QCD PHENOMENOLOGY

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Abstract
The status of QCD phenomena and open problems are reviewed

Foreword

The four lectures on "QCD Phenomenology" at the $\Sigma\epsilon\rho\nu$–$\Delta o\upsilon\mu\nu\alpha$ school, delivered in the dungeon hall of the magnificent Pylos castle were, naturally, labelled by Greek letters and dealt with

- α – the basics of QCD and its main problems,
- β – the running coupling and anatomy of the Asymptotic Freedom,
- γ – QCD partons and the rôle of colour in multiple hadroproduction and
- δ – non-perturbative corrections to QCD observables.

In the written version of the lectures I have chosen to concentrate on the *qualitative* discussion of selected QCD phenomena rather than teach you the basic perturbative QCD techniques[1]. The selection criterion was as follows. I have picked the topics that I find puzzling and/or whose importance I feel have not attracted as much attention as they rightfully deserved.

1. INTRODUCTION. ON GUESSWORKS.

The spirits were kind enough to say:
"*We have no respect of cullours*" [2]

In the late 1970s one could say "QED was 30 years *old*". In 2003 we cannot but state that "QCD is 30 years *young*". Dating great discoveries is a delicate business, though. "The spirit" in the above Charlotte Fell-Smith's narrative actually was Uriel[2], the one of four Archangels responsible for fundamental science (and physics in particular) [3]. Thus the idea of **colour invariance**, as communicated by Arch. Uriel to Dr. John Dee, may be dated back 420 years[3]! In 1582 Dr. Dee pens down Uriel's detailed instructions on arranging the conjuring table:

"*The sylk must be of diuers cullors: the most changeable that can be got*{ten}" [4].

The idea of colour symmetry could not be put forward more clearly: "*The most changeable, diuers* [diverse] *cullors*" to which quantum number "*we* [angels] *have no respect.*" However Dee here adds a confusing marginal remark,

"*The cullor was shewed red and greene interchangeably,*" Nouemb. 21. Ao 1582 [4].

This suggests it may have been $SU(2)$ rather than $SU(3)$ that Uriel was trying to deliver (unless Dee was colour blind to the blue part of the spectrum, of which we have no documented evidence). Tiny details

[1] A systematic introduction into the physics of colour, gluon radiation, parton multiplication etc. can be found in the Proceedings of another CERN–Dubna school [1].

[2] Aleph, vau, resh, yod, aleph, lamed – "Fire of God", the middle pillar of the Tree of Life and supervisor of Nature Spirits.

[3] An apparent contradiction with another angelic message, "*Note the forme of the thing seen. Note the cullour*" [4], is resolved by accepting that Dr. Dee misunderstood the angel: Uriel meant "***Not** the colour*."

aside, the key idea of the local non-Abelian symmetry had been clearly present in the angelic message (*no respect of cullours*).

Now let us leave Dr. John Dee for the time being and stress that the physics of hadrons always was, and still is, providing puzzles and inspiration. If 30–40 years ago quantum field theory (QFT) had been kept in higher respect (which it was not), some general phenomenological features of hadron interactions that were known then could have already hinted at QCD as an underlying microscopic theory of hadrons.

1.1 Hints from the past

- The fact that in high energy hadron interaction processes *inelastic* breakup typically dominates over elastic scattering hinted at proton being a loosely bound compound object:

  $\Longrightarrow$ *Constituent Quarks*

- Constancy of transverse momenta of produced hadrons, rare appearance of large-$k_\perp$ fluctuations, was signaling the weakness of interaction at small relative distances:

  $\Longrightarrow$ *Asymptotic Freedom*

- The total hadron interaction cross sections turned out to be practically *constant* with energy. *If* we were to employ the standard quantum field theory (QFT) picture of a particle exchange between interacting objects,

  $$\sigma_{\rm tot} \propto s^{J-1} \simeq \text{const},$$

  then this called for a spin-one elementary field, $J = 1$, to be present in the theory.
  Uniformity in rapidity of the distribution of produced hadrons (Feynman plateau) pointed in the same direction, *if*, once again, we were willing to link final particle production to accompanying QFT radiation.

  $\Longrightarrow$ *Vector Gluons*.

Nowadays the dossier of puzzles & hints that the hadron phenomenology has accumulated is very impressive. It includes a broad spectrum of issues ranging from unexplained regularities in hadron spectroscopy to soft "forceless" hadroproduction in hard processes. To locate and formulate a puzzle, to digest a hint, – these are the road-signs to the hadron chromodynamics construction site. We are learning to listen. And to hear.

1.2 That nasty confinement

The reason why one keeps talking, 30 years later, about *puzzles and hints*, about *constructing* QCD rather than *applying* it, lies in the conceptually new problem one faces when dealing with a non-Abelian theory with unbroken symmetry (like QCD). We have to understand how to master QFTs whose dynamics is intrinsically unstable in the infrared domain: the objects belonging to the physical spectrum of the theory (supposedly, colorless hadrons, in the QCD context) have no direct one-to-one correspondence with the fundamental fields the microscopic Lagrangian of the theory is made of (colored quarks and gluons).

In these circumstances we don't even know how to formulate at the level of the microscopic fields the fundamental properties of the theory, such as conservation of probability (unitarity) and analyticity (causality):

- What does **Unitarity** imply for confined objects?
- How does **Causality** restrict quark and gluon Green functions and their interaction amplitudes?
- What does the **Mass** of an INFO – [well] Identified [but] Non-Flying Object – mean?

The issue of quark masses is especially damaging since a mismatch between quark and hadron thresholds significantly affects predicting the yield of heavy-flavored hadrons in hadron collisions.

Understanding the confinement of colour remains an open problem. Given the present state of ignorance, one has no better way but to circle along the *Guess-Calculate-Compare* loop. There are, however, *guesses* and *guesses*.

1.3 Circling the G-C-C loop

Perturbative QCD (pQCD) is believed to govern the realm of "hard processes" in which a large momentum transfer Q^2, either time-like $Q^2 \gg 1$ GeV^2 (jets), or space-like $Q^2 \ll -1$ GeV^2 (structure functions), is applied to hadrons. pQCD controls the relevant cross sections and, to a lesser extent, the structure of final states produced in hard interactions. Whatever the hardness of the process, it is hadrons, not quarks and gluons, that hit the detectors. For this reason alone, the applicability of the pQCD approach, even to hard processes, is far from being obvious. One has to rely on plausible arguments (completeness, duality) and look for observables that are *less vulnerable* towards our ignorance about confinement.

Speaking of substituting *good guesses* for *ignorance* the following ladder emerges.

Total cross sections.

