Brane decay in curved space-time

DAN ISRAËL, IAP, UNIV. PIERRE ET MARIE CURIE

From D. I. & E. Rabinovici, JHEP **0701** (2007) D. I., JHEP **0704** (2007)

Outline of the Talk

- 1. Introduction & motivations from cosmology
- 2. Brane decay in flat space-time
- 3. Brane decay in AdS
- 4. Closed and open string emission
- 5. Open string tachyons in non-critical strings (geometric and universal)
- 6. Brane decay in non-critical strings
- 7. Conclusions

① Brane inflation & open string tachyons

✓ Natural setting of string cosmology: flux compactification of type II string theory, with stabilized moduli $\mathbb{R}^{3,1} \times \mathbb{M}_6$

generically warped throats develop

✓ AdS₅^xM₅ ✓ AdS₅ geometry, capped both in the UV (compact 6-manifold) and in the IR (tip of the throat)

① Brane inflation & open string tachyons

✓ Natural setting of string cosmology: flux compactification of type II string theory, with stabilized moduli

generically warped throats develop



✓ AdS₅ geometry, capped both in the UV (compact 6-manifold) and in the IR (tip of the throat) [Giddings, Kachru, Polchinski '03]

✓ D-brane/ anti D-brane pair in the throat: Coulombian attraction redshifted by AdS₅ metric → slow-roll inflation (inflaton $d(t, \mathbf{x})$) [Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi '03]

Tachyon condensation & Brane Reheating

 \checkmark End of inflation: D- \overline{D} annihilation \blacktriangleright open string tachyon for $d^2 < 8\pi^2 \ell_s^2$



★String theory realization of hybrid inflation

Tachyon condensation & Brane Reheating

✓ End of inflation: D- \overline{D} annihilation → open string tachyon for $d^2 < 8\pi^2 \ell_s^2$



★String theory realization of hybrid inflation

✓ Tachyon condensation: involves all the massive string modes (m > 1/ℓ_s)
 ➡ string corrections important

★For brane decay in flat spacetime one can use exact tree-level string computations ★For brane decay in flat spacetime one can use exact tree-level string computations

"Rolling tachyon" solution of boundary CFT

★For brane decay in flat spacetime one can use exact tree-level string computations
[Sen '02]

- "Rolling tachyon" solution of boundary CFT
- produces non-relativistic "tachyon dust" of massive closed strings

★For brane decay in flat spacetime one can use exact tree-level string computations
[Sen '02]

- "Rolling tachyon" solution of boundary CFT
- produces non-relativistic "tachyon dust" of massive closed strings

★How much this elementary process of string theory is universal?

★For brane decay in flat spacetime one can use exact tree-level string computations

- "Rolling tachyon" solution of boundary CFT
- produces non-relativistic "tachyon dust" of massive closed strings

★How much this elementary process of string theory is universal?
✓ Both low-energy (background geometry) and high energy data (asymptotic density of string states) play a role in the computation of closed string radiation.
✓ both regimes are related through modular invariance

★For brane decay in flat spacetime one can use exact tree-level string computations

- "Rolling tachyon" solution of boundary CFT
- produces non-relativistic "tachyon dust" of massive closed strings

★How much this elementary process of string theory is universal?
✓ Both low-energy (background geometry) and high energy data (asymptotic density of string states) play a role in the computation of closed string radiation.
✓ both regimes are related through modular invariance

► look for curved spacetimes (especially warped geometries) where the Dbrane decay can be followed exactly (at the perturbative level) σ

ίτ

2 Brane Annihilation: Flat Space-Time

✓ Decay of an unstable D-brane: equivalent to coincident $D-\bar{D}$ pair with no relative velocity (using $(-)^{F_L}$ orbifold) → solvable worldsheet string model [Sen '02]

 $\delta S = \lambda \int d\tau \exp\{X^0(\tau)/\ell_s\}$ Wick rotation of boundary Liouville theory

2 Brane Annihilation: Flat Space-Time

✓ Decay of an unstable D-brane: equivalent to coincident $D-\bar{D}$ pair with no relative velocity (using $(-)^{F_L}$ orbifold) → solvable worldsheet string model [Sen '02]

 $\delta S = \lambda \int d\tau \exp\{X^0(\tau)/\ell_s\}$ Wick rotation of boundary Liouville theory

✓ Couplings to closed strings (grav. sector) $\langle V_E \rangle_{\lambda} = (\pi \lambda)^{-iE} \frac{\pi}{\sinh \pi E}$ → time-dependent source for all closed string modes

2 Brane Annihilation: Flat Space-Time

✓ Decay of an unstable D-brane: equivalent to coincident $D-\bar{D}$ pair with no relative velocity (using $(-)^{F_L}$ orbifold) → solvable worldsheet string model [Sen '02]

 $\delta S = \lambda \int d\tau \exp\{X^0(\tau)/\ell_s\}$ Wick rotation of boundary Liouville theory

✓ Couplings to closed strings (grav. sector) $\langle V_E \rangle_{\lambda} = (\pi \lambda)^{-iE} \frac{\pi}{\sinh \pi E}$ → time-dependent source for all closed string modes

★Closed strings production (coherent state) Number of emitted strings(tree-level): $\mathcal{N} = \int \frac{dE}{2E} \rho(E) |\langle V_E \rangle_{\lambda}|^2$ [Lambert, Liu, Maldacena '03]

► exponentially growing (cf. Hagedorn transition at high temperature)

 $\bigstar In$ flat space-time, $\rho(N) \sim N^{\alpha} e^{+4\pi \sqrt{N}}$ with $E = 2\sqrt{N}/\ell_s$

✓ Density of closed strings oscillators $\rho(N)$

► exponentially growing (cf. Hagedorn transition at high temperature)

 $\bigstar {\rm In}$ flat space-time, $\rho(N) \sim N^{\alpha} e^{+4\pi \sqrt{N}}$ with $E = 2\sqrt{N}/\ell_s$

✓ Amplitude $\int \mathcal{N} \sim \int dE \ E^{2\alpha - 1} \ e^{2\pi E} \sinh^{-2}(\pi E)$

 \rightarrow divergent for D0-branes ($\alpha = 0$) (D3-branes: instable to inhomogeneous decay)

 \blacktriangleright exponentially growing (cf. Hagedorn transition at high temperature)

 $\bigstar {\rm In}$ flat space-time, $\rho(N) \sim N^{\alpha} e^{+4\pi \sqrt{N}}$ with $E = 2\sqrt{N}/\ell_s$

✓ Amplitude
$$\int \mathcal{N} \sim \int dE \ E^{2\alpha - 1} \ e^{2\pi E} \sinh^{-2}(\pi E)$$

 \rightarrow divergent for D0-branes ($\alpha = 0$) (D3-branes: instable to inhomogeneous decay)

★Divergence signals breakdown of string perturbation theory

Large gravitational back-reaction from the brane decay!

