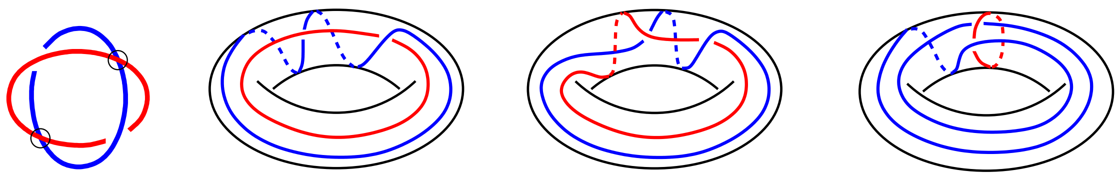
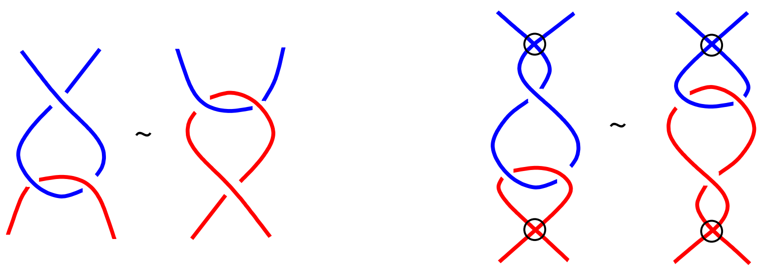
Matrix Integrals and the Generation and Counting of Virtual Tangles and Links

Integrals over matrices of size N are known to have a large N expansion in terms of Feynman graphs drawn on surfaces of increasing genus. As such, they may yield generating functions of objects of a given topology. This is for example the case of knots, links and tangles, which may be represented in projection as planar graphs with under- and over-crossings. The subclass of *alternating* links and tangles, for which one encounters alternatingly under- and over-crossings as one circulates around each thread, had been previously shown to be enumerated by the large N limit of the integral $\int D(M, M^\dagger) \exp -N\mathrm{tr}(MM^\dagger + \frac{g}{2}(MM^\dagger)^2)$ over $N \times N$ complex matrices. Counting topologically independent objects requires to quotient by the "flypes", a special variety of moves which preserve the alternating character of the links and tangles. In that way, results previously obtained by other combinatorial techniques have been reproduced.

In a second step, the subdominant terms in the large N limit have been shown to count "virtual" alternating links and tangles. Virtual links are generalizations of classical links that can be represented as embedded in a "thickened" surface $\Sigma \times I$, product of a Riemann surface of genus h with an interval. The generating functions of virtual links up to 10 crossings and up to genus 5 have been computed, under the assumption that it is necessary and sufficient to quotient by the action of the planar "flypes". This assumption has been tested by computing several independent invariants, but it remains an open conjecture.



A genus 1 virtual link in Kaufmann's notation, with virtual crossings in addition to under- and overcrossings; the same link as a Feynman diagram wrapped around a genus 1 surface, in three equivalent representations.



The flype of an ordinary and of a virtual tangle.

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