The safest bet of all is the idea of *Completeness* applied to a handful of observables that enjoy the status of "totally inclusive cross sections". Completeness of colour states may be looked upon as a *good **direct** guess*. The examples are

$$\sigma_{\rm tot}(e^+e^- \to \text{hadrons}), \quad \Gamma(\tau \to \nu_\tau + \text{hadrons}).$$

Here one replaces the probability of production of *hadrons* by a colorless current j (virtual photon, Z^0 or $W^\pm$) by that of a $q\bar{q}$ pair,

$$W(j \to \text{hadrons}) \;=\; W(j \to q\bar{q}) \otimes \mathbf{1}, \tag{1}$$

and argues that the *total* probability of the conversion of quarks into hadrons cannot be anything but **1**. This sounds fine if the momentum transfer to the hadron system, Q^2, well exceeds the hadron mass scale $\mathcal{O}\left(1\text{GeV}^2\right)$. The guess becomes less *direct* when the momentum transfer gets smaller and the final state hadronic system starts to "resonate". In particular, in the case of the τ lepton decay width where $Q^2 < m_\tau^2 \simeq (1.8\,\text{GeV})^2$ a point-by-point correspondence between the left and right hand sides of (1) is lost and some "smearing" over the invariant mass of the hadron system should be applied. There is a smart way to do this, by referring to the *analyticity* in Q^2 of the correlator of the currents, $\langle j\, j\rangle$, which follows from *causality*. By treading this path one arrives at an amazingly tight control over potentially disturbing non-perturbative effects, which makes the τ decay a legitimate source of the $\alpha_{\rm s}$ measurement at pretty small scales (for details see [5]).

DIS structure functions.

These are not "totally inclusive cross sections", as far as hadrons are concerned, simply because there is a definite hadron in the *initial* state. We are not clever enough to deduce from first principles the parton distributions inside a target hadron (PDF, or structure functions). However, the rate of their $\ln Q^2$-dependence (scaling violation) is an example of a Collinear-and-Infrared-Safe (CIS) measure and stays under pQCD jurisdiction. Here one applies a similar logic and appeals to analyticity of the virtual boson–proton scattering amplitude to translate the Bjorken-x *moments* of the inclusive Minkowskian DIS cross section (structure functions) into Euclidean space, the Operator Product Expansion (OPE) being the name of the game (a *good **indirect** guess*).

Recall that in the Bjorken limit (x = const, $|Q^2| \to \infty$, that is neglecting corrections in powers of $1/Q^2$) one can describe the pattern of the logarithmic deviations from the exact Bjorken scaling in terms of probabilistic QCD improved parton picture with cascading quarks and gluons replacing point-like partons of the original Bjorken-Feynman parton picture.

Final state; Inclusive.

The next step down our squeaking ladder of ignorance – and we arrive at inclusive characteristics of hadronic final states produced in hard processes. Oops! Here our guesses cannot be labeled other than ***wild***. There is no *a priory* reason for distributions of final hadrons to bear much resemblance to those of underlying partons[4]. As we shall discuss below, both the energy and angular distributions of hadrons *do* follow partonic ones. This fact is well established phenomenologically (for a review see [6]). It does not follow from "first principles", but rather tells us about confinement (as providing soft, local in the configuration space hadronization of partons) supporting the original wild guess known under the name of LPHD (Local Parton–Hadron Duality) hypothesis [7].

It is important to mention that the probabilistic parton evolution picture (the source of inspiration for Monte Carlo event generators) is as *approximate* as it is *limited*. Strictly speaking, it had been validated for DIS SFs [8, 9] (and single-particle inclusive distributions in e^+e^- – fragmentation functions [9]). No less but no more. Aiming at more than that with MC tools is, strictly speaking, illegitimate. However this does not mean that a probabilistic treatment cannot be somewhat extended beyond its original limits.

A famous example to the contrary is given by the so-called Angular Ordering (AO) story of early 1980s. Alfred Mueller and Victor Fadin found that quantum mechanical interferences affect soft gluon cascades in e^+e^- jets and invalidate the classical (= probabilistic) DGLAP evolution picture [8, 9]. At the same time they showed that the interference effects could be taken full care of by simply restricting gluon multiplication into successively shrinking angular regions, $\Theta_{i+1} \ll \Theta_i$, with i the parent parton and $i+1$ its softer offspring, $\omega_{i+1} \ll \omega_i$ [10]. Surprizingly, the AO was later found to work beyond the leading strong ordering approximation (Double Logarithmic Approximation of strongly ordered energies *and* angles, DLA). The most natural specification of the AO prescription, $\Theta_{i+1} \leq \Theta_i$, was shown to properly embed the next-to-leading (single logarithmic, SL) corrections [7]. This *Exact Angular Ordering* rule (in place of the DL *Strong* one) does the job and restores the probabilistic evolutionary picture for *energy* spectra of (soft) particles in jets. As far as *angular* distributions are concerned, it works, however, only *on average*. It cannot be applied to *angular correlations*. In particular, quantum-mechanical coherence plays a crucial rôle in predicting inter-jet particle flows in multi-jet events. This is the domain of the so-called string/drag phenomena, of collective radiation effects – QCD radiophysics.

Final state; Correlations.

Multi-particle correlations are obviously far more vulnerable. Even having learned and accepted the God[5]-given LPHD in single-particle distributions (inclusive particle flows), we feel at sea when correlations come onto stage. There may be some good news coming from pQCD approaches to KNO, intermittency phenomena and alike [11], but head-on perturbative attacks on correlations fail more often than not.

1.4 Substituting a good guess for ignorance

George Sterman and Steven Weinberg suggested to look for *Collinear-and-InfraRed-Safe* (CIS) observables, those which can be calculated in terms of quarks and gluons without encountering either collinear (zero-mass quark, gluon) or soft (gluon) divergences. They proclaimed such observables to be "more equal", free of large distance – confinement – effects and encouraged us to directly compare corresponding $\text{PT}^{\{1\}}$ predictions with hadronic measurements [12]. This guess ranks higher than *hypothesis*: it is rather an *ideology*.

The Sterman–Weinberg ideology gave rise to well elaborated procedures for counting jets (CIS jet finding algorithms) and for quantifying the internal structure of jets (CIS jet shape variables). They allow

[4] modulo perturbatively controlled Q^2-dependence of the Feynman-x moments of fragmentation functions, see below.

[5] Uriel?

the study of the gross features of the final states while staying away from the physics of hadronization. Along these lines one visualizes asymptotic freedom, checks out gluon spin and colour, predicts and verifies scaling violation pattern in hard cross sections, etc. These and similar checks have constituted the basic QCD tests of the past two decades.

This epoch is over. Now the HEP physics community aims at probing genuine confinement effects in hard processes to learn more about strong interactions. The programme is ambitious and provocative. Friendly phenomenology keeps it afloat and feeds our hopes of extracting valuable information about physics of hadronization. The quest is not easy, we are bound to make mistakes and are trying to avoid errors.

