Phenomenology of the Standard Model and Beyond

Lecture 2

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Other accelerator experiments

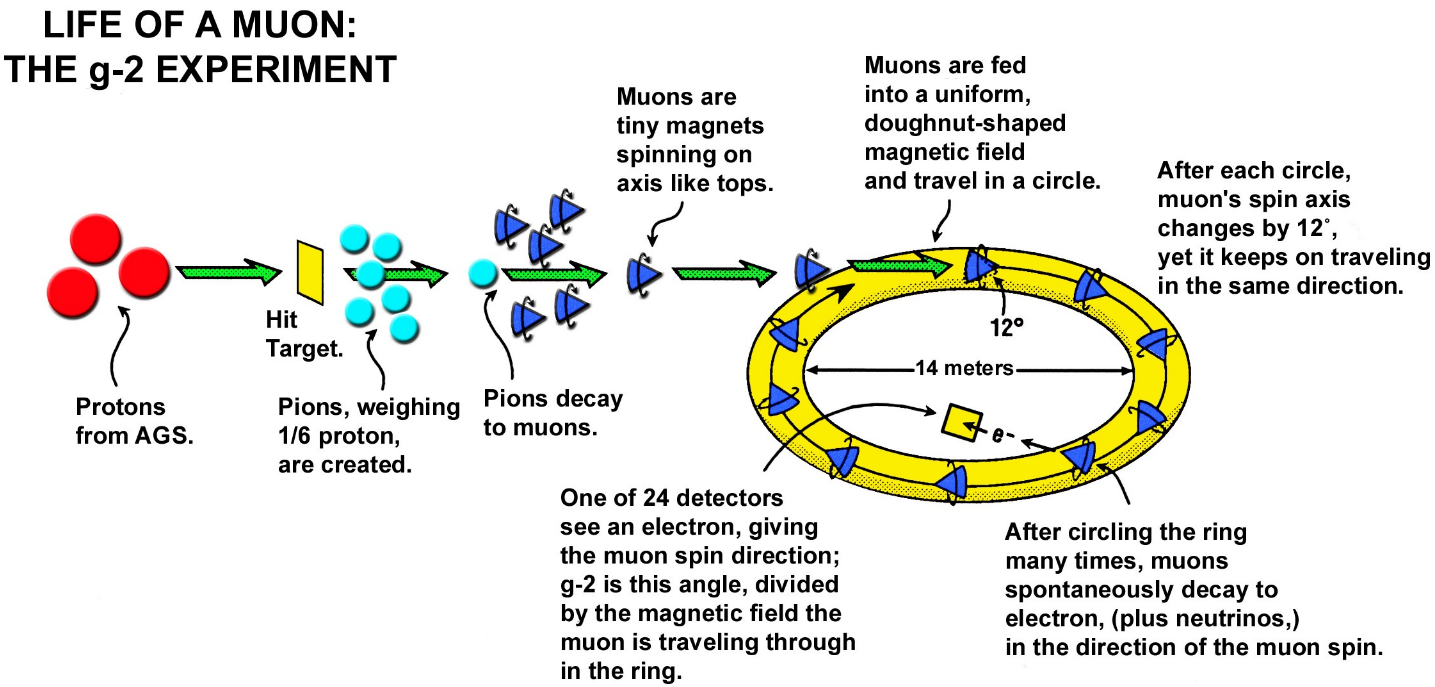
There are several smaller experiments using dedicated accelerators, e.g.:

- Muon g-2
- Rare lepton decays $\mu \to e\gamma, \mu \to 3e$
- B factory – Belle II $e^+e^- \to \Upsilon(4S)$
- MESA (beam dump) linac, O(100) MeV electrons

Muon g-2 experiment(s)

The muon g-2 is one of the original measurements confirming QFT.

For now, you may have heard that there was a long-standing discrepancy between the SM prediction and measurements.

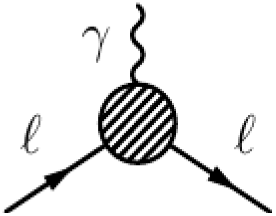


The experimental measurement was confirmed by a new Fermilab experiment that is still running. A second experiment will run at JPARC in the near future. But in the meantime, several lattice QCD groups disputed the SM prediction! Their computation is now taken as the standard.

Physics from the dipole operator

The lepton vertex function is responsible for a lot of physics:

QED diagrams



$$= \quad (-ie\mathsf{q}_e)\,\bar{u}(p')\Gamma^\mu(p,p')u(p)$$

$$\Gamma^\mu(p,p') = \gamma^\mu F_E(k^2) + i\frac{\sigma^{\mu\nu}k_\nu}{2m_\ell}F_M(k^2) + \frac{\sigma^{\mu\nu}k_\nu}{2m_\ell}\gamma_5 F_D(k^2) + \frac{k^2\gamma^\mu - k^\mu \not{k}}{m_\ell^2}\gamma_5 F_A(k^2)$$

Gauge coupling renormalisation

Anomalous magnetic moment

Electric Dipole moment

Anapole

$$i\varepsilon_\mu(k)\sigma^{\mu\nu}k_\nu \sim \partial_0 A_i \sigma^{0i} + \partial_i A_j \sigma^{ij} \sim \mathbf{E}\cdot\sigma\gamma_5 + i\mathbf{B}\cdot\sigma$$

We define $\mu \equiv g\left(\frac{q}{2m}\right)\mathbf{S}$

$$a_\ell = \frac{1}{2}(g_\ell - 2) = F_M(0)$$

Anomalous magnetic moment

$$d_\ell = -\frac{e_{\rm QED}\mathsf{q}_e}{2m_\ell}F_D(0)$$

Electric dipole moment (violates CP)

The muon g-2 is very precisely measured:

$$a_\mu^{\rm SM} = 116\,591\,810(43) \times 10^{-11}$$

Using 'R ratio'

$$a_\mu^{\rm SM,lattice} = 116592033(62) \times 10^{-11}$$

arXiv:2505.21476, review

$$a_\mu^{\rm exp} = 116\,592\,061(41) \times 10^{-11}$$

Lepton EDMs have very tight constraints from experiments:

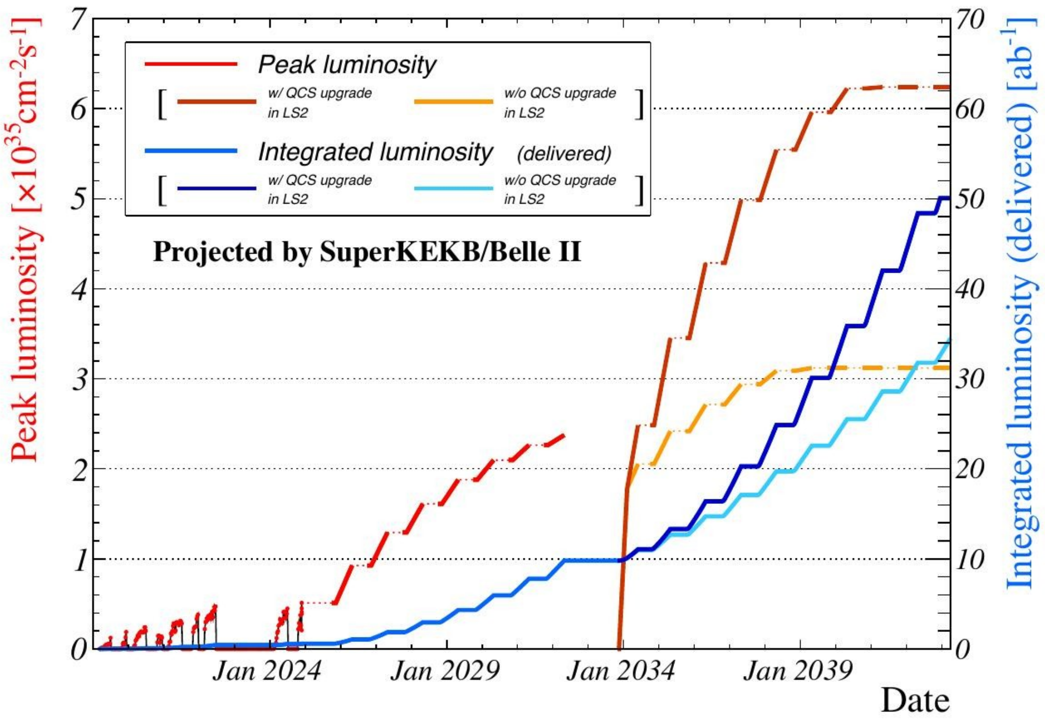
$$d_\ell^{SM} \lesssim 10^{-38}\, e\,{\rm cm}$$

