes,[11] see fig 5. I have obtained a semi-classical description of this system with D4-branes in the background of the two NS5-branes on an orbifold as described above. The orbifold action onto the Chan-Patton matrices gives an $SU(M)^N$ gauge group as in flat space orbifolds [56], while the bi-fundamental matter multiplets $Q_{\ell\to\ell+1}$ correspond to excitations transverse to the orbifold locus.

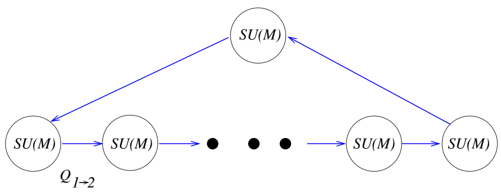


FIGURE 5. $\mathcal{N}=1$ chiral $\hat{A}_{N-1}$ affine quiver.

The low-energy quantum dynamics of this $\mathcal{N}=1$ quiver theory in its Coulomb branch is also governed by a hyper-elliptic curve, which is expressed in terms of the composite adjoint operator $\Xi = Q_{1\to2}\cdots Q_{N\to1} - \frac{1}{M}\text{Tr}\,(Q_{1\to2}\cdots Q_{N\to1})\mathbb{I}_{M\times M}$ [57]:

$$y^2 = \frac{1}{4}P_M(x)^2 - \Lambda_1^{2M}\cdots\Lambda_N^{2M} = 0\,, \quad P_M(x) = \langle \det(x-\Xi)\rangle = \sum_{\ell=0}^{M} s_\ell(\Xi)\, x^\ell\,. \tag{14}$$

As in $\mathcal{N}=2$ theories this curve has Argyres–Douglas points giving superconformal theories. Likewise their double-scaling limit can be captured by an NS5-brane wrapping the nearly-degenerate curve, which back-reacts into a non-critical string.

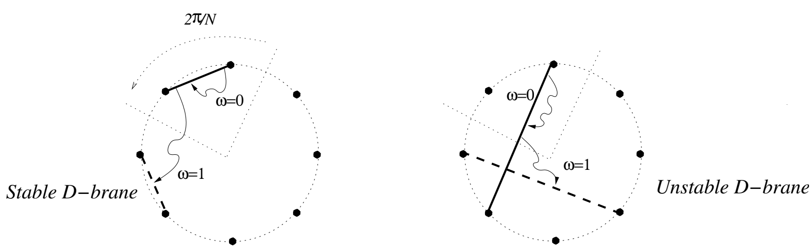


FIGURE 6. Stable and unstable D-branes on the covering space of the $\mathbb{Z}_N$ quotient.

I have shown that the holographic dual of such $\mathcal{N}=1$ theory near an Argyres–Douglas point is given by a chiral $\mathbb{Z}_N$ orbifold of type IIA on $\mathbb{R}^{3,1}\times SU(2)/U(1)|_{MN}\times\mathcal{N}=$

[11] The $U(1)$ factors are anomalous hence drop out of the low-energy action.

2 Liouville, breaking all supersymmetry from the left-movers.[12] The holographic dictionary maps bulk normalizable chiral operators in the untwisted sector to vacuum expectation values of the gauge-invariant operators $s_\ell(\Xi)$, and chiral operators in the γ-th twisted sector of the asymmetric orbifold to the VEVs of the 'baryonic' operators $\mathfrak{B}_{1\to\gamma} = \det(Q_{1\to 2}\cdots Q_{\gamma-1\to\gamma})$. Having identified the space-time R-charge allowed to predict their scaling dimensions at the superconformal fixed point. The dyons becoming massless at the singularity are also mapped to D1-branes, which are no longer BPS. Computing their annulus amplitudes one finds that a subset of these branes are stable, see fig. 6.[13] Their number matches the number of 'light' dyons predicted from the curve (14) near the Argyres–Douglas points. Interestingly the string dual gives the masses of these dyons, which cannot be computed from the $\mathcal{N}=1$ gauge theory.

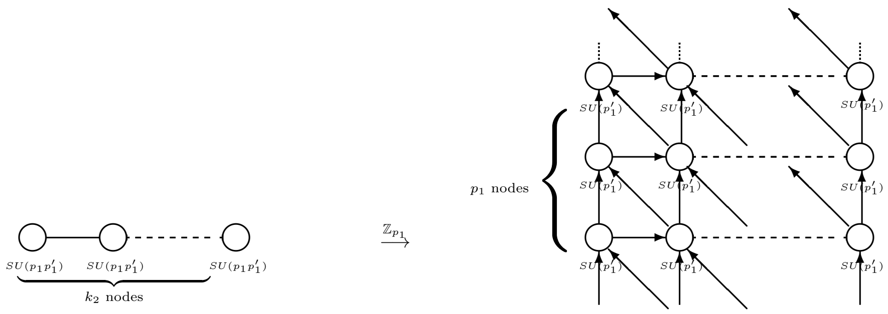


FIGURE 7. $\mathcal{N}=1$ Annular quivers from $\mathcal{N}=2$ linear quiver theories

An interesting extension of this work is to consider more generic type IIA backgrounds based on $\mathbb{R}^{3,1}\times SU(2)/U(1)|_{k_1}\times SU(2)/U(1)|_{k_2}\times\mathcal{N}=2$ Liouville *(unpublished)*. In this case the string theory is critical, allowing to write a corresponding supergravity solution (which is not very illuminating, being singular and having only two Killing vectors). By a similar reasoning it corresponds to the back-reaction of an NS5-brane wrapping the Riemann surface $x^{k_1}+y^{k_2}=\mu$. This surface can be obtained for instance as the degeneration of the Seiberg–Witten curve for a linear A_{k_2-1} quiver $\mathcal{N}=2$ gauge theory with $SU(k_1)$ nodes:

$$y^{k_2}+\sum_{r=1}^{k_2-1} y^r\left(x^{k_1}+u^{(r)}_{k_1-1}x^{k_1-1}+\cdots+v^{(r)}\right)+1=0 \tag{15}$$

near an Argyres–Douglas type of singularity.[14] Equivalently it can be obtained from an $SU(k_2)^{k_1-1}$ quiver, predicting a non-trivial equivalence between these seemingly different gauge theories in the double-scaling limit.[15] As in previous examples if $k_1 = p_1p_1'$ one can take a $\mathbb{Z}_{p_1}$ quotient of the theory, corresponding holographically to an $\mathcal{N}=1$ annular quiver where each node of the A_{k_2-1} linear quiver is 'blown-up' into an affine $\hat{A}_{p_1}$ quiver with chiral bi-fundamental matter, see fig. 7. Starting from the equivalent description of the original theory as an A_{k_1-1} quiver with $SU(k_2)$ gauge groups at the nodes, the quotient can be seen as an orbifold by a $\mathbb{Z}_{p_1}$ subgroup of the cyclic group $\mathbb{Z}_{k_1}$ under which the gauge-invariant operators $u_p^{(r)}$, associated with the r-th gauge group factor, have charge $r-1$. If $k_2 = p_2p_2'$ it is possible to consider a $\mathbb{Z}_{p_2}$

[12]The chiral $\mathbb{Z}_N$ and $\mathbb{Z}_M$ orbifolds are T-dual to each other, implying an isomorphism between $\hat{A}_{N-1}$ quivers with $SU(M)$ nodes and $\hat{A}_{M-1}$ quivers with $SU(N)$ nodes near Argyres–Douglas points.

[13]The decay of the instable dyons can be holographically captured using the non-critical models of open string tachyon condensation described in chapter 1.

[14]If one chooses an $SU(2)_{k_2}$ modular invariant from the D or E series one obtains quivers theories associated with the corresponding Dynkin diagrams.

[15]A similar equivalence was observed in [58] in the context of geometrical engineering of gauge theories from fibered non-compact Calabi-Yau three-folds.

quotient at the same time, with a similar action on the annular quiver of fig. 7. These constructions worth being explored further at the field theory level, as they provide interesting $\mathcal{N} = 1$ theories with superconformal fixed points in their moduli space.

5. Heterotic conifolds and holography

Fivebrane holography in heterotic theories is more mysterious than its type IIA or IIB counterparts that have been discussed in section 4. Heterotic fivebranes occur as the point-like limit of four-dimensional gauge instantons – more precisely of heterotic supergravity solutions sourced by them. The heterotic Bianchi identity relates indeed a singular gauge background to a magnetic point-like source for the NS-NS three-form. For instance, blowing down $SU(2)$ instantons in flat space gives the Callan, Harvey and Strominger solution [59], while blowing down a resolved A_1 singularity with a $U(1)$ instanton gives fivebranes transverse to a $\mathbb{C}^2/\mathbb{Z}_2$ orbifold [60]. The degrees of freedom associated with these singularities depends on the theory. In $Spin(32)/\mathbb{Z}_2$ heterotic strings, an S-duality gives type I D5-branes, which support $Sp(n)$ gauge groups; in $E_8 \times E_8$ theory it is understood in terms of M5-brane dynamics.

The world-volume theories associated with six-dimensional singularities are quite mysterious, apart from singularities of K3 fibrations related to four-dimensional ones. They are quite interesting as they are associated with $\mathcal{N} = 1$ theories in four dimensions. We have shown in [61] that there exist exact smooth heterotic solutions corresponding to a resolved orbifoldized conifold with flux and an Abelian gauge bundle, specified by two orthogonal 'charge vectors' $\vec{p}$ and $\vec{q}$ in the Cartan sub-algebra $\vec{\mathfrak{T}}$; see chapter 3 for details. The string frame metric in the decoupling limit reads:

$$(16)\quad ds^2 = dx^\mu dx_\mu + \frac{\alpha' k}{2r^2}\left[\frac{dr^2}{1-\frac{a^8}{r^8}} + \frac{r^2}{8}\left(d\theta_2^2 + \sin^2\theta_1\, d\phi_1^2 + d\theta_2^2 + \sin^2\theta_2\, d\phi_2^2\right)\right.$$
$$\left. + \frac{r^2}{16}(1-\tfrac{a^8}{r^8})\left(d\psi + \cos\theta_1\, d\phi_1 + \cos\theta_2\, d\phi_2\right)^2\right],$$

while the dilaton, B-field and gauge potential are:

$$e^{2\Phi} = e^{2\Phi_0}\frac{(\alpha' k)^2}{r^4},$$
$$\mathcal{B} = \frac{\alpha' k}{8}\left(\cos\theta_1\, d\phi_1 + \cos\theta_2\, d\phi_2\right)\wedge d\psi,$$
$$(17)\quad \mathcal{A} = \left[\tfrac{1}{2}\left(\cos\theta_1\, d\phi_1 - \cos\theta_2\, d\phi_2\right)\vec{p} + \left(\tfrac{a}{r}\right)^4\left(d\psi + \cos\theta_1\, d\phi_1 + \cos\theta_2\, d\tilde{\phi}_2\right)\vec{q}\,\right]\cdot\vec{\mathfrak{T}}.$$

As in Eguchi–Hanson space, there is a bolt at $r \to a^+$ since the S^1 fiber of $T^{1,1}$ shrinks to zero size there.

From a holographic perspective, the bundle specified by $\vec{p}$ determines a deformation of the UV Lagrangian, being non-normalizable, that breaks part of the flavour symmetry corresponding to the heterotic gauge group. In the blow-down limit $a \to 0$ the theory develops a strong coupling singularity, while the resolved conifold solution, specified by $\vec{q}$, is smooth; worldsheet instanton effects break then the $U(1)_R$ R-symmetry dual to translations along the fiber of $T^{1,1}$ to $\mathbb{Z}_{\vec{q}^2/2-1}$ in the infrared, see section 1 in chapter 3. The $SU(2)\times SU(2)$ isometries of the base are kept unbroken as exact worldsheet symmetries.

Having an $\mathcal{N} = 1$ smooth supergravity solution with an asymptotic linear dilaton one may wonder whether it is dual to a confining gauge theory, a sort of heterotic analogue of the Maldacena–Nuñez solution [62]. Some features as the absence of mass gap, or the R-symmetry being broken spontaneously to $\mathbb{Z}_{\vec{q}^2/2-1}$ rather than to $\mathbb{Z}_2$ by a gaugino condensate, plead against this interpretation. Nevertheless since we have a gauge theory with a rather involved matter sector none of these features is conclusive.

The confining behaviour can be probed by finding the holographic dual of a confining string. The low-energy dynamics of the $\mathrm{Spin}(32)/\mathbb{Z}_2$ solution is best captured in the type I frame in order to decouple the massive string states.[16] In the type I dual the candidate for the confining string is the fundamental string; thanks to the warp factor the metric component $-g_{tt}(r) = e^{\Phi_0-\Phi}$ has a non-vanishing minimum at the 'IR end' $r \to a^+$ of the geometry, leading to a linear potential for the separation between the string endpoints. The type I fundamental string is prone to breaking onto the D9-branes, which is expected for a gauge theory with flavour in a 'confining' phase as quark/anti-quark pairs can be created. Naturally, in the 'UV end' of the geometry $r \to +\infty$ one needs to use the heterotic frame and no such string exist any more.

Let us summarize the situation. In the blow-down limit, the holographic theory, in type I variables, is IR-free but strongly coupled in the UV. On the contrary, in heterotic variables, it is asymptotically free – at least up to a scale where the little string theory description takes over – and flows to a strong coupling singularity. An example of supersymmetric gauge theory with this behaviour is $SU(N_c)$ SQCD in the free electric phase, *i.e.* with $N_f > 3N_c$ flavours [63]. Pursuing this analogy, one could identify the resolution of the singularity with a full Higgsing of the magnetic theory. It gives a mass term to part of the electric quark multiplets, leading to an electric theory with $N_f = N_c$ flavours remaining massless. Then, below this mass scale (that is set by the VEV of the blow-up modulus) the electric theory confines. In a holographic dual of such field theory it would be problematic to write down a classical string solution with a confining behaviour. Trying to connect the putative string with the boundary, one would cross the threshold $1/a$ above which the electric theory has $N_f > 3N_c$ flavours, hence is strongly coupled at high energies and is not described in terms of free electric quarks. This exemplifies the type of gauge theory that is expected to describe the low-energy physics of the theory holographically dual to (16). Finding what this theory actually is remains a difficult open question.

[16] The ratio of the mass scale associated with the resolution of the singularity over the string mass scale is $\sqrt{\lambda/g_s}$, where λ is the double-scaling parameter defined in chapter 3 and g_s the heterotic string coupling. In the heterotic perturbative regime this ratio is necessarily large.

3

Heterotic Compactifications with torsion

Supersymmetric heterotic compactifications constitute one of the main approaches to particle physics phenomenology. Investigations of such models started nearly thirty years ago; they had well-known shortcomings, as a large number of massless scalars and unrealistic gauge groups. Recently, a renewal of interest, driven by new ideas and technical advances, promoted heterotic compactifications as good alternatives to type II compactifications with branes and Ramond–Ramond fluxes, which are only accessible in the large-volume supergravity approximation. In contrast, heterotic constructions are amenable to worldsheet techniques, allowing to explore the small-volume regime and to include string theory effects as worldsheet instantons corrections.

In the heterotic approach, the different aspects of string compactifications, such as the choice gauge group and couplings, the number of generations, moduli stabilization, are closely tied together; there is no 'step by step' approach as is sometimes used in type II. The Bianchi identity, modified by the Green-Schwarz mechanism,

$$\mathrm{d}H = \frac{\alpha'}{4}\big(\mathrm{tr}\, R \wedge R - \mathrm{Tr}\, F \wedge F\big) + \mathcal{O}(\alpha'^2) \tag{18}$$

plays a major role. The first term on the right-hand side is computed using the curvature two-form for some choice of spin connection with torsion. There is an ambiguity in the choice of connection, related to different regularization schemes in the worldsheet sigma-model [64]. Different choices are related by field redefinitions, which affect the α' corrections to the supersymmetry conditions and to the equations of motion.

The *standard embedding* amounts to choose the spin connection as a gauge connection, giving a consistent solution to eq. (18) with vanishing torsion H, while the supersymmetry conditions give a Calabi–Yau manifold; the unbroken gauge group is the commutant of the structure group.[1] Away from this standard embedding, which is unappealing phenomenologically, one faces number of difficulties. First the gauge bundle should satisfy the zero-slope limit of the *Hermitian Yang–Mills equations*, see sec. 1, which are difficult to solve apart from the special cases of Kähler manifolds or Abelian bundles. Second, the Bianchi identity (18) indicates that generically a non-standard gauge bundle requires non-vanishing flux, which has to satisfy this non-linear constraint mixing terms at different orders in α'. Third, non-zero torsion H is related by supersymmetry to non-kählerity of the manifold. The interplay between these aspects – Hermitian Yang–Mills gauge bundles, torsion and non-Kähler geometries – is the reason for both the difficulty and the interest of these solutions. The supersymmetry conditions at order α' with torsion have been known for a long time [65, 66], yet explicit solutions appeared only recently; there as been some debate about the exactness of these solutions related to the choice of connection, see [67] for a recent discussion.

My interest in the subject has been to explore new solutions with torsion, both from the supergravity and worldsheet point of views, and draw general lessons from them. With Luca Carlevaro and Marios Petropoulos, we have considered heterotic supergravity solutions conformal to Euguchi–Hanson space with line bundles, and showed

[1] At special points in the moduli space solvable worldsheet theory are available, as toroidal orbifolds or Gepner models.

that in a certain double-scaling limit they become solvable as world-sheet theories [60]; the associated exact solution of the Bianchi identity for the Chern connection was computed with Esteban Herrera–Cordero during his master thesis. I have pursued this collaboration with Luca Carlevaro, who was postdoctoral fellow at Polytechnique; we have first generalized this construction to models with a torus bundle in [68] and to supersymmetry-breaking vacua. We have then constructed a resolved conifold solution with torsion [61], both in supergravity and on the worldsheet. It is a rare instance of explicit smooth $SU(3)$-structure solution with flux in heterotic strings.