1.5 On mistakes vs. errors

mistake: smth. done wrongly, or
smth. that should not have been done.
Longman Dictionary of Contemporary English, 1987.

The original calculation of the electron loop which participates in the polarisation of QED vacuum and makes the coupling run with virtuality, $\alpha(k^2)$, produced a wrong sign (in modern words, a QCD-ish β-function). According to the Longman Dictionary this was an *error*, though not a *mistake* since it was worth making!

It took a while before young colleagues[6] pointed out to the maître that he erred. However the time span proved to be enough for Lev Landau to develop and enthusiastically discuss with Isaak Pomeranchuk and their pupils a beautiful physical picture of what is now known to us under the name of "asymptotic freedom". The seminal paper "*On the quantum theory of fields*" followed [14]. It has the sign right; the *error* had been corrected[7].

In "*Fundamental Problems*" [15] – a homage to Wolfgang Pauli – Landau discusses "'*nullification' of the theory*" which is "*tacitly accepted even by theoretical physicists who profess to dispute it.*" He remarks that "*the validity of Pomeranchuk's proofs has been doubted*." He considers the criticism but asserts that "*It therefore seems to me inappropriate to attempt an improvement in the rigor of Pomeranchuk's proofs, especially as the brevity of life does not allow us the luxury of spending time on problems which will lead to no new results.*"[8]

In the late 1950s the problem was known as "Moscow Zero": vanishing of the physical interaction (renormalized coupling) in the limit of a point-like bare interaction, $\Lambda_{\rm UV} \to \infty$. The depth of that crisis can be measured by the Dyson prophesy [16] that the correct "meson" theory – the theory of strong interactions – "*will not be found in the next hundred years*" and/or by the Landau conclusion [15] that "*the Hamiltonian method for strong interactions is dead and must be buried, although of course with deserved honour.*"

This was not an *error*. It was a *mistake*. But one well grounded. It was based on Pomeranchuk's extensive analysis of all then-known renormalizable theories – with scalar ($\lambda\phi^4$), Yukawa, four-fermion interactions [17]. In all these QFTs corresponding running couplings *increased* with momentum transfer $|k^2|$, slowly but catastrophically. Let us not forget the same behaviour of QED [14] and an unrealistic but pedagogically valuable Lee model [18]. No wonder, the situation looked desperate indeed.

We may guess that Landau and Pomeranchuk apparently understood too well that search for a "better" (asymptotically free) theory was unlikely to bear fruit. The pattern of the fall-off (*screening*) of the interaction at *large distances* (increase with momentum transfer) seemed too general to be

[6] B. Ioffe and A. Galanin.

[7] This paper, however, contains a *mistake* of a rather different nature. In a footnote we read: "*The quadratically divergent photon mass should be put equal zero.*" [So far so good. But then,] "*The presence of a finite photon mass would violate the charge conservation law.*" [Nope. Though their footnote does fully apply to the QCD *gluon* mass. What an irony!]

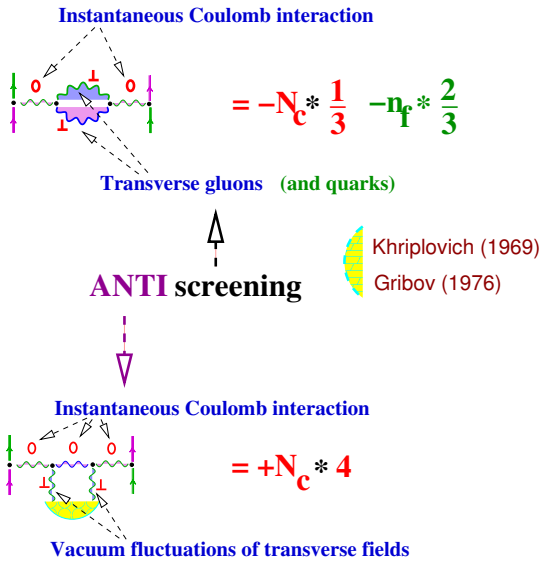
[8] This short but intensely wise paper turned out to be Landau's last.

passed by. Indeed, the vacuum polarisation loop corrections are analytic in k^2 (causality). Hence (by crossing-symmetry) the "zero-charge" sign of the β-function inevitably *follows* from positivity of the cross-channel pair production cross section being proportional (by unitarity) to the imaginary part of the loop amplitude.

Back in 1969 Yulik Khriplovich demonstrated that in a non-Abelian $SU(2)$ Yang–Mills gauge theory the coupling constant disrespects this argument [19]. Vladimir Gribov explained how it dares to do so without violating "first principles" [20].

Imagine a pair of static colour charges e.g., heavy quarks interacting via instantaneous Coulomb gluon exchange marked "0" on the adjacent picture.

In the next order in α_s there appears the standard vacuum polarisation correction due to gluon decays into "physical" quanta, either a $q\bar{q}$ pair or two transverse gluons ("$\perp$"). They both respect unitarity and give the same-sign contributions to the β-function as shown on the top part of the picture – *screen* the charge (as in QED and everywhere else).



In QCD this is not the end of the story however. There is another type of radiative corrections due to the fact that our Coulomb carrier propagates in the "external field" of vacuum fluctuations of *transverse* quanta. Coulomb gluons couple directly to transverse ones (whereas photons did not). The first non-vanishing contribution emerges in the second order in the coupling g_s (bottom part of the picture). It is large and has the opposite sign corresponding to *anti-screening*. The origin of the "opposite sign" is readily understood [20]: it is the same phenomenon that pushes *down* the ground state energy of a quantum-mechanical system in the second order in perturbation,

$$E_0' - E_0 = \sum_n \frac{|\langle 0|\delta V|n\rangle|^2}{E_0 - E_n} \; < \; 0.$$

1.6 Good guesses and bad guesses

Given our present awareness of the essential difference between mistakes and errors, an appeal for *bad guesses* (BG) would not sound provocative. In the first place, one needs BGs to have *good guesses* (GG) shining ever brighter. However to avoid slipping toward PR values we had better put forward a more serious argument: one learns by making bad guesses and confronting them with reality. QCD history is rich in BGs. Let us recall a couple of them.

A celebrated example of a BG is given by the initial parton model picture of how quarks hadronize. Feynman's original idea was that each parton converts into a bunch of hadrons (with limited transverse momenta and a uniform distribution in dx/x) – the Feynman *plateau* of rapidity length $y_{\max} \simeq \ln E$, with E the parton energy. This idea was realised in the very first (Field–Feynman) fragmentation model and was accommodated later by more advanced MC event generators like ISAJET, COJET. Such models today can be pronounced *dead* and to *be buried, although of course with deserved honour*. They lost the race to the so-called Lund string model which was based on the smart decision to take into proper consideration the *colour topology* of the underlying multi-parton system [21].

