Gepner models in a nutshell Asymmetric constructions: $K3 \times T^2$ models Simple currents and three-folds constructions Conclusions and future directions

Asymmetric Gepner models in type II superstrings

Dan Israël LPTHE, Université Pierre et Marie Curie

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D.I. and Vincent Thiéry, arXiv:1310.4116 D.I., work in progress $\begin{array}{c} \mbox{Gepner models in a nutshell} \\ \mbox{Asymmetric constructions: } K3 \times \mathcal{T}^2 \mbox{ models} \\ \mbox{Simple currents and three-folds constructions} \\ \mbox{Conclusions and future directions} \end{array}$

Introduction

Type II compactifications with fewer moduli and susy's

- Besides flux compactifications of type II SUGRA → are there models with a better grip on α' corrections ?
- NSNS three-form flux *alone* is not allowed because of the tadpole condition $\int_{\mathcal{M}_6} e^{-2\Phi} H \wedge \star H = 0$

Non-geometric compactifications/asymmetric freely-acting orbifolds

- Some of these constructions provide such examples
- However, there are build upon free theories of bosons and fermions, raising some questions:
 - \bigstar are they very specific to free CFT's ?
 - ★ do they have a (non-)geometrical interpretation in higher dimension (especially for free-fermion models)?
- Having more sophisticated examples will also teach us about quantum geometry of string theory

 $\begin{array}{l} \mbox{Gepner models in a nutshell} \\ \mbox{Asymmetric constructions: } & K3 \times T^2 \mbox{ models} \\ \mbox{Simple currents and three-folds constructions} \\ \mbox{Conclusions and future directions} \end{array}$

Asymmetric Gepner models

- Gepner models provide solvable *interacting* worldsheet theories for Calabi-Yau compactifications (in the stringy regime of negative Kähler moduli)
- As we will explain, one can construct type II asymmetric Gepner models in a rather systematic fashion
- One can expect that the asymmetric Gepner models are still understandable from a CY perspective.

A sufficient condition for non-geometry ?

- We will consider type II models whose spacetime supercharges come only from the left-movers
- SUGRA susy conditions : $\nabla_M^{\pm} \epsilon^{\pm} := (\nabla_M \pm \frac{1}{8} H_{MNP} \Gamma^{MNP}) \epsilon^{\pm} = 0$
 - one would need that ∇_+ has special holonomy (SU(3) in d = 6) but not ∇_- , giving an SU(3)-structure compactification
 - Contradicts the statement that H-flux alone is not allowed
- \star No standard SUGRA interpretation (*i.e.* geometric) is possible

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Outline Gepner models in a nutshell Asymmetric constructions: K3 × T² models Simple currents and three-folds constructions Conclusions and future directions

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Gepner models in a nutshell

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$\mathcal{N} = (2,2)$ minimal models

- \mathfrak{M}_k is an $\mathcal{N} = (2, 2)$ superconformal field theories in two dimensions with central charges $c_L = c_R = 3 \frac{6}{k}$, $k \in \{2, 3, \ldots\}$
- Splitting the Ramond and Neveu-Schwarz sectors according to the fermion number mod 2 → coset CFT (SU(2)_{k-2} × U(1)₂)/U(1)_k.
- Primary states are labelled by the triplet (j, m, s) with 2j = 0, ..., k 2, $m \in \mathbb{Z}_{2k}$ and $s \in \mathbb{Z}_4$ (s even is NS, s odd is R)
- Primaries have and R-charge and conformal dimension

$$Q_R\equiv rac{s}{2}-rac{m}{k}\mod 2\;,\quad \Delta\equiv rac{s^2}{8}+rac{4j(j+1)-m^2}{4k}\mod 1$$

Chiral rings

- ★ Antichiral primaries with m = 2j, s = 0 → $h = -\frac{1}{2}Q_R = \frac{j}{k}$
- ★ Chiral primaries with m = 2(j+1), $s = 2 \Rightarrow h = \frac{1}{2}Q_R = \frac{1}{2} \frac{j+1}{k}$

