Heterotic Torsional Backgrounds, from Supergravity to CFT

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Eurostrings@Madrid, June 2010

L .Carlevaro, D.I. and M. Petropoulos, arXiv:0812.3391
L .Carlevaro and D.I., arXiv:0910.3190
+ work in progress

Introduction

Heterotic compactifications

- $\mathcal{N}=1$ heterotic compactifications correspond to solvable perturbative worldsheet CFT only at very special points: toroidal orbifolds, Gepner models, free fermions,...
- Usually the geometrical interpretation is lost – although the topological data matches geometrical Calabi-Yau compactifications

Smooth compactifications

- Instead one can consider in supergravity a CY compactification
- One specifies a gauge bundle V on the compactification manifold. Needs to satisfy the modified Bianchi identity for the NSNS two-form
- Satified automatically for the standard embedding of the $SU(3)$ spin connection in the gauge connection ➥ however not interesting
- For more general gauge bundles, torsion usually is needed ➥ fluxes appear naturally even without trying to stabilize moduli

Heterotic Flux compactifications (torsional)

- Less understood that type II models, as *not* conformally CY, not even Kähler manifolds appear
- One needs therefore to deal with explicit examples (few are known in the compact case)
- However, perturbative heterotic compatifications are potentially accessible to worldsheet CFT methods because *(i)* no R-R flux and *(ii)* the dilaton is not stabilized perturbatively

Local models of flux compactifications

- Simpler descriptions can be found near smoothed singularities (as Klebanov-Strassler throats in type IIB) ➥ local models
- In heterotic, we find two examples with a solvable worldsheet CFT: a T^2 fibration over Eguchi-Hanson and a resolved conifold
- As in type II one may have interesting physics 'localized' in these throats due to the strong warping ➥ SUSY breaking ?

Outline

Manifolds with $SU(3)$ structure and $\mathcal{N}=1$ heterotic vacua

- $\mathcal{N}=1$ compactification with torsion : 6d manifold admitting a covariantly constant spinor w.r.t. spin connection $\omega - \frac{1}{2}\mathcal{H}$
- Defines an $SU(3)$ structure, characterized by the $SU(3)$-invariant real 2-form J and complex 3-form Ω
- (J, Ω) give a complex structure and a metric
- Susy conditions: calibrations for wrapped 5-branes

$$\begin{cases} \mathrm{d}(e^{-2\Phi}\Omega) & = 0 \\ \mathrm{d}(e^{-2\Phi} J \wedge J) & = 0 \\ \mathrm{d}(e^{-2\Phi} J) - e^{-2\Phi} \star_6 \mathcal{H} & = 0 \end{cases}$$

➥ non Kähler manifolds

- Gauge bundle has to satisfy Hermitean Yang-Mills equations:

$$\mathcal{F}^{a\bar{b}} J_{a\bar{b}} = \mathcal{F}^{ab} = \mathcal{F}^{\bar{a}\bar{b}} = 0$$

- Massive spinorial reps of the gauge group ➥ $c_1(V) \in H^2(2\mathbb{Z})$

Bianchi identity

$\mathrm{d}\mathcal{H} = \alpha' \left[\mathrm{tr} R(\omega_-) \wedge R(\omega_-) - \mathrm{Tr}_v F \wedge F \right]$ in forms

★Non-linear constraint : $R(\omega_-)$ constructed w. connection $\omega - \frac{1}{2}\mathcal{H}$

Warped Conifold I : supergravity ansatz

- Local model of heterotic $\mathcal{N}=1$ compactification in 4d
  ➥ conifold singularity
  hypersurface $z_1^2+z_2^2+z_3^2+z_4^2=0$ in $\mathbb{C}^4$
- Singular cone over a a $T^{1,1}\sim (SU(2)\times SU(2))/U(1)$ base :
  S^1 fibration over $S^2\times S^2$ ➥ $S^3\times S^2$ topology
- Singularity regularized with a finite S^2 (blow-up) or a finite S^3 (deformation)
  ➥ the latter case is used in type IIB (Klebanov-Strassler) as the compact 3-cycle can support RR 3-form flux corresponding to fractional D3-branes
- In heterotic needs instead a 2 or 4-cycle (by Hodge duality) to support magnetic flux.
- We will consider the latter, which is topologically possible for the orbifold $\textit{conifold}/\mathbb{Z}_2$

