Domain-Wall Dynamics in SYM and Duality

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Introduction

Supersymmetric Yang-Mills

- Supersymmetric non-Abelian gauge theory : shares many features with gluodynamics
  ➥ useful to understand strong coupling dynamics of QCD
- Vacuum structure understood (VY superpotential)
- Admits half-BPS domain walls, invisible at weak coupling

Domain-wall worldvolume theory

- Domain-walls : field theory D-branes ➥ worldvolume 3d theory
- String embedding ➥ $\mathcal{N}=1$ Yang-Mills Chern-Simons w/ matter
- SYM point of view predicts strong/weak duality
  ➥ how is it realized on the walls worldvolume ?

3d CFT's

- Generalizations lead to other $\mathcal{N}=1$ 3d theories with similar dualities
- In some cases non-trivial IR fixed points
- May help to understand flux compactifications via holography

Outline

$\mathcal{N} = 1$ SYM at low energies

$\mathcal{N} = 1$ SYM

- Non-Abelian $SU(N)$: gauge field A_μ + massless adjoint fermion λ
- Anomalous $U(1)_R$ symmetry broken to $\mathbb{Z}_{2N}$ by instantons
- As pure Yang-Mills, displays color confinement at low energies
- At the same time develops a gaugino condensate $\langle \lambda^2 \rangle \sim \Lambda^3 e^{\frac{2i\pi k}{N}}$ (with QCD scale $|\Lambda| \propto \exp -\frac{8\pi^2}{3Ng^2}$) ➥ breaks $\mathbb{Z}_{2N}$ to $\mathbb{Z}_2$

Effective Action

- Features described by (non-Wilsonian) effective action for the gluino bilinear superfield $S = \frac{1}{32\pi^2} \mathrm{Tr} W^2$
- Veneziano-Yankelevitz superpotential fixed by symmetries $\mathcal{W} = S \left[\log \frac{\Lambda^{3N}}{S^N} + 2i\pi n \right]$
- $\mathbb{Z}_N$ invariance of the theory ➥ sum over integer values of n

Domain walls

- Spontaneously broken discrete global symmetry with N distinct vacua ➥ expects *domain-walls* interpolating between any pair of vacua, invisible at weak coupling *(Dvali, Shifman)*
- $\mathcal{N}=1$ susy algebra admits a central extension for a 3d object
$$\{Q,Q\} = \frac{N\vec{\sigma}}{4\pi^2}\int \mathrm{d}^3x\,\vec{\nabla}\,\mathrm{Tr}\,\lambda^2$$
- Half-BPS domain-walls are found for SYM
- Energy density (tension) obtained exactly from the susy algebra
$$\varepsilon = \frac{N}{8\pi^2}\left|\langle \mathrm{Tr}\lambda^2\rangle_{\infty} - \langle \mathrm{Tr}\lambda^2\rangle_{-\infty}\right|$$

Domain walls as D-branes

- Wall tension scales like N, i.e. like $1/g_{\text{string}}$ at large N, whereas a soliton would scale like $N^2 \sim 1/g^2_{\text{string}}$
- Exactly like D-branes in string theory
- As shown by Witten using an M-theory embedding, confining strings can end on them ➥ similar to open strings in ordinary string theory
- Suggests the existence of a theory describing their worldvolume dynamics (open/closed duality)

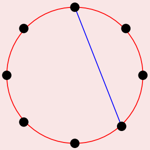
Dynamics of domain-walls stacks

- Tension of domain wall between ℓ-th and $\ell+k$-th vacua given by
$$\varepsilon_k = \frac{N^2\Lambda^3}{4\pi^2}\sin\frac{\pi k}{N}$$
- Can be viewed as a bound state of k elementary walls (non-zero binding energy)
- Expects some $U(k)$ gauge theory with $\mathcal{N}=1$ 3d susy, with at least one singlet scalar multiplet (center of mass)

Duality

- Interpolating 'clockwise' between k vacua and 'anti-clockwise' between $N-k$ vacua is equivalent by charge conjugation symmetry of the 4d theory

  ➥ does it correspond to a non-trivial duality in the 3d worldvolume theory ?