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Building type II Gepner models

- Compactification to $d = 10 2n \Rightarrow$ internal CFT with $c_L = c_R = 3n$
- Tensor product SCFT $\prod_{i=1}^{r} \mathfrak{M}_{k_i}$ with $3r 6\sum_{i=1}^{r} \frac{1}{k_i} = 3n$
- Representations of the chiral algebra associated with the characters

$$\phi_{\mu}^{\lambda(\sigma)}(q) := \prod_{i=1}^{r} C_{m_i}^{j_i(s_i)}(q) \twoheadrightarrow Q_R(\mu, \sigma) \equiv \sum_{i=1}^{r} \left(\frac{s_i}{2} - \sum_{i=1}^{r} \frac{m_i}{k_i} \right) \mod 2$$

 Type II string compactification : Q_R, Q_R ∈ 2ℤ + 1 and all chiral fermions in the same (NS or R) sector → generalized GSO projection.

Gepner modular invariant ($K3 \times T^2$ example)

$$\begin{split} Z &= \frac{1}{2^{r}} \sum_{\lambda,\mu,s_{i}} \sum_{b_{0} \in \mathbb{Z}_{k},b_{i} \in \mathbb{Z}_{2}} (-1)^{b_{0}} \frac{\Theta_{s_{0},2}(q)\Theta_{s_{0}+b_{0}+2\sum_{i=1}^{5}b_{i},2}(\bar{q})}{\tau_{2}^{2}\eta^{3}\bar{\eta}^{3}} \frac{\Theta_{s_{5},2}(q)\Theta_{s_{5}+b_{0}+2b_{5},2}(\bar{q})\Gamma_{2,2}}{\eta^{2}\bar{\eta}^{2}} \\ &\phi_{\mu}^{\lambda(\sigma)}(q) \phi_{\mu+\beta_{0}b_{0}}^{\lambda(\sigma+b_{0}\beta_{0}+2\sum_{i}b_{i}\beta_{i})}(\bar{q}) \delta^{(1)}\left(\frac{Q_{R}-1}{2}\right) \prod_{i=1}^{5} \delta^{(1)}\left(\frac{s_{0}-s_{i}}{2}\right) \end{split}$$

 $\begin{aligned} &\mathcal{K} = \mathsf{lcm} \left(4, 2k_1, \ldots, 2k_r \right), \, \beta_0 = (1, \ldots, 1), \, \beta_i = (0, \ldots, 0, 2, 0, \ldots, 0) \\ \bullet \quad \mathsf{Generalized GSO induces twisted sectors labelled by } b_0 \text{ and } \{ b_i \}. \end{aligned}$

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Geometry & moduli

- A Gepner model $\prod_{i=1}^{r} \mathfrak{M}_{k_i}$ is believed to be the IR fixed point of the Landau-Ginzburg orbifold $(W(X_i) = \sum_{i=1}^{r} X_i^{k_i})/\mathbb{Z}_N$, $N = \text{lcm}(k_i)$.
- The latter is obtained in the regime $\operatorname{Re}(t) \to -\infty$ of a U(1), $\mathcal{N} = (2,2)$ gauged-linear sigma model with FI parameter t
- best described in the regime Re(t) ≫ 1 as the Calabi-Yau sigma-model corresponding to the hypersurface W(X_i) = 0 in CPⁿ⁺¹ with complexified Kähler modulus t.
- Moduli space of non-linear sigma models on *CY*₃ spanned by the marginal chiral states (complex structure moduli space) and twisted chiral states (Kähler moduli space)
- Example: in the untwisted sector, marginal (a, a) states have $m_i = 2j_i$, $s_i = 0$ and are such that $\sum_i \frac{j_i}{k_i} = \frac{1}{2}$.