Heterotic supergravity ansatz

$$\begin{aligned} ds^2 = dx^\mu dx_\mu + \tfrac{3}{2}\, H(r) \Big[\tfrac{dr^2}{f^2(r)} + \tfrac{r^2}{6}\left(d\theta_1^2 + \sin^2\theta_1\, d\phi_1^2 + d\theta_2^2 + \sin^2\theta_2\, d\phi_2^2\right) \\ + \tfrac{r^2}{9} f(r)^2 \left(d\psi + \cos\theta_1\, d\phi_1 + \cos\theta_2\, d\phi_2\right)^2 \Big] \end{aligned}$$

$$\mathcal{H}_{[3]} = \tfrac{\alpha' k}{6}\, g_1(r)^2 \left(\Omega_1 + \Omega_2\right) \wedge \left(d\psi + \cos\theta_1\, d\phi_1 + \cos\theta_2\, d\phi_2\right)$$

- The S^1 fiber is deformed by the squashing factor $f(r) \leqslant 1$
  ➥ non-Kähler warped conifold with torsion
- Unlike in type II the $\mathbb{R}^{3,1}$ space-time is unwarped (in string frame)
- We will find below a 'bolt' for some $r = a$ ($f(a) = 0$), as in Eguchi-Hanson space
  ➥ conical singularity removed by the orbifold $\psi \sim \psi + 2\pi$

Susy, Bianchi and HYM

- The SUSY calibration conditions give the system

$$\begin{cases} f^2 H' = -2\alpha' k g_1^2 / r^3 \\ r^3 H f f' + 3r^2 H (f^2 - 1) + \alpha' k g_1^2 = 0 \end{cases}$$

- The HYM equations are satisfied with a line bundle ($\vec{p}$, $\vec{q}$ gives embedding in the Cartan $\vec{T}$):

Gauge bundle

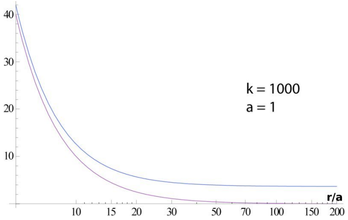
$$\mathcal{A} = \tfrac{1}{2}\vec{p}\cdot\vec{T}(\cos\theta_1\,\mathrm{d}\phi_1 - \cos\theta_2\,\mathrm{d}\phi_2) + \vec{q}\cdot\vec{T}\left(\tfrac{a}{r}\right)^4(\mathrm{d}\psi + \cos\theta_1\,\mathrm{d}\phi_1 + \cos\theta_2\,\mathrm{d}\phi_2)$$

- Second magnetic field ($\vec{q}$) responsible for the resolution of the conifold singularity ➥ free parameter a (blow-up parameter)

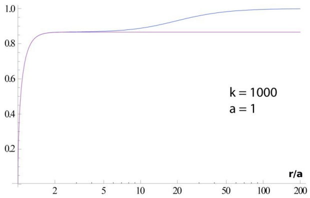
- In the large charges limit ($\vec{p}^2$,$\vec{q}^2 >> 1$) the $\mathcal{R}^2$ term in Bianchi identity is negligible (checked 'on-shell')
- $\mathrm{d}\mathcal{H} = \alpha' \mathrm{Tr}\mathcal{F}^2$ ➥ $g_1^2(r) = \frac{3}{4}(1 - (\frac{a}{r})^4)$ and $k = \vec{p}^{\,2} = 4\vec{q}^{\,2}$

Numerical solution

- One can solve numerically the susy equations for $f(r)$ and $H(r)$
- Smooth and weakly coupled everywhere
- For $r \to \infty$ one finds the usual Ricci-flat conifold and constant dilaton, however with non-zero NSNS and magnetic charges
- For $r \to a$, the S^1 fiber degenerates (bolt)
  ➥ one finds a non-Ricci flat conifold resolved by a 4-cycle



Conformal factor $H(r)$



Resolution function $f(r)$

both numerical (blue) and near-horizon (purple) solutions are plotted

Analytical solution in the double-scaling limit

- String coupling diverges in the blow-down limit (point-like instanton)
  ➥ One can define a double-scaling limit $g_s \to 0$ with $g_s \alpha' / a^2$ fixed
- It isolates the near-bolt region $a^2 \leqslant r^2 \ll \alpha' k$ from the asymptotically Ricci-flat region
- In this regime, one can solve analytically the susy equations