String theory embedding (I) : large N transition

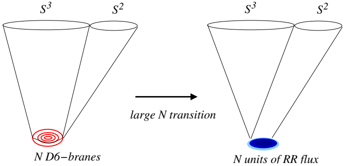
$\mathcal{N}=1$ SYM from string theory

- Strong coupling dynamics of susy field theories accessible at large N through string theory constructions with branes
- Several type IIA/IIB string and $\mathcal{M}$-theory constructions related through dualities ➥ geometrical engineering, Hanany-Witten setups,...

D6-branes on the conifold

- Conifold : singular Ricci-flat cone over a 5d base $T^{1,1} \sim SU(2)^2/U(1)$ (Calabi-Yau$_3$) : $\sum_{i=1}^4 z_i^2 = 0$
- Deformed conifold : non-collapsing 3-sphere at the tip of the cone $\sum_{i=1}^4 z_i^2 = \rho$
- Wrap N D6-branes in type IIA around this compact susy cycle ➥ At low energies, reduces to $\mathcal{N}=1$ $SU(N)$ SYM (no adjoint scalars as the D6-branes cannot move)

D4-branes and domain walls



Vafa's large-N transition

- Large N: branes replaced by their *backreaction* in 10d IIA supergravity, giving a completely smooth solution (holography)
- One obtains a *resolved conifold* with N units of Ramond-Ramond 2-form flux through the non-vanishing two-sphere at the tip (transverse to the $D6$'s)
- Has features of strongly coupled SYM (vacua, symmetries)
- IIA flux superpotential reproduces the VY field theory answer (where S is the complex Kähler modulus of the $\mathbb{CP}^1$)
- SYM confining string realized as the fundamental string in this background,

Domain walls

- One can look for an analogue of SYM domain-walls in the IIA background *after* the large N transition
- Wrapping k D4-branes around the blown-up two-sphere preserves 2 supercharges
- At low energies : 3d $\mathcal{N}=1$ gauge theory
  ➥ $\mathcal{N}=2$ $U(k)$ SYM in 3d with an $\mathcal{N}=1$ Chern-Simons interaction for the gauge field at level N, obtained from the DBI-CS action for D4-branes, in the presence of RR 2-form flux.
- Fundamental strings end on them as for any D-brane ➥ identified with $\mathcal{N}=1$ SYM domain walls

✔ This string theory construction allows to obtain a worldvolume theory for SYM domain walls. One of the requested features of this theory should be the duality outlined above.

Acharya-Vafa field theory

✔ The low-energy theory on k D4-branes wrapped on the two-cycle in the resolved conifold with RR two-form flux should give the worldvolume theory for the SYM domain wall $\ell \to \ell + k$

Action of the AV theory

- $\mathcal{N}=1$ theory of an $U(k)$ vector multiplet (A,χ) with a massless adjoint real scalar multiplet (ϕ,ψ)
- $\mathcal{N}=1$ Chern-Simons term at level N, breaks explicitely $\mathcal{N}=2$ susy
- As usual implies that the vector multiplet is massive ($m_{\rm CS}=gN$)
- $S=\frac{1}{4g^2}\int d^3x\,\mathrm{Tr}\left(-F^2+i\bar\chi \not{D}\chi+(D\phi)^2+i\bar\psi\not{D}\psi+2i\bar\chi[\phi,\psi]\right)$ $+\frac{N}{4\pi}\int \mathrm{Tr}(AdA+\frac{2}{3}A^3)-\frac{N}{4\pi}\int d^3x\,\mathrm{Tr}\chi\bar\chi$

✔ Classically, one has a moduli space spanned by gauge-invariant polynomials in ϕ