 \blacktriangleright mapped to the monomial deformations $X_1^{2j_1} \dots X_r^{2j_r}$ of $W(X_i)$

Some generalizations

- One has chosen the *diagonal SU*(2)_{ki-2} modular invariant for each minimal model
 → one can use any of the ADE modular invariants
- New models by orbifoldizing a subgroup of the discrete global symmetry of the model (∏_i ℤ_{ki})/ℤ_N → CY orbifolds
- *Discrete torsion, i.e.* extra phases in the twisted sectors, can be added to the orbifold partition function.
- Conveniently described in the *simple current* formalism (more later)
- In the heterotic case, many such asymmetric constructions have been explored (Schellekens & collaborators)
- Orientifolds compactifications based on Gepner models have been studied also in depth (Brunner, Hori, Hosomichi & Walcher)
- Suprisingly, the type II asymmetric models that we shall present now were not investigated (as far as I know)

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Asymmetric constructions: $K3 \times T^2$ models

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The idea

• This first construction relies on a very simple observation:

Minimal models characters $C_m^{j(s)}(q)$ behave as $\chi^j(q)\Theta_{s,2}(q)\Theta_{m,k}(\bar{q})$ under modular transformations, as far as the (j, m, s) labels are concerned

- Comes from the implicit definition of minimal model characters $\chi^j \Theta_{s,2} = \sum_{m \in \mathbb{Z}_{2k}} C_m^{j(s)} \Theta_{m,k}$, where χ^j is an $SU(2)_{k-2}$ character.
- One can then replace, in the $K3 \times T^2$ partition function, the contribution of $S^1 \times \mathfrak{M}_{k_i}$ (the circle being at radius $\sqrt{\alpha' k_i}$) as

$$\begin{split} \frac{\Gamma_{1,1}(q,\bar{q})}{|\eta|^2} \ C_{m_i}^{j_i(s_i)}(q) \ C_{m_i+b_0}^{j_i(s_i+b_0+2b_i)}(\bar{q}) &\longrightarrow \\ & \frac{1}{|\eta|^2} C_{m_i}^{j_i(s)}(q) \left(\Theta_{m_i+b_0,k_i} \ \chi^{j_i}(\bar{q}) \ \Theta_{s_i+b_0+2b_i,2}(\bar{q})\right) \\ &= \sum_{\bar{m}_i \in \mathbb{Z}_{k_i}} \frac{\Theta_{m_i+b_0,k_i}(q)\Theta_{\bar{m}_i,k_i}(\bar{q})}{|\eta|^2} \ C_{m_i}^{j_i(s_i)}(q) \ C_{\bar{m}_i}^{j_i(s_i+b_0+2b_i)}(\bar{q}) \end{split}$$

• Sort of asymmetric freely-acting orbifold

Gepner models in a nutshell Asymmetric constructions: $K3 \times 7^2$ models Simple currents and three-folds constructions Conclusions and future directions

Models

- For each of the 14 Gepner models for K3 ➡ one has two chose which pair of minimal models is 'twisted'
- List of 62 inequivalent asymmetric Gepner models for $K3 \times T^2$ (last two are twisted)

 $\begin{array}{l} (2, 3, 10, 15), (2, 10, 3, 15), (2, 15, 3, 10), (3, 10, 2, 15), (3, 15, 2, 10), (10, 15, 2, 3)\\ (2, 3, 8, 24), (2, 8, 3, 24), (2, 24, 3, 8), (3, 8, 2, 24), (3, 24, 2, 8), (8, 24, 2, 3)\\ (2, 3, 9, 18), (2, 9, 3, 18), (2, 18, 3, 9), (3, 9, 2, 18), (3, 18, 2, 9), (9, 18, 2, 3)\\ (2, 3, 7, 42), (2, 7, 3, 42), (2, 42, 3, 7), (3, 42, 2, 7), (7, 42, 2, 3), (3, 7, 2, 42)\\ (2, 4, 6, 12), (2, 6, 4, 12), (2, 12, 4, 6), (4, 12, 2, 6), (6, 12, 2, 4), (4, 6, 2, 12)\\ (2, 4, 5, 20), (2, 5, 4, 20), (2, 20, 4, 5), (5, 20, 2, 4), (4, 5, 2, 20), (4, 20, 2, 5)\\ (3, 3, 4, 12), (3, 4, 3, 12), (3, 12, 3, 4), (4, 12, 3, 3)\\ (2, 5, 5, 10), (2, 10, 5, 15), (5, 5, 2, 10), (5, 10, 2, 4)\\ (3, 4, 4, 6), (3, 6, 4, 4), (4, 4, 3, 6), (4, 6, 3, 4)\\ (3, 6, 6), (3, 6, 3, 6), (6, 6, 3, 3)\\ (2, 6, 6, 6), (6, 6, 2, 6)\\ (4, 4, 4, 4) \end{array}$