$$|d_e| < 1.1 \times 10^{-29}\ e\text{–cm} \qquad @\,90\%\,{\rm CL}$$

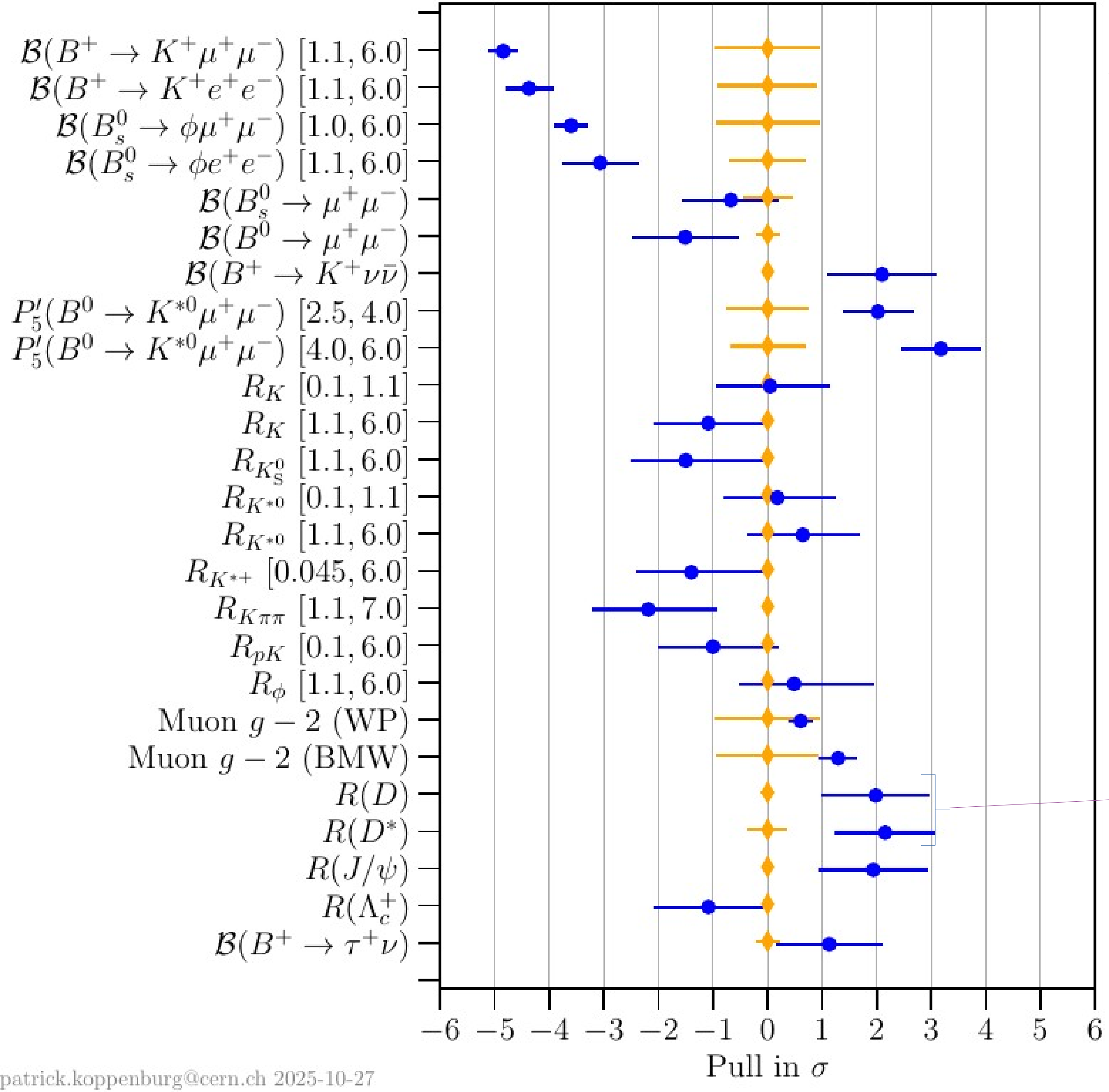
$$|d_\mu| < 1.5 \times 10^{-19}\ e\text{–cm} \qquad @\,90\%\,{\rm CL}$$

B factories

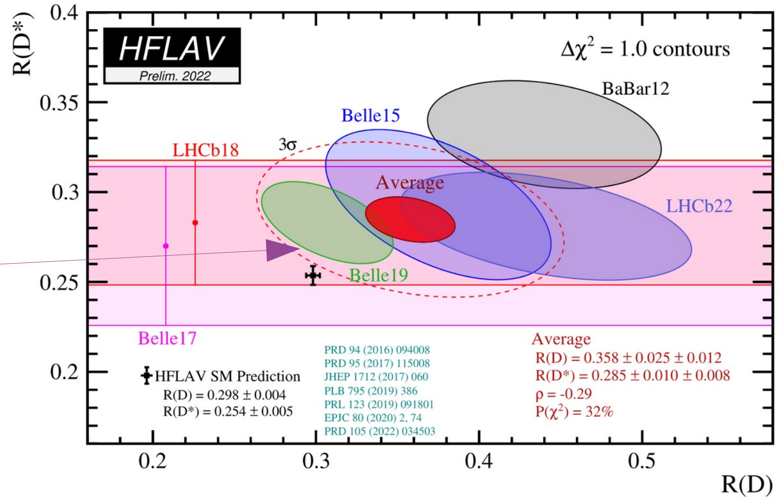
- By tuning accelerator energies, can produce copious amounts of particles *on resonance* (see later).
- Mesons containing b-quarks have many interesting properties and several experiments have produced them.
- Huge theoretical industry in making predictions.
- BaBar was fed by the SLC linac, produced 514 fb^{-1}
- Belle ran at KEK (Japan) from 1999 to 2010, yielded 988 fb^{-1}
- Belle II started collecting data in 2019, aims at 50 ab^{-1} by 2040!
- Interesting because of persistent deviations from SM predictions (although two of them disappeared at the end of 2022).