Perturbative potential

- $\mathcal{N}=1$ susy in 3d does not protect the moduli space from corrections (no holomorphy constraints or R-symmetry)
- Split adjoint scalar $\Phi = \Phi_0 + \hat{\Phi}$ as $\mathfrak{u}(k) \simeq \mathfrak{u}(1) + \mathfrak{su}(k)$
- For $k=2$ (two elementary Walls) Coleman-Weinberg 2-loop potential for $\hat{\phi}$ has been computed at large N *(Armoni,Hollowood)* $V \sim \frac{1}{N}\frac{u}{1+u}$ (with $u = \mathrm{Tr}\,\hat{\phi}^2/m_{\mathrm{CS}}^2$) ➥ loop-generated mass $m_{\mathrm{LOOP}} = m_{\mathrm{CS}}/N$
- Binding energy matches large N limit of 2-wall tension
- Free massless $U(1)$ multiplet remains : center of mass degrees of freedom of the 2-walls bound state

✔ As hinted in the introduction, the theory should contain *more* than the free dynamics of the free massless scalar multiplet

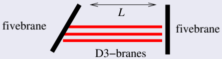
➥ geometrical engineering is not the easiest way to obtain the strong/weak duality expected (fluxed geometry)

Brane engineering of the AV theory

✔ A different string realization of the AV theory is achieved with brane engineering. It gives a more intuitive picture of symmetries and possible transitions (although the link with 4d SYM is less obvious).

3d Yang–Mills-Chern–Simons theories from branes

- via T-duality, D4-branes around the S^2 mapped to D3-branes ending a pair of fivebranes at distance L



- NS-Fivebranes being extremely heavy ➥ non-dynamical objects, impose *boundary conditions* (Dirichlet-type) for the low-energy d.o.f. on D3-branes that end on them (*i.e.* to $\mathcal{N}=4$ SYM in 4d)
- Replacing one fivebrane by its bound state with N D5-branes ➥ twisted boundary conditions
- Susy preserved classically by the configuration depends on NS5 and $(N,1)$5-brane relative orientations
- At energies $\ll 1/L$ Kaluza-Klein modes decouple ➥ 3d gauge theory with (dimensionful) coupling $g^2 = g_s/L$
- Twisted boundary conds. give a supersymmetric CS term at level N

Getting the domain walls theory

- NS5-brane along dimensions $x^{0,1,2,3,4,5}$
- $(N,1)$-fivebrane along $x^{0,1,2,3,8}$ at distance L along x^6, and at angle $\theta = \text{Arctan}\,(g_s N)$ in $(5,9)$-plane
  (angle fixed by susy once everything else is chosen)
- k D3-branes along $x^{0,1,2,6}$, ending on 5-branes along x^6
  ➥ $U(k)$ YM-CS theory at level N
- Accidental common direction x^3 not dictated from $\mathcal{N}=1$ susy (special tuning of angles in $(3,7)$-plane)

✔ Ab-initio construction of Acharya-Vafa theory ➥ not exactly T-dual to Vafa's construction
➥ indeed, the RR flux from the D5-branes in the bound state does not have the orientation expected from the dual of the RR two-form flux, in the T-dual of the resolved conifold

★ However at low energies one gets the same field theory ➥ 2 different UV completions

Dynamics of the AV brane setup

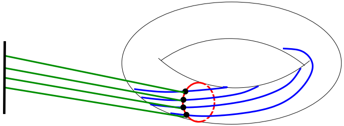
Vacua of AV theory from the branes picture

- Low-energy dynamics: dimensional reduction of $\mathcal{N}=4$ SYM in 4d on an interval with $\mathcal{N}=1$-preserving boundary conds.
- $\mathcal{N}=1$ $U(k)$ YM-CS w. massless adjoint & $\mathcal{N}=1$ CS term
- Classically arbitrary number of D3-branes suspended between fivebrane and $(N,1)$5-brane ➥ contradiction with domain walls expectations ($k \leqslant N$ as N coincident elementary walls are equivalent to the vacuum)
- In this tree-level string construction, motion of D3's are free along common direction x^3 ➥ no potential for adjoint scalar Φ