The supergravity approach is of limited use for compact manifolds with flux given that the Bianchi identity (18) generically forbids to take a large-volume limit. As the relevant worldsheet superconformal field theories are not explicitly known, it is convenient to use some $(0,2)$ supersymmetric gauge theories in two-dimensions, *gauged linear sigma-models*, that are expected to admit non-trivial infrared fixed points. The recent discovery of models with torsion in target-space [69] is a very important development. I have used these techniques in [70] to study T-duality in torsional compactifications and associated topology changes, giving exact results. I work currently with Stefan Groot Nibbelink on more general compact models with and without fivebranes.

Worldsheet theories allow to obtain the four-dimensional effective action which is important for phenomenological applications. In particular, the one-loop threshold corrections to the gauge couplings can be computed using a by-product of the $(0,2)$ elliptic genus, even if the two-dimensional theory is not known explicitly at the super-conformal point, given that this quantity is topological. With Luca Carlevaro we have obtained these corrections for the local model of an Eguchi–Hanson singularity [71]; this computation was very interesting mathematically as it involved *mock-modular forms*. During his master thesis, Matthieu Sarkis has computed the elliptic genera of some compact and non-compact torsional gauged linear sigma-models under my supervision, which will be the matter of a forthcoming publication.

1. Local models of torsional compactifications

Local models of heterotic compactifications with torsion are non-compact four- or six-dimensional supersymmetric solutions that can be viewed as an approximate description of a genuine compactification in some region of the manifold, as the neighbourhood of a resolved or deformed singularity. If there exists a *decoupling limit*, one can rigorously obtain the full non-compact solution from a compact model; it typically leads to a holographic duality between the localized degrees of freedom in this region and the non-compact back-reacted gravitational background.

These solutions are easier to study as one can write explicit metrics for these manifolds that have several Killing vectors, unlike compact Calabi–Yau and their non-Kähler generalizations. One can also avoid the 'tadpole conditions' coming from the integrated Bianchi identity and expand the solution in a large charges limit. In type II flux compactifications, such local models, as the Klebanov–Strassler solution [72], have proven to be very useful in studying a variety of phenomena, from supersymmetry breaking to inflation models; the same is expected in heterotic strings. The first known class of such non-compact solutions in heterotic supergravity consists in two-torus bundles over a warped resolved A_1 singularity [73].[2]

1.1. Supergravity solutions. Four-dimensional $\mathcal{N} = 1$ compactifications are neatly characterized in the language of G-structures. The supersymmetry variation of the gravitino at leading order in α' implies the existence of a nowhere vanishing spinor,

[2]It is a particular example of a more general construction of torus bundles over Calabi–Yau two-fold bases, that was found first from duality with type IIB orientifolds [74].

covariantly constant w.r.t. the connection with torsion $\omega_{-}{}^{A}{}_{B} = \omega^{A}{}_{B} - \frac{1}{2}H^{A}{}_{B}$, or equivalently the existence of globally defined real two-form J and complex three-form Ω satisfying the $SU(3)$-structure relations.[3] The supersymmetry conditions can be recast as calibration conditions [75],

$$d(e^{-2\Phi}\Omega) = 0 \quad d(e^{-2\Phi}J \wedge J) = 0\,, \tag{19}$$

while the three-form flux is given as $H = i(\bar{\partial} - \partial)J$. The holomorphic vector bundle V satisfies the zero-slope limit of the Hermitian-Yang–Mills equation, namely

$$F^{(2,0)}(V) = F^{(0,2)}(V) = 0\,, \quad J \lrcorner F(V) = 0\,. \tag{20}$$

Finally, these data are tied together by the Bianchi identity (18).

In type II supergravities, one usually looks for exact solutions of the supersymmetry conditions and Bianchi identities, expecting them to be corrected order by order in a α'^3/V large-volume expansion. In heterotic supergravity the situation is different as terms at different orders in α' are mixed together and some requirements seem impossible to meet at the same time. First, given that $dH = 2i\partial\bar{\partial}J$, the right-hand side of the Bianchi identity should involve only $(2,2)$ forms. This suggests that, *generically*, obtaining non-Kähler solutions requires that one uses the *Chern connection* for the holomorphic tangent bundle, which has precisely this property, in the Bianchi identity (18). Second, the supersymmetry variations receive no corrections at order α' if we use the *Hull connection* $\omega_{+} = \omega + \frac{1}{2}H$ in the Bianchi identity [76,77]. Third the supersymmetry equations and Bianchi identity imply the equations of motion at order α' iff the spin connection in Bianchi is an $SU(3)$ instanton, namely satisfies (20) as the gauge bundle [78]; according to [79] the Hull connection has this property at lowest order, given the exchange identity $R_{ijkl}(\omega_{+}) - R_{klij}(\omega_{-}) = \frac{1}{2}(dH)_{ijkl}$ and that dH is of order α' thanks to the Bianchi identity. We will illustrate these points in non-compact models which have a well-defined dimensionless expansion parameter (rather than the ill-defined α' expansion).

1.1.1. Warped Eguchi–Hanson space. A simple class of $\mathcal{N} = 2$ non-compact smooth non-Kähler solutions is given by principal two-torus bundles over Eguchi-Hanson space $T^2 \hookrightarrow \mathcal{M}_6 \xrightarrow{\pi} T^\star\mathbb{P}^1$ [73]. The ansatz for the Hermitian form is

$$J = e^u J_{EH} + \frac{i}{2}\theta \wedge \bar{\theta}\,, \tag{21}$$

where J_{EH} is the Kähler form deriving from $K = \sqrt{r^4 + a^4} + a^2 \log \frac{r^2}{\sqrt{r^4+a^4}+a^2}$. The curvatures F of the Abelian gauge bundle and $G = d\theta$ of the torus bundle are both proportional to the pull-back of the normalizable anti-self-dual $(1,1)$-form on Eguchi–Hanson (Poincaré dual to the $\mathbb{P}^1$ of size a). The gauge bundle is characterized by the charges $\vec{p}$ in the Cartan of the gauge group, while the torus bundle depends on two Kaluza–Klein charges (w^1, w^2). The integrated Bianchi identity gives the tadpole condition:

$$Q_5 = \frac{U_2}{2T_2}|w^1 + T w^2|^2 + \vec{q}^{\,2} - 6\,, \tag{22}$$

where T and U are the complex structure and complex Kähler modulus of the torus. Finally the warp factor $e^u = e^{2\Phi}/g_s^2$ is set by the Bianchi identity (18). Fu, Tseng and Yau solved this problem in [73] using the Chern connection, following their analysis of models with a K3 base [80], and obtained the dilaton as a series in $\alpha'/(g_s^2 a^2)$, i.e. in a 'large resolution' limit with mild effects of the torsion. They have argued that a vanishing five-brane charge Q_5 is necessary for regularity. I have studied this system

[3] We have $\Omega \wedge \bar{\Omega} = -\frac{4i}{3}J^{\wedge 3}$ and $J \wedge \Omega = 0$. The three-form determines an almost complex structure, such that Ω is a holomorphic $(3,0)$ form, hence the manifold is necessarily complex.

with Esteban Herrera–Cordero and we first observed that an integration constant can relieve this constraint, allowing any positive value of Q_5.

In the *large charge* regime $Q_5 \gg 1$ one can consistently ignore the curvature term in the Bianchi identity at lowest order; a good dimensionless expansion parameter of the solution is indeed $1/Q_5$. One finds then a simple asymptotically ALE solution

$$\frac{1}{g_s^2}\, e^{2\Phi} = 1 + \frac{2\alpha' Q_5}{a^2\sqrt{1+\frac{r^4}{a^4}}}\,, \tag{23}$$

which reduces in the blow-down limit to a singular background of five-branes transverse to $\mathbb{C}^2/\mathbb{Z}_2$, as the instantons collapse to zero-size. A regular decoupling limit of the system can be achieved by considering a novel *double scaling limit* $g_s \to 0$ with $\lambda = g_s\sqrt{\alpha'}/a$ fixed [60]. One obtains a smooth 'near-bolt' metric which is independent of the blow-up parameter, while λ is the effective coupling in this asymptotically linear dilaton background. A similar limit can be in principle defined more generally in order to isolate the dynamics of collapsable two-cycles supporting Abelian instantons in torsional compactifications.[4]

I have shown with Esteban Herrera–Cordero that there exists in this limit a solution to the Bianchi identity with the Chern connection, obtained as a series expansion converging for $Q_5 > 6$. As discussed above the equations of motion at order α' (i.e. with the term from $\alpha' \mathrm{tr}|R|^2$) are satisfied iff the tangent bundle connection fulfils eq. (20). It amounts here to an independent equation for the warp factor, whose solution $\exp(u) = r^2/(a^2+\sqrt{a^4+r^4})$ fits neither with the Fu–Tseng–Yau result nor with our double-scaling limit answer. Nevertheless the violation of the Einstein equation is a sub-leading $1/Q_5$ correction.

As we shall see this solution has an associated worldsheet conformal field theory, hence should define an exact background also for finite Q_5; we have examined in [60] whether the Hull connection leads to an exact solution of the Bianchi identity at order α'. Given that $\mathrm{tr}\, R\wedge R$ has components only along the base it is automatically of Hodge type $(2,2)$. Next, because of the exchange identity, the curvature $R(\Omega_-)$ associated with the Hull connection is anti-self-dual only if $dH = 0$. This happens for $\vec{q} = \vec{0}$, and an $SU(2)$ standard embedding becomes possible; this does not come as a surprise since these models have type II counterparts [60, 79]. There exists an exact T-duality between such heterotic sigma-models, with torus bundles and no Abelian gauge bundle, and models with the torus and gauge bundles exchanged, see sec. 2 for a proof. It is intriguing that in these T-dual models no supersymmetric standard embedding is possible, thus the Einstein equations are necessarily violated by $1/Q_5$ corrections.

We have learnt an interesting lesson from this study: getting exact solutions of the order α' equations is a strategy that usually fails in heterotic. Rather it is useful to find a dimensionless expansion parameter and solve the equations to a given order.

1.1.2. Warped resolved conifold. Our knowledge of $SU(3)$-structure heterotic solutions with torsion is very limited. For a long time only the aforementioned $T^2 \hookrightarrow \mathcal{M}_6 \to K3$ solutions were known. We have obtained in [61] a new class of non-compact solutions, given by a warped resolved $\mathbb{Z}_2$ orbifold of the conifold with Abelian bundles.[5]

[4]Heterotic compactifications with line bundles are of significant interest as they give rise to interesting phenomenology and are relatively tractable, see e.g. [81, 82].

[5]They can be seen as a generalization of the previous solutions; the $\mathbb{Z}_2$ orbifold of the conifold admits a 'double resolution' by a four-cycle, such that an S^1 degenerates at a bolt as in Eguchi–Hanson space.

The ansatz we started with is a warped squashed conifold with three-form flux:

$$\text{(24a)}\qquad ds^2 = \frac{3}{2g_s}e^{\Phi}\left(\frac{dr^2}{f(r)^2} + \frac{r^2}{6}\sum_{\imath=1}^{2} ds^2(\mathbb{P}^1_\imath) + \frac{r^2}{9}f(r)^2 d\varpi\right),$$

$$\text{(24b)}\qquad H = \frac{\alpha' Q_5}{3} g(r)^2 d\varpi \wedge \sum_{\imath=1}^{2} \mathrm{Vol}\,(\mathbb{P}^1_\imath)\,,$$

where ϖ is the connection form of the S^1 fibration over $\mathbb{P}^1_1 \times \mathbb{P}^1_2$. An analysis of the torsion classes indicates that this manifold is conformally Kähler. The Abelian gauge bundle curvature has components proportional to the two harmonic two-forms on the $\mathbb{Z}_2$ orbifold of the conifold resolved by a four-cycle, namely $\sum_{\imath=1}^{2} \mathrm{Vol}\,(\mathbb{P}^1_\imath)$ and $\frac{a^4}{4\pi}\, d(\varpi/r^4)$, to which we assign two orthogonal charge vectors $\vec{p}$ and $\vec{q}$ in the Cartan sub-algebra.[6]

The calibration conditions (19) lead to a first-order system for f, g and Φ, while the Bianchi identity (18) provides another relation between these functions. As for the warped Eguchi–Hanson geometry one considers a large charge limit $Q_5 \gg 1$ allowing to neglect the curvature contribution to the latter equation. In this regime g is determined and one can show that the geometry has a bolt for $r \to a^+$, in keeping with the $\mathbb{Z}_2$ orbifold. It also sets $\vec{p}^2 = \vec{q}^2 = 2Q_5 + \mathcal{O}(1)$. We have then solved the system numerically and found asymptotically locally Ricci-flat solutions, see fig. 1. We have checked

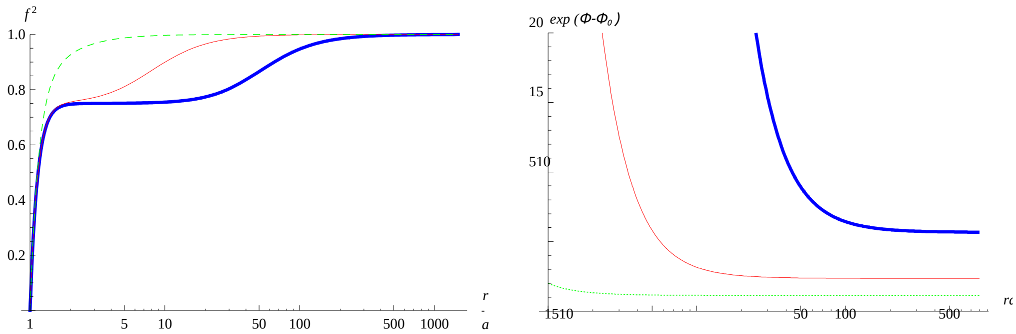


FIGURE 1. Numerical solution for f^2 and the conformal factor e^{Φ}/g_s, with $Q_5 = 5000$ and $a^2/\alpha' k = \{0.0001, 0.01, 1\}$, respectively thick, thin and dashed lines.

that the $\mathrm{tr}\, R \wedge R$ term, evaluated 'on-shell' on the solution, is consistently subleading w.r.t. the other terms in the Bianchi identity. Given that the two-form associated with $\vec{p}$ is not normalizable, one has a tadpole condition at infinity $6Q_5 = 3\vec{p}^2 - 4$.

In the blow-down limit $a \to 0^+$ one finds that the Abelian instanton associated with the normalizable harmonic two-form collapses to zero-size and the solution develops a strong coupling singularity. As for the Eguchi–Hanson solution one can define a *double scaling limit*, given by $g_s \to 0$ with $\lambda = g_s \alpha'/a^2$ fixed, that decouples the 'near-bolt' geometry, in a regime where the blow-up parameter a is hierarchically smaller than the scale $\sqrt{\alpha' Q_5}$ at which the effects of torsion and warping kick in. One obtains a smooth analytical solution that was already given in chapter 2, see eq. (16), where we analysed its interesting holographic interpretation.

The previous class of heterotic supergravity backgrounds corresponding to torus bundles over Calabi–Yau two-folds can be convincingly associated with genuine backgrounds of heterotic string theory given that it can be obtained by duality from type IIB orientifolds. There is no such duality frame for the conifold solutions, therefore one may wonder whether they lift to actual solutions beyond the first term in the $1/Q_5$ expansion. As we shall see below, it is the case at least in the double-scaling limit.

[6]In the $\mathrm{Spin}(32)/\mathbb{Z}_2$ theory charge quantization indicates that $\vec{p} \pm \vec{q}$ have either all even or all odd entries, corresponding to bundles with or without vector structure respectively.

One can reasonably expect that these near-horizon solutions can be extended to the full asymptotically Ricci-flat geometries.

1.2. Worldsheet models. In the double-scaling limit the warped Eguchi–Hanson space of 1.1.1 and the warped conifold of 1.1.2 are exact solutions of the heterotic string, since there exist superconformal field theories whose target spaces correspond precisely to these background fields [60, 61]. They are more easily understood if one starts with their blow-down limits.

1.2.1. Singular limits. The blow-down limit of these solutions give singular five-brane backgrounds. The warped Eguchi–Hanson solution degenerates to a $\mathbb{Z}_2$ orbifold of the Callan–Harvey–Strominger five-brane background, namely to the $\mathcal{N} = (0,1)$ superconformal field theory $\mathbb{R}_{\mathfrak{Q}} \times SU(2)_{2Q_5}/\mathbb{Z}_2$, where the background charge is $\mathfrak{Q} = 1/\sqrt{Q_5}$; the $\mathbb{Z}_2$ action in the $Spin(32)/\mathbb{Z}_2$ or $E_8 \times E_8$ lattice is determined by the Abelian instanton that has become point-like in the blow-down limit. In the case of the conifold, one obtains the tensor product of a linear dilaton with a conformal field theory corresponding to a non-Einstein metric on $T^{1,1}/\mathbb{Z}_2$ with torsion. This $T^{1,1}$ is the target space of a right-gauged $\mathcal{N} = (0,1)$ Wess–Zumino–Witten model $(SU(2)_{2Q_5} \times SU(2)_{2Q_5})/U(1)_{\text{RIGHT}}$. This theory is classically not gauge-invariant, but gauge invariance can be restored in the quantum theory by adding left-moving charged fermions that give rise to a gauge anomaly. Assigning a charge p_i to each of the sixteen Weyl fermions one gets a consistent theory for $\vec{p}^{\,2} = 2Q_5$. As before the $\mathbb{Z}_2$ action in the gauge lattice is dictated by the gauge instanton of the smooth solution.