Bo Andersson, Gösta Gustafson and Carsten Peterson[9] chose to view a gluon radiated off a primary $q\bar{q}$ pair in $e^+e^- \to q\bar{q}$ as a system of a fake quark and antiquark, $q_f\bar{q}_f$. Then with good accuracy (modulo $1/N_c^2 \sim 10\%$ colour-suppressed correction) each of the two "pairs" $\bar{q}q_f$ and $\bar{q}_f q$ finds itself in

[9] Theory Department of the Lund University, Sweden. Hence the name of the model.

a *colour singlet* state. They suggested to treat each "pair" according to the Field–Feynman prescription (but in its proper cms!). This seemingly harmless modification had dramatic consequences. For one thing, the multiplicity of additional hadrons originating from emission of a hard gluon turned out to be a function of the gluon *transverse momentum* (with respect to the primary $q\bar{q}$) rather than its cms *energy*, $y_{\max} \simeq \ln k_\perp$. The crucial rôle of colour topology, both for multi-jet event multiplicities and for the pattern of particle flows between jets – the so-called string effect(s) [21], was later confirmed by purely perturbative QCD considerations [22].

Another prominent though less known example is provided by the story of the EEC (energy–energy correlation) measure in e^+e^- annihilation [23]. It was the first CIS observable to have been experimentally studied at e^+e^- accelerator PETRA in DESY (Hamburg). And with disastrous results. The "ideology of infrared stability" I was praising so above seemed to have failed. The discrepancy between the pQCD quark–gluon prediction and the measured hadron–hadron energy weighted inclusive correlation was found to be *substantial*. Worse than that, it turned out to be *stubborn* as it refused to go away with increase of the annihilation energy Q^2, defying ideology.

Now we understand what has happened. The EEC in the back-to-back kinematics turned out to be particularly strongly contaminated by non-perturbative effects [24].

2. PERTURBATIVE QCD AT WORK. WHY?

> *He* [Dr. Dee] *deprecates any kind of traffic with unauthorised or unreliable spirits, and acknowledges again the only Source of wisdom. But since he has so long and faithfully followed learning, he does think it of importance that he should know more. The blessed angels, for instance, could impart to him things of at least as much consequence as when the prophet told Saul, the son of Kish, where to find a lost ass or two!* [2]

In recent years pQCD has helped us to collect an impressive number of "*lost asses*" indeed. However one cannot help wondering why the pQCD treatment works so surprizingly well in some cases and fails miserably in others (often of a similar nature, residing on the same plank of our ladder of ignorance)?

It seems the messages are being sent. To grasp them we have to separate, scrutinize and try to classify the "good" and "bad" cases. But first we'd better agree on the vocabulary.

2.1 Words, words, words ...

Speaking of "perturbative QCD" can have two meanings.

- $\{1\}$ In a narrow, strict sense of the word, *perturbative approach* implies representing an answer for a (calculable) quantity in terms of series in a (small) expansion parameter $\alpha_s(Q)$, with Q the proper hardness scale of the problem under consideration.
- $\{2\}$ In a broad sense, *perturbative* means applying the language of quarks and gluons to a problem, be it of perturbative (short-distance, small-coupling) or even non-perturbative nature.

The former definition $\{1\}$ is doomed: the perturbative series so constructed are known to diverge. In QCD these are asymptotic series of a kind that cannot be "resummed" into an analytic function in a unique way. For a given calculable (collinear-& -infrared-safe; CIS) observable [12] the nature of this nasty divergence can be studied and quantified as an intrinsic uncertainty of pQCD series, in terms of so-called *infrared renormalons* [5]. Such uncertainties are non-analytic in the coupling constant and signal the presence of non-perturbative (large-distance) effects. For a CIS observable, non-perturbative physics enters at the level of power-suppressed corrections $\exp\{-c/\alpha_s(Q)\} \propto Q^{-p}$, with p an observable-dependent positive integer[10] number.

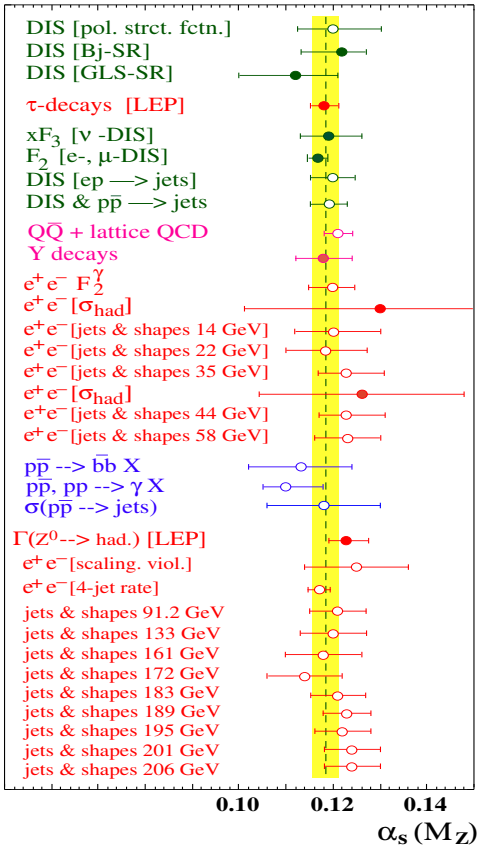
[10] usually, though not necessarily [24]

Meanwhile the broader definition $\{2\}$ of being "perturbative" is bound to be right. At least as long as we aim at eventually deriving the physics of hadrons from the quark-gluon QCD Lagrangian.

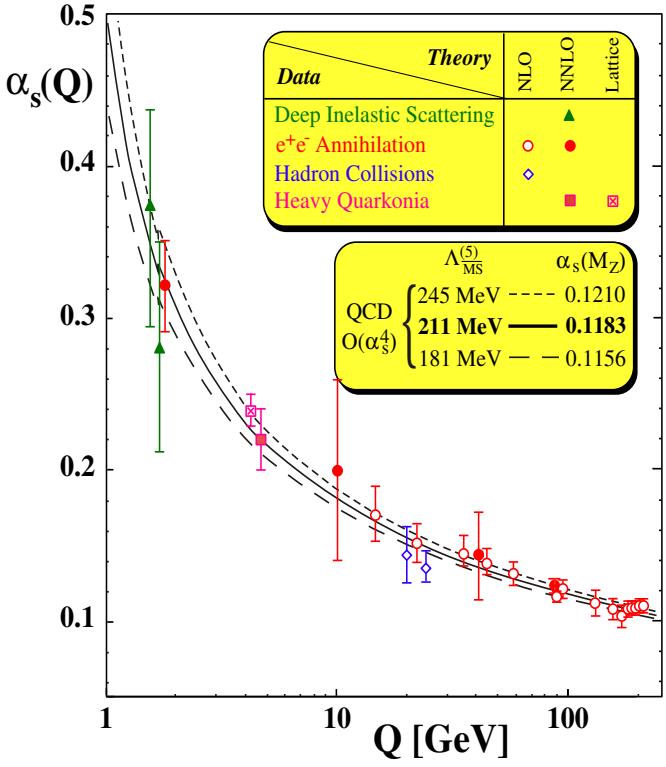
To distinguish between the two meanings, in what follows we will supply the word *perturbative* with a superscript $^{\{1\}}$ or $^{\{2\}}$. Thus, when discussing the strong interaction domain in terms of quarks and gluons in what follows we will be actually speaking about perturbatively$^{\{1\}}$ probing non-perturbative$^{\{1\}}$ perturbative$^{\{2\}}$ effects.