➥ clearly one needs to take into account quantum effects in the string setup

Witten index and susy breaking

- For well-defined counting of vacua needs to compactify on a torus
- T-duality along x^2 and lift to $\mathcal{M}$-theory ➥ fivebranes become a pair of M5-branes intersecting on a two-torus (x^2, x^{10}) N times
- D3-branes mapped to M2-branes, that need to end at M5 intersections by supersymmetry
- Using the *s* − *rule* for brane ending on branes, only one M2-brane is allowed at each intersection *(Hanany,Witten; Ohta)*
  ➥ related to the Pauli principle in D0/D8 systems *(Bachas, Green, Schwimmer)*
- Then susy is preserved only for $k \leqslant N$, otherwise spontaneous susy breaking ➥ matches expectations from domain walls
- Number of configurations easy to find : $\binom{N}{k}$ ➥ gives Witten index of AV field theory

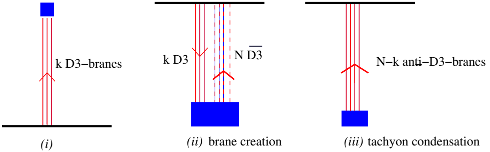


Forces between D3-branes

- Supersymmetry dictates an *attractive force* ➥ otherwise one would get a susy configuration from any number of D3-branes, separated along the x^3 direction common to all fivebranes
- Compatible with brane creation effect *(Hanany-Witten)*
- Hard to obtain directly the binding potential from the $\mathcal{M}$-theory description (quantum dynamics of the M5-branes)
- However in the low-energy & large N limit captured by the perturbative field theory result

✔ Therefore a generalization of the usual rules of brane constructions (s-rule, brane creation,...) to systems with 2 supercharges fits with the expectations from the domain-wall worldvolume picture

Seiberg duality



Brane creation effect

- One can move the $(N, 1)$5-brane along x^6 freely
  ➥ IR dynamics on D3-branes invariant (L is not a field theory parameter at low energies)
- As the $(N, 1)$5-brane crosses the fivebrane, N extra D3-branes should be created to ensure smooth susy dynamics
- k of them annihilate with original D3's that are carried along hence change orientation
  ➥ this is possible only if $k \leqslant N$, consistently with the s-rule

Seiberg-like duality on the domain walls

- Along the Hanany-Witten transition (fivebranes crossing) one expects no phase transition in the theory on the D3-branes, hence one should get equivalent IR dynamics
- Low-energy limit of field theory on D3-branes *after* the transition ➥ $U(N-k)$ Acharya-Vafa theory, also at level N
- Therefore gives a Seiberg-like duality for the domain-wall worldvolume theory between $U(k)$ and $U(N-k)$ both at level N
- Remark : usual Seiberg duality in 4d $\mathcal{N}=1$ SQCD can be found by similar methods *(Elitzur, Giveon, Kutasov)*

✔ From the point of view of domain-walls in 4d SYM this duality is a simple consequence of charge conjugation symmetry

➥ however from the 3d point of view it is far from trivial

Comments on low-energy dynamics of AV theory

Deep IR limit

- At energies below the dynamically generated mass $m_{\text{LOOP}} = m_{\text{CS}}/N$ for the $SU(k)$ adjoint scalar multiplet, all fields are massive, (except the free center-of-mass multiplet)
- It gives purely bosonic $U(N-k)$ Chern-Simons theory at level k, which is topological
  ➥ duality reduces to well-known *level-rank duality* in Chern-Simons and WZW models :
  $$SU(k)_{N-k} \longleftrightarrow SU(N-k)_k$$

✔ Did we find eventually something trivial ? is the duality dynamical or purely topological ?