1.2.2. Conformal field theories for the resolved geometries. A non-singular worldsheet theory is obtained by adding an exactly marginal deformation that screens the strong coupling region and is compatible with the symmetries and charges of the solutions. Let us illustrate this point with the case of the torus bundle over warped Eguchi–Hanson. We denote by $\rho(z,\bar{z})$, $Y_R(\bar{z})$, $Z_L^\iota(z)$ and $X_L^a(z)$ the canonically normalized bosons corresponding respectively to the linear dilaton, the right-moving affine $SU(2)_{2Q_5}$ Cartan generator, the left-moving Cartan generators of the gauge group and the left-moving part of the T^2 coordinates. We consider then the $(0,2)$ superpotential:

$$\delta S = \mu \int d^2z\, \bar{G}_{-1/2}\, e^{-\sqrt{2Q_5}(\rho(z,\bar{z})+iY_R(\bar{z}))} e^{ip_{a,L}X_L^a(z)+i\vec{q}\cdot\vec{Z}_L(z)} + \text{c.c.}. \tag{25}$$

Demanding the existence of the marginal operator that appears in (25) in the physical spectrum contains a wealth of information:

- It is part of the physical spectrum only if we considered a $\mathbb{Z}_2$ orbifold of S^3 as it belongs to the corresponding twisted sector; this mirrors the conditions of no conical singularity at the bolt in the Eguchi–Hanson geometry.
- Compatibility with the right-moving GSO projection indicates that the entries of $\vec{q}$ are either integer or half-integer, corresponding respectively to bundles with or without vector structure. Moreover it gives the condition $\sum_\iota q_\iota \equiv 0 \mod 2$. It reproduces the generic 'K-theory constraint' $c_1(V) \in H^2(\mathcal{M}_6, 2\mathbb{Z})$ known to avoid global anomalies in heterotic sigma-models [83, 84].
- The two-torus operator has only left-moving momentum $p_{a,L}$ and vanishing right-moving one. The existence of such holomorphic operators in the T^2 theory give strong constraints on the T and U moduli. They should be such that this $c = 2$ theory is a *rational CFT*; we will discuss this again in section 2.
- It is marginal iff one satisfies the condition $Q_5 = \frac{U_2}{2T_2}|w^1 + T w^2|^2 + \vec{q}^2 - 1$, which is a finite correction to the supergravity tadpole condition (22).[7]

In the conifold models the analysis and the conclusions are quite similar.

[7] The shift from -6 to -4 comes from the double-scaling limit that changes the asymptotics of the metric, while the extra shift by 3 units of charge comes from α' corrections.

1.2.3. Gauged WZW model description. The singular solutions perturbed by asymmetric Liouville potentials like (25) are expected to provide a string completion of the smooth supergravity solutions given in 1.1. A more direct relation is found by using an $\mathcal{N}=(0,2)$ analogue of mirror symmetry between $\mathcal{N}=2$ Liouville and the axial coset $SL(2,\mathbb{R})/U(1)$; indeed both resolved models are given as asymmetrically gauged WZW models, respectively $(SU(2)_{2Q_5}/\mathbb{Z}_2 \times SL(2,\mathbb{R})_{2Q_5})/U(1)_{\text{RIGHT}}$ and $(T^{1,1}/\mathbb{Z}_2 \times SL(2,\mathbb{R})_{Q_5})/U(1)_{\text{RIGHT}}$, where the $T^{1,1}$ theory was described above. Gauge-invariance is restored in the quantum theory by minimally coupling the sixteen left-moving fermions, with charge vector $\vec{p}$, and the left-moving bosons of the two-torus as well in the Eguchi–Hanson case. In both models, integrating out classically the gauge fields – which requires bosonising the fermions first to restore classical gauge-invariance – gives heterotic non-linear sigma-models whose background fields correspond precisely to the supergravity solutions in the double-scaling limit for the torus bundles over Eguchi–Hanson and for the conifold, see eqs. (16,17), at least at leading order in $1/Q_5$.

Given that the solutions are formulated as gauged Wess–Zumino–Witten models, it is possible to compute their quantum effective action and derive their exact background fields, using methods developed by Tseytlin [85]. I have performed this computation for both the warped Eguchi–Hanson and warped conifold and found that there are no corrections to the solution, at any order in α' *(unpublished)*.[8][9] This result is naturally dependent of field redefinitions in the string effective action, or different choices of renormalisation schemes in the sigma-model; it would be interesting to know in which scheme this statement is exact, an in particular with which choice of connection in the Bianchi identity. We may ask whether there is a connection with torsion $\omega_\mu = \omega + \frac{1}{2}\mu H$ such that the solution solves the order α' Bianchi identity exactly; this occurs if $\mu^3+\mu=6$ which does not correspond to any natural choice of connection.

In conclusion, Eguchi–Hanson space with a warp factor $e^u = 2\alpha' Q_5(a^4+r^4)^{-1/2}$ and the corresponding three-form flux, dilaton and Abelian gauge field is an exact solution of heterotic string theory; this is a consequence of the enlarged superconformal symmetry, $\mathcal{N}=(0,4)$ in this case. The same is true for the $\mathcal{N}=(0,2)$ warped conifold solution in the double-scaling limit, given by eqs (16,17). Worldsheet non-perturbative effects are captured by the Liouville potential (25), which comes indeed from instanton corrections in the gauged linear sigma-model approach of [46].

1.2.4. Massless spectra. The conformal field theory description of these heterotic local models with torsion allowed us to obtain their full spectra, by computing the torus partition function. The spectrum of string states whose wave-function is localized near the bolt (discrete representations) is especially interesting as it is expected that these localized degrees of freedom capture part of those of the genuine torsional compactification before having taken the double-scaling limit. We have computed the massless spectra for the warped Eguchi–Hanson space with various choices of gauge charges $\vec{q}$, which contain 'localized' hypermuliplets in various representations of the unbroken gauge group. In addition, there exists one Abelian gauge multiplet, corresponding to the instanton turned on, which becomes massive thanks to the Green-Schwarz mechanism. The corresponding localized string state is of mass $\alpha' m^2 = 4/\sqrt{\vec{q}^2-1}$. In the tadpole-free models, i.e. with $\vec{q}^2=6$, we have compared our spectra with those of resolved $\mathbb{C}^2/\mathbb{Z}_2$ orbifolds with Abelian gauge bundles [86]. We did not find any agreement, which is not surprising given that it was assumed there a regular orbifold limit in the blow-down, while in our case one obtains a five-brane theory with new degrees of freedom. We have similarly computed the spectra of warped conifolds and found the

[8]Technically, this result is a consequence of the absence of quadratic terms like $\mathrm{Tr}(A\bar{A})$ in the worldsheet quantum effective action of this model.

[9]There may be a constant rescaling of the metric and three-form depending on the way the fivebrane charge has been defined.

associated massless chiral multiplets. This is useful in understanding the interesting $\mathcal{N} = 1$ holographic duality of these solutions, discussed in chapter 2.

1.3. Supersymmetry breaking. Supersymmetric breaking in warped throat geometries is interesting as one can lower the supersymmetry breaking scale at will and one can make explicit computations. Metastable vacua obtained by adding anti-D3-branes to the Klebanov–Strassler geometry are used in numerous phenomenological models, yet have caused considerable controversy regarding their validity, see *e.g.* [87]. It is interesting to investigate whether (tree-level) supersymmetry breaking is possible using local models of heterotic flux compactifications such as those described above;[10] given that these non-compact solutions have an underlying worldsheet theory it would be possible to address possible issues, as the existence of singularities, that has been found in type II. I will summarize below some investigations on the subject (*unpublished*).

There is a general no-go theorem [88] that forbids continuous supersymmetry breaking in compact models with a worldsheet superconformal field theory description. For non-compact solutions one can bypass this result by considering *non-normalizable* deformations, as they do not fit in unitary representations of the superconformal algebra. Such non-tachyonic supersymmetry-breaking deformations are simple to write for singular five-brane solutions [89]. We have found that they are compatible with the blowing-up of the singularity to Eguchi–Hanson only if the deformation parameter is quantized; they reduce in this case to a freely-acting supersymmetry-breaking orbifold, which gives a worldsheet model for the solutions presented in [90].

Warped Eguchi–Hanson solutions with torus bundles (or more generally non-compact solutions with $dH = 0$) exist also in type II supergravity as we have noticed already. They have a single $SU(3)$-structure, *i.e.* for instance $\nabla(\omega_+)$ has $SU(3)$-structure but $\nabla(\omega_-)$ does not. Starting with such type II solution, one obtains a heterotic solution by identifying ω_- as the connection on the gauge bundle, *i.e.* a generalized standard embedding, while keeping ω_+ as the connection on the tangent bundle. As we have seen $R(\omega_-)$ satisfies the Hermitian-Yang–Mills equations (20) as it should. A similar prescription, known as the *Gepner map*, is rather standard in worldsheet theories [91]. A different heterotic background can be obtained from the *same* type II solution by considering the opposite embedding of ω_+ as a gauge connection. In the presence of three-form flux this breaks supersymmetry.[11] Performing the corresponding 'wrong' Gepner map on the associated worldsheet theories is rather straightforward. Besides type II solutions with torsion, which are necessarily non-compact, the same method can be applied to the non-geometric compactifications described in chapter 4. It would be interesting to investigate the stability of these models.

The resolved Eguchi–Hanson and conifold solutions are completely smooth and do not have source terms. It is nevertheless possible to add extra five-branes sources supersymmetrically. For instance smeared five-branes at the bolt of warped Eguchi–Hanson space modify the warp factor as

$$\frac{1}{g_s^2}\, e^{2\Phi} = 1 + \frac{2\alpha' Q_5}{a^2\sqrt{1+r^4/a^4}} + \frac{\alpha'\tilde{Q}_5}{a^2} \log \frac{\sqrt{1+r^4/a^4}-1}{\sqrt{1+r^4/a^4}+1}\,. \tag{26}$$

A double-scaling limit of the solution with sources exists, and keep the singular nature of the supergravity background. In the regime where $\tilde{Q}_5 \ll Q_5$, one can consider the sources as a small perturbation of the asymptotic geometry and expand the solutions in terms of $\tilde{Q}_5/Q_5$. It can be interesting to consider the effect of changing the

[10] Naturally, non-perturbative mechanisms related to gaugino condensation are also possible.

[11] If one considers the full ALE geometry rather than the double-scaling limit, one observes that supersymmetry-breaking is localized near the bolt as both connections $\omega_\pm$ converge to the Levi–Civita connection asymptotically.

sign of the charge $\tilde{Q}_5$, corresponding to a small number of anti-five-branes in a flux background corresponding to blown-up fivebranes. The associated worldsheet theory may not be singular as a single five-brane worldvolume theory is not expected to be strongly coupled. If such a mechanism is possible, it would be a metastable sort of supersymmetry breaking; a supersymmetry background with the same asymptotics can be obtained by decreasing the instanton number of the gauge bundle. These claims should be supported by explicit computations that constitute an interesting project for the near future.

2. Gauged linear sigma-models with torsion

Heterotic supergravity is very useful for finding non-compact solutions with flux, as one can reach a large-charge regime where the α' corrections are under control, as we have seen in sec. 1. Compact models are a lot more difficult to deal with, given that large-charge limits are not allowed. In the well-studied example of a two-torus bundle over a K3 base, which generalizes the warped Eguchi–Hanson solution of 1.1.1, there exists a tadpole condition quite similar to (22) with $Q_5 = 0$; it implies that the volume of the two-torus is necessarily at the string scale. This is a consequence of the Bianchi identity (18), that usually forbids the existence of a large volume limit in the presence of three-form flux. Moreover, generic supersymmetric compactifications with flux are not conformally Kähler, but only conformally balanced, hence making any statement about their geometry is rather difficult.

Thankfully, a worldsheet description of heterotic compactifications with torsion may in principle be within reach, given that there are no Ramond–Ramond fluxes to deal with by definition. A first strategy is to look directly for $(0,2)$ superconformal field theories with the requested properties in terms of central charge and R-charges spectrum. This approach is certainly worthwhile but the geometric characterization of such abstractly defined theories may be very difficult or impossible. Another strategy is to generalize the *gauged linear sigma-models* to compactifications with torsion. While such models do not allow to solve the worldsheet theory completely, they provide proof of existence of – or at least very strong evidence for – a large class of compactifications, and can be used to compute associated topological quantities.

Usual gauged linear sigma-models are Abelian $\mathcal{N} = (2,2)$ or $\mathcal{N} = (0,2)$ Abelian gauged theories coupled to charged chiral matter [92]. The D-term constraints typically indicate that the vacuum manifold is a weighted projective space and the superpotential carves out of this manifold, if the charges are well-chosen, a complete intersection Calabi-Yau variety.[12] Such model is expected to flow in the infrared to a superconformal field theory which is a non-linear sigma model on the corresponding Calabi–Yau, whose Kähler moduli correspond to the Fayet–Ilioupoulos parameters of the gauge theory. Models with $\mathcal{N} = (0,2)$ are usually obtained by deforming the gauge bundle – which is described using the monad construction – continuously away from the $\mathcal{N} = (2,2)$ locus. This allows to solve easily the various anomaly constraints that a consistent model should satisfy.[13] $\mathcal{N} = (0,2)$ supersymmetry certainly accommodates much more possibilities, some of which will correspond to compactifications with flux.

2.1. T-dualities in torsional compactifications. Without much surprise, the first type of gauged linear sigma-model for compactifications with flux that was described was a

[12]Many generalizations are possible, such as more general toric varieties as ambient spaces.

[13]Not only the gauge anomaly should vanish, but there should exist also a non-anomalous right-moving $U(1)$ that will play the role of the R-symmetry at the superconformal fixed point. There should exist also a a non-anomalous left-moving $U(1)$, or at least a non-anomalous $\mathbb{Z}_2$ subgroup, in order to define properly the left-moving GSO projection.

world-sheet model for the first known class of flux compactifications, the torus bundles over a warped K3 base [69]. Let us illustrate this construction with the simplest class of $\mathcal{N} = (0,2)$ gauged linear sigma models. We consider a $U(1)$ gauge theory coupled to chiral multiplets of charges Q_i and Fermi multiplets of charge q_a; a generic model would have gauge anomalies. Under a gauge transformation, parametrised by a chiral superfield Ξ, the effective Lagrangian is shifted as (for a $U(1)$ gauge group):

$$\delta_\Xi \mathcal{L}_{\text{eff}} = \frac{\mathfrak{A}}{8}\int d\theta^+ \, \Xi \Upsilon + \text{h.c.}\,, \quad \text{with} \quad \mathfrak{A} = Q_i Q^i - q_n q^n\,, \tag{27}$$

where $\Upsilon = \bar{D}_+(\partial_- \mathcal{A} + i\mathcal{V})$ is the field-strength chiral superfield. As a starting point for constructing the torsional models we consider a gauged-linear sigma model with a K3 target-space whose gauge bundle is such that the anomaly coefficient $\mathfrak{A}$ is positive.

2.1.1. Torus bundles and torsion. In order to cancel the quantum anomaly, one adds to the theory a pair of chiral superfields Ω_a, $a = 1,2$, whose shift-symmetry is gauged: $\Omega_a \to \Omega_a + i\mathfrak{w}_a \Xi$. Unlike what happens with $(2,2)$ supersymmetry, such superfields can be supersymmetrically coupled axially to gauge fields through a field-dependent Fayet–Iliopoulos term, which is classically non-gauge invariant:

$$\mathcal{L}_{\text{FI}} = -\frac{ih^a}{4}\int d\theta^+ \, \Omega_a \Upsilon + \text{h.c.}\,, \quad \delta_\Xi \mathcal{L}_g = \frac{h^a}{4}\mathfrak{w}^a \int d\theta^+ \, \Xi \Upsilon + \text{h.c.}, \tag{28}$$

where the coupling h should be an integer to have a well-defined path-integral in any instanton sector. The full theory is gauge-invariant when $\mathfrak{A} + 2h^a \mathfrak{w}_a = 0$; the cancellation of quantum anomalies by classically non-gauge invariant terms is a reminiscent of the right coset conformal field theories that we discussed in section 1. This condition is actually a worldsheet avatar of the Bianchi identity (18).[14]

So far the model describes a $(\mathbb{C}^\star)^2$ bundle. A compact model is obtained by decoupling the non-compact real part of the multiplets Ω_a. This is compatible with worldsheet supersymmetry only if the torus moduli are properly quantized, such that only left-moving currents are gauged. We consider a generic torus of complex structure T and Kähler modulus U, and define a complex charge $\mathfrak{w} = \mathfrak{w}^1 + T\mathfrak{w}^2$. I have shown in [70] that these conditions reduce to a pair of quantization conditions for the moduli

$$\frac{U_2}{T_2}\text{Re}(\mathfrak{w}) - U_1 \mathfrak{w}_2 = -h^1 \in \mathbb{Z}\,, \quad \frac{U_2}{T_2}\text{Re}(T^\star \mathfrak{w}) + U_1 \mathfrak{w}_1 = -h^2 \in \mathbb{Z}\,, \tag{29}$$

implying also a tadpole equation $Q_i Q^i = \frac{2U_2}{T_2}|\mathfrak{w}|^2 + q_n q^n$ similar to (22). These conditions were known in supergravity, except from the role of the constant B-field U_1 which is not obvious to understand from this point of view; as we shall see it is important for consistency of T-duality transformations.

