Exercises For Part 2

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1 Effective action

Recall that for matrices:

$$
\begin{aligned}
0 =& \frac{\partial}{\partial x}(M^{-1}M) = \frac{\partial M^{-1}}{\partial x} M + M^{-1}\frac{\partial M}{\partial x} \\
\longrightarrow \frac{\partial M^{-1}}{\partial x} =& M^{-1}\frac{\partial M}{\partial x} M^{-1}
\end{aligned}
\tag{1.1}
$$

And also

$$
\begin{aligned}
\phi_{cl}(x) =& - \frac{\delta E}{\delta J(x)}, \\
\frac{\delta \Gamma}{\delta \phi_{cl}(x)} =& - J(x), \\
\frac{\delta \phi_{cl}}{\delta J(y)} =& - \frac{\delta^2 E}{\delta J(x) \delta J(y)} = iD(x,y)
\end{aligned}
\tag{1.2}
$$

1. Derive the general expression for the three-point correlator $\langle \phi(x_1)\phi(x_2)\phi(x_3)\rangle$ in terms of propagators $D(x-y)$ and derivatives of the effective action $\Gamma[\phi_{cl}]$, similar to the expressions in (2.31) and (2.32) in the notes.

2. Derive the expression for the four-point correlator $\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle$

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2 Coleman-Weinberg effective potential

2.1 One scalar

Consider the theory

$$\mathcal{L}_{\text{light}} \supset \frac{1}{2}(\partial\phi_L)^2 - \frac{1}{2}m^2\phi_L^2 - \frac{\lambda}{24}\phi_L^4.$$

1. Replace $\phi_L \to \phi_{cl,L}$ in the lagrangian and take the classical field constant.

2. Take the second derivatives to obtain $m^2_{eff}(\phi_{cl,L})$.

3. Write down the one-loop effective potential.

4. Expand the effective potential to *second order* in ϕ_L.

2.2 Two scalars

Consider next the theory

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu\phi_L)^2 + \frac{1}{2}(\partial_\mu\phi_H)^2 - \frac{1}{2}M^2\phi_H^2 - \frac{1}{2}m^2\phi_L^2 - \frac{a}{2}\phi_H\phi_L^2 - \frac{\lambda}{24}\phi_L^4$$

1. Replace $\phi_H, \phi_L \to \phi_{cl,H}, \phi_{cl,L}$ in the lagrangian and take the classical fields constant.

2. Take the second derivatives to obtain the *matrix* $m^2_{eff}(\phi_{cl,L}, \phi_{cl,H})$.

3. Set $\lambda = 0$ and $\phi_H = 0$ and compute the eigenvalues (if we were matching the theories we would have to put $\phi_H = -\frac{a}{2M^2}\phi_L^2$; we are *not* doing that here).

4. Write down the one-loop effective potential.

5. Expand the eigenvalues to second order in ϕ_L.

6. Expand the effective potential to second order in ϕ_L.

7. You have computed a finite correction to the mass of the field ϕ_L; it is equivalent to $\Pi_{\phi_L\phi_L}(0)$.

3 EFT bases

Consider the lagrangian consisting of a single complex scalar field:

$$\mathcal{L} \supset |\partial\Phi|^2 - m^2|\Phi|^2 - \lambda|\Phi|^4. \tag{3.1}$$

1. Write down all possible effective operators at dimension 6.
2. Determine the set that are independent after integration-by-parts identities are applied: choose a minimam "Green Basis."
3. Apply equation-of-motion identities to find a minimal basis for the effective field theory at dimension 6.