IR dynamics beyond the topological regime

- Non-trivial dynamics is expected in the energy range $m_{\text{LOOP}} \ll E \ll m_{\text{CS}}$ where the YM kinetic term can be dropped from the action (at least in the large N when the two-loop computation is trustable)
- Duality is of the strong/weak type : dimensionless 't Hooft coupling mapped as $\frac{k}{N} \leftrightarrow \frac{N-k}{N}$
- Remember the domain-wall's tension formula :
  $\varepsilon_k = \frac{N^2\Lambda^3}{4\pi^2} \sin \frac{\pi k}{N}$
  ➥ explicitely invariant under the proposed duality $k \leftrightarrow N - k$
- It shows that the duality contains *more* than only topological information
  ➥ indeed the binding energy is a dynamical quantity (CW potential)

✔ This strong/weak dual pair although non-trivial reduces in the extreme IR to a pair of topological theories ➥ it would be interesting to generalize these methods to theories with non-trival IR fixed points

3d conformal field theories and AdS_4 vacua

AdS/CFT duals of flux compactifications ?

- Most superstring compactifications with flux have an AdS_4 ground state with $\mathcal{N} = 1$ or no 4d susy
- Should be holographically dual to 3d conformal field theories with at most $\mathcal{N} = 1$ 3d susy ↪ would help to understand non-perturbative dynamics of the compactification landscape
- Can we use our new understanding to build examples of such CFT's ?

- A priori Chern-Simons theories with matter (as the YM kinetic term is not conformal hence plays little role in the IR dynamics)
- Difficult to get non-topological theories in the extreme IR (masses are not quantum protected because of low supersymmetry hence generically dynamically generated)
- As for 4d SQCD, Seiberg-like duality of the sort discussed here helps to find a range of parameter giving a CFT

A non-trivial CFT candidate

- Start with k D3-branes stretched between n coincident NS5-branes and a $(N,1)$ fivebrane with the same relative orientation
- Extra adjoint scalar multiplet X, with superpotential $\text{Tr}X^{n+1}$
  ➥ this coupling is classically irrelevant for $n>3$
- Supersymmetric vacuum : if and only if $k/N \leqslant n$ (using similar arguments as before)
  ➥ upper value for 't Hooft coupling $\lambda_t = k/N$ above which susy is spontaneously broken in the presence of the tree-level superpotential
- Above some critical coupling $\lambda_t^\star < n$ the X^{n+1} interaction becomes necessarily relevant in the IR, acquiring a large anomalous dimension
- Below this value, one gets only pure CS in the deep IR

Seiberg-dual description

- Seiberg-dual theory : $U(nN-k)$ at level N with same tree superpotential, weakly coupled at large 't Hooft coupling $k/N \to n$
  ➥ gets a similar upper bound on the dual 't Hooft coupling below which the tree superpotential is IR-irrelevant (as the dual theory is weakly coupled then)
- It corresponds to a lower bound $\lambda_t^{\star\star}$ for the original 't Hooft coupling above which the dual description becomes topological in the IR (CS for the dual gauge group)
- From this picture one expects to get a *conformal window* $[\lambda_t^\star, \lambda_t^{\star\star}]$ between these two critical values. Inside, the two dual descriptions are strongly coupled in the IR
  ➥ One expects a non-trivial IR fixed point for any λ_t inside the window, giving interacting superconformal field theories

✔ It would be hard to find evidence for this statement purely in terms of the 3d field theory

Conclusions

- $\mathcal{N}=1$ SYM and massive QCD admit domain walls (only visible at strong coupling)
- Worldvolume theory on the walls obtained by various string setups
- Seiberg-like duality in the 3d AV theory compatible with expectations from the walls picture
- Generalizing the string constructions one can get new interacting 3d CFT's ➥ may help to understand non-perturbatively string flux vacua via AdS_4/CFT_3

Perspectives

- Is there a similar 5d/6d embedding of 4d Seiberg duality where the duality trivializes ?
- Extend the construction to domain walls of 4d SYM with other classical gauge groups, using orientifolds ➥ however the AV theory is not really known
- Construct other 3d CFTs, relation with AdS_4 flux vacua