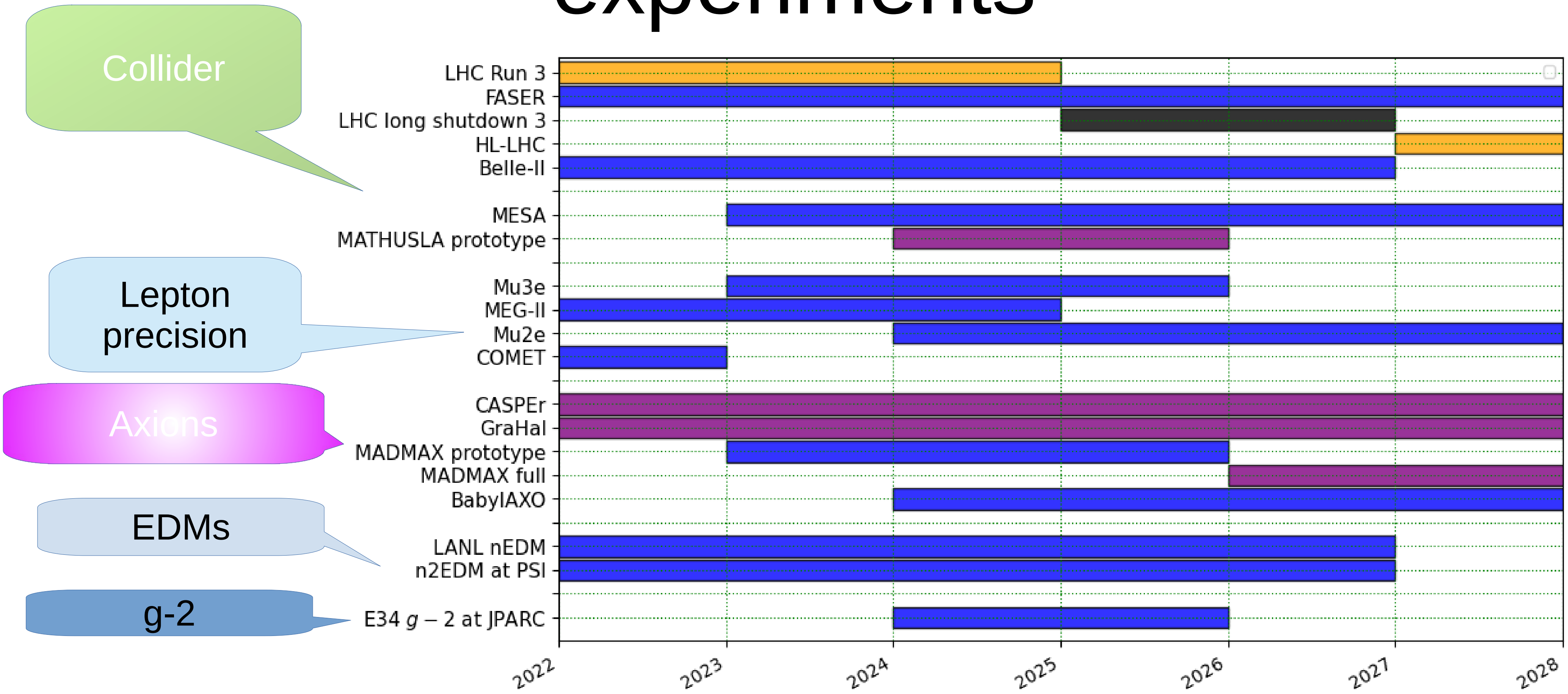
belle2.org



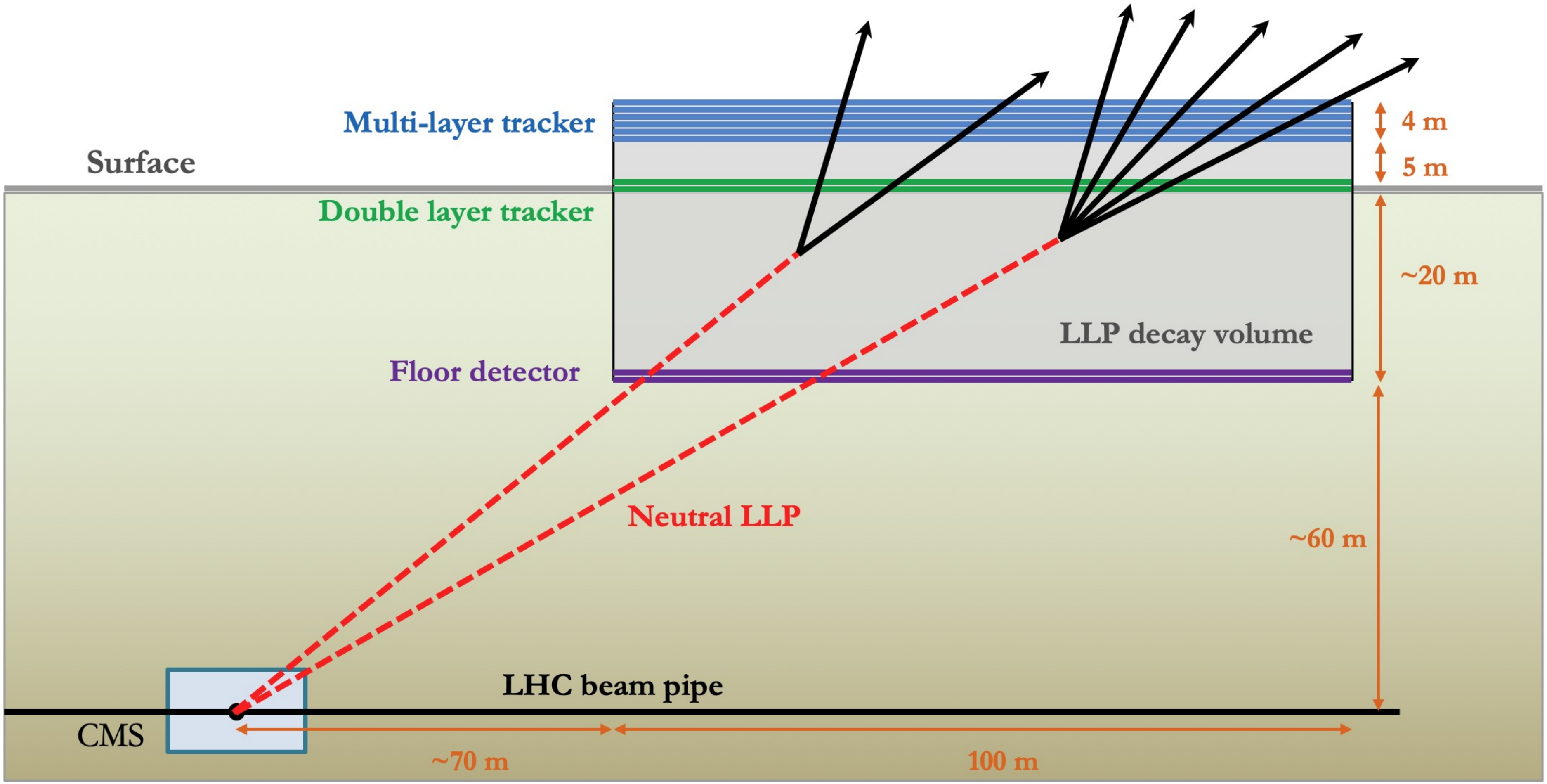
A summary of the B-physics anomalies maintained by an experimentalist at https://www.nikhef.nl/~pkoppenb/anomalies.html



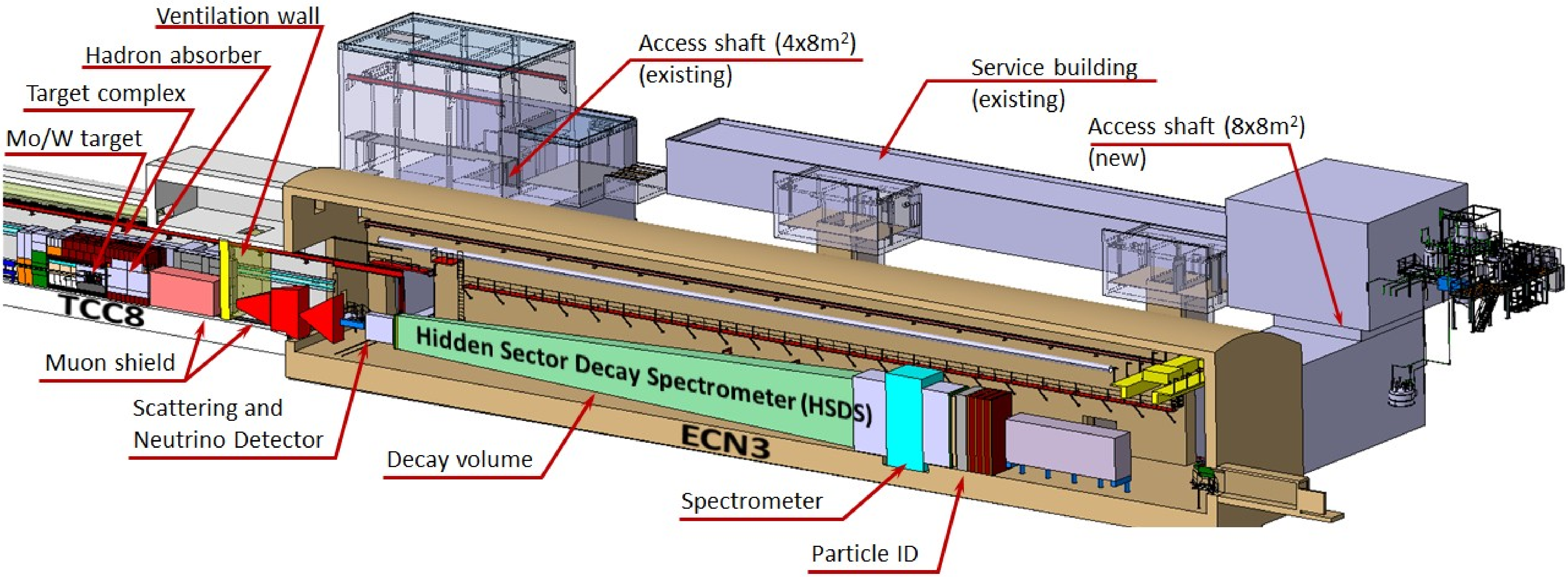
Timeline of some current and future experiments