► exponentially growing (cf. Hagedorn transition at high temperature)

 $\bigstar \mbox{In flat space-time, } \rho(N) \sim N^{\alpha} e^{+4\pi \sqrt{N}}$ with $E = 2\sqrt{N}/\ell_s$

✓ Amplitude
$$\int \mathcal{N} \sim \int dE \ E^{2\alpha - 1} \ e^{2\pi E} \sinh^{-2}(\pi E)$$

 \rightarrow divergent for D0-branes ($\alpha = 0$) (D3-branes: instable to inhomogeneous decay)

★ Divergence signals breakdown of string perturbation theory

Large gravitational back-reaction from the brane decay!

 \star mass of a D0-brane $m_{
m D0} \propto 1/\ell_s g_s$

energy conservation not "built-in" the (tree-level) computation

 \blacktriangleright exponentially growing (cf. Hagedorn transition at high temperature)

 $\bigstar {\rm In}$ flat space-time, $\rho(N) \sim N^{\alpha} e^{+4\pi \sqrt{N}}$ with $E = 2\sqrt{N}/\ell_s$

✓ Amplitude
$$\int \mathcal{N} \sim \int dE \ E^{2\alpha - 1} \ e^{2\pi E} \sinh^{-2}(\pi E)$$

 \rightarrow divergent for D0-branes ($\alpha = 0$) (D3-branes: instable to inhomogeneous decay)

★Divergence signals breakdown of string perturbation theory

Large gravitational back-reaction from the brane decay!

 \star mass of a D0-brane $m_{
m D0} \propto 1/\ell_s g_s$

energy conservation not "built-in" the (tree-level) computation

V One needs a UV cutoff at $E \sim m_{\rm D0}$

 \star fraction of total energy in strings of mass $m \sim$ cst. (up to $m_{
m D0}$)

 \blacktriangleright most energy in strings $m \sim m_{\rm D0}$, non-relativistic ($p \propto 1/\ell_s \sqrt{g_s}$): tachyon dust

1. The closed string description of the brane decay breaks down after $t \sim \ell_s \sqrt{g_s}$ \Rightarrow all energy is converted into *tachyon dust* of massive closed strings

- 1. The closed string description of the brane decay breaks down after $t \sim \ell_s \sqrt{g_s}$ \Rightarrow all energy is converted into *tachyon dust* of massive closed strings
- However the open string description of the process remains valid
 ➡ may be spoiled by open string pair production (more later)

- 1. The closed string description of the brane decay breaks down after $t \sim \ell_s \sqrt{g_s}$ \Rightarrow all energy is converted into *tachyon dust* of massive closed strings
- However the open string description of the process remains valid
 ➡ may be spoiled by open string pair production (more later)
- 3. The open string description is *holographically dual* to the closed strings description, hence is *complete*

- 1. The closed string description of the brane decay breaks down after $t \sim \ell_s \sqrt{g_s}$ \Rightarrow all energy is converted into *tachyon dust* of massive closed strings
- However the open string description of the process remains valid
 ➡ may be spoiled by open string pair production (more later)
- 3. The open string description is *holographically dual* to the closed strings description, hence is *complete*
- 4. One can use the tachyon low-energy effective action $S_{\rm T} = \int d^d x \cosh(T/\sqrt{2})^{-1} \sqrt{-\det(\eta_{\mu\nu} + \partial_{\mu}T\partial_{\nu}T + \cdots)} \Rightarrow \text{late-time "dust"}$

- 1. The closed string description of the brane decay breaks down after $t \sim \ell_s \sqrt{g_s}$ \Rightarrow all energy is converted into *tachyon dust* of massive closed strings
- However the open string description of the process remains valid
 ➡ may be spoiled by open string pair production (more later)
- 3. The open string description is *holographically dual* to the closed strings description, hence is *complete*
- 4. One can use the tachyon low-energy effective action $S_{\rm T} = \int d^d x \cosh(T/\sqrt{2})^{-1} \sqrt{-\det(\eta_{\mu\nu} + \partial_{\mu}T\partial_{\nu}T + \cdots)} \Rightarrow \text{late-time "dust"}$
- 5. Conjecture has been checked in 2D string theory

✓ Cosmological context: D/D in a curved space-time (e.g. capped AdS₅)
 ➡ is the physics of the decay similar? (in string theory, UV-IR relation)

✓ Cosmological context: D/D in a curved space-time (e.g. capped AdS₅)
 ➡ is the physics of the decay similar? (in string theory, UV-IR relation)

✓ In particular cancellation between asymptotic density of closed string states & closed string emission amplitude may not be true anymore ★In a CFT with minimal conformal dimension Δ_m , $\rho(E) \sim \exp\{\sqrt{1 - \Delta_m} 2\pi E\}$ → UV finite?

✓ Cosmological context: D/D in a curved space-time (e.g. capped AdS₅)
 ➡ is the physics of the decay similar? (in string theory, UV-IR relation)

✓ In particular cancellation between asymptotic density of closed string states & closed string emission amplitude may not be true anymore ★In a CFT with minimal conformal dimension Δ_m , $\rho(E) \sim \exp\{\sqrt{1 - \Delta_m} 2\pi E\}$ → UV finite?