T-duality along torus principal bundles in the presence of three-form flux is an interesting case of study of topology change in string theory, as B-field and off-diagonal components of the metric mix together; it has also an important role in non-geometric backgrounds, see chap. 4. These dualities have been studied geometrically in [93] and using doubled geometry in [94], investigating in particular possible obstructions. These methods were applied to the present heterotic geometries in [95] and generalized to more general $O(d, d+n)$ dualities mixing the torus and gauge bundles. Using the gauged linear sigma-model construction I was able to prove that these dualities are exact symmetries of the worldsheet theory [70].

[14] Indeed, the anomaly $\mathfrak{A} = Q_i Q^i - q_n q^n$ is a measure of $ch_2(V) - ch_2(T)$, while the B-field is associated with the imaginary part of the Fayet–Ilioupoulos term.

2.1.2. Perturbative dualities as (0,2) quotients. Perturbative T-duality transformations of the gauged linear-sigma models with torsion were obtained using the method of quotients [96] adapted to the present context. The shift symmetry of one of the torus superfields, say Ω_1, is minimally coupled to new gauge superfields $(\tilde{\mathcal{A}}, \tilde{\mathcal{V}})$ without kinetic term. One can also couple axially the superfield Ω_2 to the new gauge superfields as in (28). Finally a new chiral superfield Θ, which is neutral w.r.t. the 'tilded' gauge symmetry and with no kinetic term is also coupled axially to the field-strength superfield $\tilde{\Upsilon}$; it plays the role of a Lagrange multiplier. Gauge-invariance of the action indicates that Θ should be charged under the original gauge transformations parametrised by Ξ. If one integrates first the chiral superfield Θ, one obtains $\tilde{\Upsilon} = 0$ hence it gives back the original action.[15] Instead, one can integrate out exactly the superfields $(\tilde{\mathcal{A}}, \tilde{\mathcal{V}})$, as they have no kinetic term; it gives the dual action where Θ plays the role of the dual coordinate.

One may wonder whether there exist non-perturbative corrections to this duality transformation. Mirror symmetry of gauged linear sigma models is corrected by gauge instanton contributions [97]. Nothing similar happens here as we gauge a shift symmetry rather than a phase symmetry; the relevant theory after gauge fixing is a massive Abelian gauge theory that do not admit instantons. Hence, these results about T-duality are non-perturbatively exact in α'.

Combinations of minimal and axial couplings to $(\tilde{\mathcal{A}}, \tilde{\mathcal{V}})$ allow to generate the full $O(2,2,\mathbb{Z})$ duality group of the two-torus. The first $PSL(2,\mathbb{Z})$ acts on the complex structure, and the charges $(\mathfrak{w}^1, \mathfrak{w}^2)$ transform as a doublet. The second $PSL(2,\mathbb{Z})$ acts on the Kähler modulus U and transform the torus charges in an interesting way.[16] Under a $U \to -1/U$ duality, the complex charge of the torus bundle transforms as $\mathfrak{w} \to -\bar{U}\mathfrak{w}$. In order to get a consistent theory for generic $\mathfrak{w}$, the transformed charge should belong to the same lattice $\mathbb{Z} + T\mathbb{Z}$ as the original one; this requirement is highly non-trivial.

The elliptic curve $E_T = \mathbb{C}/(\mathbb{Z} + T\mathbb{Z})$ associated with the torus complex structure should indeed admit a non-trivial endomorphism $\phi : z \in E_T \to -\bar{U}z$. This property, known as *complex multiplication*, is only shared by elliptic curves whose complex structure T is valued in an *imaginary quadratic number field* $\mathbb{Q}(\sqrt{D})$, namely

$$T \in \mathbb{Q} + \sqrt{D}\,\mathbb{Q} \quad \text{with} \qquad D = b^2 - 4ac < 0 \quad , \qquad a, b, c \in \mathbb{Z}\,, \tag{30}$$

implying also that U belongs to the same field $\mathbb{Q}(\sqrt{D})$. These elliptic curves occur in several contexts in string theory [98]. Interestingly, two-torus conformal field theories whose moduli belong to an imaginary quadratic number field are rational [99].[17] This moduli quantization is in agreement with the supergravity analysis of the dual type IIB orientifolds with fluxes and with their lifts to M-theory and F-theory.

2.1.3. Heterotic dualities. These heterotic solutions have a larger duality group which is of the form $O(2, 2+r, \mathbb{Z})$, depending on the gauge bundle. These additional transformations are interesting as they mix the torus bundle with Abelian components of the gauge bundle, hence modify the topology of space-time. Such transformations have been studied in [95] from a target-space point of view, and can be obtained as well in the gauged linear sigma-model approach. It it difficult to perform directly the duality, as it exchanges quantum anomalies with classical gauge non-invariant terms.

[15] If the theory is defined on the torus or on higher genera Riemann surfaces, there are global issues that are taken care of as in the usual case of toroidal duality.

[16] To be complete, the first of the two extra $\mathbb{Z}_2$ symmetries correspond to T-duality along the first torus coordinate and maps $\mathfrak{w} \to \frac{U_2}{T_2}\mathfrak{w}$ while the second one, parity transformation, acts as $\mathfrak{w} \to -\bar{\mathfrak{w}}$.

[17] In the non-compact Eguchi–Hanson model of sec. 1 the interpretation of this result is quite obvious. If the T^2 theory is rational, it contains some holomorphic generators of an extended chiral algebra; one of them is requested to write down the asymmetric Liouville potential (25).

As usual in this situation, one should 'classicalise' the problem by bosonising the fermions. There is no $(0,2)$ superspace bosonisation of Fermi multiplets, as their on-shell degrees of freedom are those of a left-moving Weyl fermion; one can instead embed the corresponding chiral compact boson as the left-moving imaginary part of a $(0,2)$ chiral multiplet. If the duality transformation leaves uninteracting the spurious degrees of freedom they can be decoupled safely from the dual theory. A Fermi multiplet of charge q_ℓ is a section of a line bundle over the K3 base. After bosonisation one gets a chiral multiplet B_ℓ whose shift symmetry is gauged with charge $-q_\ell$, namely $B_\ell \to B_\ell - iq_\ell\Xi$, and with an axial coupling $h^\ell = q_\ell/2$. This identification of parameters ensures that the gauge variation stays the same.[18]

The previous method can be used if one considers an 'auxiliary torus' made with a bosonised Fermi multiplet B_1 of charge q_1 and one of the torus chiral multiplets. Let us consider for illustration a model with vanishing shift-charge $\mathfrak{w}$, i.e. an ordinary $K3 \times T^2$ with an Abelian component in the gauge bundle. We perform a $u \to -1/u$ duality transformation on the 'torus' (Ω_1, B_1). In order to be able to decouple the real part of the dual multiplet and re-fermionise its imaginary part, the torus moduli of the original theory needed to satisfy $U = 2T$. Whenever this is the case one obtains a dual theory with a different topology, having a torus bundle characterized by the charges $(\tilde{\mathfrak{w}}_1 = q_1/2, 0)$. If the original torus was orthogonal, i.e. with $T_1 = 0$, the gauge bundle of the dual model has one Abelian factor less. If it was tilded ($T_1 \neq 0$) one obtains also a Wilson line, which is promoted to an Abelian bundle over the total space if one dualizes a model with a non-trivial torus bundle initially.

In the previous paragraph we have established a perturbative duality between two $(0,2)$ sigma-models corresponding to different target-space topologies. If we view these sigma-models as world-sheets of the heterotic string the situation is a little bit more subtle because of the GSO projection. The bosonised fermions are not independent but subject to a diagonal $\mathbb{Z}_2$ quotient, which remains after the duality as residual large gauge transformations; hence the GSO projection acts on the torus fiber in the dual model. This seems however consistent with the analysis of global anomalies in these gauged linear sigma models [100], which indicates that the torus superfields Ω_a should be charged under the left global $U(1)$ symmetry that contains this $\mathbb{Z}_2$ as a subgroup, in order to get an anomaly-free symmetry.

2.2. More general compact models with torsion. The previous construction of gauged linear sigma models with torsion was very specific to compactifications with torus bundles. One can generalize the idea by considering non-gauge-invariant field-dependent Fayet–Ilioupolos terms for ordinary charged chiral multiplets. This is possible with a logarithmic coupling [101, 102]

$$\mathcal{L}_{\text{FI}} = -\frac{in_a}{4}\int d\theta^+\, \Upsilon \log \Phi_a + \text{h.c.}\,, \tag{31}$$

which does or does not lead to infrared singularities depending on the D-terms structure. It is not obvious though that a logarithmic term makes sense in the UV Lagrangian of the theory; it may be more appropriate to obtain such terms as one-loop contributions from massive multiplets in a low-energy effective action [103].

We have started to investigate some examples with Stefan Groot Nibbelink. In order to use intuition from the most common $(2,2)$ models, and from the non-compact solutions that we have described at the beginning of this chapter, we have considered simple 'localized' non-Kähler modifications of $(2,2)$ models. Let us consider a K3 example, starting with the $(2,2)$ gauged linear sigma-model for the hypersurface $G(z_i) = z_1^6 + z_2^6 + z_3^3 + z_4^3 = 0$ in $\mathbb{P}_{1122}$, namely a $U(1)$ theory coupled to five $(2,2)$ chiral

[18] We consider here models with all the q_ℓ even, corresponding to bundles with vector structure.

multiplets with a superpotential $W = PG(Z_i)$. This hypersurface has three $\mathbb{Z}_2$ quotient singularities inherited from the ambient space. They can be resolved by adding another $U(1)'$ gauge symmetry, whose Fayet–Ililopoulos coupling is the blow-up parameter, and an extra chiral superfield X; the charges of the fields, written in $(0,2)$ multiplets are[19]

(32)

	$Z_{1,2}$	$\Lambda_{1,2}$	$Z_{3,4}$	$\Lambda_{3,4}$	P	Λ_P	X	Λ_X	Σ	Σ'
$U(1)$	1	1	2	2	-6	-6	0	0	0	0
$U(1)'$	1	1	0	0	0	0	-2	-2	0	0

,

and the hypersurface is modified to $\tilde{G}(x, z_i) = x^3(z_1^6 + z_2^6) + z_3^3 + z_4^3 = 0$. In a patch containing a resolved singularity Z_1, Z_2 and X locally describe $\mathbb{P}_{11-2}$, i.e. Eguchi–Hanson space. It indicates that one can study alternative $(0,2)$ resolutions with line bundles and/or 'blown-up' fivebranes, while keeping the 'bulk' gauge bundle close to the $(2,2)$ locus. Possibilities are severely restricted by the gauge anomalies; there exists only one strong departure from the standard embedding which consists in replacing Λ_X by four Fermi multiplets $\Gamma_{1,\dots,4}$ of charges $q_a = \pm 1$, while dropping the superpotential term $\int d\theta^+ P\Lambda_X \partial_X \tilde{G}$. Left and right global $U(1)$ symmetries are also anomaly-free provided that $\sum q_a = -2$; it is expected to flow to a superconformal fixed point with the requested central charges. The elliptic genus of this model will be discussed in section 3.

A resolution supporting non-trivial flux is possible if one relaxes the condition of gauge invariance for the $U(1)'$ factor by allowing generic charges $\vec{q}$ for some Fermi multiplets $\vec{\Gamma}$, and introduces a logarithmic coupling of the form (31) for X. Otherwise it can be generated at one-loop if one adds a chiral superfield Y and a Fermi superfield Ψ together with a superpotential $\mathcal{L}_T = -\frac{m}{\sqrt{2}} \int d\theta^+ \Psi XY$ that gives to both a mass whenever $\langle x \rangle \neq 0$. A seemingly consistent example, with a five-brane charge equal to $\vec{q}^2/4 - 1$, is given by the field content:

(33)

	$Z_{1,2}$	$\Lambda_{1,2}$	$Z_{3,4}$	$\Lambda_{3,4}$	P	Λ_P	X	$\vec{\Gamma}$	Y	Ψ	Σ
$U(1)$	1	1	2	2	-6	-6	0	0	0	0	0
$U(1)'$	1	1	0	0	0	0	-2	$\vec{q}$	$\vec{q}^2/4$	$2-\vec{q}^2/4$	0

.

We are also considering models allowing separate resolutions of the singularities.

3. Elliptic genera and threshold corrections

Worldsheet models of heterotic compactifications allow to compute the corresponding four-dimensional effective action and explore its phenomenological properties. In an ideal world one knows the $(0,2)$ superconformal field theory explicitly; the massless spectrum is obtained from the torus partition function and tree-level interactions are computed from the N-point functions on the sphere. The torus amplitude gives also the one-loop corrections to the gauge couplings [104]. If only an ultraviolet description of the worldsheet is known, one can still carry part of this program. The massless spectra of $(0,2)$ theories can be computed in a Landau–Ginzburg phase [105], if there is any. The twisted Ramond–Ramond sector partition function, or *elliptic genus*, is invariant under RG flow; the threshold corrections to the gauge couplings are by-products of this quantity. The elliptic genus is easily computed in a Landau-Ginzburg model [106]. It was found recently that it can be computed in any gauged linear sigma model [107].

The elliptic genus is a topological index that encodes the topological properties of the manifold corresponding to the $(0,2)$ sigma-model and of its gauge bundle. Elliptic genera of heterotic strings $K3 \times T^2$ compactifications have been well studied; by a suitable expansion they allow to compute the D-instanton contributions to the dual type II

[19]The chiral superfield P and the Fermi superfield Γ_P appear in the $(2,2)$ superpotential term $\int d\theta^+(\Lambda_P + P\Lambda_i\partial_i)G(Z_i)$, while the chiral superfields Σ and Σ' come from the $(2,2)$ vector multiplets.

orientifolds [108]. We were interested in understanding how this index is modified in the presence of three-form flux, and what are the consequences for the effective action. The elliptic genus is also a very efficient tool in identifying potential superconformal theories associated to a given UV Lagrangian, or conversely to check if several microscopic theories lie in the same universality class, for instance a strong/weak dual pair of two-dimensional gauge theories.

We have computed the contributions to threshold corrections of the warped Eguchi-Hanson geometries in [71]. The mathematical aspects of this computation are very interesting too as it involves *mock modular forms* which have attracted lot of attention recently. During his master thesis, Matthieu Sarkis has computed the elliptic genera of some of the gauged linear sigma models with torsion discussed in section 2.

3.1. Threshold corrections in local models with flux. In section 1 we have constructed local heterotic models, that can be consistently decoupled from a full compactification. In this regime degrees of freedom supported at the resolved singularity survive, giving a discrete spectrum containing massless hypermultiplets, while a continuous spectrum, which is an artefact of the double-scaling limit, appears. One expects that the discrete spectrum encodes part of the physics of the original compactification; the one-loop integral in this limit should give the contribution of the localized states to the threshold corrections to the gauge couplings. There is however a potential issue: this discrete spectrum is anomalous from the four-dimensional point of view, corresponding only to part of the degrees of freedom of the full compactification. From the two-dimensional point of view, it should translate naively into a modular anomaly in the threshold correction. As we shall see the theory takes care efficiently of this problem.

The one-loop correction to the gauge couplings, setting apart a universal contribution independent of the gauge group, is given by the 'new supersymmetric index' [109] with an insertion of the corresponding regularized Casimir operator ($\vec{J}$ being the Cartan) [110]:[20]

(34)

$$\Lambda_a = \int \frac{d^2\tau}{8\tau_2}\Gamma_{2,2}(T,U)\mathcal{A}_a \, , \quad \mathcal{A}_a = \frac{1}{\eta^4}\text{Tr}_{\tilde{R}}\left(\left[Q_a^2 - \frac{k_a}{4\pi\tau_2}\right]\bar{J}_0^R e^{i\pi\bar{J}_0^R} q^{L_0-\frac{5}{6}}\bar{q}^{\bar{L}_0-\frac{1}{4}}e^{2i\pi\vec{\nu}\cdot\vec{J}_0}\right)\Big|_{\vec{\nu}=\vec{0}},$$

where Q_a^2 is the quadratic casimir of the gauge factor $\hat{g}_a$ at level k_a. The index $\mathcal{A}_a$ in eq. (34) is obtained from the elliptic genus of warped Eguchi–Hanson, that we have computed first. We have obtained a first contribution from the localized states, corresponding to BPS representations of the $\mathcal{N}=2$ superconformal algebra:

(35)

$$G_L(\tau,\nu) = \frac{1}{2\ell}\sum_{j=1}^{k/2}(\chi^{j-1}+\chi^{k/2-j})(\tau)q^{-\frac{(j-\frac{\ell+1}{2})^2}{k}}\sum_{N=0}^{\ell-1}e^{-\frac{i\pi N}{\ell}}A_1\left(\frac{\nu+\frac{\ell+1}{2}-j}{\ell}\tau+N,\frac{\nu}{\tau}\Big|\tau\right)\frac{i\vartheta_1(\tau,\nu)}{\eta^3}\,.$$

This expression involves affine $SU(2)$ characters χ^j, reflecting the $SU(2)$ symmetry of Eguchi–Hanson space, and an *Appell-Lerch sum* A_1 which is a well-known example of *mock modular form*; it signals the failure of the index (35) to be a holomorphic Jacobi form. In the present context this modular anomaly is related to the contribution of non-BPS states in the continuum, that have been ignored so far.[21]

A mock-modular form $h(\tau)$ of weight r is a holomorphic function on the upper half-plane which 'almost' transforms as a modular form of the same weight. It is such

[20]In ordinary $K3\times T^2$ compactifications this quantity is uniquely determined by its modular properties and anomaly cancellation in space-time [111].