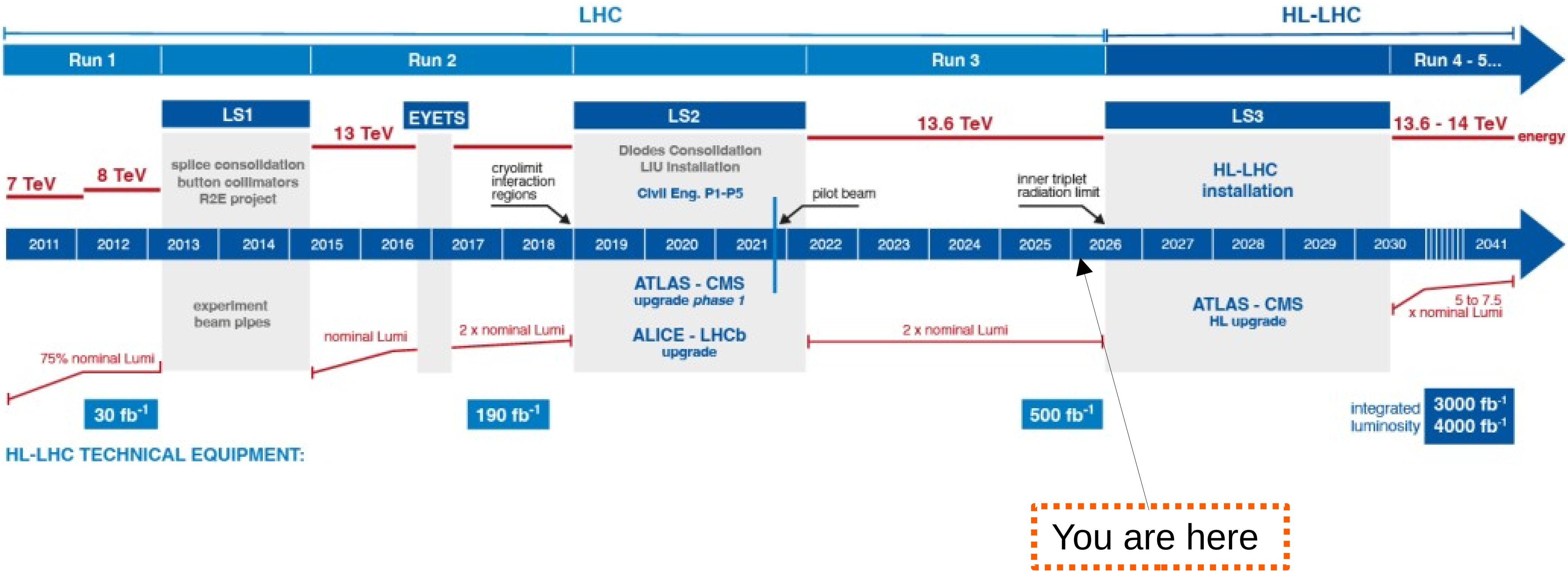
MATHUSLA experiment proposal



SHiP experiment (being built, 2031)

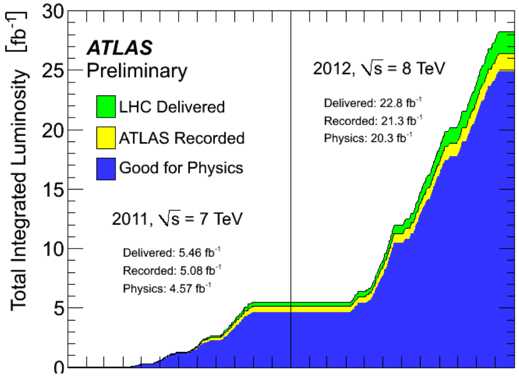


The LHC: where are we now?

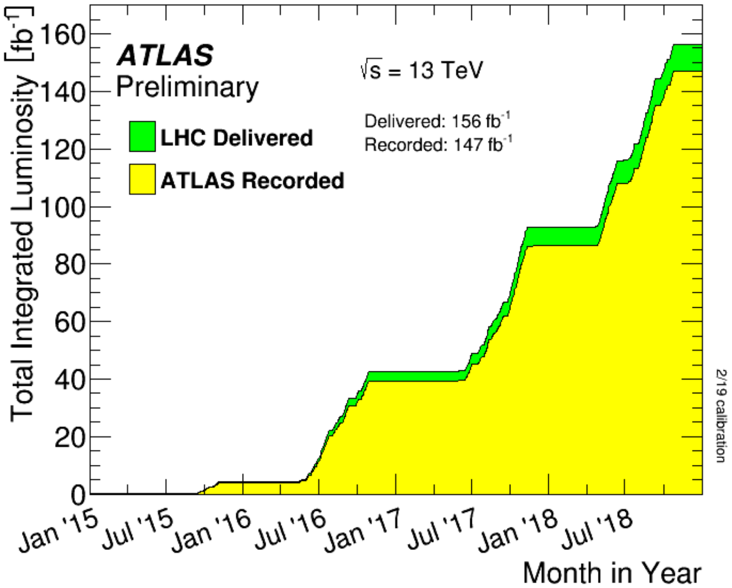


Recall expected events = cross-section x luminosity
Total ‘inelastic’ cross-section of pp collisions @ the LHC is measured in microbarn
‘Typical’ BSM cross-sections in picobarn or femtobarn

Search for BSM @ the LHC

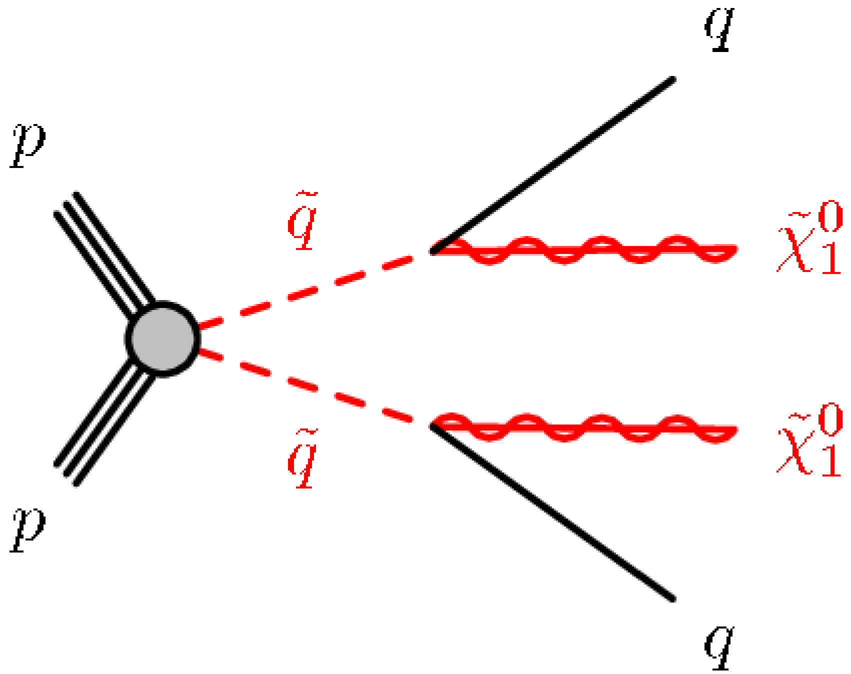


Big jump from Tevatron to LHC run 1 in energy: limits on heavy particles with small backgrounds



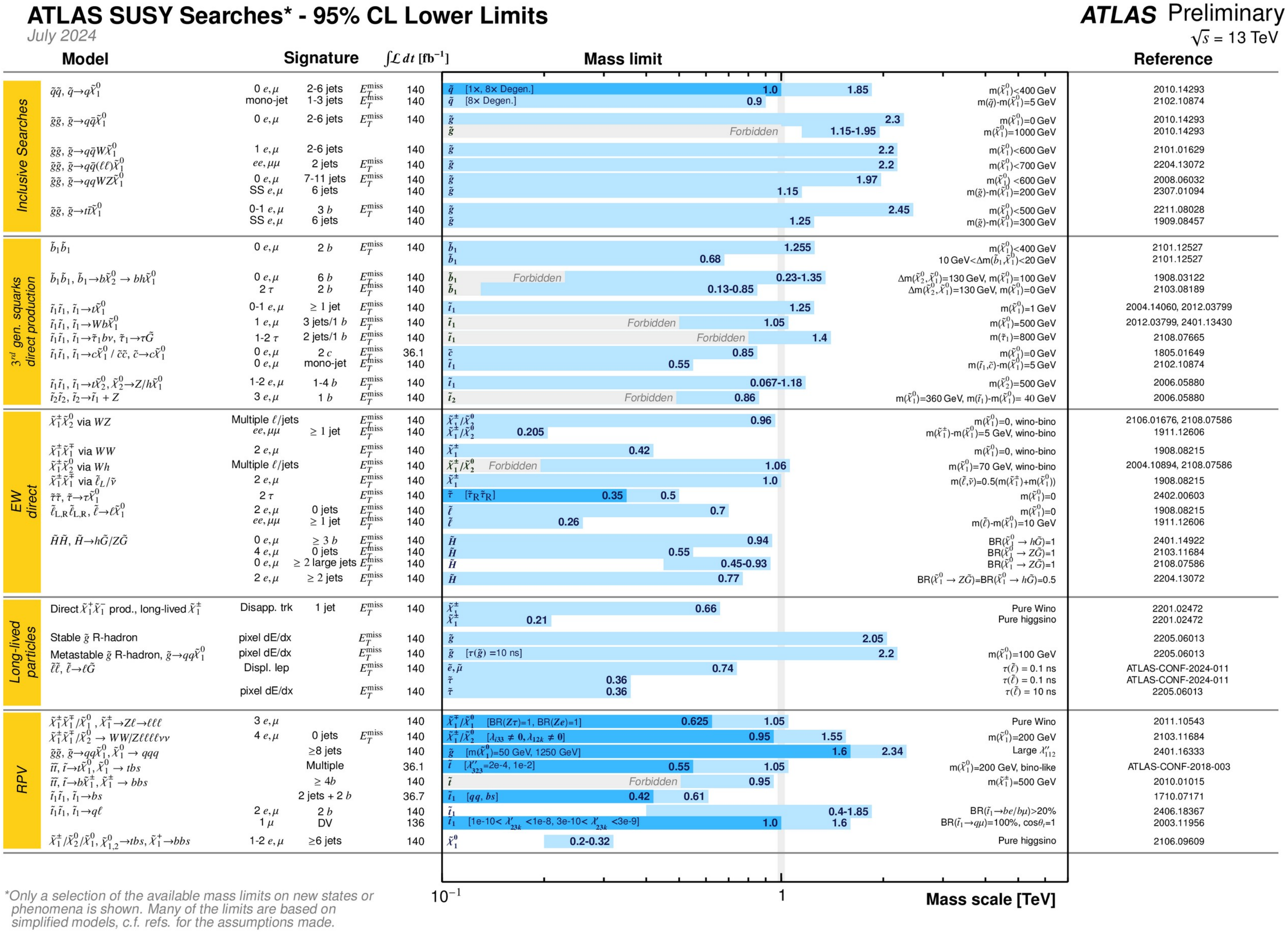
Run 2 gave another jump in energy, and a big jump in luminosity: can look for ‘sore thumbs’ and also some processes with larger backgrounds

