TD: Practical QFT, solutions

Mark D. Goodsell*

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1 Vertices

2 Processes in QFT

2.1 Decays

2.1.1 Fermion to fermion scalar

Let's use the example Lagrangian from earlier:

$$\mathcal{L}_4 \supset c_L \Phi \overline{\Psi}_1 P_L \Psi_2 + c_R \Phi \overline{\Psi}_1 P_R \Psi_2$$

1. Derive the matrix element for $\mathcal{M}(\Psi_2 \rightarrow \Psi_1 + \Phi)$ where all particles are massive.
2. Compute the spin-averaged square matrix element $\frac{1}{2}\sum_{\text{spins}} |\mathcal{M}|^2$.
3. Write down the decay width.

Here we have

$$i\mathcal{M} = ic_L \overline{u}_1 P_L u_2 + ic_R \overline{u}_1 P_R u_2. \tag{2.1}$$

Then

$$\begin{aligned}
\frac{1}{2}\sum_{\text{spins}} |\mathcal{M}|^2 =& \frac{1}{2}\sum_{\text{spins}} \overline{u}_1 (c_L P_L + c_R P_R) u_2 [\overline{u}_2 (c_L^* P_R + c_R^* P_L) u_1] \\
=& \frac{1}{2}\text{tr}\left[(\not{p}_1 + m_1)(c_L P_L + c_R P_R)(\not{p}_2 + m_2)(c_L^* P_R + c_R^* P_L)\right] \\
=& \frac{1}{4}(|c_L|^2 + |c_R|^2)\text{tr}[\not{p}_1 \not{p}_2] + \frac{1}{2}(c_L c_R^* + c_L^* c_R) 2 m_1 m_2 \\
=& (|c_L|^2 + |c_R|^2) p_1 \cdot p_2 + m_1 m_2 (c_L c_R^* + c_L^* c_R) \\
=& (|c_L|^2 + |c_R|^2)\frac{1}{2}(m_1^2 + m_2^2 - m_S^2) + m_1 m_2 (c_L c_R^* + c_L^* c_R).
\end{aligned} \tag{2.2}$$

*goodsell@lpthe.jussieu.fr

This is independent of $\cos\theta$ so we have

$$\begin{aligned}\Gamma =&\frac{4\pi}{2m_1}\times\frac{1}{32\pi^2}\frac{K^{1/2}(m_1^2,m_2^2,m_S^2)}{m_1^2}\\ =&\frac{1}{16\pi m_1^3}K^{1/2}(m_1^2,m_2^2,m_S^2)\times\left[\frac{1}{2}|\mathcal{M}|^2\right].\end{aligned}\tag{2.3}$$

2.1.2 Massive vector to a pair of massless fermions

Using the interaction between a massive vector and a massless fermion:

$$\mathcal{L}\supset c_L Z_\mu \overline{\Psi}\gamma^\mu P_L\Psi \tag{2.4}$$

1. Derive the matrix element for $\mathcal{M}(Z\to 2\Psi)$ where Z is massive with mass m_Z.
2. Compute the spin-averaged square matrix element $\frac{1}{3}\sum_{\text{spins}}|\mathcal{M}|^2$.
3. Write down the decay width.
4. If the uncertainty on the measured value of the Z boson width is less than 2.3 MeV and $m_Z = 91.2$ GeV, how big could c be?

Here

$$i\mathcal{M} = ic_L\overline{v}(p_2)\gamma^\mu P_L u(p_1)\varepsilon_\mu(q) \tag{2.5}$$

and

$$\begin{aligned}\frac{1}{3}\sum_{\text{spins}}|\mathcal{M}|^2 =&\frac{1}{3}|c_L|^2\text{tr}\left[\gamma^\mu P_L\not{p}_1\gamma^\nu P_L\not{p}_2\right]\times -\eta_{\mu\nu}\\ =&\frac{2}{3}|c_L|^2\text{tr}(P_L\not{p}_1\not{p}_2)\\ =&\frac{4}{3}|c_L|^2 p_1\cdot p_2 = \frac{2}{3}|c_L|^2 m_V^2.\end{aligned}\tag{2.6}$$

Then for our case $K(m_V^2,m_1^2,m_2^2) = m_V^4$ so

$$\begin{aligned}\Gamma =&\frac{1}{16\pi m_V}\frac{K^{1/2}}{m_V^2}\times\frac{1}{3}\sum_{\text{spins}}|\mathcal{M}|^2\\ =&\frac{1}{16\pi m_V}\times\frac{2}{3}|c_L|^2 m_V^2 = \frac{1}{24\pi}|c_L|^2 m_V.\end{aligned}\tag{2.7}$$

So if this is equal to 2.3×10^{-3} we have

$$|c_L|\lesssim\sqrt{\frac{24\pi\Gamma}{m_V}}\lesssim 0.04. \tag{2.8}$$