[21]The usual argument that prevents the non-BPS representations to contribute to the elliptic genus fails if there is a continuum of states, as bosons and fermions can have a different density of states [112].

that one can associate to h a *shadow* g, which is a holomorphic modular form of weight $2-r$, allowing to complete h into a harmonic weak Maaß form $\hat{h}= h + g^\star$, where $g^\star = (\frac{i}{2})^{r-1}\int_{-\bar\tau}^{i\infty} dz\, \bar{g}(-\bar\tau)(z+\tau)^{-r}$, transforming as a modular form of weight r. The shadow encodes the failure of $\hat{h}$ to be holomorphic. These forms have played an important role in BPS black hole entropy [113]. In the present context, the Appell-Lerch sum A_1 in the index (35) can be completed to a non-holomorphic Jacobi form $\hat{A}_1 = A_1 + g^\star$ giving an elliptic genus with the requested modular behaviour.[22] The shadow g adds a non-holomorphic contribution from the non-BPS continuous modes that propagate in the semi-infinite throat geometry. A direct path integral computation of the elliptic genus of $\mathcal{N}=2$ Liouville theory [114], which enters in our computation, proved that this analysis is physically consistent.

The non-holomorphic elliptic genus can be used then to compute the indices $\mathcal{A}_a$ in (34) for the unbroken gauge group factors. It is worth noticing that the (unique) non-holomorphic completion is universal, i.e. independent of the gauge group factor. In order to obtain the moduli dependence of the threshold corrections one needs to perform the modular integral over the fundamental domain in Λ_a. This can be done by the 'unfolding technique' [104], convenient for understanding the type I dual interpretation, which consists in trading the sum over winding modes in the T^2 lattice $\Gamma_{2,2}(T,U)$ for an unfolding of the fundamental domain. The modular integral in the zero orbit of $PSL(2,\mathbb{Z})$ can be performed directly. For terms in the lattice sum lying in degenerate orbits one can unfold to the upper half-strip; in the type I dual they correspond to perturbative contributions to the threshold correction. The contribution of discrete representations to these corrections is of the same type as in compact models, and compatible with the contributions of the localized hypermultiplets to the gauge coupling beta-function. The contributions of non-BPS states are also computable; they are exponentially suppressed w.r.t. the discrete part in the large T^2 limit. The terms corresponding to non-degenerate orbits can be unfolded to the upper half-plane, and are mapped to D-instantons contributions in type I [108]. Similarly the term from BPS-states is quite similar to the $K3\times T^2$ case, while the non-BPS terms lead to an infinite series of corrections to the dimension-eight operators.

Computing the contribution to threshold corrections in local models of resolved singularities with flux was another incarnation of UV/IR mixing in string theory. As they capture only part of the consistent low-energy spectrum of the full compact theory the one-loop amplitude exhibited a modular anomaly which required 'anomaly inflow' from the bulk, leading to a contribution of massive non-BPS representations that drastically affects the physics. It would be quite interesting to understand the meaning of this result from the holographic perspective of heterotic little string theory.

3.2. Elliptic genera of torsional models. For compact models with torsion there are no conformal field theories available so far. Fortunately it is possible to compute the elliptic genera of gauged linear sigma-models using localisation techniques. For ordinary theories, i.e. without couplings like (31), the whole Lagrangian of the model is Q-exact for an appropriate choice of supercharge. This allows to compute the elliptic genus as a one-loop computation around a free saddle point [107].

With Matthieu Sarkis we have used these expressions to compute the elliptic genera of some gauged linear sigma models with torsion. In order to understand better the relation between the local models of section 1 and the compact models of section 2, in particular for models with resolved singularities like (33), it was useful first to reconsider the warped Eguchi–Hanson space with line bundles from a similar point of

[22]The first Appell-Lerch sum is $A_1(u,v|\tau) = e^{i\pi u}\sum_n (-)^n q^{n(n+1)/2} e^{2i\pi n v}/(1-e^{2i\pi u}q^n)$ and its shadow $g^\star(u,v|\tau) = -\frac{i}{2}\theta_1(v|\tau)\sum_{n\in\mathbb{Z}+1/2}(-)^{n-1/2}\left[\text{sign}(n) - E\left((n+\frac{v_2}{\tau_2})\sqrt{2\tau_2}\right)\right] e^{-i\pi v n} q^{-n^2/2}$, E being the error function.

view. Eguchi–Hanson space is identified with the projective space $\mathbb{P}_{11-2}$, which is a simple example of non-compact toric Calabi–Yau. A corresponding $(0,2)$ model with line bundle and non-zero fivebrane charge is essentially given by the second line of eq. (33). A set of Fermi multiplets Γ^i give a sum of line bundles[23] while the multiplets Y and Ψ, once integrated out, provide the gauge-variant Fayet–Ililopoulos term in the two-dimensional effective action. The computation indicates that none of the subtleties discussed in the previous example are present. The elliptic genus, which should be regularized of an infrared infinite-volume divergence by turning on some chemical potential, is holomorphic. It suggests that this model does not flow in the infrared to the warped geometry in the double-scaling limit, but rather to the asymptotically locally flat one with the conformal factor (23). It would be interesting to analyse the model with the logarithmic coupling (31) in the UV Lagrangian, as this term is not Q-exact hence may lead to the same model as the superconformal one.

The same computation was carried out for the full model (33), which embeds resolved warped Eguchi–Hanson geometries in a compactification. It paves the way to an understanding of more realistic compactification on six-manifolds with flux, in particular of embeddings of the warped conifold geometries in compact models.

[23]The chiral multiplet Σ, that restricts the sum of line bundles to the tangent bundle in the $(2,2)$ models, is not present here.

4

Beyond geometric compactifications

Considering compactifications far from the large-volume and geometric regimes is extremely interesting as it unveils the most unusal aspects of string theories. In some settings, as heterotic compactifications with fluxes studied in chapter 3, this is forced upon us given that there does not exist any large volume limit. In a broader perspective, one expects that generic compactifications of any of the string theories are non-geometric, which calls for a better understanding of string geometry.[1]

There are two related classes of phenomena stemming from the quantum geometry of strings. In the small volume limit the notion of target-space becomes ambiguous and several seemingly different geometries and even topologies become equivalent at certain points in the moduli space. Discrete unbroken subgroups of these enlarged symmetries are responsible for an enhancement of the symmetry group by perturbative dualities away from these special locii in moduli space, which can be used to twist the compactification [115].

D-branes can be considered as more refined probes of the geometry than the string themselves in the perturbative regime, at least when no three-form flux is present.[2] With Matthias Gaberdiel and Eliezer Rabinovici, we have investigated in [116] D-branes at multicritical points where several components in the moduli space of conformal field theories, associated with different target-spaces, meet; we have found a non-trivial mapping between the branes in the two equivalent descriptions.

I have constructed recently a new class of non-geometric backgrounds that consist in asymmetric Gepner models for $\mathsf{K3} \times \mathsf{T}^2$ [117] in type II that preserve $\mathcal{N} = 2$ space-time supersymmetry. The massless spectra were computed with Vincent Thiéry during his master thesis. These models allow in particular to get at low energies STU supergravity without any other massless multliplet.

I have started to investigate the algebraic and geometric structure underlying these models. They consist in K3 bundles over two-torii with non-trivial monodromies involving a new kind of symmetry of Calabi–Yau quantum sigma-models in the Landau–Ginzburg regime. Quotienting the world-sheet theory by this symmetry gives a two-dimensional model corresponding to the *fractional mirror* quantum geometry; they are isomorphic as quantum field theories. This duality provides interesting relations between Calabi–Yau and non-Calabi–Yau compactifications, as I have shown in [118].

1. D-branes at multicritical points

It is expected that the moduli space of D0-branes in a given conformal field theory gives the 'quantum geometry' of the target-space. In general the same space can be probed by D-branes of different dimensonalities. In torus compactifications they are all related to the D0-brane by T-duality hence they probe the associated T-dual geometries. In a more general setting, the moduli space of all D-branes associated to some

[1]Understanding the behavior of string theory near space-like singularities is also a very important subject, however I will focus here on aspects of string compactifications.

[2]In the presence of fluxes D-branes exhibit some 'fuzziness'; an example was given in chap 1, sec. 1.

conformal field theory contains also typically several components and one may wonder how they are related to different geometrical interpretations. An interesting case of study for testing these ideas is given by *multicritial points*, which are defined as points in the moduli space of conformal field theories where several marginal lines intersect.[3] At such points there exist different geometrical descriptions of the same theory, each of them coming with its own 'canonical' set of D-brane probes – for instance Dirichlet or Neumann branes for a free boson – that are embedded in a larger D-brane moduli space. A more fondamental question is whether the D-brane completions of the different conformal field theories actually coincide at the multicritical point. It may seem that the answer of this question is obviously positive, given that a conformal field theory on the disk can be entirely solved by the data of the theory on the sphere. The story is actually slightly more subtle. One associates to the theory a set of boundary states that satisfy several consistency conditions; we construct this set such that it contains the 'canonical' branes associated to the theory. It is not known in general whether several consistent sets of boundary states associated to the same 'bulk' theory exist, and if it were the case it may be that the set containing the 'canonical' brane of one description at the multicritical point do not match the 'canonical' set in the other descriptions.

1.1. Conformal branes. The multicritical point of $c = 1$ theories is described either as a free boson on a circle of radius $R = \sqrt{2}$, *i.e.* twice the self-dual radius, or as the orbifold $S^1/\mathbb{Z}_2$ at the self dual radius. The usual branes of an S^1 theory have Dirichlet or Neumann boundary conditions and preserve a $\hat{u}(1)$ affine symmetry.[4] Whenever the radius is rational in self-dual units they are embedded in a larger moduli space of branes that preserve only the conformal symmetry. Highest weight $\hat{u}(1)$ representations of charge $q = \sqrt{2}m$ with m half-integer are reducible into irreducible Virasoro representations as $\mathcal{V}^{u(1)}_m = \oplus_{\ell=0}^{\infty} \mathcal{V}^{\mathfrak{Vir}}_{(\ell+m)^2}$. One can associate to any such Virasoro representation of highest weight $\Delta = j^2$ coming from the $\hat{u}(1) \times \hat{u}(1)$ representations of charges $(\sqrt{2}m, \sqrt{2}\bar{m})$ an Ishibashi state $|jm\bar{m}\rangle\rangle$. In the present case ($R = \sqrt{2}$) they come from even-momentum primary states.

It has been shown in [119] that the associated conformal boundary states, which are linear combinations of those, are labeled by $SU(2)$ elements g up to the identification $g \sim \mathrm{Ad}_{\sigma_3} g$.[5] In other words the associated moduli space is $SU(2)/\mathbb{Z}_2$; it contains the Neumann branes for $g = i\sigma_1 e^{i\sigma_3 \phi}$. The fixed point set of this quotient, $g = e^{i\sigma_3\theta}$, corresponds to a pair of antipodal Dirichlet branes that should be resolved into *fractional branes*, that are simply individual Dirichlet boundary states in this theory. The picture of the D-brane moduli space from the circle theory point if view, is the following. A single Dirichlet brane moduli space is isomorphic to the circle at radius $R = \sqrt{2}$. The moduli space of Neumann branes is embedded in a larger $SU(2)/\mathbb{Z}_2$ moduli space; at the fixed point set this moduli space is glued onto another branch, the moduli space of two Dirichlet branes.

The $S^1/\mathbb{Z}_2$ orbifold at the self-dual radius gives an isomorphic but physically distinct picture. The new ingredients are the twist fields associated with the fixed points $y_0 = 0, \pi R$, to which we associate new Dirichlet Ishibashi states $|D; y_0\rangle\rangle_{tw}$. It is well-known that Dirichlet branes sitting at one of the fixed points are resolved into fractional branes as $|D, y_0; \pm\rangle = \frac{1}{2}|D; y_0\rangle_{orb} \pm 2^{-1/4}|D; y_0\rangle\rangle_{tw}$; the same is true for Neumann branes at the T-dual fixed points. Conformal branes in the orbifold theory are also labeled by $g \in SU(2)$ up to $g \sim \mathrm{Ad}_{\sigma_1} g$. This time a single Dirichlet brane, away from

[3]Naturally one can study also multicritical lines at the intersection of marginal planes, etc....

[4]There is in addition for generic radii a continuous family of boundary states that couple only to the vacuum representation.

[5]This structure is inherited from the enhanced $SU(2)_1 \times SU(2)_1$ symmetry at the self-dual radius.

the fixed points, is fundamental. The fixed point set $g = e^{i\sigma_1\psi}$ contains both regular Dirichlet branes at the fixed points and Neumann branes at the T-dual fixed points. The construction of generic conformal branes associated with the fixed points goes as follows. One decomposes the twisted $\mathfrak{u}(1)$ representations, built upon the twist fields σ^{y_0}, into non-degenerate Virasoro representations, to which we associate Ishibashi states $|(\ell+1/2)^2/4, y_0\rangle\rangle_{\mathrm{Vir}}$, $\ell = 0, 1, \ldots, \infty$. We have then obtained the 'interpolating' boundary states for the fractional branes of the full fixed point circle, reproducing the Dirichlet and Neumann fractional branes at special values, as linear combinations of these Ishibashi states. Thus from the $S^1/\mathbb{Z}_2$ orbifold point of view the moduli space of regular D-branes is isomorphic to $SU(2)/\mathbb{Z}_2$ and contains both individual Dirichlet and Neumann branes. The fixed point set is resolved into a pair of one-parameter family of branes that interpolate between Dirichlet and Neumann fractional branes.

1.2. Superconformal branes. A second multicritical point occurs in the moduli space of $\hat{c} = 1$ superconformal theories. The first line of theories corresponds to a 0B-type modular invariant for free bosons and fermions; the multicritical point occurs at $R = 1$. There are two possible superconformal boundary conditions, for $\eta = \pm 1$:

$$(36) \qquad \left(L_n - \bar{L}_{-n}\right)|B;\eta\rangle = 0\,, \qquad \left(G_r + i\eta\,\bar{G}_{-r}\right)|B;\eta\rangle = 0\,.$$

As we shall see the two boundary conditions will lead to rather different pictures. In the NS–NS sector even-momenta $\hat{\mathfrak{u}}(1)$ representations are degenerate and decompose into infinitely many irreducible super-Virasoro representations. The associated Ishibashi states $|jm\bar{m};\eta\rangle\rangle_{\mathrm{NS}}$ have only integer values of j, hence the moduli space of NS branes is $SO(3)/\mathbb{Z}_2$. The analysis is more subtle in the Ramond–Ramond sector because of the fermionic zero modes; the GSO parity of the superconformal Ishibashi states $|jm\bar{m};\eta\rangle\rangle_{\mathrm{R}}$ is indeed $-\eta(-)^{m-\bar{m}}$. The full boundary states are obtained by putting together the two contributions. With $\eta = +1$ and at $R = 1$ the Ramond Ishibashi states are all GSO-odd, hence there is only an NS–NS component and the moduli space is $SO(3)/\mathbb{Z}_2$. When g is diagonal the brane is not fundamental and corresponds to two antipodal non-BPS D0-branes, while an off-diagonal g is the superposition of a Neumann brane and a Neumann anti-brane. If $\eta = -1$ the RR boundary state is GSO-even thus the moduli space is enlarged to $SU(2)/\mathbb{Z}_2$. In this case a diagonal g is resolved into antipodal D0-branes with opposite R–R charge while off-diagonal g corresponds to a single non-BPS Neumann brane.

The second line of theories which intersect the circle line is the line of *super-orbifold* theories; they are defined as a standard $\mathbb{Z}_2$ orbifold of the *super-affine* theories.[6] To begin with we have constructed the boundary states in the latter theory which, for $R = 1$, has the maximal $\widehat{\mathfrak{so}}(3)_1 \times \widehat{\mathfrak{so}}(3)_1$ super-Kač–Moody symmetry. NS-NS boundary states span a $SO(3)$ moduli space while for $\eta = -1$ the R–R boundary state is also GSO-invariant and the moduli space is enhanced to $SU(2)$. The super-orbifold theory is then a quotient by $\mathcal{I} : (X, \psi) \mapsto (-X, -\psi)$. Away from the fixed locus the boundary states are obtained by adding the super-affine boundary state and its image under the orbifold. At the fixed points infinite sets of extra super–Virasoro representations built on the twisted states appear, as well as corresponding Ishibashi states. Interestingly, the Ramond–Ramond ground-state Ishibashi states associated with the two fixed points have opposite GSO parities, because one of them lies in the untwisted sector of the $\mathcal{S}$-orbifold and the other one in its twisted sector; it will lead eventually to an interesting picture. The superconformal boundary states with $\eta = +1$ have only an NS–NS contribution, hence the moduli space $SO(3)/\mathbb{Z}_2$ has two fixed point sets. For diagonal g we get in particular a regular Dirichlet brane sitting at the fixed point $y = 0$;

[6]The super-affine line obtained from the circle line with the orbifold by $\mathcal{S} = s(-)^{F_s}$, where s is a half-shift around the compact circle and F_s denotes the left-moving space-time fermion number.

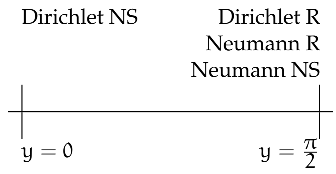


FIGURE 1. *Twisted sector branes for* $\eta = +1$ *in the super-orbifold theory.*

it is resolved using the associated Dirichlet twisted NS Ishibashi states. There exists also a Neumann point which is suprisingly resolved using only Neumann twisted NS Ishibashi states associated with the other fixed point $y = \pi/2$. The other fixed-point set, off-diagonal g, contains a Dirichlet point, associated with the fixed point $y = \pi/2$, and a Neumann point as well, which are both resolved using *only* twisted Ramond superconfomral Ishibashi states associated with the twist field at $y = \pi/2$, see fig 1. A related story goes for superconformal boundary states with $\eta = -1$, which involve also Ramond superconformal Ishibashi states; the moduli space is then $SU(2)/\mathbb{Z}_2$. The unique fixed point set contains a regular Dirichlet brane at $y = \pi/2$ which is resolved both by twisted R and NS Ishibashi states attached to the same fixed point, while the Neumann branes get resolved by a contribution from each, see fig. 2.