2.2 QCD coupling

Loke unto thy charge truely:
Thow art yet dead. Thow shallt be revyved. [4]



The pictures of the properly measured and properly running perturbative$^{\{1\}}$ QCD coupling [25] are soothing.



Experimenters (as well as theorists) shy away from looking below 1 GeV. And for a good reason too: how to discuss the strength of interaction between colored objects – quarks and gluons – that supposedly "don't exist" at "large" distances corresponding to $Q < 1\,\mathrm{GeV}$? We may eventually have to.

Recall what the Renormalization Group teaches us about the change of renormalized coupling $\alpha_s(\mu_R^2)$ with the renormalization scale μ_R. This teaching, however, is of limited value. The momentum variation of α_s is determined by the β-function,

$$\frac{d}{d\ln\mu_R}\left(\frac{\alpha_s(\mu_R^2)}{2\pi}\right)^{-1} = \beta(\alpha_s(\mu_R^2))\,, \qquad \beta(\alpha) \;=\; \beta_0 + \beta_1\frac{\alpha}{2\pi} + \ldots\,.$$

Beyond two loops the coefficients β_n with $n \geq 2$ turn out to be *scheme-* (and in some schemes even *gauge*)-dependent; in other words, arbitrary. Therefore, the large-momentum behaviour of the running coupling $\alpha_s(Q^2)$ cannot be uniquely fixed beyond two loops. The reason for that is pretty simple. Universality is inherited from the basic property of *ultraviolet renormalizability* of the theory, and it is only the first two loops that are truly dominated by the UV region, by small-distance physics.

Indeed, the one-loop radiative corrections contain the standard logarithmically divergent integral

$$\int \frac{d^4q}{q^4} \propto \ln \Lambda_{\rm UV} \;=\; \infty \quad \Longrightarrow \quad \beta_0 \,.$$

Hiding infinity under the carpet produces β_0, the first coefficient in the PT β-function expansion. In the next step we supply our loop with an additional internal gluon. Now we have two independent loop-momenta to integrate over, q_1 and q_2. Integration regions $q_{1(2)} \ll q_{2(1)} \ll \Lambda_{\rm UV}$ could have produced $(\ln \Lambda_{\rm UV})^2$ contributions. These get suppressed by renormalizing the *internal* propagators and vertices at the one-loop level, the result being a single-logarithmic integral determined by the region $q_1 \sim q_2 \ll \Lambda_{\rm UV}$,

$$\int \frac{d^4q}{q^4} \alpha_{\rm s}(\mu_R^2) \;\propto\; \alpha_{\rm s}(\mu_R^2) \ln \Lambda_{\rm UV} \;=\; \infty \quad \Longrightarrow \quad \beta_1 \,. \tag{2}$$

This is how the usual story goes, order by order in perturbation theory. We can do better, however, by taking into consideration that the coupling in (2) runs with the internal momentum. This means reorganising the PT series so as to incorporate into the two-loop diagram the higher order effects which result in substituting the running $\alpha_{\rm s}(q^2)$ for the constant $\alpha_{\rm s}(\mu_R^2)$. By doing so we obtain a contribution which is still UV-divergent, though modified by the logarithmic decrease of the coupling at large momenta:

$$\int \frac{d^4q}{q^4} \alpha_{\rm s}(q^2) \propto \ln \ln \Lambda_{\rm UV} \;=\; \infty \quad \Longrightarrow \quad \beta_1 \,.$$

Renormalizing it out gives rise to β_1. Starting from the third loop (twointernal gluons) the situation however changes drastically: the UV-region is no longer dominant, and we get

$$\int \frac{d^4q}{q^4} \left(\alpha_{\rm s}(q^2)\right)^2 \;=\; \text{finite} \quad \Longrightarrow \quad \beta_{n\geq 2} \text{ depend on the } \textbf{infrared} \text{ physics!}$$

Thus starting from the $\alpha_{\rm s}^3$ (next-to-next-leading) level, a purely perturbative[{1}] treatment may become intrinsically ambiguous because of an interconnection between small and large distances. There is no way of unambiguously defining the QCD coupling $\alpha_{\rm s}$ (beyond two loops) without solving the Theory in the infrared, that is without understanding the physics of colour confinement.

2.3 Where is confinement?

The quark–gluon picture works rather well across the board. Moreover, in many cases it seems to work *too well*. This is another worry: too good to be true ain't good enough.

Too early?

The way the differential large angle $2 \to 2$ particle scattering cross sections should scale with energy (momentum transfer) was envisaged by the so-called "quark counting rules" [26].

$$\frac{d\sigma}{dt} = \frac{f(\Theta)}{s^{K-2}}; \qquad \frac{t}{s} = \text{const},$$

with K the number of *elementary fields* (quarks, photons, leptons, etc.) among / inside the initial and final particles.

For example, in the case of the deuteron break-up by a photon, $\gamma + D \to p + n$, we have $K = 1 + 6 + 6 = 13$ (a photon and 6 quarks inside the initial deuteron and another 6 in the final proton and neutron). So, the differential cross section is expected to fall with s, *asymptotically*, as $s^{-11} = E_{\rm c.m.}^{-22}$. The key word *asymptotically* always provided an excuse for unnerved HEP theorists in their encounters with angered experimenters. The JLAB plot in Fig. 1 which I borrowed from Paul Hoyer's talk [27] seems to be telling us that this standard excuse is unnecessary here. However, it is again unnerving but

for precisely opposite reason, if you take my meaning. Indeed, it is *very difficult* to digest how the naive asymptotic regime manage to settle *that* early! The lab. energy $1\,\mathrm{GeV}$ of the incident photon, where the scaling behaviour starts, is just *too* low.