✓ Is the process still well-described by the curved background generalization of the open string tachyon effective action?

 $S_{\mathrm{T}} = \int \mathrm{d}^{p+1}x \sqrt{-g} \cosh(\frac{T}{\sqrt{2}})^{-1} \sqrt{-\det\{(g+B+2\pi\ell_s^2 F)_{\mu\nu}+\partial_{\mu}T\partial_{\nu}T\}} + \int W(T)\mathrm{d}T \wedge C_{[p]}$

✓ Cosmological context: D/D in a curved space-time (e.g. capped AdS₅)
 ➡ is the physics of the decay similar? (in string theory, UV-IR relation)

✓ In particular cancellation between asymptotic density of closed string states & closed string emission amplitude may not be true anymore ★In a CFT with minimal conformal dimension Δ_m , $\rho(E) \sim \exp\{\sqrt{1 - \Delta_m} 2\pi E\}$ → UV finite?

✓ Is the process still well-described by the curved background generalization of the open string tachyon effective action? $S_{\rm T} = \int d^{p+1}x \sqrt{-g} \cosh(\frac{T}{\sqrt{2}})^{-1} \sqrt{-\det\{(g+B+2\pi\ell_s^2 F)_{\mu\nu}+\partial_{\mu}T\partial_{\nu}T\}} + \int W(T) dT \wedge C_{[p]}$

 \star In particular, if all the brane energy is not radiated into massive closed strings, the whole picture may be challenged

 \checkmark Despite recent progress AdS₅ string theory not solvable

 \checkmark Despite recent progress AdS₅ string theory not solvable

✓ Solvable "toy model": three-dimensional AdS → conformal field theory on the string worldsheet: Wess-Zumino Witten model for the group manifold SL(2,ℝ) $ds^2 = \ell_s^2 k \left[d\rho^2 + \sinh^2 \rho d\phi^2 - \cosh^2 \rho d\tau^2 \right]$, with a B-field $B = \ell_s^2 k \cosh 2\rho d\tau \wedge d\phi$

 \checkmark Despite recent progress AdS₅ string theory not solvable

✓ Solvable "toy model": three-dimensional AdS → conformal field theory on the string worldsheet: Wess-Zumino Witten model for the group manifold SL(2,ℝ) $ds^2 = \ell_s^2 k \left[d\rho^2 + \sinh^2 \rho d\phi^2 - \cosh^2 \rho d\tau^2 \right]$, with a B-field $B = \ell_s^2 k \cosh 2\rho d\tau \wedge d\phi$



Two types of string modes:

short strings trapped in AdS (exponentially decreasing wave-functions) long strings, macroscopic solutions winding w-times around ϕ

 \checkmark Despite recent progress AdS₅ string theory not solvable

✓ Solvable "toy model": three-dimensional AdS → conformal field theory on the string worldsheet: Wess-Zumino Witten model for the group manifold SL(2,ℝ) $ds^2 = \ell_s^2 k \left[d\rho^2 + \sinh^2 \rho d\phi^2 - \cosh^2 \rho d\tau^2 \right]$, with a B-field $B = \ell_s^2 k \cosh 2\rho d\tau \wedge d\phi$



Two types of string modes:

short strings trapped in AdS (exponentially decreasing wave-functions) long strings, macroscopic solutions winding w-times around ϕ

✓ Unstable D0-brane of type IIB superstrings in $AdS_3 \times M_7$: localized at the origin $\rho = 0$ (infrared) → decay of the brane solvable (equivalent to D- \overline{D} annihilation)

Closed Strings Emission by the brane decay

✓ Open string sector on the D0-brane: tachyon + tower of string modes built on the *identity representation* of $SL(2, \mathbb{R})$

Closed Strings Emission by the brane decay

✓ Open string sector on the D0-brane: tachyon + tower of string modes built on the *identity representation* of SL(2, ℝ)
 ➡ writing SL(2, ℝ) ~ SL(2, ℝ)/U(1) × ℝ^{0,1} the dynamics involves only the time direction (up to an orbifold)

Closed Strings Emission by the brane decay

✓ Open string sector on the D0-brane: tachyon + tower of string modes built on the *identity representation* of $SL(2, \mathbb{R})$

writing $SL(2,\mathbb{R}) \sim SL(2,\mathbb{R})/U(1) \times \mathbb{R}^{0,1}$ the dynamics involves only the time direction (up to an orbifold)

 \blacktriangleright decay given by same deformation as in flat space $\lambda \int_{\partial \Sigma} dt \, \mathbb{I} \times \exp\{\sqrt{k/2} \, \tau(t)\}$
Closed Strings Emission by the brane decay

✓ Open string sector on the D0-brane: tachyon + tower of string modes built on the *identity representation* of $SL(2, \mathbb{R})$

writing $SL(2,\mathbb{R}) \sim SL(2,\mathbb{R})/U(1) \times \mathbb{R}^{0,1}$ the dynamics involves only the time direction (up to an orbifold)

⇒ decay given by same deformation as in flat space $\lambda \int_{\partial \Sigma} dt \, \mathbb{I} \times \exp\{\sqrt{k/2} \, \tau(t)\}$ ★One gets the D-brane couplings to closed string modes, e.g. for long strings with radial momentum p_{ρ} and winding w:

$$\left| \langle V_{p_{\rho},w,E} \rangle_{\lambda} \right| \propto \sqrt{\frac{\sinh 2\pi p_{\rho} \sinh \frac{2\pi p_{\rho}}{k}}{\cosh 2\pi \rho + \cos \pi (E - kw)}} \frac{1}{|\sinh \frac{\pi E}{\sqrt{2k}}|} \text{ with } E = \frac{kw}{2} + \frac{2}{w} \left[\frac{p_{\rho}^2 + \frac{1}{4}}{k} + N + \cdots \right]$$