• As we will see, 33 of them give exactly the same low-energy spectrum

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Space-time susy & Ramond ground states

Relevant piece of the partition function for the discussion

$$\prod_{i=1}^{2} C_{m_{i}}^{j_{i}(s_{i})} \bar{C}_{m_{i}+b_{0}}^{j_{i}(s_{i}+b_{0}+2b_{i})} \prod_{i=3}^{4} \sum_{\bar{m}_{i} \in \mathbb{Z}_{k_{i}}} \Theta_{m_{i}+b_{0},k_{i}} \bar{\Theta}_{\bar{m}_{i},k_{i}} C_{m_{i}}^{j_{i}(s_{i})} \bar{C}_{\bar{m}_{i}}^{j_{i}(s_{i}+b_{0}+2b_{i})} \times \delta^{(1)} \left(\sum \frac{s_{i}}{4} - \sum \frac{m_{i}}{2k_{i}} - \frac{1}{2}\right)$$

- On the left-moving side
 the generalized GSO projection onto odd-integer R-charge still provides two spacetime supersymmetries
- On the right moving side, since m
 _{3,4} are unconstrainted, the right R-charge is not integer-valued
 → no spacetime susy from the right
- Explicitely, the right Ramond ground states get a mass shift from the twisted *T*² lattice
- The right gravitinos are in the sector j_i = 0, m_i = 0, s_i = 1 mod 2 and b₀ = 1 → mass M = √(1/α'k₃ + 1/α'k₄)
- All the models have $\mathcal{N}=2$ space-time supersymmetry, and no massless RR states.

Gepner models in a nutshell Asymmetric constructions: $K3 \times T^2$ models Simple currents and three-folds constructions Conclusions and future directions

Massless spectra

• On the left, one still looks for chiral states with $Q_R = \sum_i \frac{2j_i}{k_i} = 1$, but there exists an important new constraint : the mass shift from the twisted torus lattice should vanish

 $2j_3 + b_0 \equiv 0 \mod 2k_3$, $2j_4 + b_0 \equiv 0 \mod 2k_4$

- For each such left chiral states, one looks for right states of dimension 1/2 (not necessarily chiral in principle)
 one finds that only the subset of the original massless states satifying the above conditions are massless (both untwisted & twisted)
- The theory has a right $SU(2)_{k_3-2} \times SU(2)_{k_4-2}$ affine symmetry, leaving invariant the massless states but acting on the massive spectrum
- As the $U(1)^2$ affine currents of the T^2 CFT are not lifted, one can move away from this extended symmetry point

• generically gravitini masses are $M = \sqrt{\frac{T_2}{U_2} + \frac{(T_1 \pm 1)^2}{U_2 T_2}}$.

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Scan of all models

- All the models have an ${\cal N}=2$ supergravity multiplet, and three vector multiplets (STU) corresponding to the torus moduli and to the axio-dilaton
- Out of the 62 inequivalent asymmetric Gepner models, 33 don't have any other multiplets

 \blacktriangleright STU supergravity at low energies

- The other 29 models have some sporadic hypermultiplets corresponding to the surviving K3 moduli, with no regular pattern
- Unlike type II compactifications on CY₃, the dilaton sits in a vector multiplet
 the hypermultiplets moduli space receives no quantum corrections
- Are there other descriptions under STU triality (*e.g.* in heterotic) ?
 ★Heterotic duals, if any, should have some nonperturbative nature (as U and S are exchanged).