The "counting rules" invite us to view a fast deuteron as a system of six comoving valence quarks. One of them is punched by the photon. The other five we have to properly push ourselves so as to make them fit into two outgoing nucleons. This is done by exchanging five gluons between the quarks in the *scattering amplitude* so that the *cross section* acquires the factor α_s^{10}. The picture makes sense as long as 1) the deuteron is indeed *fast* and 2) typical momentum transfers q^2 between quarks are large enough to allow us to use the concept of gluon exchange and of the QCD{1} coupling $\alpha_s(q^2)$ for that matters. None of these conditions holds for $E_\gamma \simeq 1\,\mathrm{GeV}$.

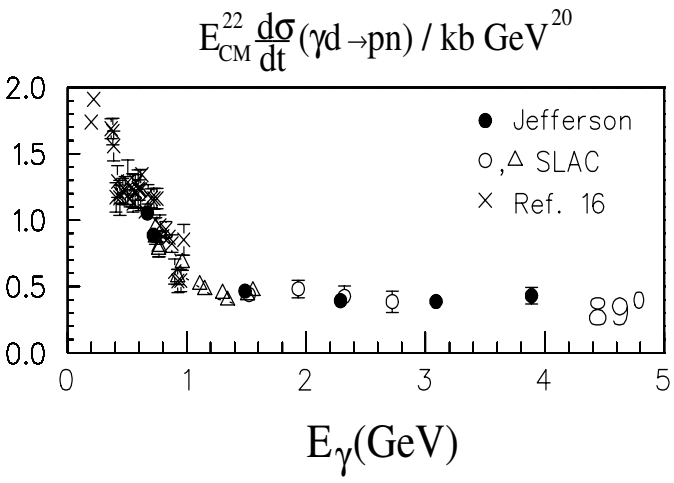


Fig. 1: Large angle γ-disintegration of a deuteron [28].

Nonetheless we would have had every right to feel happy about Fig. 1 provided we could convincingly answer but one question: why is such precocious scaling not seen for simpler systems and in particular for the simplest of them all – the electromagnetic form factor of a pion?

Too smooth?

HERA measurements of the DIS proton structure function $F_2(x, Q^2)$ in a wide range of photon virtualities,

$$0.1\,\mathrm{GeV}^2 < Q^2 < 35\,\mathrm{GeV}^2,$$

are compiled in Fig. 2. The data are plotted as a function of the simple variable

$$\xi = \log\frac{0.04}{x}\log\left(1+\frac{Q^2}{0.5\,\mathrm{GeV}^2}\right)$$

proposed by Dieter Haidt [29].

Being surprisingly smooth, they show no sign of a "phase transition" when going from large virtualities (perturbative{1} regime) downto very small scales where non-perturbative{1} physics should dominate.

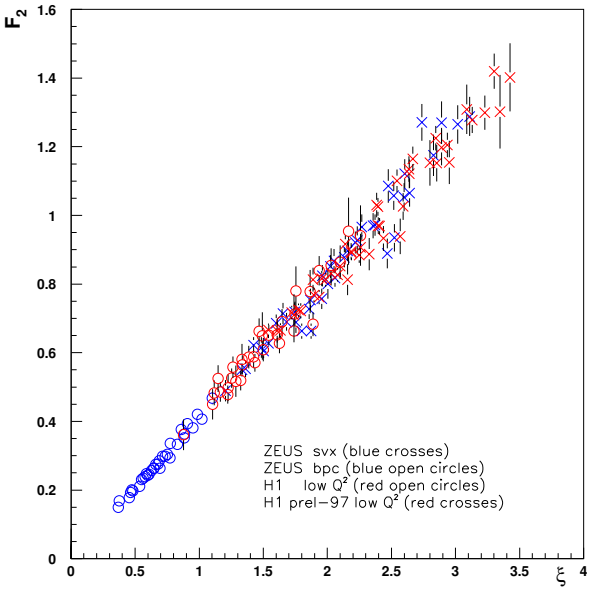


Fig. 2: F_2 for $x \leq 10^{-3}$, $Q^2 \geq 0.1\,\mathrm{GeV}^2$ [29].

Too soft?

As we will discuss below in great detail, the perturbatively{1} predicted inclusive energy spectra of relatively soft, $x_p \ll 1$, *partons* (mostly gluons) were found at LEP, HERA, Tevatron and elsewhere to be *mathematically similar* to those of charged hadrons (mostly pions), thus confirming the LPHD hypothesis of soft confinement. The p_T distribution in $\ln 1/x_p$ has a characteristic shape which follows from coherent gluon cascades (abovementioned AO). The predicted position of the hump for gluons coincides with the maximum of the pion spectrum and lies, typically, below $p = 1\,\mathrm{GeV}$!

The same story with angular distributions of interjet soft particle flows in multi-jet ensembles (numerous string/drag effects). The worry is, that these in-between jets particles are in reality but $100 - -300\,\mathrm{MeV}$ pions which for some reason beyond our apprehension still choose to obediently follow the pattern of underlying colour fields. The message is strange but clear: whatever the ultimate solution of the confinement problem may be, it had better be gentle in transforming the quark-gluon Poynting-vector into the Poynting-vector of the final state hadrons.

Is proton really bound?

HERA taught us that proton is fragile. It suffices to kick it with 1 GeV momentum transfer, or even less, to blow it to pieces. It seems that what keeps a proton together is not any strong forces between the quarks but merely quantum mechanics: the proton just happened to be the ground state with a given well conserved quantum number (baryon charge). It is interesting to see how easy it is to break a proton. To achieve that it is not even necessary to kick it hard. A soft scratch (or rather two) is enough to do the job.

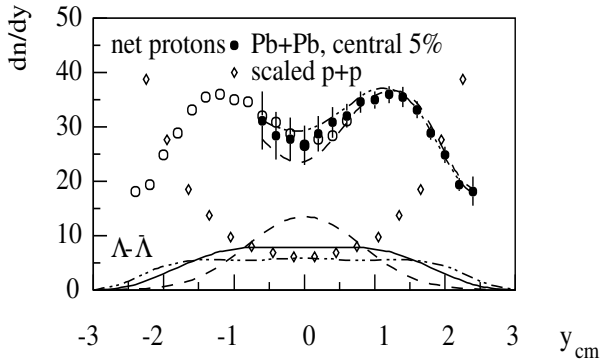


Fig. 3: Proton "stopping" as seen by NA-49 (1999)

There is no sign of advocated fragility in a normal (minimum bias, soft) high energy proton-proton scattering. The famous leading particle effect shows that a projectile proton stays intact in the final state and carries away a major fraction of the incident momentum (diamonds for "scaled $p+p$" in Fig. 3). This should not surprise us. In a typical pp interaction it is only one of the valence quarks of the proton that scatters. Internal coherence of the spectator quark pair remains undisturbed. In these circumstances the proton splits into a triplet quark and a spectator diquark which is in a colour anti-triplet state. At the hadronization stage, the former picks up an antiquark and turns into a meson carrying, roughly, $z \simeq \frac{1}{3}$ of the initial proton momentum, while the diquark (colour equivalent of a $\bar{q}$) picks up a quark forming a leading baryon with $z \simeq \frac{2}{3}$. It may be, for example, a Λ–baryon as shown in Fig.4a. More often it will be a proton, neutron or Δ. What is important, however, is that the *baryonic* quantum number moves forward – stays close in rapidity to the projectile proton.