Closed Strings Emission by the brane decay

✓ Open string sector on the D0-brane: tachyon + tower of string modes built on the *identity representation* of $SL(2, \mathbb{R})$

writing $SL(2,\mathbb{R}) \sim SL(2,\mathbb{R})/U(1) \times \mathbb{R}^{0,1}$ the dynamics involves only the time direction (up to an orbifold)

⇒ decay given by same deformation as in flat space $\lambda \int_{\partial \Sigma} dt \, \mathbb{I} \times \exp\{\sqrt{k/2} \, \tau(t)\}$ ★One gets the D-brane couplings to closed string modes, e.g. for long strings with radial momentum p_{ρ} and winding w:

$$\left| \langle V_{p_{\rho},w,E} \rangle_{\lambda} \right| \propto \sqrt{\frac{\sinh 2\pi p_{\rho} \sinh \frac{2\pi p_{\rho}}{k}}{\cosh 2\pi \rho + \cos \pi (E - kw)}} \frac{1}{|\sinh \frac{\pi E}{\sqrt{2k}}|} \text{ with } E = \frac{kw}{2} + \frac{2}{w} \left[\frac{p_{\rho}^2 + \frac{1}{4}}{k} + N + \cdots \right]$$

$$\Rightarrow \text{ also coupling to discrete states (i.e. localized strings)}$$

Closed Strings Emission by the brane decay

✓ Open string sector on the D0-brane: tachyon + tower of string modes built on the *identity representation* of $SL(2, \mathbb{R})$

writing $SL(2,\mathbb{R}) \sim SL(2,\mathbb{R})/U(1) \times \mathbb{R}^{0,1}$ the dynamics involves only the time direction (up to an orbifold)

⇒ decay given by same deformation as in flat space $\lambda \int_{\partial \Sigma} dt \, \mathbb{I} \times \exp\{\sqrt{k/2} \, \tau(t)\}$ ★One gets the D-brane couplings to closed string modes, e.g. for long strings with radial momentum p_{ρ} and winding w:

$$\left| \langle V_{p_{\rho},w,E} \rangle_{\lambda} \right| \propto \sqrt{\frac{\sinh 2\pi p_{\rho} \sinh \frac{2\pi p_{\rho}}{k}}{\cosh 2\pi \rho + \cos \pi (E - kw)}} \frac{1}{|\sinh \frac{\pi E}{\sqrt{2k}}|} \text{ with } E = \frac{kw}{2} + \frac{2}{w} \left[\frac{p_{\rho}^2 + \frac{1}{4}}{k} + N + \cdots \right]$$

also coupling to discrete states (i.e. localized strings)

 \star Total number of emitted closed strings given by the imaginary part of the annulus one-loop amplitude, using *optical theorem* + *open/closed channel duality*

$$\mathcal{N} = \mathrm{Im} \left[\int rac{\mathrm{d}s}{2s} \mathrm{Tr}_{\mathsf{open}} e^{-\pi s \mathcal{H}}
ight]$$

$$\bigstar E \sim \frac{2N}{w} \rightarrowtail \rho(E) \sim E^{\alpha} \exp\{2\pi \sqrt{(1-\frac{1}{2k})wE}\} \text{ (while } |\langle V_E \rangle|^2 \sim \exp\{-\sqrt{\frac{2}{k}\pi E}\}\text{)}$$

 $\star E \sim \frac{2N}{w} \rightarrowtail \rho(E) \sim E^{\alpha} \exp\{2\pi \sqrt{(1-\frac{1}{2k})wE}\} \text{ (while } |\langle V_E \rangle|^2 \sim \exp\{-\sqrt{\frac{2}{k}\pi E}\}\text{)}$ $\star \text{Like a 2D field theory (cf. AdS_3/CFT_2)}$

 $\star E \sim \frac{2N}{w} \blacktriangleright \rho(E) \sim E^{\alpha} \exp\{2\pi \sqrt{(1-\frac{1}{2k})wE}\} \text{ (while } |\langle V_E \rangle|^2 \sim \exp\{-\sqrt{\frac{2}{k}\pi E}\}\text{)}$ $\star \text{Like a 2D field theory (cf. AdS_3/CFT_2)}$

 \star for given winding w, long strings emission is (exponentially) UV-finite!



 $\star E \sim \frac{2N}{w} \blacktriangleright \rho(E) \sim E^{\alpha} \exp\{2\pi \sqrt{(1-\frac{1}{2k})wE}\} \text{ (while } |\langle V_E \rangle|^2 \sim \exp\{-\sqrt{\frac{2}{k}\pi E}\}\text{)}$ $\star \text{Like a 2D field theory (cf. AdS_3/CFT_2)}$

 \star for given winding w, long strings emission is (exponentially) UV-finite!



Displacement of ⟨p_ρ⟩ due to non-perturbative corrections in ℓ²_s (worldsheet instantons)
 ➡ not seen in semi-classical limit

 $\star E \sim \frac{2N}{w} \blacktriangleright \rho(E) \sim E^{\alpha} \exp\{2\pi \sqrt{(1-\frac{1}{2k})wE}\} \text{ (while } |\langle V_E \rangle|^2 \sim \exp\{-\sqrt{\frac{2}{k}\pi E}\}\text{)}$ $\star \text{Like a 2D field theory (cf. AdS_3/CFT_2)}$

 \star for given winding w, long strings emission is (exponentially) UV-finite!



- Displacement of ⟨p_ρ⟩ due to non-perturbative corrections in ℓ²_s (worldsheet instantons)
 → not seen in semi-classical limit
- For large w, $\langle E \rangle \sim k w$

 $\star E \sim \frac{2N}{w} \blacktriangleright \rho(E) \sim E^{\alpha} \exp\{2\pi \sqrt{(1-\frac{1}{2k})wE}\} \text{ (while } |\langle V_E \rangle|^2 \sim \exp\{-\sqrt{\frac{2}{k}\pi E}\}\text{)}$ $\star \text{Like a 2D field theory (cf. AdS_3/CFT_2)}$

 \star for given winding w, long strings emission is (exponentially) UV-finite!