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Simple currents and three-folds constructions

 $\begin{array}{l} \mbox{Gepner models in a nutshell} \\ \mbox{Asymmetric constructions: } K3 \times T^2 \mbox{ models} \\ \mbox{Simple currents and three-folds constructions} \\ \mbox{Conclusions and future directions} \\ \end{array}$

Simple current extensions & discrete torsion

As often, finding generalizations requires a bit of formalism

- A simple current J is a primary such that by fusion with a generic primary (in particular with themselves) gives a single primary : J ★ φ_μ = φ_ν
- In a rational CFT a simple current J_i generates an final Abelian group \mathbb{Z}_{n_i} , n_i being the *length* of the simple current
- The action of a simple current on a primary defines its monodromy charge $Q_i(\mu) = \Delta(\phi_\mu) + \Delta(J_i) \Delta(J_i \star \phi_\mu) \mod 1$
- Two-currents are mutually local if $Q_i(J_j) = 0$

Minimal models simple currents

• Simple currents $J_{m,s}$ of the minimial models : primaries of quantum numbers (j = 0, m, s)

$$J_{m,s} \star V_{m'}^{j'(s')} = V_{m'+m}^{j'(s'+s)}.$$

• $\mathfrak{L}_{m,s}$, the length of the orbit generated by the simple current $J_{m,s}$, is the smallest integer such that (for odd k)

$$\mathfrak{L}_{m,s}s\equiv 0 \mod 4$$
, $\mathfrak{L}_{m,s}m\equiv 0 \mod 2k$

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• The diagonal partition function of a given RCFT can be extended by a set of simple currents $\{J_i, i = 1, \dots, M\}$ as

$$Z = \sum_{\mu} \prod_{i=1}^{M} \sum_{b_i \in \mathbb{Z}_{n_i}} \phi_{\lambda}(q) \phi_{\lambda + \sum_{i=1}^{M} \beta_i b_i}(\bar{q}) \delta^{(1)}(Q_i(\mu) + X_{ij}b_j),$$

with β_i such that $J_i \star \phi_\mu = \phi_{\mu+\beta_i}$.

• The selection rule depends on the (Q-valued) $M \times M$ matrix X_{ij} whose symmetric part is determined by the relative monodromies of the currents:

$$X_{\imath\jmath} + X_{\jmath\imath} = Q_\imath(J_\jmath) \mod 1$$

• The antisymmetric part, called *discrete torsion*, should merely be such that for all entries of *X*

$$gcd(n_i, n_j) X_{ij} \in \mathbb{Z}$$

• If the left and right Kernels of X are different, the simple-current extended modular invariant is asymmetric.

Gepner models in a nutshell Asymmetric constructions: $K3 \times T^2$ models Simple currents and three-folds constructions Conclusions and future directions

$K3 \times T^2$ asymmetric models as simple current extensions

Aim of the exercise : asymmetric $K3 \times T^2$ models from a simple current extension

- One starts with an ordinary Gepner model for K3 × T² with torus moduli U = iα'√k₃k₄, T = i√^{k₄}/_{k₃}
 ➡ rational torus U(1)_{k₃} × U(1)_{k₄} with extended chiral algebra
- A generic simple current of the model has quantum numbers (α, β) left $\mathbb{Z}_{2k_3} \times \mathbb{Z}_{2k_4}$ charges of the T^2) $(j_1 = 0, \dots, j_4 = 0; s_0, \dots, s_5; m_1, \dots, m_4; \alpha, \beta)$
- The usual Gepner model is a simple current extension with

$$\begin{array}{rcl} J_0 & = & (0, \ldots, 0; 1, \ldots, 1; 1, \ldots, 1; 0, 0) \\ J_i & = & (0, \ldots, 0; 2, 0, \ldots, 0, 2, 0, \ldots, 0; 0, \ldots, 0; 0, 0) \ , & i = 1, \ldots, 4 \end{array}$$