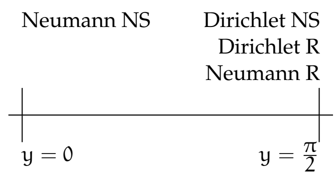


FIGURE 2. *Twisted sector branes for* $\eta = -1$ *in the super-orbifold theory.*

To conclude we have obtained for both multicritical points a perfect agreement between the two full sets of boundary states coming from the two alternative descriptions of the theory; it led to some suprises related to the relation between the brane moduli space and the target-space geometry, especially regarding the fixed-points resolutions.

2. Asymmetric Gepner models and non-geometric backgrounds

Ambiguities in the geometry as viewed by string probes, and the associated dualities, can be used in a more active fashion in *non-geometric* compactifications. One can consider compactifications with a fibration structure, and allow the perturbative duality transformations to appear in the transition functions [115]. When the fibers are toroidal typical examples are *T-folds*, where the fiber is glued onto its T-dual while going around a non-contractible loop of the base space. In more complicated examples, whenever the theory is given for instance as an abstract two-dimensional conformal field theory it is not obvious to distinguish between, say, non-geometric compactifications and geometric ones in the small-volume limit. In type II superstring models with no Ramond-Ramond flux, there is a simple sufficient criterion for non-geometry. Let us consider a *compact* model preserving different amount of supersymmetry coming from the left- and right-movers of the worldsheet. If a *geometric* solution of this sort existed, the two torsionful connections $\nabla(\omega \pm \frac{1}{2}H)$ that appear in the gravitini supersymmetry variations would be characterized by different G-structures; this would require non-vanishing NS-NS three-form flux H. For compact supergravity solutions this is forbidden by the tadpole condition $\int e^{-2\Phi} H \wedge \star H = 0$ following from the equations of motion. Hence any string theory without Ramond–Ramond fluxes, with a discrete spectrum in its internal space and asymmetric space-time supersymmetry is necessarily non-geometric.

Little is known about possible non-geometric compactifications besides the toroidal and free-fermionic cases; they are potentially very interesting because they allow for lifting many massless moduli without resorting to the use of orientifold flux compactifications which are only accessible in the large-volume limit. A good starting point is provided by *Gepner models* [91], that are non-trivial interacting superconformal field theories that describe Calabi–Yau compactifications in the deep quantum regime of negative Kähler moduli. As we shall see one can construct new non-geometric backgrounds by 'twisting' a K3 at a Gepner point over a two-torus.

A Gepner model for a Calabi–Yau n-fold is obtained from a tensor product of r $\mathcal{N}=2$ minimal models whose central charges satisfy $\sum_{n=1}^{r} c_i = \sum_{n=1}^{r}(3-6/k_i) = 3n$. In order to construct an appropriate worldsheet theory for a type II superstring compactification one needs to project the theory onto states with integer left- and right-moving R-charges, before a final GSO projection. A modular-invariant partition function with these properties has been constructed by Gepner, as a $\mathbb{Z}_K$ diagonal orbifold, with $K = \text{lcm}(k_1,\ldots,k_r)$, followed by a chiral $\mathbb{Z}_2\times\mathbb{Z}_2$ quotient. The building blocks are the minimal $\mathcal{N}=2$ characters, labeled by the triplet (j,m,s), with $s\equiv 0 \bmod 2$ (resp $1 \bmod 2$) in the Neveu–Schwarz (resp. Ramond) sector. Using the identification between minimal models and supersymmetric $SU(2)_k/U(1)$ cosets, these characters are implicitly defined from $SU(2)_k$ characters by the branching relation:

$$\chi^j\Theta_{s,2} = \sum_{m\in\mathbb{Z}_{2k}} C_m^{j\,(s)}\Theta_{m,k}\,, \tag{37}$$

where the theta-functions at level 2 and k correspond respectively to a free Weyl fermion and a $U(1)_k$ free chiral boson.

The construction of asymmetric Gepner models starts from a simple observation. Relation (37) indicates that a minimal model character $C_m^{j\,(s)}(q)$ has the same modular transformations as the character combination:

$$\chi^j(q)\frac{\Theta_{s,2}(q)}{\eta(q)}\frac{\Theta_{m,k}(\bar q)}{\eta(\bar q)} = \sum_{n\in\mathbb{Z}_{2k}} C_n^{j\,(s)}(q)\frac{\Theta_{n,k}(q)\Theta_{m,k}(\bar q)}{\eta(q)\eta(\bar q)}\,. \tag{38}$$

The last factor in the last expression comes from a free boson at radius $\sqrt{\alpha' k}$ with an asymmetrically shifted lattice. One can start with *any* K3$\times$ Gepner model times a two-torus and replace the contributions of the torus and of two minimal models by two factors like (38), spoiling neither modular invariance, nor worldsheet supersymmetry.

These compactifications have unusual features. Because the Gepner generalized GSO projection is blind to the sum over n in eq. (38), the left-moving R-charges are generically fractional, hence space-time supersymmetry from the left-movers is completely broken.[7] One checks indeed explicitely from the partition function that the two associated gravitini become massive, as is the whole Ramond–Ramond sector. The theory, initially defined for an orthogonal torus of radii $\sqrt{\alpha' k_1}$ and $\sqrt{\alpha' k_2}$ can be deformed to any torus moduli given that the associated worldsheet operators are still marginal.[8] The supersymmetry-breaking of $\mathcal{N}=4$ to $\mathcal{N}=2$ is associated with the gravitini masses (T and U being the complex structure and Kähler modulus of T^2):

$$M^{\pm}_{\Psi_\mu}(T,U) = \sqrt{\frac{T_2}{U_2}+\frac{(T_1\pm 1)^2}{U_2T_2}}\,. \tag{39}$$

[7]As explained in chapter 3, section 1, models like those can be mapped to interesting non-supersymmetric heterotic solutions.

[8]This point is characterized by an enhanced $SU(2)_{k_1}\times SU(2)_{k_2}$ left-moving affine symmetry. However all the massless states of the asymmetric Gepner model are in the trivial representation of both.

The spectrum of massless operators can be inferred from the one-loop partition function. Right-moving massless operators are necessarily chiral or anti-chiral operators of R-charge $|\bar{Q}_R| = 1$ as in the usual case. This is not obviously true on the left as fractional R-charges are present. We have scanned the spectra of all asymmetric models of this type; from the 14 ordinary K3 Gepner models we obtain 62 inequivalent asymmetric models. Among them we have found that, in 33 models, all the massless moduli of K3 are lifted, except the volume mode. The low-energy effective action of these theories is therefore the well-known $\mathcal{N} = 2$ STU supergravity without any other massless multiplet. The T and U vector multiplets come from the unlifted torus moduli and Kaluza–Klein gauge fields while S is the usual axio-dilaton multiplet. In the remaining 29 models some K3 moduli remain as massless hypermultiplets in the low-energy theory; their number is model-dependent. Interestingly, because the dilaton is part of a vector multiplet, the associated hypermultiplet moduli space receives no corrections.

As explained in the introduction of this chapter these solutions are intrisically non-geometric as all space-time supersymmetry come froms the right-movers. As expected in such situation the number of massless moduli is rather low; I am not aware of another way of obtaining STU supergravity *exactly* in string theory, at least without Ramond–Ramond fluxes. Given that the masses of the gravitini can be taken hierachically smaller than the string scale, a low-energy interpretation of this construction is to consider some gauging of $\mathcal{N} = 4$ supergravity with 22 vector multiplets, whose vacua break spontaneously supersymmetry to $\mathcal{N} = 2$ and whose massless spectra in these vacua match the worldsheet calculations. This is an interesting ongoing project.

Besides the supersymmetry argument it would be better to have a more intrisic characterisation of the non-geometric nature of these asymmetric Gepner models. I will study this underlying structure in the next section and find, surprisingly, that these constructions are associated with twists involving new perturbative symmetries of Calabi–Yau quantum sigma-models.

3. New symmetries in Calabi–Yau moduli space

The structure of the models constructed in section 2 becomes more transparent if we use the formalism of *simple currents* [120, 121] that we shall review very briefly. It will provide the seed for the constructions of new asymmetric models and of fractional mirror symmetry that I have introduced in [118].

We consider some conformal field theory characterized by a chiral algebra, whose representations are labelled by μ. A *simple current* J is a primary such that its fusion with a generic primary of the chiral algebra gives a single primary : $J \star \phi_\mu = \phi_\nu$. This action defines the *monodromy charge* of the primary w.r.t. the current, $Q_\imath(\mu) = \Delta(\phi_\mu) + \Delta(J_\imath) - \Delta(J_\imath \star \phi_\mu) \mod 1$. Two simple currents are said *mutually local* if $Q_\imath(J_\jmath) = 0$. Provided that the simple currents action has no fixed points, the extended modular-invariant partition function is given by:

$$Z = \sum_{\mu} \prod_{\imath=1}^{M} \sum_{b^\imath \in \mathbb{Z}_{n_\imath}} \chi_\mu(q)\, \chi_{\mu+\beta_\jmath b^\jmath}(\bar{q})\, \delta^{(1)}\left(Q_\imath(\mu) + X_{\imath\jmath} b^\jmath\right) , \tag{40}$$

with $\beta_\imath$ such that $J_\imath \star \phi_\mu = \phi_{\mu+\beta_\imath}$, giving the twisted sectors. The selection rule in (40), expressed as a modulo one Kronecker symbol, depends on the ($\mathbb{Q}$-valued) $M \times M$ matrix $X_{\imath\jmath}$ whose symmetric part is determined by the relative monodromies:

$$X_{\imath\jmath} + X_{\jmath\imath} = Q_\imath(J_\jmath)\,. \tag{41}$$

There is some freedom in the choice of antisymmetric part, or *discrete torsion*; it should be such that for all entries of X

$$\gcd(n_\imath, n_\jmath)\, X_{\imath\jmath} \in \mathbb{Z}\,, \tag{42}$$

where $n_\imath$ is the length of the simple-current $J_\imath$, that generates an Abelian group isomorphic to $\mathbb{Z}_{n_\imath}$. If the left and right kernels of X are different the extended theory is asymmetric.

3.1. Asymmetric extensions of Gepner models. The simple currents of $\mathcal{N} = (2,2)$ minimal models are all primary operators with $j = 0$, hence are labelled by $(m_\imath, s_\imath)$. In a three-fold Gepner model they can be grouped together into the $(2r+1)$-dimensional current $J = (\sigma_0 | m_1, \ldots, m_r | s_1, \ldots, s_r)$ where σ_0 is the $\mathbb{Z}_4$ charge carried by the free complex fermion. The Gepner construction can be rephrased as a simple current extension with the current $J_0 = (1|1,\ldots,1|1,\ldots,1)$, that ensures the projection onto states with odd-integer R-charges, and the currents $J_\imath = (2|0,\ldots,0|0,\ldots,0,2,0,\ldots,0)$, $\imath = 1,\ldots,r$, which enforce worldsheet supersymmetry. These currents are mutually local.

Simple current extensions of Gepner models preserving space-time and world-sheet supersymmetry should be mutually local with respect to J_0 and $\{J_\imath\}$, see [122, 123]. Let us consider a simple current of the form $\mathfrak{J} = (0|2\rho_1,\ldots,2\rho_r|0,\ldots,0)$, preserving world-sheet supersymmetry. Space-time supersymmetric extensions correspond to simple currents that are mutually local w.r.t. J_0, hence satisfy $\sum_\imath \rho_i/k_i \in \mathbb{Z}$. Extending a Gepner model by *all* such simple currents (without discrete torsion) gives the mirror Gepner model, which is such that the right R-charge has opposite sign. This is the basis of the construction of mirror symmetry by Greene and Plesser [124].

The non-geometric quotients of Gepner models that we aim to construct correspond instead to extensions with simple currents that are *not* mutually local w.r.t. J_0. A generic simple current $\mathfrak{J}$ as above generates an Abelian group isomorphic to $\mathbb{Z}_N$ with $N = \mathrm{lcm}\,(\mathrm{lcm}\,(\rho_1,k_1)/\rho_1,\ldots,\mathrm{lcm}\,(\rho_r,k_r)/\rho_r)$. The relative monodromies of the simple currents are no longer vanishing. Being non-local w.r.t. J_0, the simple current $\mathfrak{J}$ 'perturbs' the projection onto integral R-charges in the partition function (40) hence space-time supersymmetry is generically broken. There exist a specific choice of discrete torsion, consistent with the condition (42) for any set of integers $\{\rho_i\}$, which brings down the X matrix to a lower-triangular form, whose only non-zero entries are:

$$X_{\mathfrak{J}\mathfrak{J}} = \sum_{i=1}^{r} \frac{\rho_i^2}{k_i}\,, \quad X_{\mathfrak{J}0} = \sum_{i=1}^{r} \frac{\rho_i}{k_i}\,. \tag{43}$$

The modular-invariant partition function of the model is explicitly[9]:

$$Z = \frac{1}{\tau_2^2|\eta|^4}\frac{1}{2^r}\sum_{\lambda,\mu}\sum_{b_0\in\mathbb{Z}_K}(-1)^{b_0}\,\delta^{(1)}\left(\frac{Q_R-1}{2}\right)\sum_{B\in\mathbb{Z}_N}\delta^{(1)}\left(\sum_{i=1}^{r}\frac{\rho_i(m_i+b_0+\rho_i B)}{k_i}\right)$$
$$\prod_{i=1}^{r}\sum_{b^i\in\mathbb{Z}_2}\delta^{(1)}\left(\frac{s_0-s_i}{2}\right)\chi^\lambda_\mu(q)\chi^\lambda_{\mu+\beta_0 b_0+\beta_j b^j+\beta_I B}(\bar{q})\,, \tag{44}$$

where $\chi^\lambda_\mu = \Theta_{\sigma_0,2}/\eta \times \prod_{i=1}^r C^{j_i\,(s_i)}_{m_i}$. As $X_{0\mathfrak{J}} = 0$ the projection onto integer R-charges is restored in the left-moving sector. However twisted sectors associated with the

[9] If some levels k_n are even, there may be fixed points under the simple current action, and multiplicity factors need to be added accordingly to the partition function. For simplicity of presentation we assume that we do not encounter this situation, which does not change the salient features of the construction; for instance one can take all the levels k_n to be odd.

$\mathfrak{J}$-extension (labelled by $B \neq 0$) have fractional right-moving R-charge hence space-time supersymmetry from the right-movers is generically broken while supersymmetry from the left-movers is preserved. Therefore these models fit in the category of non-geometric compactifications, following the discussion at the beginning of this section.

One can check that in general models the gravitino coming from the NSR sector is indeed massive; furthermore the twisted sectors ($B \neq 0$) contain massless operators which are neither chiral nor antichiral on the right, while they are still chiral or antichiral on the left. Studying the whole set of such extensions is interesting, but we will concentrate below on an important class of examples, that are related to the models of section 2.

3.2. Asymmetric $K3 \times T^2$ models revisited. A slight extension of this formalism provides the appropriate reformulation of the $K3 \times T^2$ models of section 2. One starts with an orthogonal torus with radii $\sqrt{\alpha' k_1}$ and $\sqrt{\alpha' k_1}$. The simple currents associated with the full world-sheet theory are of the form $J' = (\sigma_0, \sigma_1 | m_1, \ldots, m_r | s_1, \ldots, s_r | n_1, n_2)$ where $n_{1,2}$ are the $\hat{u}(1)_{k_1} \times \hat{u}(1)_{k_2}$ charges of the two-torus theory.[10] As in the previous discussion we consider simple currents that are not mutually local with the current J_0 that ensures R-charges integrality. Explicitely we take

$$\mathfrak{J}_1 = (0,0|2,0,\ldots,0|0,\ldots,0|2,0)\,, \quad \mathfrak{J}_2 = (0,0|0,2,0\ldots,0|0,\ldots,0|0,2)\,. \tag{45}$$

There exists also a choice of discrete torsion bringing X to a lower-diagonal form, as in eq. (43), hence restoring space-time supersymmetry attached to the left-movers, while the other half is still broken. This construction reproduces precisely the models studied in section 2.

3.3. Fractional mirror symmetry in Calabi-Yau compactifications. The simple current reformulation of asymmetric $K3 \times T^2$ Gepner models helps to understand the underlying symmetry that has been used. The Calabi-Yau sigma-model at the Gepner point is twisted by the symmetries corresponding to the currents (45) truncated to their K3 part. As we shall see extensions of this type have remarkable properties.

Extending the partition function of a three-fold Gepner model with a current $\mathfrak{J} = (0|2,0,\ldots,0|0,\ldots,0)$ and discrete torsion amounts, while taking into account the twisted sectors, to replace in the original partition function the anti-holomorphic character for the first minimal model with the character of opposite $\mathbb{Z}_{k_1}$ charge, namely

$$C^{j_1\,(s_1+b_0+2b_1)}_{m_1+b_0}(\bar{q}) \xrightarrow{\mathfrak{J}_1\text{-ext.}} C^{j_1\,(s_1+b_0+2b_1)}_{-m_1-b_0}(\bar{q})\,. \tag{46}$$

In the right NS sector it is equivalent to change the sign of the right R-charge associated with the first minimal model; in the R sector one reaches the same conclusion if one changes the right-moving space-time chirality at the same time. As superconformal field theories the original model and the new one are isomorphic, hence the two theories are dual.[11]

Let us start with a type IIA Calabi–Yau compactification at a Gepner point. A simple current extension by $\mathfrak{J} = (0|2,0,\ldots,0|0,\ldots,0)$ with discrete torsion gives a type IIB theory on a Gepner model whose right-moving R-charge associated with the first minimal model has ben reversed; with respect to the original right-moving diagonal

[10] A free boson at radius $\sqrt{\alpha' k}$ is a $\hat{u}(1)_k$ theory with primary fields $\phi_n(z) = e^{i\frac{n}{\sqrt{\alpha' k}}X(z)}$, $n \in \mathbb{Z}_{2k}$.