Displacement of ⟨p_ρ⟩ due to non-perturbative corrections in ℓ²_s (worldsheet instantons)
→ not seen in semi-classical limit
For large w, ⟨E⟩ ~ kw

✓ Summation over spectral flow: $\mathcal{N}_{\text{long}} \sim \sum_{w=1}^{\infty} \frac{1}{w} > \text{divergence at high energies}$

 $\star E \sim \frac{2N}{w} \blacktriangleright \rho(E) \sim E^{\alpha} \exp\{2\pi \sqrt{(1-\frac{1}{2k})wE}\} \text{ (while } |\langle V_E \rangle|^2 \sim \exp\{-\sqrt{\frac{2}{k}\pi E}\}\text{)}$ $\star \text{Like a 2D field theory (cf. AdS_3/CFT_2)}$

 \star for given winding w, long strings emission is (exponentially) UV-finite!



Displacement of ⟨p_ρ⟩ due to non-perturbative corrections in ℓ²_s (worldsheet instantons)
→ not seen in semi-classical limit
For large w, ⟨E⟩ ~ kw

✓ Summation over spectral flow: $\mathcal{N}_{\text{long}} \sim \sum_{w=1}^{\infty} 1/w$ → divergence at high energies ★Needs non-perturbative UV cutoff: • $w \leq 1/g_s^2$ (NS-NS charge conservation) • $w \leq 1/g_s$ (energy conservation)

 $\star E \sim \frac{2N}{w} \blacktriangleright \rho(E) \sim E^{\alpha} \exp\{2\pi \sqrt{(1-\frac{1}{2k})wE}\} \text{ (while } |\langle V_E \rangle|^2 \sim \exp\{-\sqrt{\frac{2}{k}\pi E}\}\text{)}$ $\star \text{Like a 2D field theory (cf. AdS_3/CFT_2)}$

 \star for given winding w, long strings emission is (exponentially) UV-finite!



Displacement of ⟨p_ρ⟩ due to non-perturbative corrections in ℓ²_s (worldsheet instantons)
 → not seen in semi-classical limit
 For large w ⟨E⟩ = kw

• For large w, $\langle E \rangle \sim k w$

✓ Summation over spectral flow: N_{long} ~ ∑_{w=1}[∞] 1/w → divergence at high energies
 ★ Needs non-perturbative UV cutoff: • w ≤ 1/g_s² (NS-NS charge conservation)
 • w ≤ 1/g_s (energy conservation)
 ★ On the contrary, emission of short strings (localized strings) stays finite





★Closed string emission fails to converge because of non-pert. effects in $\alpha' = \ell_s^2$



★Closed string emission fails to converge because of non-pert. effects in $\alpha' = \ell_s^2$ ★Production of short strings negligible in the perturbative regime $g_s \ll 1$ (since it does not depend on the coupling constant)



★Closed string emission fails to converge because of non-pert. effects in $\alpha' = \ell_s^2$ ★Production of short strings negligible in the perturbative regime $g_s \ll 1$ (since it does not depend on the coupling constant)

✓ AdS_3/CFT_2 correspondence string theory on AdS_3 dual to a symmetric product 2D CFT → dual description of tachyon decay?



★Closed string emission fails to converge because of non-pert. effects in $\alpha' = \ell_s^2$ ★Production of short strings negligible in the perturbative regime $g_s \ll 1$ (since it does not depend on the coupling constant)

✓ AdS_3/CFT_2 correspondence string theory on AdS_3 dual to a symmetric product 2D CFT → dual description of tachyon decay? ★For AdS_5/CFT_4 , corresponds to sphaleron decay [Drukker,Gross,Itzhaki] ★Difficult here since CFT is singular (unstable to fragmentation \leftrightarrow long strings emission)

✓ Open string point of view: time-dependent Hamiltonian → pair production Mini-superspace limit : $\left[\partial_t^2 + \lambda e^t + \mathbf{p}^2 + N - 1\right]\psi(t) = 0$ [Gutperle, Strominger '03]

✓ Open string point of view: time-dependent Hamiltonian → pair production Mini-superspace limit : $\left[\partial_t^2 + \lambda e^t + \mathbf{p}^2 + N - 1\right]\psi(t) = 0$ [Gutperle, Strominger '03]

★String theory naturally "chooses" (from Liouville theory) the $|out\rangle$ vacuum: $\psi \propto H^{(2)}_{-2iE}(2\sqrt{\lambda}e^{t/2}) \overset{t \to -\infty}{\sim} e^{-iEt} + R(E)e^{iEt}$ (R(E): reflection coefficient)

✓ Open string point of view: time-dependent Hamiltonian → pair production Mini-superspace limit : $\left[\partial_t^2 + \lambda e^t + \mathbf{p}^2 + N - 1\right]\psi(t) = 0$ [Gutperle, Strominger '03]

★String theory naturally "chooses" (from Liouville theory) the $|out\rangle$ vacuum: $\psi \propto H^{(2)}_{-2iE}(2\sqrt{\lambda}e^{t/2}) \stackrel{t \to -\infty}{\sim} e^{-iEt} + R(E)e^{iEt}$ (R(E): reflection coefficient)

Bogolioubov coefficient $\gamma = \frac{\beta_E}{\alpha_E} \leftrightarrow$ open string two-point function $\langle e^{iEt(\tau)}e^{-iEt(\tau')} \rangle$

★ Tension with Sen's conjecture in flat space?