• The J₀-extension enforces the generalized GSO projection, while the J_i extensions align the all fermions in the Ramond or Neveu-Schwarz sector

Gepner models in a nutshell Asymmetric constructions: $K3 \times T^2$ models Simple currents and three-folds constructions Conclusions and future directions

• We add a simple current extension that is *not mutually local with the Gepner simple current* J₀, but local with respect to the simple currents {J_i}

$$J_{\alpha} = (0, \ldots, 0; 0, \ldots, 0; 0, 0, 2, 0; 2, 0)$$

- $J_{\beta} = (0, \ldots, 0; 0, \ldots, 0; 0, 0, 0, 2; 0, 2)$
- One checks that $Q_0(J_\alpha) = 1/k_3$ and $Q_0(J_\beta) = 1/k_4$ • $X_{0\alpha}^{sym} = \frac{1}{2k_1}$ and $X_{0\beta}^{sym} = \frac{1}{2k_2}$
- This non-locality messes up the GSO projection, which is shifted in the twisted sector as $\delta^{(1)}\left(\frac{Q_R-1}{2} + X_{0\alpha}B_{\alpha} + X_{0\beta}B_{\beta}\right)$
- A specific choice of discrete torsion, such that $X_{0\alpha} = X_{0\beta} = 0$, preserves projection onto $Q_R \in 2\mathbb{Z} + 1$ states on the left

$$Z = \frac{1}{\tau_2^2 |\eta|^4} \frac{1}{2^r} \sum_{\lambda,\mu} \sum_{b_0 \in \mathbb{Z}_K} (-1)^{b_0} \delta^{(1)} \left(\frac{Q_R - 1}{2}\right) \sum_{r_\alpha \in \mathbb{Z}_{k_3}} \delta^{(1)} \left(\frac{m_3 - n_\alpha + \mathbf{b_0}}{k_3}\right) \sum_{r_\beta \in \mathbb{Z}_{k_4}} \delta^{(1)} \left(\frac{m_4 - n_\beta + \mathbf{b_0}}{k_4}\right) \delta^{(1)} \left(\frac{m_4 - n_\beta + \mathbf{b_0}}{k_4}$$

$$\sum_{B_{\alpha} \in \mathbb{Z}_{k_{3}}, B_{\beta} \in \mathbb{Z}_{k_{4}}} \prod_{i=1}^{r+1} \sum_{b_{i} \in \mathbb{Z}_{2}} \delta^{(1)} \left(\frac{s_{0}-s_{i}}{2}\right) \phi_{\mu}^{\lambda}(q) \phi_{\mu+\beta_{0}b_{0}+\sum_{i=1}^{r+1} \beta_{i}b_{i}+\mathbf{B}_{\alpha}\mathbf{b}_{3}+\mathbf{B}_{\beta}\mathbf{b}_{4}}^{(\bar{q})}$$
(1)

★ However right R-charges no longer odd integers in the new twisted sectors (B_α ≠ 0, B_β ≠ 0)
 ★ X_{α0} = ¹/_{k₃}, X_{β0} = ¹/_{k₄} ➡ right gravitini get a mass from the T² lattice.

 $\begin{array}{l} \mbox{Gepner models in a nutshell} \\ \mbox{Asymmetric constructions: } {\cal K} 3 \times {\cal T}^2 \mbox{ models} \\ \mbox{Simple currents and three-folds constructions} \\ \mbox{Conclusions and future directions} \end{array}$

Asymmetric CY three-folds

• Start with a Gepner model for a CY 3-fold. We also consider an extension with a simple current that is *not* mutually local with J₀

$$I = (j_1 = 0, \dots, j_5 = 0; s_0 = 0, \dots, s_5 = 0; m_1 = \nu_1, \dots, m_5 = \nu_5)$$