A classic ‘sore thumb’ is a heavy squark pair decaying to quarks + light invisible particles:



The squarks are strongly coupled, so have a large cross-section even if they are heavy, and their decays produce lots of missing energy

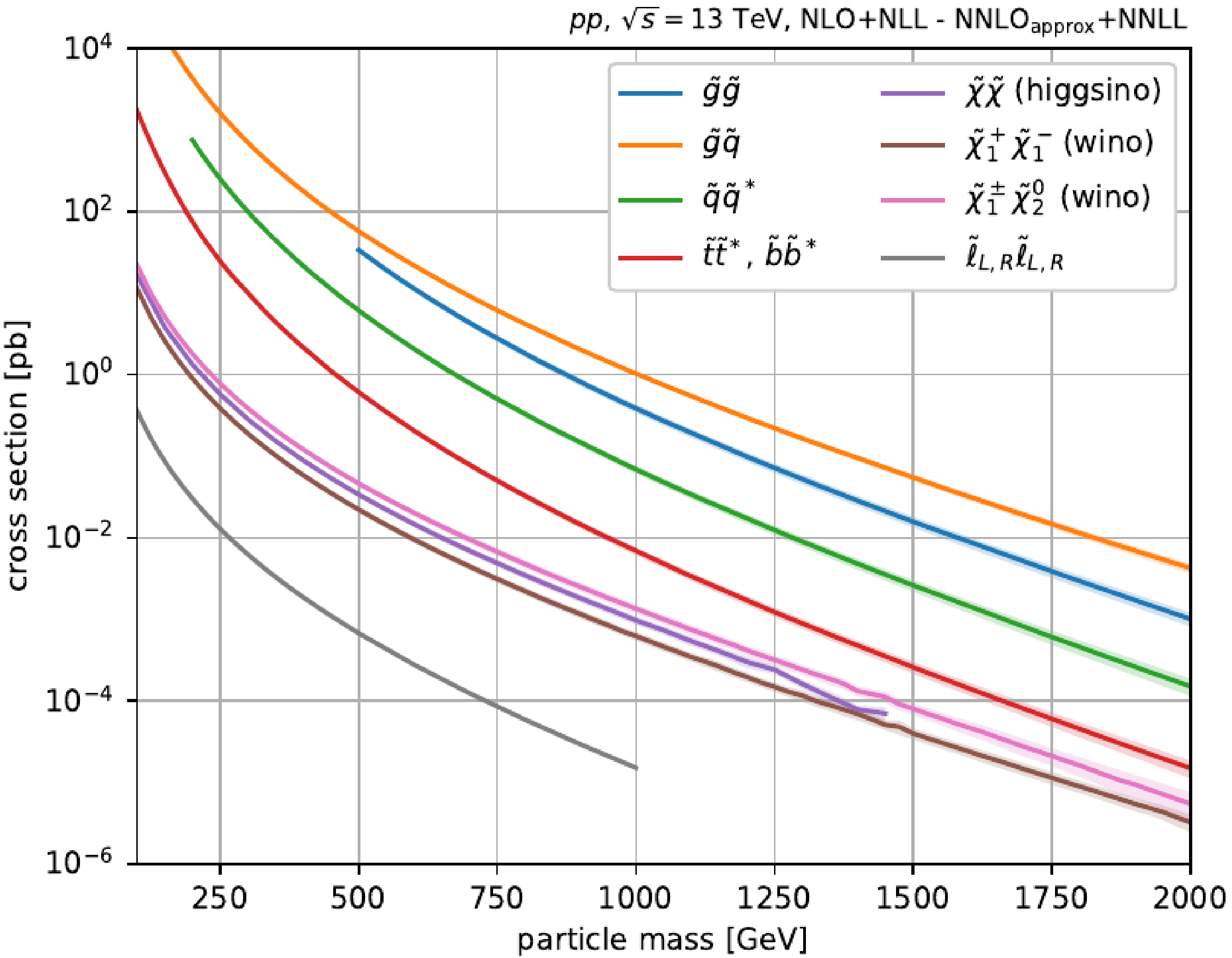
Here are some ATLAS headline numbers



For run 2 we have 140 inverse fb, which means a process with cross-section of about 10^{-5} pb would have produced one event (in a poisson distribution).

E.g. for SUSY particles:

Electroweak-coupled particles would still be produced at the LHC, but to have large cross-sections they must be lighter – and then the backgrounds are much bigger:



EW-charged particles are where we might hope to find new physics during your PhDs

The future of collider experiments

- Current European plans are to build an ee collider to follow the LHC …
- … and a pp collider to follow that.
- The strategy to adopt (what we, as a community, will ask for) is being discussed as part of the EPPSU process, but this seems largely fixed now.

- An **International Linear Collider** (ILC) proposed in Japan is unlikely to be built.
- The US community is putting its weight behind a proposed **muon collider.**
- There were hopes that China would build a lepton collider (**CEPC),** but this has been put on hold (probably because of FCC.)

FCC IN A NUTSHELL

Timeline

- **2025**: Release of the FCC Feasibility Study report
- **2028**: Decision by CERN Member States and international partners

Tunnel

- **90.7 km** circumference
- **180 – 400 m** depths for access shafts
- **8 surface sites** (7 in France, 1 in Switzerland)

Two stages

- **FCC-ee** (precision measurements) about 15 years from the **late 2040s**
- **FCC-hh** (high energy) about 25 years from the **2070s**

Costs/benefits

- **15 billion CHF**, spread over about **12 years** for FCC-ee with four experiments
- **Positive** socio-economic benefit–cost ratio
- About **800 000** person-years of employment created

Let’s get to work

Basics of collision physics

Rate of an interaction is given by luminosity x cross-section

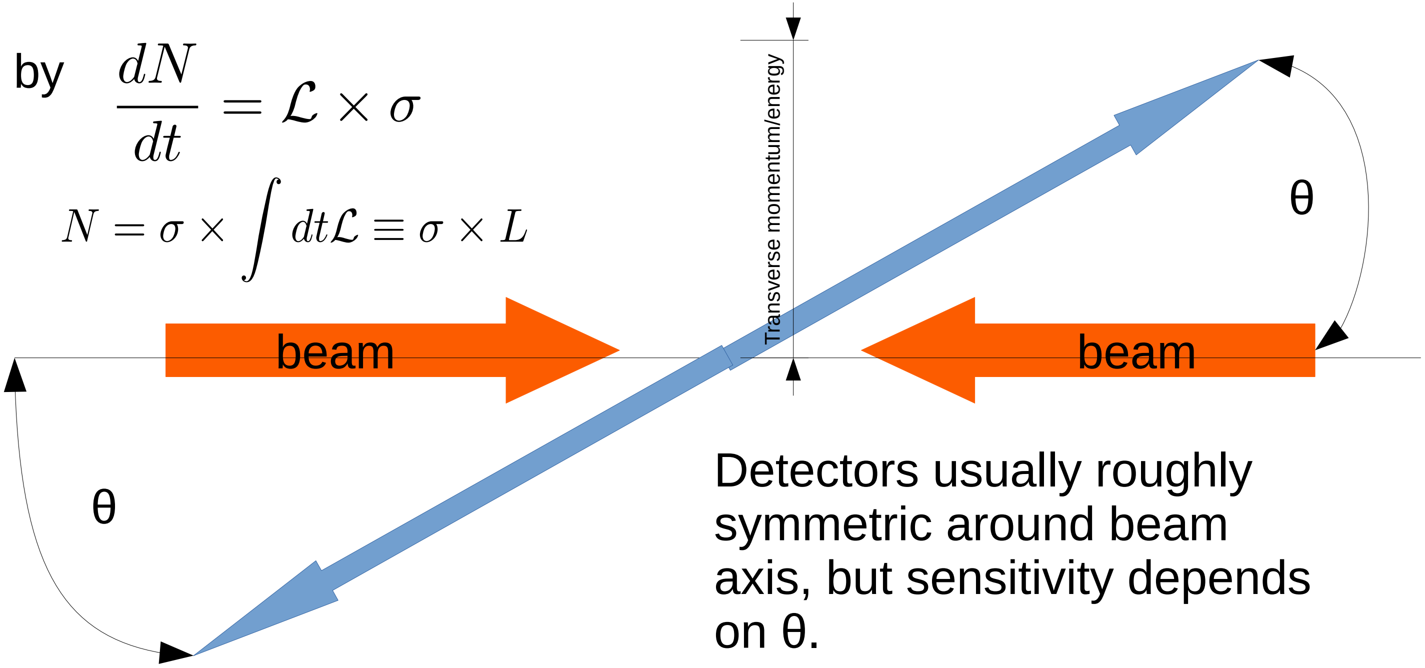
$$\frac{dN}{dt} = \mathcal{L} \times \sigma$$