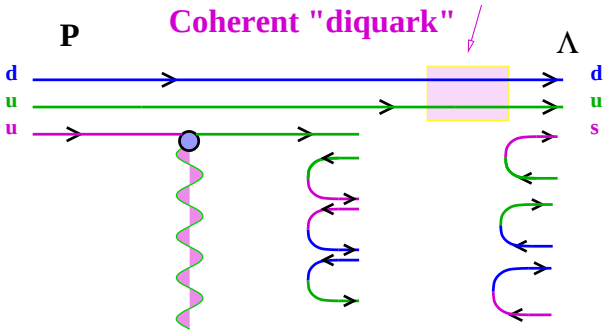


Fig. 4a: Gluon exchange produces a leading baryon.

It suffices, however, to organize a *double* scattering within a life-time of the intrinsic proton fluctuation in order to destroy the proton coherence completely (including that of the diquark which remains intact after the first scratch). Now the three quark-splinters of the proton separate as independent triplet charges and normally convert in the final state into three leading mesons carrying $z \simeq \frac{1}{3}$ each as Fig. 4b suggests, with the baryon quantum number sinking into the sea.

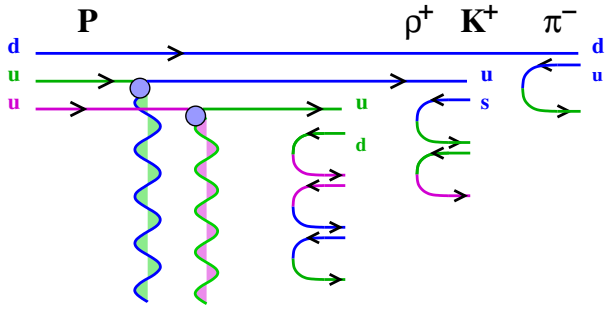


Fig. 4b: Double exchange "breaks up" the proton.

This is what seems to be going on in the lead–lead scattering, see Fig. 3. Disappearance of leading protons is known as "stopping" in the literature. This I believe is an inadequate name: there is no

way to *stop* an energetic particle, especially in soft interaction(s). Relativistic quantum field theory is more tolerant to *changing particle identity* than to allowing a large transfer of energy-momentum (recall relativistic Compton where the *backward* scattering dominates: an electron turns into a forward photon, and vice versa).

If this heretic explanation of the "stopping" as proton instability is correct, the same phenomenon should be seen in the proton hemisphere of proton-nucleon collisions and even in pp. As we know, in pp there are leading protons. However, this is true *on average*. Even in pp collisions one can enforce multiple scattering (and thus full proton breakup) by selecting rear events, e.g. with larger than average final state multiplicity.

In all these cases (pp, pA, AB) "proton decay" should be accompanied by an enhanced strangeness production.

2.4 Perturbative$^{\{1\}}$ quark confinement?

> *A spirit afterwards told him* [John Dee] *that ignorance was the nakedness wherewith he was first tormented, and "the first plague that fell unto man was the want of science."* [2]

Soft hadronization, likely absence of strong inter-parton forces, fragile proton – can it be reconciled with confinement in the first place? To the best of my knowledge, the Super-Critical Light-Quark Confinement theory (GSCC) suggested by V.N. Gribov in early 90s [30] is the only scenario to offer a natural explanation to the puzzling phenomenology of multi-hadron production discussed above.

As a result of the search for a possible solution of the confinement puzzle Gribov formulated for himself the key ingredients of the problem and, correspondingly, the lines to approach it:

- The question of interest is not of "a" confinement, but that of "the" confinement in the real world, namely, in the world with two very light quarks (u and d) whose Compton wave lengths are much larger than the characteristic confinement scale ($m_q \sim 5 - 10\,\mathrm{MeV} \ll 1\,\mathrm{GeV}$).
- No mechanism for binding massless *bosons* (gluons) seems to exist in QFT, while the Pauli exclusion principle may provide means for binding together massless *fermions* (light quarks).
- The problem of ultraviolet regularization may be more than a technical trick in a QFT with apparently infrared-unstable dynamics: the ultraviolet and infrared regimes of the theory may be closely linked. Example: the pion field as a Goldsone boson emerging due to spontaneous chiral symmetry breaking (short distances) and as a quark bound state (large distances).
- The Feynman diagram technique has to be reconsidered in QCD if one goes beyond trivial perturbative correction effects. Feynman's famous $i\epsilon$ prescription was designed for (and is applicable only to) the theories with stable perturbative vacua. To understand and describe a physical process in a confining theory, it is necessary to take into consideration the response of the vacuum, which leads to essential modifications of the quark and gluon Green functions[11].

There was a deep reason for this turn, which Gribov formulated in the following words:

> "I found I don't know how to bind massless *bosons*"

(read: how to dynamically construct *glueballs*).

As for fermions, there is a corresponding mechanism provided by the Fermi-Dirac statistics and the concept of the "Dirac sea". Spin-$\frac{1}{2}$ particles, even massless which are difficult to localize, can be held together simply by the fact that, if pulled apart, they would correspond to the free-fermion states that are *occupied* as belonging to the Dirac sea.

[11] The proper technology lies in a generalisation of the Keldysh diagram technique designed to describe kinetics out of equilibrium.

Thus, light quarks are crucial for GSCC. It is clear without going into much mathematics that the presence of light quarks is sufficient for preventing the colour forces from growing real big: dragging away a heavy quark we soon find ourselves holding a blanched D-meson instead. The light quark vacuum is eager to screen any separating colour charges.

The question becomes quantitative: how strong is strong? How much of a tension does one need to break the vacuum and organize such a screening?

In a pure perturbative (non-interacting) picture, the empty fermion states have *positive energies*, while the *negative-energy* states are all filled. With account of interaction the situation may change, *provided* two *positive-energy* fermions (quarks) were tempted to form a bound state with a *negative* total energy. In such a case, the true vacuum of the theory would contain *positive kinetic energy* quarks hidden inside the *negative energy* pairs, thus preventing positive-energy quarks from flying free.

A similar physical phenomenon is known in QED under the name of super-critical binding in ultra-heavy nuclei. Dirac energy levels of an electron in an external static field created by the large point-like electric charge $Z > 137$ become *complex*. This means instability. Classically, the electron "falls onto the centre". Quantum-mechanically, it also falls, but into the Dirac sea.