✓ Open string point of view: time-dependent Hamiltonian → pair production Mini-superspace limit : $\left[\partial_t^2 + \lambda e^t + \mathbf{p}^2 + N - 1\right]\psi(t) = 0$ [Gutperle, Strominger '03]

★String theory naturally "chooses" (from Liouville theory) the $|out\rangle$ vacuum: $\psi \propto H^{(2)}_{-2iE}(2\sqrt{\lambda}e^{t/2}) \stackrel{t \to -\infty}{\sim} e^{-iEt} + R(E)e^{iEt}$ (R(E): reflection coefficient)

► Bogolioubov coefficient $\gamma = \frac{\beta_E}{\alpha_E} \leftrightarrow$ open string two-point function $\langle e^{iEt(\tau)}e^{-iEt(\tau')}\rangle$

★ Tension with Sen's conjecture in flat space?
 Rate of pair production W = -Re ln⟨out|in⟩ ~ ∫ dEρ(E)e^{-2πE}
 ▶ power-law convergent only (divergent for D_{p>22} in bosonic strings)

 \checkmark High energy behavior of open string pair production in AdS₃

 \star For open strings with angular momentum r, one gets (orbifold construction)

 $|R(E)| = \left| \frac{\sinh \pi (E + r/\sqrt{k}) \sinh \pi (E - r/\sqrt{k})}{\sinh^2 2\pi E} \right| \implies \text{same large } E \text{ asymptotics as in flat space}$

 \checkmark High energy behavior of open string pair production in AdS₃

 \star For open strings with angular momentum r, one gets (orbifold construction)

 $|R(E)| = \left| \frac{\sinh \pi (E + r/\sqrt{k}) \sinh \pi (E - r/\sqrt{k})}{\sinh^2 2\pi E} \right| \Rightarrow \text{same large } E \text{ asymptotics as in flat space}$

$$\star$$
Density of states smaller ($\Delta_{\min} > 0$): $ho(E) \sim \exp\{2\pi \sqrt{1 - 1/2k} \, \ell_s E\}$

 \blacktriangleright open string production rate exponentially convergent for very massive open strings on the D0-brane in AdS₃

★One gets that open string perturbative string (field) theory remains a valid description (despite the disappearance of the brane!)

✓ Non-critical superstrings: superstrings in spacetime dimension d < 10→ extra ($\mathcal{N} = 2$) Liouville (super-)field ϕ

✓ Non-critical superstrings: superstrings in spacetime dimension d < 10
 ⇒ extra (N = 2) Liouville (super-)field φ
 ★ Corresponds to string theory near genuine CY singularities

✓ Non-critical superstrings: superstrings in spacetime dimension d < 10
 → extra (N = 2) Liouville (super-)field φ
 ★ Corresponds to string theory near genuine CY singularities
 ★ Example: C²/Z_k orbifold singularity → by T-duality, equivalent to k NS5-branes distributed on a transverse circle

✓ Non-critical superstrings: superstrings in spacetime dimension d < 10
 ⇒ extra (N = 2) Liouville (super-)field φ
 ★ Corresponds to string theory near genuine CY singularities
 ★ Example: C²/Z_k orbifold singularity ⇒ by T-duality, equivalent to k NS5-branes distributed on a transverse circle

Universal and geometric tachyons

 \star BPS D*p*-brane halfway between two NS5-branes at a maximum of its potential (attracted by both fivebranes) \rightarrowtail geometric open string tachyon on its worldvolume whose dynamics is very similar to the usual "universal" open string tachyon [Kutasov]

✓ Non-critical superstrings: superstrings in spacetime dimension d < 10
 ⇒ extra (N = 2) Liouville (super-)field φ
 ★ Corresponds to string theory near genuine CY singularities
 ★ Example: C²/Z_k orbifold singularity ⇒ by T-duality, equivalent to k NS5-branes distributed on a transverse circle

Universal and geometric tachyons

★BPS D*p*-brane halfway between two NS5-branes at a maximum of its potential (attracted by both fivebranes) **⇒** geometric open string tachyon on its worldvolume whose dynamics is very similar to the usual "universal" open string tachyon [Kutasov] ★The universal tachyon appears on non-BPS D(*p*+1)-branes stretched between the NS5-branes





 \star Both are solutions of the D(p+1)-brane open string field theory (Dp is a kink)



★Both are solutions of the D(p+1)-brane open string field theory (Dp is a kink) ★Sen conjectured than these two solutions merge for $R < R_c$ [Sen'07] → only true for one or two fivebranes, which leads to strong curvatures and require some non-renormalization of the tachyonic DBI effective action → is there an exact string description? BPS Dp-brane non-BPS D(p+1) brane

★Both are solutions of the D(p+1)-brane open string field theory (Dp is a kink) ★Sen conjectured than these two solutions merge for $R < R_c$ [Sen'07] → only true for one or two fivebranes, which leads to strong curvatures and require some non-renormalization of the tachyonic DBI effective action → is there an exact string description?

 \checkmark We prove the conjecture in the double scaling limit: $g_s \rightarrow 0$, $\frac{L}{q_s \ell_s}$ fixed

BPS Dp-brane non-BPS D(p+1) brane

★Both are solutions of the D(p+1)-brane open string field theory (Dp is a kink) ★Sen conjectured than these two solutions merge for $R < R_c$ [Sen'07] → only true for one or two fivebranes, which leads to strong curvatures and require some non-renormalization of the tachyonic DBI effective action → is there an exact string description?

✓ We prove the conjecture in the double scaling limit: $g_s \rightarrow 0$, $\frac{L}{g_s \ell_s}$ fixed ★Gives near-horizon geometry of k = 2 close parallel fivebranes in non-compact transverse space ➡ six-dimensional non-critical string \star Start from the large k weakly curved regime (near-horizon geometry of k NS5branes in non-compact transverse space), where one can trust the geometrical interpretation

\star For any $k \ge 2$ an exact worldsheet CFT description is available

 \star Start from the large k weakly curved regime (near-horizon geometry of k NS5branes in non-compact transverse space), where one can trust the geometrical interpretation

\star For any $k \ge 2$ an exact worldsheet CFT description is available

 \blacktriangleright take k as the (discrete) control parameter

 \star Start from the large k weakly curved regime (near-horizon geometry of k NS5branes in non-compact transverse space), where one can trust the geometrical interpretation

\star For any $k \ge 2$ an exact worldsheet CFT description is available

 \blacktriangleright take k as the (discrete) control parameter

 \star One finds a transition at k = 2 where the D*p*-brane with a geometric tachyon and the non-BPS suspended D(*p*+1)-brane boundary states become identical



 \star Proof of geometric/universal tachyon equivalence including all α' corrections.