- Abelian group isomorphic to \mathbb{Z}_N with $N = \text{lcm}\left(\frac{\text{lcm}(\nu_i, 2k_i)}{\nu_i}\right)$
- One checks that $Q_0(I) = rac{1}{2} \sum_i rac{
 u_i}{k_i} \mod 1$
- As before one can restore the left GSO projection by adding a specific discrete torsion, giving a lower-triangular X-matrix with non-zero entries

$$X_{II} = rac{1}{4} \sum_{i} rac{
u_{i}^{2}}{k_{i}} \quad , \qquad X_{I0} = rac{1}{2} \sum_{i} rac{
u_{i}}{k_{i}}$$

• Defines a consistent model as long as $\nu_i := 2\rho_i \in 2\mathbb{Z}$, $i = 1, \dots, 5$.

 $\begin{array}{l} \mbox{Gepner models in a nutshell} \\ \mbox{Asymmetric constructions: } K3 \times T^2 \mbox{ models} \\ \mbox{Simple currents and three-folds constructions} \\ \mbox{Conclusions and future directions} \\ \end{array}$

The partition function for the CY₃ Gepner model extended with the simple current *I* reads then, with $\beta_I = (2\rho_1, \dots, 2\rho_5)$

Asymmetric CY₃ Gepner model

$$Z = \frac{1}{\tau_2^2 |\eta|^2} \frac{1}{2^5} \sum_{\lambda,\mu} \sum_{b_0 \in \mathbb{Z}_K} (-1)^{b_0} \,\delta^{(1)} \left(\frac{Q_R - 1}{2}\right) \sum_{B \in \mathbb{Z}_N} \delta^{(1)} \left(\sum_{i=1}^r \frac{\rho_i(\mathbf{m}_i + \mathbf{b}_0 + \rho_i \mathbf{B})}{\mathbf{k}_i}\right)$$
$$\prod_{i=1}^r \sum_{b_i \in \mathbb{Z}_2} \delta^{(1)} \left(\frac{s_0 - s_i}{2}\right) \,\Theta_{s_0,2}(q) \,\Theta_{s_0 + b_0 + \sum_i b_i,2}(\bar{q}) \,\phi^{\lambda}_{\mu}(q) \,\phi^{\lambda}_{\mu + \beta_0 b_0 + \sum_{i=1}^{r+1} \beta_i b_i + \beta_i \mathbf{B}}(\bar{q})$$

- On the left there is still a projection onto states with odd integral R-charge bone gravitino
- On the right the R-charge of a generic state is non-integer $\bar{Q}_R = 1 + 2B \sum_{i=1}^r \frac{\rho_i}{k_i} \mod 2$
- The would-be massless 'right' gravitino would have $2j_i = m_i = s_i = B = 0$ and $b_0 = 1$ but the projection $\sum_{i=1}^r \frac{\rho_i}{k_i} = 0$ mod 1 is not satisfied generically.
- \star Type II compactifications with $\mathcal{N}=1$ space-time susy

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Conclusions and future directions

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Conclusions

- Somewhat suprisingly, there were still unexplored corners of Gepner model constructions nearly 30 years after their discovery.
- The asymmetric Gepner model construction provides examples of type II vacua whose space-time supersymmetry comes only from the left-movers on the worldsheet
- It indicates, as argued earlier, that these compactifications cannot have a standard geometrical interpretation
- At least in the $K3 \times T^2$ case (for which the relevant computations have been done) the (freely-acting) quotient lifts many, if not all, the moduli of the underlying CY compactification
- In these constructions, there are also no Ramond-Ramond fluxes available
- Standard orientifold compactifications (Pardisi-Sagniotti-Stanev open descendants) are not compatible with the discrete torsion

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Directions for future work

- Analyze the asymmetric three-folds in detail
- Understand the asymmetric models from the four-dimensional effective supergravity of CY compactifications with Q or R gaugings
- Is there a ten-dimensional picture in terms of generalized geometry ?
- Can this formulation being extended to heterotic models, for which the asymmetric constructions are ubiquitous ?
- Are there more sophisticated orientifold parity symmetries that are consistent with these models ? (remark : no RR tadpoles here)