For given data-taking period have an 'integrated luminosity'

$$N = \sigma \times \int dt \mathcal{L} \equiv \sigma \times L$$

Differential information is usually very useful, since most scattering will be ~semi-elastic at small angles. And since detectors are not always 4π

$$\text{e.g.} \ \frac{d\sigma}{d\Omega}, \frac{d\sigma}{dEd\Omega}$$



Detectors usually roughly symmetric around beam axis, but sensitivity depends on θ.

Variables

Experimentalists define a lot of variables that capture some physics. Also useful nowadays for feeding into ML.

Preference for boost-covariance:

- Transverse momentum
- Transverse energy
- Missing transverse momentum
- Rapidity
- Pseudorapidity
- Angular "distance" between objects

$$\mathbf{p}_T \equiv (p_x, p_y, 0),\ \ p_T \equiv \sqrt{p_x^2 + p_y^2}$$

$$E_T \equiv E \sin\theta$$

$$E_T^{\mathrm{miss}} \equiv -\sum \mathbf{p}_T$$

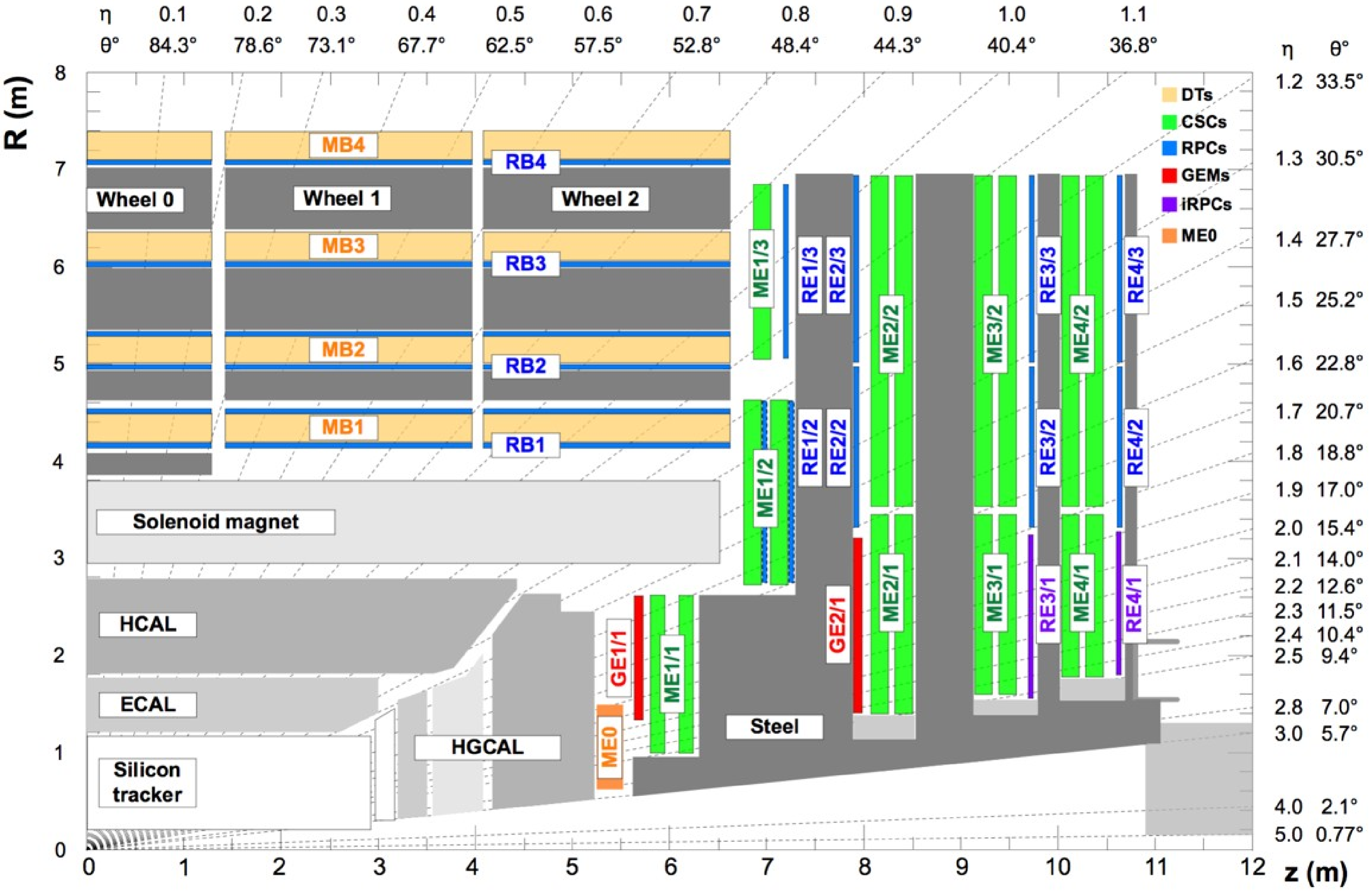
$$y = \frac{1}{2} \log \frac{E + p_z}{E - p_z}$$

$$\eta \equiv -\log \tan \frac{\theta}{2}$$

$$\Delta R \equiv \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}$$

To help get a feel for the η variable:

Typically can measure charged particles with |η| < 2.5, and hadrons with |η| < 5



Exercises

- Show that the rapidity transforms under boosts as

$$y \to y + \frac{1}{2} \log \frac{1-\beta}{1+\beta}$$

- Show that the pseudorapidity equals the rapidity for massless particles
- Heavy particles tend to have a low rapidity at the LHC. Why?
- Why should we care about boost invariance?
- Why might missing transverse energy be important?

Resonances

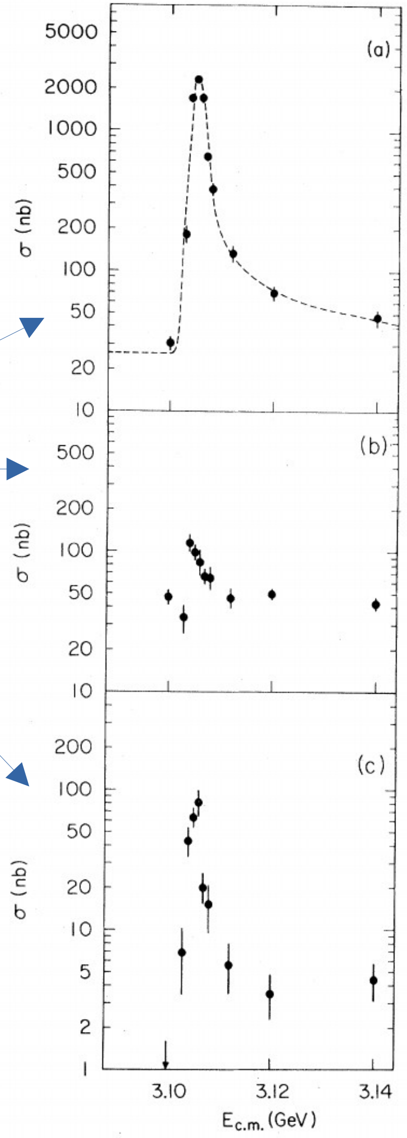
Particle discovery at colliders

Collider experiments produced discoveries of new long-lived particles detected by placing emulsion plates etc at distances perpendicular to the collision point.