[11] One can define an R-current in the new theory $\bar{J}_R = -\bar{J}_{R,(1)} + \sum_{\iota=2}^{r} \bar{J}_{R,(\iota)}$ with respect to which the R-charges are integer, hence can be expontentiated to a spin field mutually local with the full spectrum. In other words there exists another realization of space-time supersymmetry in the dual model.

R-current the spectrum of R-charges is not integer-valued hence the model is not associated with a Calabi–Yau. Put it differently the quotient does not preserve the holomorphic three-form. As we have observed these two models are isomorphic as conformal field theories hence describe the same physics. This transformation can be repeted stepwise and once the right-moving R-charges of all minimal models have be reversed one finds the mirror Gepner model in the usual sense; these new symmetries can be described as *fractional mirror symmetries*.

A minimal model can be obtained as the infrared fixed point of a Landau–Ginzburg model for a chiral superfield with superpotential $W = Z^k$. The mirror of the minimal model, obtained by a $\mathbb{Z}_k$ quotient, is a Landau–Ginzburg model for a twisted chiral superfield $\tilde{Z}$ with a twisted superpotential $\tilde{W} = \tilde{Z}^k$. In the present context we are considering a similar quotient acting on a Landau–Ginzburg orbifold, with a discrete torsion that disantangles the two orbifolds – the diagonal one ensuring R-charge integrality and the $\mathbb{Z}_{k_1}$ quotient giving the fractional mirror. We end up with a 'hybrid' Landau–Ginzburg orbifold containing both a twisted chiral superfield $\tilde{Z}_1$ and chiral superfields $Z_{2,\dots,r}$, hence cannot be related to a Calabi–Yau gauged linear sigma model. More generally, it cannot be obtained as a 'phase' of an ordinary GLSM as only chiral superfields, and not twisted chiral ones, can be minimally coupled to a $(2,2)$ vector multiplet. Hence the fractional mirrors of a given Calabi–Yau are not expected to be in any Calabi–Yau moduli space.

A more geometrical characterization of the fractional mirrors for a given Calabi–Yau manifold can be obtained following the approach of Hori and Vafa to mirror symmetry [97]. Starting with a $(2,2)$ gauged linear sigma-model for a Calabi–Yau one can dualize only part of the chiral superfields. It gives a 'hybrid' GLSM with both minimally coupled chiral superfields and twisted chiral superfields axially coupled to the vector multiplets, with a supplementary twisted superpotential coming from instanton corrections, *i.e.* vertices. Some mysteries remain, as the role of the 'P superfield' and how the orbifold of the hybrid Landau–Ginzburg models that appear in the fractional mirrors emerges in the dual model. From the supergravity point of view, it may be possible to understand these dualities from the point of view of generalized geometry [125], or some extension thereof.

Fractional mirror symmetry occurs only at specific loci of the moduli space where some 'accidental' discrete symmetries are manifest, unlike mirror symmetry, that is valid everywhere in the moduli space of Calabi–Yau sigma-models. Let us illustrate this point with the quintic three-fold. As part of the chiral ring is preserved by the asymmetric quotient, as can be seen from (44), one expects that for every hypersurface in $\mathbb{P}^4$ of the form $z_1^5 + \sum \alpha_{abc} z_2^a z_3^b z_4^c z_5^{5-a-b-c} = 0$, a fractional mirror acting on the chiral superfield Z_1 exists. In other words all the complex structure deformations that preserve the $\mathbb{Z}_5$ symmetry $z_1 \to e^{2i\pi/5} z_1$ are compatible with this duality. However a Kähler deformation out of the Landau–Ginzburg phase is not allowed, as can be seen explicitly at the Gepner point, excluding the existence of a large-volume limit. When these conditions are met the $\mathcal{N} = 2$ superconformal algebra can be split into the superconformal algebra coming from the Landau–Ginzburg model $W = Z_1^5$ and from the Landau–Ginzburg model for the other multiplets. One expects that other types of fractional mirrors exist whenever the world-sheet theory can be expressed as an orbifold of the tensor product of several $\mathcal{N} = 2$ theories.

Finally it would be interesting to find whether these new symmetries are related to the mysterious *Mathieu moonshine*, that indicates that quantum K3 sigma–models have an underlying symmetry corresponding to the Mathieu group M_{24}, whose origin is not presently fully understood [126].

3.4. Asymmetric $K3 \times T^2$ models as 'monodrofolds'. Let us come back a last time to the asymmetric Gepner models on $K3 \times T^2$ in light of this new understanding. The situation is actually very similar to the original construction of T-folds from toroidal compactifications [115], as one considers a reduction on a two-torus with non-trivial monodromies belonging to the (non-geometric) discrete symmetry group of the K3 fiber.

The quotient of the K3 Landau–Ginzburg model at the Gepner point is given by $Z_1 \to e^{2i\pi/k_1} Z_1$; this orbifold is also characterized by a specific choice of discrete torsion. This $\mathbb{Z}_{k_1}$ rotation is accompanied with a shift along the first torus coordinate of radius R_1, namely $x_1 \to x_1 + 2\pi R_1/k_1$. In the same way an order k_2 twist acting as $Z_2 \to e^{2i\pi/k_2} Z_2$ comes with a shift along the second torus coordinate. This perspective explains why all the fields that are not invariant under these symmetries become massive, since, as shown in [115], the orbifold point captures the minimum of the generalized Scherk–Schwarz potential, which is a fixed point of the twist.

One may argue that the twist in the K3 fiber is geometric, as the discrete torsion only plays a role in the twisted sectors. This is not actually correct, as the tensor product of minimal models becomes a K3 sigma-model only after the Gepner orbifold (corresponding to the J_0 and J_i extensions), has been implemented, including the twisted sectors. The discrete torsion has an effect *both* in the twisted sectors of the J_0 extension – hence giving the compactification a non-geometric nature – and in the twisted sectors of the new $\mathfrak{J}_i$ extension corresponding to the generalized Scherk-Schwarz compactification.[12] In particular it has a different action on twisted chiral multiplets, *i.e.* on Kähler moduli, compared to the corresponding geometric orbifold.

Finally one can understand the breaking of space-time supersymmetry as follows. Both the original K3 sigma–model and its fractional mirror preserve the same amount of space-time supersymmetry, however the corresponding spin fields are obtained by exponentiating two different integrated $U(1)$ currents in the right-moving sector. An asymmetric $K3 \times T^2$ model is an interpolating orbifold that gives back the original Calabi–Yau two-fold in the $\text{Im}(U) \to +\infty$ limit (*i.e.* decompactification of the two-torus), and it 'half-mirror' in the $\text{Im}(U) \to 0$ limit. Given that there exists no global choice of current with integer-valued charges there is no preserved space-time supersymmetry from the right-movers along the way.

[12]To be concrete, the $\mathfrak{J}_i$ extension of a Gepner model imposes the constraint $m_i + b_0 \equiv 0 \mod k_i$ in its untwisted sectors, see eq. (44) while without discrete torsion one would have $2m_i + b_0 \equiv 0 \mod 2k_i$.

5

Perspectives

Among the various themes that have been developed in this thesis, there are several aspects that I wish to explore in the near future.

First I would like to understand better the new Calabi–Yau dualities, especially regarding their target-space interpretation. In the framework of $\mathcal{N} = 4$ supergravity, there should exist some gaugings having $\mathcal{N} = 2$ vacua reproducing the $\mathsf{K3} \times \mathsf{T}^2$ asymmetric models; finding the map between both approaches would be illuminating as there are some general classifications relating the choice of gauging to geometric and non-geometric fluxes [127]. Whether there exists a generalized geometry approach to fractional mirror symmetry is a very interesting open question. I would like also to explore the orientifolds, if any, of the non-geometric Calabi–Yau Gepner models, extending the known orientifolds of their geometric quotients [45, 123]; they may be interesting starting points for phenomenology, having reduced moduli spaces.

Second I would like to develop an understanding of non-geometric compact models in heterotic strings, that are presumably much more numerous than their type II counterparts [128]. Possible heterotic duals of the type II $\mathsf{K3} \times \mathsf{T}^2$ asymmetric models can provide interesting examples. More generally, it would be important to find a more algebraic characterization of compactifications with torsion and (non-)geometric fluxes; as we have seen the distinction between for instance torsional and non-torsional compactifications is not always sharply defined.

In a broader perspective it is essential to explore the phenomenological aspects of heterotic world-sheet models with torsion. During his PhD thesis under my supervision, Matthieu Sarkis will study methods for computing the massless spectra of $(0, 2)$ gauged linear sigma-models without resorting to a Landau–Ginzburg phase, that does not exist in most (interesting) cases. If this project is successful, it would allow in particular to study moduli stabilization from the world-sheet perspective, which is necessary as, in the context of heterotic compactifications with torsion, the large volume limit does not exist in general. The dynamics of non-Abelian $(0, 2)$ gauge theories is also promising for enlarging the class of known supersymmetric compactifications. It would be very helpful to have $(0, 2)$ analogues of the Gepner points accessible to conformal field theory methods.[1]

I plan to explore more non-compact flux heterotic solutions, using supergravity and gauged-linear sigma-models, in order to find new gauge/gravity dualities, in particular of models with broken supersymmetry. One can learn also about non-perturbative dynamics of heterotic compactifications using this approach.

In a longer term there are some projects that I had in mind for some time and would like to develop further. For instance, constructing non-critical type 0 string theories with Ramond–Ramond flux in a covariant way, which are expected to be holographically dual to pure Yang–Mills theories, or investigating about N=1 quiver gauge theories and their rich infrared dynamics. I wish also to study more examples of dualities in non-supersymmetric field and string theories.

[1]For Calabi–Yau compactifications with gauge groups departing from the standard embedding, such models have been obtained in [129–131].

A

Curiculum vitae

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RESEARCH POSITIONS

IAP (2006-2013) then **LPTHE (2013-)**, Université Pierre et Marie Curie, Paris

- CNRS delegation contract **Sept. 2009 - janv. 2011**
- Maître de conférences **Oct. 2006 - today**

Racah Institute for Physics, Hebrew University, Jérusalem, Israël

- Post-doctoral fellowship **Oct. 2004 - Oct. 2006**

EDUCATION

École Normale Supérieure, Paris, France

- *Thèse de doctorat* defended on September the 20th, 2004 (first class honours), under the supervition of C. Kounnas **Sept. 2001 - Sept. 2004**
- *Diplôme d'Études Approfondies de Physique Théorique*, Université Pierre et Marie Curie/École Normale Supérieure, with first class honours **sept. 2000 - june 2001**

École Supérieure de Physique et de Chimie (ESPCI), Paris, France

- Diplôme d'ingénieur physicien (first in my year) **Sept. 1997 - June 2000**

PROJECTS AND AWARDS

- ANR *programme blanc*, PI A. Dabholkar (LPTHE) **2013 - 2017**
- CNRS delegation contract **2009 - 2010**

- ANR *programme blanc*, PI D. Langlois (APC) **2009 - 2013**
- *Marie Curie* individual fellowship of the European Union **2005 - 2006**
- *Daniel Guinier* Prize of the Société Française de Physique for the best PhD in physics **2004**

Publications

My research work led to 24 publications in the best peer-reviewed journals in the field plus one under review, which have been cited 468 times.

Invited seminars (since 2005)

○ *Two-dimensional strings, black holes and matrix models*, Racah Institute, Jerusalem, Jan. 2005
○ *D-branes in NS5-branes and F1 backgrounds*, Spinoza Institute, Utrecht, Apr. 2005
○ *D-Branes in NS5-branes and F1 backgrounds* CPHT, École Polytechnique, Apr. 2005
○ *D-branes in Little String Theory*, DAMTP, Cambridge, June 2005
○ *D-Branes in NS5-branes and F1 backgrounds*, CERN, July 2005
○ *Aspects of little string theories in various dimensions* SPHT, CEA, Sept. 2005
○ *Cordes non-critiques et holographie* LPTHE, Jussieu, Sept. 2005
○ *Non-critical strings and supersymmetric gauge theories*, Ben Gurion U., Beer Sheva, Nov. 2006
○ *Non-critical strings duals of N=1 gauge theories*, Niels Bohr Institute, Copenhague, Jan. 2006
○ *Cordes non-critiques et théories de jauge supersymétriques*, Université de Tours, Apr. 2006
○ *D-branes on CY, non-critical string and gauge theories*, *Liouville, Integrability and Branes* conference, Seoul University and APCTP, June 2006
○ *Brane annihilation in curved space-time*, *High Energy, Cosmology and Strings* conference, Institut Henri Poincaré, Dec. 2006
○ *Brane decay in curved space-time*,conférence *Eurostrings*, Crete, June 2007
○ *Perturbative Stability in type 0 string theories*, Tel-Aviv University, Oct. 2007
○ *Moduli Space of Branes at Multicritical Points*. CPHT, École Polytechnique, Nov. 2007
○ *Brane decay in curved space-time*, Swansea University, Jan. 2008
○ *Non-Supersymmetric Seiberg duality & non-critical strings*, *String Phenomenology and Dynamical Vacuum Selection* conference, Liverpool University, Mars 2008
○ *Duality in orientifold QCD & type 0B non-critical strings*, IAS, Jerusalem, Apr. 2008
○ *Double-Scaling Limit of Heterotic Bundles and Dynamical Deformation in CFT*, Institut Henri Poincaré, Feb. 2009
○ *Aspects of SYM domain-walls dynamics*, APC, Université Denis Diderot, June 2009
○ *Seiberg Duality Beyond the Planar Limit*, conférence LargeN@Swansea, Swansea University, Jul. 2009
○ *Domain wall dynamics in SYM and duality*, Institut Henri Poincaré, Sept. 2009
○ *Heterotic torsional backgrounds, from supergravity to CFT*, Kavli Institute for Theoretical Physics, Santa Barbara, Sept. 2009
○ *Heterotic torsional backgrounds, from supergravity to CFT*, Tel Aviv University, Sept. 2009
○ *Heterotic conifolds with torsion*, IPHT, CEA Saclay, May 2010
○ *Heterotic conifolds with torsion*, European String Conference, Madrid, June 2010
○ *Tachyon condensation in brane/antibrane systems*, Institut d'été, LPTENS, Aug. 2011
○ *Heterotic Torsional Backgrounds, Threshold Corrections and Mock Modular Forms*, Mc Gill University, Montreal, June 2012
○ *Asymmetric Gepner models*, Université de Munich, Apr. 2014

◦ *Fractional mirror symmetry*, Institut Henri Poincaré, Oct. 2014
◦ *Fractional mirror symmetry*, Université de Lyon, Nov. 2014

ACADEMIC AND ADMINISTRATIVE DUTIES

- Appointed member of the Scientific Board, Institut Lagrange de Paris **2014 -**
- Elected member of the Laboratory Board, LPTHE **2014 -**
- Elected member of the Commitee of Experts, 29th section, UPMC **2011 -**
- Head of the Gravity and Cosmology Group (GRECO) at IAP (12 permament researchers), and appointed member of the Laboratory Board **2009 - 2013**
- Reviewer for *Journal of High Energy Physics*, *Physical Review*, *International Journal of Modern Physics* on a regular basis.

