# TD: Practical QFT, solutions 2 (incomplete)

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## 1 Processes in QFT

### 1.1 Scalar to a pair of fermions

#### 1.1.1 Question

Now consider a simple decay of a real scalar:

$$\mathcal{L} \supset y\Phi\overline{\Psi}\Psi$$

Let us pretend that the fermion is massless.

1. Derive the matrix element for $\mathcal{M}(\phi \to \overline{\psi}\psi)$.
2. Compute the spin-summed square matrix element $\sum_{\text{spins}} |\mathcal{M}|^2$.
3. Write down the decay width.

#### 1.1.2 Solution

As done in class:

$$i\mathcal{M} = iy\overline{u}(p_1)v(p_2) \tag{1.1}$$

So for massless fermions:

$$\begin{aligned}\sum |\mathcal{M}|^2 =& |y|^2 \sum \overline{u}(p_1)v(p_2)\overline{v}(p_2)u(p_1) \\ =& |y|^2 \mathrm{tr}(\not{p}_1 \cdot \not{p}_2) \\ =& 4|y|^2 p_1 \cdot p_2 \end{aligned} \tag{1.2}$$

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Then we use $m$ for the mass of the scalar:

$$\begin{aligned} p_0 =& p_1 + p_2 \\ m^2 =& (p_1 + p_2)^2 = 2p_1 \cdot p_2 \end{aligned} \tag{1.3}$$

so

$$\sum |\mathcal{M}|^2 = 2|y|^2 m^2. \tag{1.4}$$

Then the width is given by

$$\begin{aligned} \Gamma =& \frac{1}{2m} \int d\Pi_2 \sum |\mathcal{M}|^2 \\ =& |y|^2 m \int d\Pi_2 \\ =& \frac{|y|^2 m}{8\pi} \frac{K^{1/2}(s,0,0)}{s} \\ =& \frac{|y|^2 m}{8\pi}. \end{aligned} \tag{1.5}$$