Double-scaling limit of the warped orbifoldized deformed conifold

$$\mathrm{d}s^2 = \frac{\alpha' k}{2r^2}\left[\frac{\mathrm{d}r^2}{1-\frac{a^8}{r^8}} + \frac{r^2}{8}\left(\mathrm{d}\theta_2^2 + \sin^2\theta_1\,\mathrm{d}\phi_1^2 + \mathrm{d}\theta_2^2 + \sin^2\theta_2\,\mathrm{d}\phi_2^2\right) + \frac{r^2}{16}\left(1-\frac{a^8}{r^8}\right)\left(\mathrm{d}\psi + \cos\theta_1\,\mathrm{d}\phi_1 + \cos\theta_2\,\mathrm{d}\phi_2\right)^2\right]$$

$$e^{2\Phi} = e^{2\Phi_0}\,\frac{(\alpha' k)^2}{r^4}$$

$$\mathcal{A} = \left[\frac{1}{2}\left(\cos\theta_1\,\mathrm{d}\phi_1 - \cos\theta_2\,\mathrm{d}\phi_2\right)\vec{p} + \left(\frac{a}{r}\right)^4\left(\mathrm{d}\psi + \cos\theta_1\,\mathrm{d}\phi_1 + \cos\theta_2\,\mathrm{d}\tilde{\phi}_2\right)\vec{q}\,\right]\cdot\vec{H}$$

$$\mathcal{B}_{[2]} = \frac{\alpha' k}{8}\left(\cos\theta_1\,\mathrm{d}\phi_1 + \cos\theta_2\,\mathrm{d}\phi_2\right)\wedge\mathrm{d}\psi$$

➥ Smooth weakly coupled background without sources (as $r \geqslant a$)

Worldsheet CFT in the blow-down limit

- In the blow-down limit one gets a linear dilaton $\times$ $T^{1,1}$
  ➥ Non-Einstein $[SU(2)^2]/U(1)$ coset w. torsion and & line bundle
- Obtained in the worldsheet CFT as a $\boxed{U(1)_{\text{L}} \backslash^{SU(2)_k \times SU(2)_k}}$ asymmetrically gauged $\mathcal{N}=(1,0)$ WZW model (left action)
- Classical 'anomaly' of the gauging cancelled by an action in the $\widehat{SO(32)}_1$ or $(\widehat{E_8} \times \widehat{E_8})_1$ anti-holomorphic CFT
- Specified by a 16-dim vector of charges $\vec{p}$, with $k = \vec{p}^2$
  ➥ in space-time, Abelian magnetic field

Worldsheet CFT in the double-scaling limit

- The heterotic orbifoldized warped conifold resolved by a 4-cycle has a worldsheet CFT *also* in the double scaling limit
- Asymmetrically gauged WZW model :

$$\frac{SL(2)_{k/2}\times U(1)\backslash SU(2)_k \times SU(2)_k}{U(1)_{\mathrm{L}} \times U(1)_{\mathrm{R}}}$$

➥ Needs gauging in $\widehat{G}_1$ specified by $\vec{q}$, with $k=\vec{p}^2=4\vec{q}^2-4$

- One can extract from the gauged WZW model the background fields to all orders in α' ➥ exact solution to Bianchi !
- Left $\mathcal{N}=2$ SCA ➥ $\mathcal{N}=1$ 4d SUSY in spacetime as expected

Spectrum (from one-loop partition function)

- Discrete $SL(2)$ representations (localized bound states)
  ➥ massive $U(1)$ gauge field along $\vec{q}\cdot\vec{T}$ and set of massless 4d chiral multiplets localized at the bolt
- Unbroken gauge bosons and graviton ➥ non-normalizable massless modes and continuum of massive modes

Worldsheet instantons and Liouville potentials

- So far, the $\mathbb{Z}_2$ orbifold of $T^{1,1}$ put 'by hand' in the CFT ➥ is there an analogue of no-conical singularity condition of sugra ?
- Condition on the bundle $c_1(V) \in H^2(2\mathbb{Z})$ ➥ role in worldsheet CFT ?
- $\frac{SL(2)}{U(1)}$ corrected by worldsheet non-perturbative effects *(Fateev, Zamolodchikov)*

Liouville-like potentials for Abelian bundles

- In our heterotic coset, the dynamically generated $\mathcal{N}=(2,0)$ Liouville potential reads $\mu_L \int (\psi^\rho + \psi^3) e^{-\frac{\sqrt{\vec{q}^2-4}}{2}(\rho + iY^3_L) - \frac{i}{2}\vec{q}\cdot\vec{X}_R} + c.c.$
  ($\bar{\partial} X^n_R = \bar{\psi}^{2n-1}\bar{\psi}^{2n}$ is the bosonized Cartan)
- Belongs to the twisted sector of the $\mathbb{Z}_2$ orbifold
- Orbifold and (right) GSO invariant only if $c_1(V) \in H^2(2\mathbb{Z})$:
  Explicitely $\sum_\ell (p_\ell \pm q_\ell) \equiv 0 \mod 4$