STUDENTS SUPERVISION

- PhD of Matthieu Sarkis at LPTHE about heterotic gauged linear sigma-models with torsion, ED564 **2014-2017**
- PhD of Flavien Kiefer at IAP on tachyon condensation in string theory and cosmology, joint advisor (90%) (with Costas Kounnas), ED107 (Defended in June 2012 at UPMC). Two publications associated with this PhD. **2008-2012**
- Master thesis of Matthieu Sarkis (CFP master program, ENS) about elliptic genera of flux compactifications. One publication in preparation **Jan.-Feb. 2014**
- Master thesis of Vincent Thiéry (CFP master program, ENS) about asymmetric Gepner models. One publication (Israël and Thiéry, 2013). **Jan.-Feb. 2013**
- Master thesis of Esteban Herrera-Cordero (HEP master program, École Polytechnique) on torsional geometry in heterotic supergravity. **Feb.-Aug. 2012**

RECENT TEACHING EXPERIENCE

- Statistical physics (MP002) and quantum mechanics (MP014) tutorials, master degree in physics at UPMC **Sept. 2012-today**
- Lectures, tutorials and practical work in *Mathematics for physicists* (LP206), physics degree at UPMC **Sept. 2011-today**
- Lectures, tutorials and practical work in *Mechanical and optical waves* (LP201), physics degree at UPMC **sept. 2006-2011**
- Tutorials and practical work in mathematics for physicists : LP207 et LP213, UE LP311 at UPMC **Sept. 2007-Jul. 2009**
- Practical work in astrophysics, master degree at UPMC **2006-today**

GENERAL PUBLIC

- Interviews in popular science journals (Sciences et Avenir, Sciences et Vie)
- Invited general public conference at Université de Troyes about black holes **2011**
- Invited general public conference at Ambassade de France d'Ottawa, about primordial cosmology **2012**
- Invited general public conference at Société d'Astronomie de Nantes, about black holes **2015**

B

List of publications (after PhD)

Peer-reviewed journals

- **Heterotic strings on homogeneous spaces**
  D. Israël, C. Kounnas, D. Orlando and P. M. Petropoulos.
  hep-th/0412220
  Fortsch. Phys. **53**, 1030 (2005)
  We construct heterotic string backgrounds corresponding to families of homogeneous spaces as exact conformal field theories. They contain left cosets of compact groups by their maximal tori supported by NS-NS 2-forms and gauge field fluxes. We give the general formalism and modular-invariant partition functions, then we consider some examples such as $SU(2)/U(1) \sim S^2$ (already described in a previous paper) and the $SU(3)/U(1)^2$ flag space. As an application we construct new supersymmetric string vacua with magnetic fluxes and a linear dilaton.
- **D-branes in little string theory**
  D. Israël, A. Pakman and J. Troost.
  hep-th/0502073
  Nucl. Phys. B **722**, 3 (2005)
  We analyze in detail the D-branes in the near-horizon limit of NS5-branes on a circle, the holographic dual of little string theory in a double scaling limit. We emphasize their geometry in the background of the NS5-branes and show the relation with D-branes in coset models. The exact one-point function giving the coupling of the closed string states with the D-branes and the spectrum of open strings is computed. Using these results, we analyze several aspects of Hanany-Witten setups, using exact CFT analysis. In particular we identify the open string spectrum on the D-branes stretched between NS5-branes which confirms the low-energy analysis in brane constructions, and that allows to go to higher energy scales. As an application we show the emergence of the beta-function of the N=2 gauge theory on D4-branes stretching between NS5-branes from the boundary states describing the D4-branes. We also speculate on the possibility of getting a matrix model description of little string theory from the effective theory on the D1-branes. By considering D3-branes orthogonal to the NS5-branes we find a CFT incarnation of the Hanany-Witten effect of anomalous creation of D-branes. Finally we give an brief description of some non-BPS D-branes.
- **D-branes in Lorentzian AdS(3)**
  D. Israël.
  hep-th/0502159
  JHEP **0506**, 008 (2005)
  We study the exact construction of D-branes in Lorentzian AdS(3). We start by defining a family of conformal field theories that gives a natural Euclidean version of the SL(2,R) CFT and does not correspond to H(3)+, the analytic continuation of AdS(3). We argue that one can recuperate the exact CFT results of Lorentzian AdS(3), upon an analytic continuation in the moduli space of these conformal field theories. Then we construct exact boundary states for various symmetric and symmetry-breaking D-branes in AdS(3).

- **Non-critical string duals of N=1 quiver theories**
  D. Israël.
  hep-th/0512166
  JHEP **0604**, 029 (2006)
  We construct N=1 non-critical strings in four dimensions dual to strongly coupled N=1 quiver gauge theories in the Coulomb phase, generalizing the string duals of Argyres-Douglas points in N=2 gauge theories. They are the first examples of superstrings vacua with an exact worldsheet description dual to chiral N=1 theories. We identify the dual of the non-critical superstring using a brane setup describing the field theory in the classical limit. We analyze the spectrum of chiral operators in the strongly coupled regime and show how worldsheet instanton effects give non-perturbative information about the gauge theory. We also consider aspects of D-branes relevant for the holographic duality.
- **Rolling tachyon in anti-de Sitter space-time**
  D. Israël and E. Rabinovici.
  hep-th/0609087
  JHEP **0701**, 069 (2007)
  We study the decay of the unstable D-particle in three-dimensional anti-de Sitter space-time using worldsheet boundary conformal field theory methods. We test the open string completeness conjecture in a background for which the phase space available is only field-theoretic. This could present a serious challenge to the claim. We compute the emission of closed strings in the AdS(3) x S^3 x T^4 background from the knowledge of the exact corresponding boundary state we construct. We show that the energy stored in the brane is mainly converted into very excited long strings. The energy stored in short strings and in open string pair production is much smaller and finite for any value of the string coupling. We find no missing energy problem. We compare our results to those obtained for a decay in flat space-time and to a background in the presence of a linear dilaton. Some remarks on holographic aspects of the problem are made.
- **Orientifolds in N=2 Liouville Theory and its Mirror**
  D. Israël and V. Niarchos.
  hep-th/0703151 [HEP-TH]
  JHEP **0711**, 093 (2007)
  We consider unoriented strings in the supersymmetric SL(2,R)/U(1) coset, which describes the two-dimensional Euclidean black hole, and its mirror dual N=2 Liouville theory. We analyze the orientifolds of these theories from several complementary points of view: the parity symmetries of the worldsheet actions, descent from known AdS_3 parities, and the modular bootstrap method (in some cases we can also check our results against known constraints coming from the conformal bootstrap method). Our analysis extends previous work on orientifolds in Liouville theory, the AdS_3 and SU(2) WZW models and minimal models. Compared to these cases, we find that the orientifolds of the two dimensional Euclidean black hole exhibit new intriguing features. Our results are relevant for the study of orientifolds in the neighborhood of NS5-branes and for the engineering of four-dimensional chiral gauge theories and gauge theories with SO and Sp gauge groups with suitable configurations of D-branes and orientifolds. As an illustration, we discuss an example related to a configuration of D4-branes and O4-planes in the presence of two parallel fivebranes.
- **Comments on geometric and universal open string tachyons near fivebranes**
  D. Israël.
  hep-th/0703261
  JHEP **0704**, 085 (2007)
  In a recent paper (hep-th/0703157), Sen studied unstable D-branes in NS5-branes backgrounds and argued that in the strong curvature regime the universal open string tachyon (on D-branes of the wrong dimensionality) and the geometric tachyon (on D-branes that are BPS in flat space

but not in this background) may become equivalent. We study in this note an example of a non-BPS suspended D-brane vs. a BPS D-brane at equal distance between two fivebranes. We use boundary worldsheet CFT methods to show that these two unstable branes are identical.

- **Tree-Level Stability Without Spacetime Fermions: Novel Examples in String Theory**
  D. Israël and V. Niarchos.
  arXiv:0705.2140 [hep-th]
  JHEP **0707**, 065 (2007)
  Is perturbative stability intimately tied with the existence of spacetime fermions in string theory in more than two dimensions? Type 0′B string theory in ten-dimensional flat space is a rare example of a non-tachyonic, non-supersymmetric string theory with a purely bosonic closed string spectrum. However, all known type 0′ constructions exhibit massless NSNS tadpoles signaling the fact that we are not expanding around a true vacuum of the theory. In this note, we are searching for perturbatively stable examples of type 0′ string theory without massless tadpoles in backgrounds with a spatially varying dilaton. We present two examples with this property in non-critical string theories that exhibit four- and six-dimensional Poincare invariance. We discuss the D-branes that can be embedded in this context and the type of gauge theories that can be constructed in this manner. We also comment on the embedding of these non-critical models in critical string theories and their holographic (Little String Theory) interpretation and propose a general conjecture for the role of asymptotic supersymmetry in perturbative string theory.
- **Non-Supersymmetric Seiberg Duality, Orientifold QCD and Non-Critical Strings**
  A. Armoni, D. Israël, G. Moraitis and V. Niarchos.
  arXiv:0801.0762 [hep-th]
  Phys. Rev. D **77**, 105009 (2008)
  We propose an electric-magnetic duality and conjecture an exact conformal window for a class of non-supersymmetric $U(N_c)$ gauge theories with fermions in the (anti)symmetric representation of the gauge group and N_f additional scalar and fermion flavors. The duality exchanges N_c with N_f -$N_c \mp 4$ leaving N_f invariant, and has common features with Seiberg duality in N=1 SQCD with SU or SO/Sp gauge group. At large N the duality holds due to planar equivalence with N=1 SQCD. At finite N we embed these gauge theories in a setup with D-branes and orientifolds in a non-supersymmetric, but tachyon-free, non-critical type 0B string theory and argue in favor of the duality in terms of boundary and crosscap state monodromies as in analogous supersymmetric situations. One can verify explicitly that the resulting duals have matching global anomalies. Finally, we comment on the moduli space of these gauge theories and discuss other potential non-supersymmetric examples that could exhibit similar dualities.
- **D-branes at multicritical points**
  M. R. Gaberdiel, D. Israël and E. Rabinovici.
  arXiv:0803.0291 [hep-th]
  JHEP **0804**, 086 (2008)
  The moduli space of c=1 conformal field theories in two dimensions has a multicritical point, where a circle theory is equivalent to an orbifold theory. We analyse all the conformal branes in both descriptions of this theory, and find convincing evidence that the full brane spectrum coincides. This shows that the equivalence of the two descriptions at this multicritical point extends to the boundary sector. We also perform the analogous analysis for one of the multicritical points of the N=1 superconformal field theories at c=3/2. Again the brane spectra are identical for both descriptions, however the identification is more subtle.
- **Double-Scaling Limit of Heterotic Bundles and Dynamical Deformation in CFT**
  L. Carlevaro, D. Israël and P. M. Petropoulos.
  arXiv:0812.3391 [hep-th]
  Nucl. Phys. B **827**, 503 (2010)

We consider heterotic string theory on Eguchi-Hanson space, as a local model of a resolved A_1 singularity in a six-dimensional flux compactification, with an Abelian gauge bundle turned on and non-zero torsion. We show that in a suitable double scaling limit, that isolates the physics near the non-vanishing two-cycle, a worldsheet conformal field theory description can be found. It contains a heterotic coset whose target space is conformal to Eguchi-Hanson. Starting from the blow-down limit of the singularity, it can be viewed as a dynamical deformation of the near-horizon fivebrane background. We analyze in detail the spectrum of the theory in particular examples, as well as the important role of worldsheet non-perturbative effects.

- **Brane Dynamics and 3D Seiberg Duality on the Domain Walls of 4D N=1 SYM**
  A. Armoni, A. Giveon, D. Israël and V. Niarchos.
  arXiv:0905.3195 [hep-th]
  JHEP **0907**, 061 (2009)
  We study a three-dimensional U(k) Yang-Mills Chern-Simons theory with adjoint matter preserving two supersymmetries. According to Acharya and Vafa, this theory describes the low-energy worldvolume dynamics of BPS domain walls in four-dimensional N=1 SYM theory. We demonstrate how to obtain the same theory in a brane configuration of type IIB string theory that contains threebranes and fivebranes. A combination of string and field theory techniques allows us to re-formulate some of the well-known properties of N=1 SYM domain walls in a geometric language and to postulate a Seiberg-like duality for the Acharya-Vafa theory. In the process, we obtain new information about the dynamics of branes in setups that preserve two supersymmetries. Using similar methods we also study other N=1 CS theories with extra matter in the adjoint and fundamental representations of the gauge group.
- **Heterotic Resolved Conifolds with Torsion, from Supergravity to CFT**
  L. Carlevaro and D. Israël.
  arXiv:0910.3190 [hep-th]
  JHEP **1001**, 083 (2010)
  We obtain a family of heterotic supergravity backgrounds describing non-Kahler warped conifolds with three-form flux and an Abelian gauge bundle, preserving N=1 supersymmetry in four dimensions. At large distance from the singularity the usual Ricci-flat conifold is recovered. By performing a Z_2 orbifold of the $T^{1,1}$ base, the conifold singularity can be blown-up to a four-cycle, leading to a completely smooth geometry. Remarkably, the throat regions of the solutions, which can be isolated from the asymptotic Ricci-flat geometry using a double-scaling limit, possess a worldsheet CFT description in terms of heterotic cosets whose target space is the warped resolved orbifoldized conifold. Thus this construction provides exact solutions of the modified Bianchi identity. By solving algebraically these CFTs we compute the exact tree-level heterotic string spectrum and describe worldsheet non-perturbative effects. The holographic dual of these solutions, in particular their confining behavior, and the embedding of these fluxed singularities into heterotic compactifications with torsion are also discussed.
- **String Theory on Warped AdS_3 and Virasoro Resonances**
  S. Detournay, D. Israël, J. M. Lapan and M. Romo.
  arXiv:1007.2781 [hep-th]
  JHEP **1101**, 030 (2011)
  We investigate aspects of holographic duals to time-like warped AdS_3 space-times–which include Gödel's universe–in string theory. Using worldsheet techniques similar to those that have been applied to AdS_3 backgrounds, we are able to identify space-time symmetry algebras that act on the dual boundary theory. In particular, we always find at least one Virasoro algebra with computable central charge. Interestingly, there exists a dense set of points in the moduli space of these models in which there is actually a second commuting Virasoro algebra, typically with different central charge than the first. We analyze the supersymmetry of the backgrounds, finding related enhancements, and comment on possible interpretations of these results. We also perform an asymptotic symmetry analysis at the level of supergravity, providing additional support for the worldsheet analysis.

- **Rolling tachyons for separated brane-antibrane systems**
  D. Israël and F. Kiefer.
  arXiv:1108.5763 [hep-th]
  Phys. Rev. D **85**, 106002 (2012)
  We consider tachyon condensation between a D-brane and an anti-D-brane in superstring theory, when they are separated in their common transverse directions. A simple rolling tachyon solution, that describes the time evolution of the process, is studied from the point of view of boundary conformal field theory. By computing the boundary beta-functions of the system, one finds that this theory is conformal, hence corresponds to an exact solution of the string theory equations of motion. By contrast, the time-reversal-symmetric rolling tachyon is not conformal. Using these results we study space-time effective actions that can describe the system in the vicinity of these exact solutions.
- **Gauge Threshold Corrections for N = 2 Heterotic Local Models with Flux, and Mock Modular Forms**
  L. Carlevaro and D. Israël.
  arXiv:1210.5566 [hep-th]
  JHEP **1303**, 049 (2013)
  We determine threshold corrections to the gauge couplings in local models of smooth heterotic compactifications with torsion, given by the direct product of a warped Eguchi-Hanson space and a two-torus, together with a line bundle. Using the worldsheet cft description previously found and by suitably regularising the infinite target space volume divergence, we show that threshold corrections to the various gauge factors are governed by the non-holomorphic completion of the Appell-Lerch sum. While its holomorphic Mock-modular component captures the contribution of states that localise on the blown-up two-cycle, the non-holomorphic correction originates from non-localised bulk states. We infer from this analysis universality properties for heterotic local models with flux, based on target space modular invariance and the presence of such non-localised states. We finally determine the explicit dependence of these one-loop gauge threshold corrections on the moduli of the two-torus, and by S-duality we extract the corresponding string-loop and E1-instanton corrections to the Kähler potential and gauge kinetic functions of the dual type i model. In both cases, the presence of non-localised bulk states brings about novel perturbative and non-perturbative corrections, some features of which can be interpreted in the light of analogous corrections to the effective theory in compact models.
- **T-Duality in Gauged Linear Sigma-Models with Torsion**
  D. Israël.
  arXiv:1306.6609 [hep-th]
  JHEP **1311**, 093 (2013)
  (0,2) gauged linear sigma models with torsion, corresponding to principal torus bundles over warped CY bases, provide a useful framework for getting exact statements about perturbative dualities in the presence of fluxes. In this context we first study dualities mapping the torus fiber onto itself, implying the existence of quantization constraints on the torus moduli for consistency. Second, we investigate dualities mixing the principal torus bundle with the gauge bundle, relating the torsional GLSMs to ordinary ones corresponding to CY compactifications with non-standard embeddings, namely two classes of models with different target-space topologies.
- **Asymmetric Gepner models in type II**
  D. Israël and V. Thiéry.
  arXiv:1310.4116 [hep-th]
  JHEP **1402**, 011 (2014)
  We describe new four-dimensional type II compactifications with N=2 supersymmetry, based on asymmetric Gepner models for K3 x T^2. In more than half of these models, all the K3 moduli are lifted, giving at low energies N=2 supergravity with the STU vector multiplets and no hypermultiplets.

- **Fractional Mirror Symmetry**
  D. Israël.
  arXiv:1409.5771 [hep-th]
  under review.
  Mirror symmetry relates type IIB string theory on a Calabi–Yau 3-fold to type IIA on the mirror CY manifold, whose complex structure and Kähler moduli spaces are exchanged. We show that the mirror map is a particular case of a more general quantum equivalence, *fractional mirror symmetry*, between Calabi–Yau compactifications and non-CY ones. As was done by Greene and Plesser for mirror symmetry, we obtain these new dualities by considering orbifolds of Gepner models, of asymmetric nature, leading to superconformal field theories isomorphic to the original ones, but with a different target-space interpretation. The associated Landau–Ginzburg models involve both chiral and twisted chiral multiplets hence cannot be lifted to ordinary gauged linear sigma-models.

Proceedings

- **Strings and D-branes in holographic backgrounds**
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- **Gödel Universe in Heterotic String Theory**
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Abstract

This habilitation thesis summarizes several research topics in string theory, using the world-sheet two-dimensional perspective as a unifying theme. The first part of this manuscript describes new solutions of open string tachyon condensation and their physical consequences. In the second part several examples of gauge theory dynamics at strong coupling are probed using brane configurations. The third part is devoted to heterotic compactifications with flux and their symmetries. Finally, the last part covers aspects of the quantum geometry of string compactifications.

Résumé

Cette thèse d'habilitation résume des travaux de recherche en théorie des cordes ayant pour fil conducteur l'approche bidimensionnelle de la feuille d'univers. Dans la première partie du manuscrit, de nouvelles solutions de condensation du tachyon de cordes ouvertes sont présentées, et leur conséquences physiques discutées. La deuxième partie couvre plusieurs aspects des relations entre théories de jauge à couplage fort et configurations de branes. Dans la troisième partie sont abordées les compactifications hétérotiques avec flux ainsi que leurs symétries. Finalement, la quatrième et dernière partie illustre par plusieurs exemples la géométrie quantique des compactifications de la théorie des cordes.