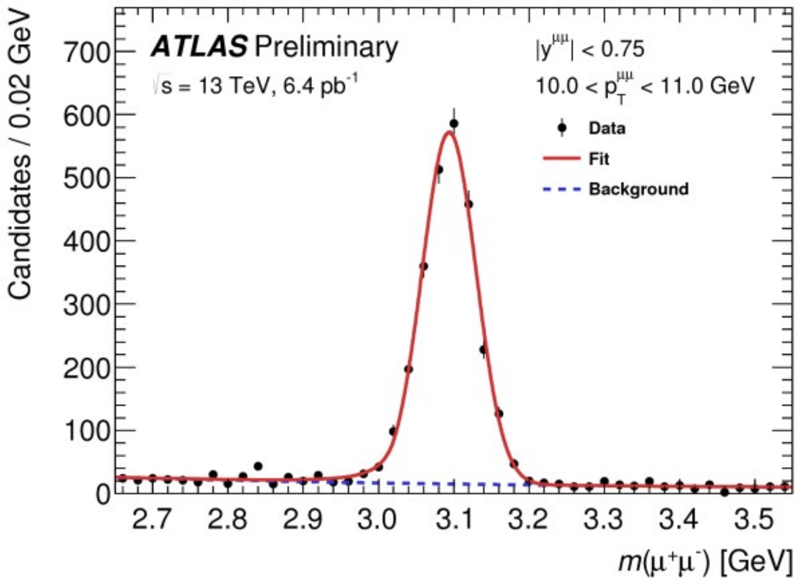
E.g. charmed mesons/baryons in the 70s with lifetimes of $\sim 10^{-13} s$

Most particles are too short-lived, however, and are discovered as ephemeral ‘resonances’

E.g. J/Psi discovery in 1974 (from SLAC) in e+e- → hadrons, electrons, muons+others



To nowadays: it is used for calibration at ATLAS



(Was simultaneously discovered in
p + Be → e+ e- + X collisions)

J/Ψ at BESIII

To bring this up to date, the BESIII experiment measured the J/Ψ width in 2022: arXiv:2206.13674

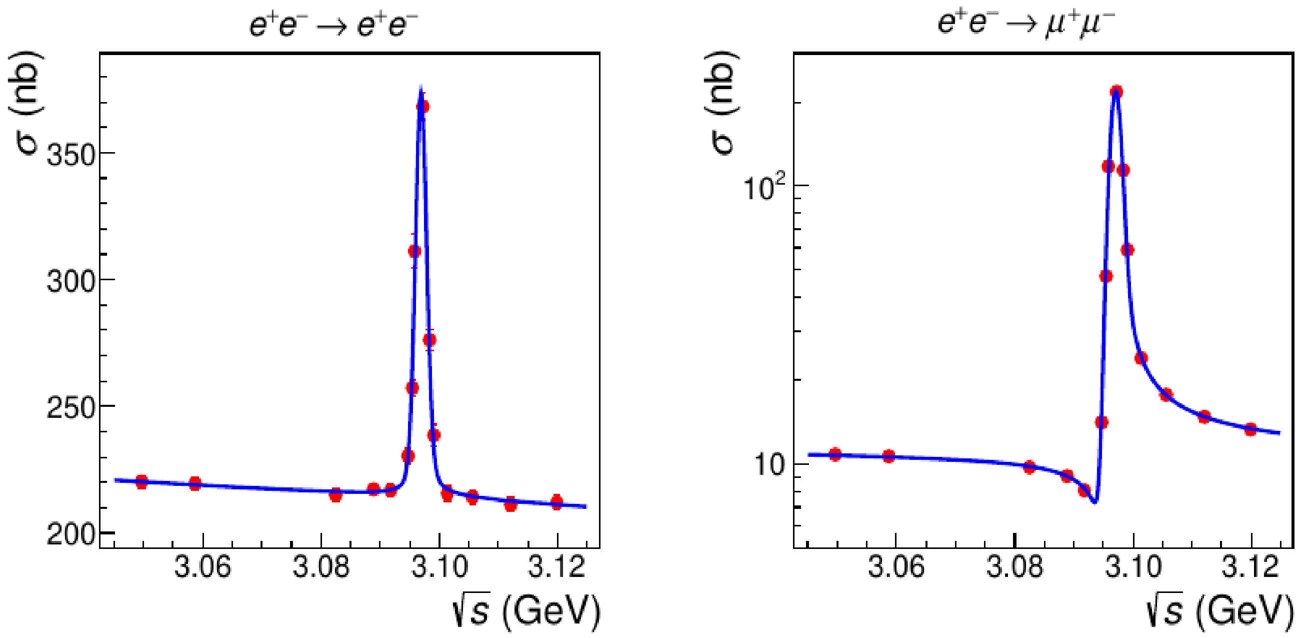


Fig. 3: Simultaneous fit to the measured, CM energy dependent cross sections of the processes $e^+e^- \to e^+e^-$ and $e^+e^- \to \mu^+\mu^-$. For clear display of the interference effect [denoted by σ_0^I in Eq. (3) and mainly illustrated by the small dip in front of the peak], the plot for the process $e^+e^- \to \mu^+\mu^-$ is drawn with a logarithmic vertical axis, while the plot for $e^+e^- \to e^+e^-$ is drawn with a linear vertical axis, because in this process the interference effect is less noticeable due to the existence of the scattering channel. In the plots, the red points with error bars and blue curves are from data and fitting, respectively.

Fermi golden rule from QM

We assume we can decompose the system in terms of energy eigenstates as

$$\Psi(t) = \sum_m a_m(t) e^{-iE_m t} \psi_m,$$

$$H_{mn} = \int \psi_m^* H \psi_n$$

Schroedinger eqn. →

$$i\dot{a}_m = \sum_{m \neq n} H_{nm} e^{-i(E_m - E_n)t} a_m.$$

If we start in a single state 0, then it decays $a_0(t) = e^{-t/2\tau} = e^{-\Gamma t/2}$ so that $|a_0|^2 = e^{-t/\tau}$

We can plug this in to get the population of a different state f at time t:

$$a_f = H_{f0} \frac{e^{-i(E_0 - E_f - i\Gamma/2)t} - 1}{E_0 - E_f - i\Gamma/2}$$

$$\xrightarrow[t\to\infty]{} \frac{H_{f0}}{E_f - E_0 + i\Gamma/2}.$$

The total probability for the particle to decay is therefore:

$$1 = \sum_{n \in f} |a_n|^2 = \int_{-\infty}^{\infty} dE \, \frac{|H_{f0}|^2}{(E - E_0)^2 + \Gamma^2/4} n_f(E)$$

where n_f is the density of states for a given E. This function is strongly peaked around E_0,

$$\begin{aligned} I = \int_{-\infty}^{\infty} f(s) \frac{1}{(s-m)^2 + \Gamma^2/4} &\approx 2f(m) \int_0^{\infty} \frac{1}{s^2 + \Gamma^2/4} \\ =& 2f(m)(2/\Gamma) \int_0^{\pi/2} du, \qquad s = \Gamma/2 \tan u \\ =& 2\pi f(m)/\Gamma \end{aligned}$$

and this leads to Fermi's golden rule:

$$\Gamma = 2\pi |H_{f0}|^2 n_i(E_0).$$

Breit Wigner from QM

Now imagine a resonance being formed as an unstable intermediate state:

$$a + b \to X^* \to f.$$

We split the states into 'incoming' and 'outgoing' and imagine a steady process where our incoming states 'i' produces state '0' which then decays to 'f':

$$i\dot{a}_0 = \sum_{m\in\{i\}} H_{0m} e^{-i(E_m - E_0)t} a_m(t) + \sum_{m\in\{f\}} H_{0m} e^{-i(E_m - E_0)t} a_m(t)$$
$$= -\, i(\Gamma/2)a_0 + \sum_{m\in\{i\}} H_{0m} e^{-i(E_m - E_0)t} a_m(t).$$

integrate

$$a_0(t) = \frac{H_{01} e^{-i(E_1 - E_0)t}(1 - e^{-\Gamma t/2})}{E_1 - E_0 + i\Gamma/2} \to |a_0(t)|^2 \underset{t\to\infty}{=} \frac{|H_{01}|^2}{(E_1 - E_0)^2 + \Gamma^2/4}.$$

But since we are in a steady state, we must have production of '0' is equal to its rate of decay:

$$\frac{dN}{dt} = -\Gamma N$$

$$\text{production of state } 0 = \Gamma |a_0|^2$$

$$= \frac{|H_{01}|^2 \Gamma}{(E_1 - E_0)^2 + \Gamma^2/4}$$

$$\text{production of state } 0 = \sigma(i \to 0) \times \text{particle flux.}$$

$$\Gamma \times |a_0|^2 = \sigma(i \to 0) \times \frac{v}{V},$$

Now we eliminate the amplitude using Fermi's golden rule for the decay, and end up with

$$\sigma(i \to X^*) = \frac{\pi}{k^2} \frac{\Gamma \Gamma_i}{(E_1 - E_0)^2 + \Gamma^2/4}$$

If we sum over the spins of our state ‘0’ and average over the spins of our initial particles we get the **Breit-Wigner formula:**

$$\sigma(i \to X^* \to f) = \frac{2s_X + 1}{(2s_1+1)(2s_2+1)} \frac{\pi}{k^2} \frac{\Gamma_i \Gamma_f}{(E_1 - E_0)^2 + \Gamma^2/4}$$

This is really for the case of a light particle hitting a heavy one: for the case of two moving particles we need a factor of 2 from the relative speed.