✓ Mass gap in the closed string spectrum $\ell_s m > \sqrt{8 - d/2}$ in the closed string sector (δ -normalizable states)

lower density of states $\rho(E) \sim \exp\{2\pi \sqrt{1 - \frac{8-d}{16}E}\}$ (higher Hagedorn temp.)
✓ Mass gap in the closed string spectrum $\ell_s m > \sqrt{8 - d/2}$ in the closed string sector (δ -normalizable states)

▶ lower density of states $\rho(E) \sim \exp\{2\pi \sqrt{1 - \frac{8-d}{16}E}\}$ (higher Hagedorn temp.)

From these considerations, it has been suggested that closed string emission in non-critical string is UV-FINITE
[Karczmarek,Liu,Hong,Maldacena,Strominger]

✓ Mass gap in the closed string spectrum $\ell_s m > \sqrt{8 - d/2}$ in the closed string sector (δ -normalizable states)

 \blacktriangleright lower density of states $\rho(E) \sim \exp\{2\pi \sqrt{1 - \frac{8-d}{16}E}\}$ (higher Hagedorn temp.)

✓ From these considerations, it has been suggested that closed string emission in non-critical string is UV-FINITE [Karczmarek,Liu,Hong,Maldacena,Strominger]

- would raise a puzzle: what is the leftover of the brane mass? $(\ell_s m_{
 m D} \sim 1/g_s^{
 m local})$
- would challenge Sen's conjecture ("universality" of DBI tachyonic action)

✓ Brane localized in the strong coupling end in $\mathcal{N} = 2$ Liouville (*cf.* ZZ *brane*) ★Dp with a geometric tachyon/suspended non-BPS D(p+1) seen before

✓ Brane localized in the strong coupling end in N = 2 Liouville (cf. ZZ brane)
 ★ Dp with a geometric tachyon/suspended non-BPS D(p + 1) seen before
 ★ Discrete spectrum built on the *identity representation* of N = 2 Liouville
 ★ identity I is a normalizable state

✓ Brane localized in the strong coupling end in N = 2 Liouville (cf. ZZ brane)
 ★ Dp with a geometric tachyon/suspended non-BPS D(p + 1) seen before
 ★ Discrete spectrum built on the *identity representation* of N = 2 Liouville
 ★ identity I is a normalizable state
 ★ D-brane has an open string tachyon built on the identity

 \blacktriangleright decay corresponds to $\delta S = \lambda \sigma^1 \oint dx G_{-1/2} \mathbb{I} \times e^{X_0(x)/\sqrt{2}\ell_s}$

✓ Brane localized in the strong coupling end in N = 2 Liouville (cf. ZZ brane)
 ★ Dp with a geometric tachyon/suspended non-BPS D(p + 1) seen before
 ★ Discrete spectrum built on the *identity representation* of N = 2 Liouville
 ★ identity I is a normalizable state
 ★ D-brane has an open string tachyon built on the identity

• decay corresponds to
$$\delta S = \lambda \sigma^1 \oint \mathrm{d} x \, G_{-1/2} \, \mathbb{I} imes e^{X_0(x)/\sqrt{2}\ell_s}$$

✓ One-point function for the rolling tachyon: $\langle V \rangle_{\lambda} \propto \frac{\sinh 2\pi p_{\phi} \sinh \pi p_{\phi}}{\cosh \pi p_{\phi} + \cos \pi s} \frac{(\pi \lambda)^{2iE}}{\sinh \pi E}$

✓ Brane localized in the strong coupling end in N = 2 Liouville (cf. zz brane)
 ★ Dp with a geometric tachyon/suspended non-BPS D(p + 1) seen before
 ★ Discrete spectrum built on the *identity representation* of N = 2 Liouville
 ★ identity I is a normalizable state

 \star D-brane has an open string tachyon built on the identity

 \blacktriangleright decay corresponds to $\delta S = \lambda \sigma^1 \oint \mathrm{d}x \, G_{-1/2} \, \mathbb{I} \times e^{X_0(x)/\sqrt{2}\ell_s}$

✓ One-point function for the rolling tachyon: $\langle V \rangle_{\lambda} \propto \frac{\sinh 2\pi p_{\phi} \sinh \pi p_{\phi}}{\cosh \pi p_{\phi} + \cos \pi s} \frac{(\pi \lambda)^{2iE}}{\sinh \pi E}$

★ Gives closed strings production $\mathcal{N} \sim \int dE \, dp_{\phi} \, d\mathbf{p} \, \sum_{N} \rho(N) \, \left| \langle V_{p_{\phi} E \mathbf{p} s} \rangle_{\lambda} \right|^{2} \, \delta(E^{2} - p_{\phi}^{2} - 2N - \mathbf{p}^{2} + d/8)$

✓ Brane localized in the strong coupling end in N = 2 Liouville (cf. zz brane)
 ★ Dp with a geometric tachyon/suspended non-BPS D(p + 1) seen before
 ★ Discrete spectrum built on the *identity representation* of N = 2 Liouville
 ★ identity I is a normalizable state

 \star D-brane has an open string tachyon built on the identity

 \blacktriangleright decay corresponds to $\delta S = \lambda \sigma^1 \oint \mathrm{d}x \, G_{-1/2} \, \mathbb{I} \times e^{X_0(x)/\sqrt{2}\ell_s}$

✓ One-point function for the rolling tachyon: $\langle V \rangle_{\lambda} \propto \frac{\sinh 2\pi p_{\phi} \sinh \pi p_{\phi}}{\cosh \pi p_{\phi} + \cos \pi s} \frac{(\pi \lambda)^{2iE}}{\sinh \pi E}$

★ Gives closed strings production $\mathcal{N} \sim \int dE \, dp_{\phi} \, dp \, \sum_{N} \rho(N) \, \left| \langle V_{p_{\phi} E \mathbf{p} s} \rangle_{\lambda} \right|^{2} \, \delta(E^{2} - p_{\phi}^{2} - 2N - \mathbf{p}^{2} + d/8)$ $\Rightarrow \rho(N)$ smaller than in flat space, but $\int dp_{\phi}$ gives UV divergent production of closed strings as in flat space (due to non-pert. α' corrections)