T^2 fibrations over Eguchi-Hanson and worldsheet CFT

- One can get also type IIA/B or heterotic local models of $T^2 \to \mathcal{M}_6 \to K3$ with torsion

Fibered EH solution in the double scaling limit

$$ds^2 = \frac{U_2}{T_2}\left|dx^1 + q_1\mathcal{A}_1 + T(dx^2 + q_2\mathcal{A}_2)\right|^2 + \frac{\alpha' k}{r^2}\left[\frac{dr^2}{1-\frac{a^4}{r^4}} + \sigma_1^2 + \sigma_2^2 + \left(1-\frac{a^4}{r^4}\right)\sigma_3^2\right]$$

- $\mathcal{A}_{1,2} \sim \sqrt{k}\frac{\sigma_3 a^2}{2r^2}$
- Bianchi identity ➥ $k = \frac{4U_2}{T_2}\left|q_1 + Tq_2\right|^2$
- The worldsheet theory only well defined for rational T^2 CFT: T, U valued in the same *imaginary quadratic number field*:
  $K = \mathbb{Q}(\sqrt{D}) \sim \mathbb{Q} + \mathbb{Q}\sqrt{D}$ w. $D = b^2 - 4ac < 0$ & $gcd(a,b,c) = 1$
  ➥ Neat worldsheet understanding of moduli stabilization

Conclusions

- We studied local models of flux heterotic compactifications to four dimensions, with line bundles ➥ new resolved warped orbifoldized conifold solutions and torsional T^2 fibrations over EH
- One can define *double scaling limits* in order to isolate the throat regions (similar to KS throats in IIB flux compactifications)
- Remarkably these throats admit weakly coupled solvable worldsheet CFT descriptions, valid *even* for large curvatures
- Susy breaking possible (work in progress)

Future directions

- Computing all α' corrections using the gauged WZW model
- Holographic understanding ➥ confining $\mathcal{N}=1$ theories, $\mathcal{N}=2$ theories with compact Coulomb bramches,...
- Precise embedding of local models in heterotic flux compactifications ➥ T^2 fibrations at Gepner points
- Description of non-Abelian bundles

SUSY breaking in heterotic throats: generalities

- Is it possible to break SUSY at tree-level in the smooth throat solutions using a *normalizable* deformation (cf. debate about KS) ?
- In all models, the worldsheet CFT predicts a discrete spectrum of normalized modes near the tip
  ➥ The Banks and Dixon theorem (SUSY breaking for an internal $\mathcal{N}=2$ CFT with compact target space) applies e
- So no 'small' susy breaking by normalizable deformations is possible (hard breaking by orbifold still possible)
- Continuum of delta-fct normalizable modes ? ➥ all massive (non-chiral operators of $\mathcal{N}=2$)
- Remains one possibility : use non-normalizable operators, that correspond to non-unitary representations of the $\mathcal{N}=2$ algebra.

Example with T^2 fibration over EH

★ *Only* normalizable two-form of EH is the $(1,1)$ a.s.d.

➥ non-normalizable connection unavoidable

SUSY-breaking T^2 fibration over EH (with a square torus)

$$\mathrm{d}s^2 = \frac{k}{r^2}\left[\frac{\mathrm{d}r^2}{1-\frac{a^4}{r^4}} + \frac{r^2}{4}\left(\sigma_1^2+\sigma_2^2+\frac{(1-\frac{a^4}{r^4})(1-\lambda^2)}{1-\lambda^2\frac{a^4}{r^4}}\sigma_3^2\right)\right] + \frac{U_2}{T_2}\left|\mathrm{d}x^1 + \mathcal{A}_1 + T(\mathrm{d}x^2+\mathcal{A}_2)\right|^2$$

with $\mathcal{A}_1 = \lambda\frac{\sqrt{k}}{R_1}\frac{1-\frac{a^4}{r^4}}{1-\lambda^2\frac{a^4}{r^4}}\sigma_3$ and $\mathcal{A}_2 = \frac{\sqrt{k}}{R_2}\frac{a^2}{r^2}\frac{1-\lambda^2}{1-\lambda^2\frac{a^4}{r^4}}\sigma_3$

- Torus moduli become also base-dependent :
  $T = \frac{R_2}{R_1}(\lambda\frac{a^2}{r^2} + i\sqrt{1-\lambda^2\frac{a^2}{r^2}})$ and $U = R_1R_2(\lambda\frac{a^2}{r^2} + i\sqrt{1-\lambda^2\frac{a^2}{r^2}})$
- Constraint in the CFT : $\sqrt{k}R_1\lambda \in \mathbb{Z}$
  ➥ not a continuous deformation for given T^2 moduli
- Worldsheet CFT still solvable ➥ no tachyons found