For elastic scattering at the peak (when the initial and final widths equal the total width) we get:

$$\sigma \simeq \frac{4\pi}{p_{\mathrm{cm}}^2} \frac{2s_X + 1}{(2s_1+1)(2s_2+1)}$$

We can use this to measure the spin!

Exercise

Here we have (positively) charged pions scattering on protons

Take the peak to be at 190 MeV of around 190 mb

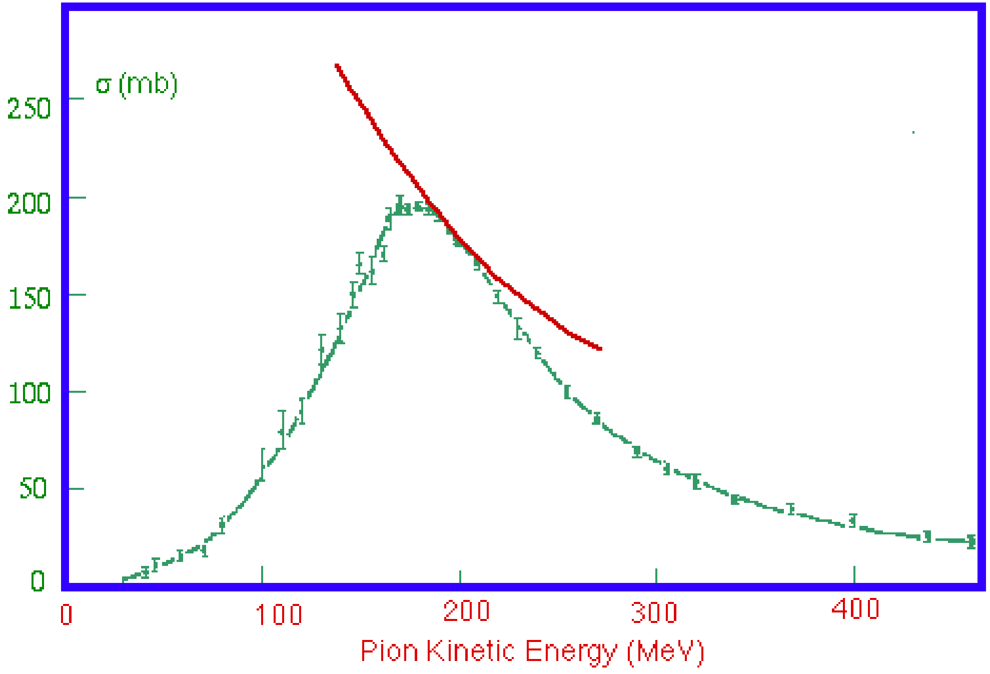
$$m_\pi = 139 \text{ MeV}, m_p = 938 \text{ MeV}$$

Show that the mass of the resonance is 1232 MeV

Then calculate its spin!

$$\hbar c = 0.1973269602(77) \text{GeV fm}$$

$$eV = 1.6021766209(98) \times 10^{-19} J$$



$$E_k \equiv E_\pi - m_\pi$$

J/Ψ at KEDR

The KEDR experiment measured the J/Ψ production in leptonic channels in arXiv:1110.0328

(see exercises on board)

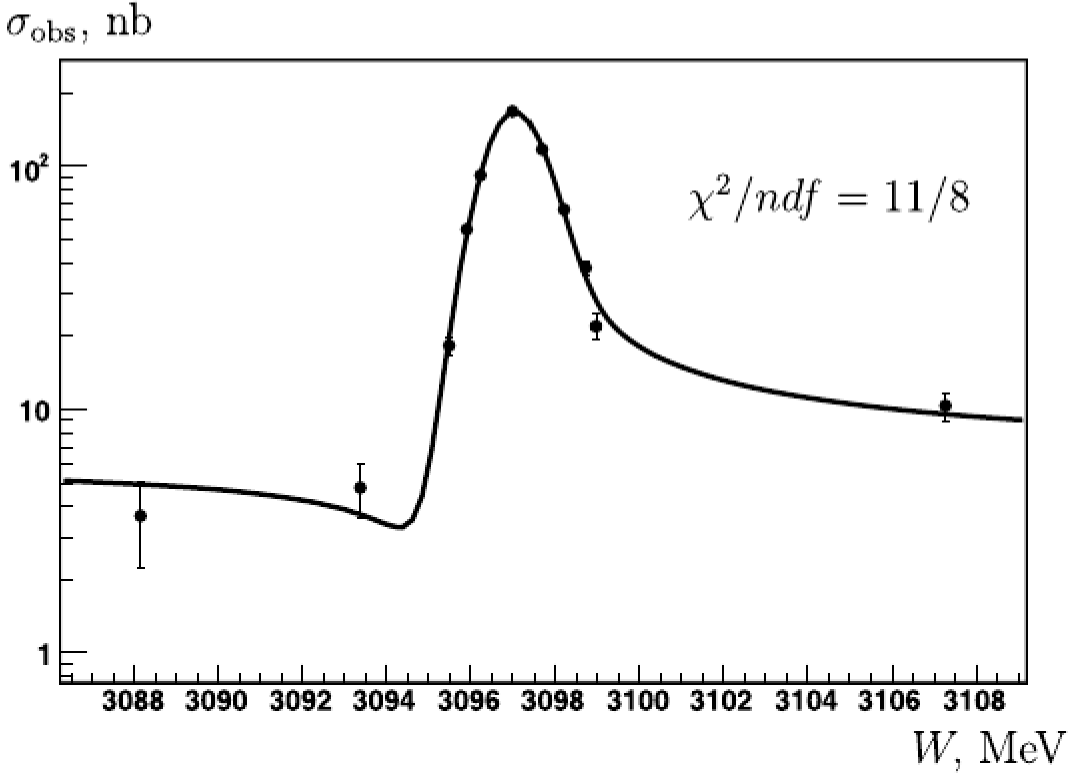


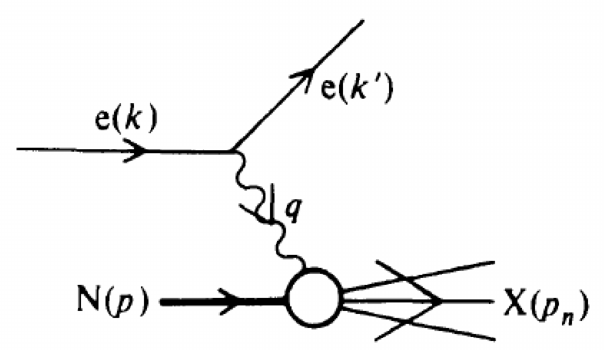
Fig. 5. Fit to experimental data for $e^+e^- \to \mu^+\mu^-$ process in the J/ψ energy range.

Collider processes

Important collider processes

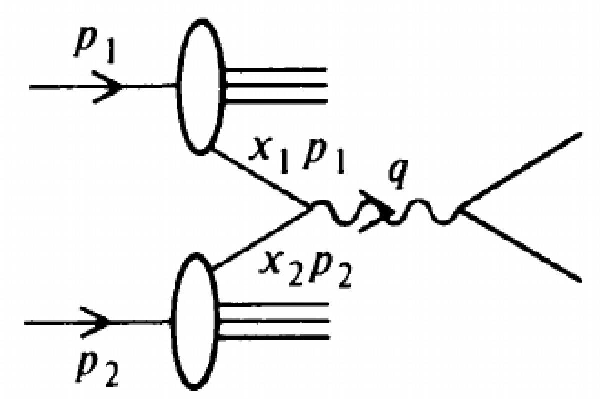
Historically very important

Deep inelastic scattering

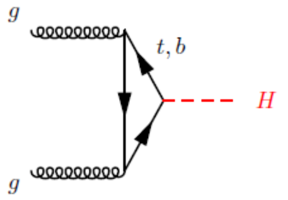
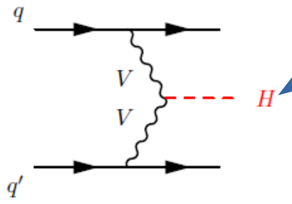
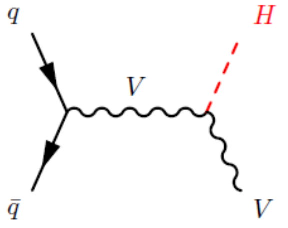
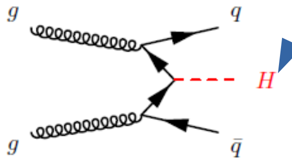


Will need a lot of QFT to understand these!

Drell-Yan



Important collider processes 2: discovery of the Higgs (and beyond?)

Production mode	LO diagram
ggF production	
VBF production	
VH production	
$q\bar{q}H$ production	

Can understand something already with QM!

These involve a resonance

Very much QFT here