✓ Brane extended along the dilaton gradient in $\mathcal{N} = 2$ Liouville (*cf.* FZZT *brane*)

✓ Brane extended along the dilaton gradient in $\mathcal{N} = 2$ Liouville (*cf.* FZZT *brane*) ★Continuous spectrum (δ -norm) above a gap

 \blacktriangleright vertex operators: $V_p(x) = \exp\{-(\sqrt{1-d/8} + ip_{\phi})\phi(x) + p_{\mu}X^{\mu}(x) + \cdots\}$

✓ Brane extended along the dilaton gradient in N = 2 Liouville (cf. FZZT brane)
 ★ Continuous spectrum (δ-norm) above a gap
 ➡ vertex operators: V_p(x) = exp{-(√1-d/8+ip_φ)φ(x) + p_µX^µ(x) +···}

✓ Non-BPS D-brane (or D/\bar{D} pair): open string tachyon of mass $\ell_s m = i\sqrt{d}/4$

✓ Brane extended along the dilaton gradient in $\mathcal{N} = 2$ Liouville (*cf.* FZZT *brane*) ★Continuous spectrum (δ -norm) above a gap

• vertex operators: $V_p(x) = \exp\{-(\sqrt{1-d/8} + ip_\phi)\phi(x) + p_\mu X^\mu(x) + \cdots\}$

✓ Non-BPS D-brane (or D/\bar{D} pair): open string tachyon of mass $\ell_s m = i\sqrt{d}/4$ ★Homogeneous decay: $\delta S = \lambda \sigma^1 \oint dx \ G_{-1/2} \ e^{-\sqrt{1-d/8} \phi(x) + \frac{\sqrt{d}}{4\ell_s} X_0(x)}$

not a known conformal field theory (non-trivial along the dilaton gradient)

✓ Brane extended along the dilaton gradient in $\mathcal{N} = 2$ Liouville (*cf.* FZZT *brane*) ★Continuous spectrum (δ -norm) above a gap

→ vertex operators: $V_p(x) = \exp\{-(\sqrt{1-d/8} + ip_\phi)\phi(x) + p_\mu X^\mu(x) + \cdots\}$

✓ Non-BPS D-brane (or D/\bar{D} pair): open string tachyon of mass $\ell_s m = i\sqrt{d}/4$ ★Homogeneous decay: $\delta S = \lambda \sigma^1 \oint dx \ G_{-1/2} \ e^{-\sqrt{1-d/8} \phi(x) + \frac{\sqrt{d}}{4\ell_s} X_0(x)}$

not a known conformal field theory (non-trivial along the dilaton gradient)

✓ One could instead deform the worldsheet with $\delta S = \lambda \sigma^1 \oint dx \ G_{-1/2} \ \mathbb{I} \times e^{\frac{X_0(x)}{\sqrt{2}\ell_s}}$

✓ Brane extended along the dilaton gradient in $\mathcal{N} = 2$ Liouville (*cf.* FZZT *brane*) ★Continuous spectrum (δ -norm) above a gap

→ vertex operators: $V_p(x) = \exp\{-(\sqrt{1-d/8} + ip_\phi)\phi(x) + p_\mu X^\mu(x) + \cdots\}$

✓ Non-BPS D-brane (or D/\bar{D} pair): open string tachyon of mass $\ell_s m = i\sqrt{d}/4$ ★Homogeneous decay: $\delta S = \lambda \sigma^1 \oint dx \ G_{-1/2} \ e^{-\sqrt{1-d/8} \phi(x) + \frac{\sqrt{d}}{4\ell_s} X_0(x)}$

not a known conformal field theory (non-trivial along the dilaton gradient)

✓ One could instead deform the worldsheet with $\delta S = \lambda \sigma^1 \oint dx \ G_{-1/2} \ \mathbb{I} \times e^{\frac{X_0(x)}{\sqrt{2}\ell_s}}$ ★However the identity \mathbb{I} is not normalizable on the extended brane in Liouville theory (measure $\propto d\phi \ e^{\sqrt{4-d/2}\phi}$)

✓ Brane extended along the dilaton gradient in $\mathcal{N} = 2$ Liouville (*cf.* FZZT *brane*) ★Continuous spectrum (δ -norm) above a gap

► vertex operators: $V_p(x) = \exp\{-(\sqrt{1-d/8} + ip_\phi)\phi(x) + p_\mu X^\mu(x) + \cdots\}$

✓ Non-BPS D-brane (or D/\bar{D} pair): open string tachyon of mass $\ell_s m = i\sqrt{d}/4$ ★Homogeneous decay: $\delta S = \lambda \sigma^1 \oint dx \ G_{-1/2} \ e^{-\sqrt{1-d/8} \phi(x) + \frac{\sqrt{d}}{4\ell_s} X_0(x)}$

not a known conformal field theory (non-trivial along the dilaton gradient)

✓ One could instead deform the worldsheet with $\delta S = \lambda \sigma^1 \oint dx \ G_{-1/2} \ \mathbb{I} \times e^{\frac{X_0(x)}{\sqrt{2}\ell_s}}$ ★ However the identity \mathbb{I} is not normalizable on the extended brane in Liouville theory (measure $\propto d\phi \ e^{\sqrt{4-d/2}\phi}$)

 \blacktriangleright does not represent the decay of the open string tachyon but changes the boundary conditions at $\phi \rightarrow -\infty$ (however leads to a UV-finite result)

Conclusions

- D-brane decay involves all the tower of string modes
- Non-perturbative α' effects & asympt. density of states are crucial ingredients
- String theory clever enough to convert all brane mass into closed strings although the detailed outcome may be different
 Sen's conjecture seems universal !
- However, perturbative string theory leaves many issues open (backreaction)
- Future work: tachyon condensation and D/\overline{D} scattering, relevant for cosmology