In QFT the instability develops when the energy ϵ of an empty atomic electron level falls, with increase of Z, below $-m_e c^2$. An e^+e^- pair pops up from the vacuum, with the vacuum electron occupying the level: the super-critically charged ion decays into an "atom" (the ion with the smaller charge, $Z-1$) and a real positron

$$A_Z \implies A_{Z-1} + e^+ , \qquad \text{for } Z > Z_{\text{crit.}}$$

Thus, the ion becomes unstable and gets rid of an excessive electric charge by emitting a positron [31] In the QCD context, the increase of the running quark-gluon coupling at large distances replaces the large Z of the QED problem.

Gribov generalised the problem of super-critical binding in the field of an infinitely heavy source to the case of two massless fermions interacting via Coulomb-like exchange. He found that in this case the super-critical phenomenon develops much earlier. Namely, a *pair of light fermions* interacting in a Coulomb-like manner develops super-critical behaviour if the coupling hits a definite critical value

$$\frac{\alpha}{\pi} > \frac{\alpha_{\text{crit}}}{\pi} = 1 - \sqrt{\frac{2}{3}} \,. \tag{3}$$

In QCD one has to account for the colour Casimir operator. Then the value of the coupling above which restructuring of the PT vacuum leads to chiral symmetry breaking and, likely, to confinement ([30] and references therein), translates into

$$\frac{\alpha_{\text{crit}}}{\pi} = C_F^{-1} \left[1 - \sqrt{\frac{2}{3}} \right] \simeq 0.137 \,. \tag{4}$$

This number, apart from being easy to memorize, has another important quality: it is numerically small. Gribov's ideas, being understood and pursued, offer an intriguing possibly to address all the diversity and complexity of the hadron world from within the field theory with a reasonably small effective interaction strength (read: not only *perturbatively*$^{\{2\}}$ but *perturbatively*$^{\{1\}}$).

3. MULTIPLE HADROPRODUCTION: ASCENDING THE LADDER

We have already carefully measured our steps down the ladder of ignorance. Now let us ascend another one, looking for indirect and then direct evidences in favour of quark-gluon dynamics in multiple hadroproduction.

High energy e^+e^- annihilation, DIS, production in hadron-hadron collisions of massive lepton pairs, heavy quarks and their bound states, of large transverse momentum jets and photons are classical

examples of hard processes. Copious production of hadrons is typical for all of them. On the other hand, at the microscopic level, multiple quark-gluon "production" is to be expected as a result of QCD bremsstrahlung – gluon radiation accompanying abrupt creation/scattering of colour partons.

3.1 Scaling violation pattern (indirect evidence)

Indirect evidence that gluons are there, and that they behave, can be obtained from the study of the scaling violation pattern. QCD quarks and gluons are not point-like particles, as the orthodox parton model once assumed. Each of them is surrounded by a proper field coat – a coherent virtual cloud consisting of gluons and "sea" $q\bar{q}$ pairs. A hard probe applied to such a dressed parton breaks coherence of the cloud. Constituents of these field fluctuations are then released as particles accompanying the hard interaction.

The harder the hit, the larger an intensity of bremsstrahlung and, therefore, the fraction of the energy-momentum of the dressed parton that the bremsstrahlung quanta typically carry away. Thus we should expect, in particular, that the probability that a hit "bare" core quark carries a large fraction $x \sim 1$ of the energy of its dressed parent will decrease with increase of Q^2. And so it does. The logarithmic scaling violation pattern in DIS structure functions is well established and meticulously follows the QCD prediction based on the parton evolution picture.

In DIS we look for a "bare" quark inside a target dressed one. In e^+e^- hadron annihilation at large energy $s = Q^2$ the chain of events is reversed.

Here we produce instead a bare quark with energy $Q/2$, which then "dresses up". In the process of restoring its proper field-coat our parton produces (a controllable amount of) bremsstrahlung radiation which leads to formation of a hadron jet. Having done so, in the end of the day it becomes a constituent of one of the hadrons that hit the detector. Typically, this is the leading hadron. However, the fraction x of the initial energy $Q/2$ that is left to the leader depends on the amount of accompanying radiation and, therefore, on Q^2 (the larger, the smaller).

In fact, the same rule (and the same formula) applies to the scaling violation pattern in e^+e^- fragmentation functions (time-like parton evolution) as to that in the DIS parton distributions (space-like evolution).

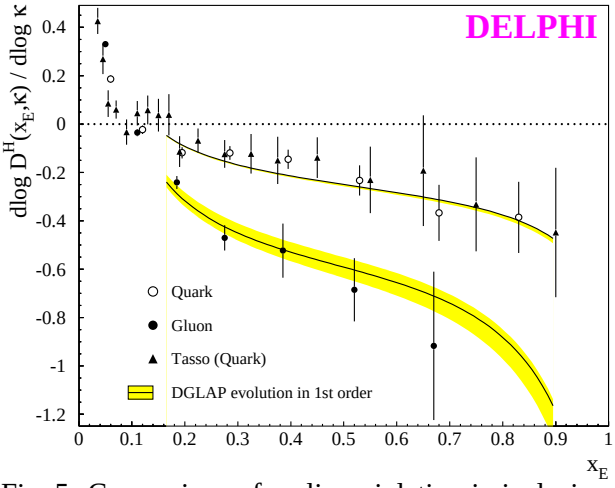


Fig. 5: Comparison of scaling violation in inclusive hadron distributions from gluon and quark jets [32].

The e^+e^- annihilation experiments have become so sophisticated as to provide us with a near-to-perfect separation between quark- and gluon-initiated jets (the latter being extracted from heavy-quark-tagged three-jet events).

In Fig. 5 a comparison is shown of the scaling violation rates in the hadron spectra from gluon and quark jets, as a function of the hardness scale κ that characterizes a given jet [32].

For large values of $x \sim 1$ the ratio of the logarithmic derivatives is predicted to be close to that of the gluon and quark "colour charges", $C_A/C_F = 9/4$. Experimentally, the ratio was measured to be

$$\frac{C_A}{C_F} = 2.23 \pm 0.09_{\text{stat.}} \pm 0.06_{\text{syst.}}. \tag{5}$$

3.2 Bremsstrahlung parton vs. hadron multiplicities (global direct evidence)

Since accompanying QCD radiation seems to be there, we can make a step forward by asking for a *direct* evidence: what is the fate of those gluons and sea quark pairs produced via multiple initial gluon bremsstrahlung followed by parton multiplication cascades?

Let us look at the Q-dependence of the mean hadron multiplicity, the quantity dominated by relatively soft particles with $x \ll 1$. This is the kinematical region populated by accompanying QCD radiation.

Fig. 6 demonstrates that the hadron multiplicity increases with the hardness of the jet proportional to the multiplicity of secondary gluons and sea quarks.

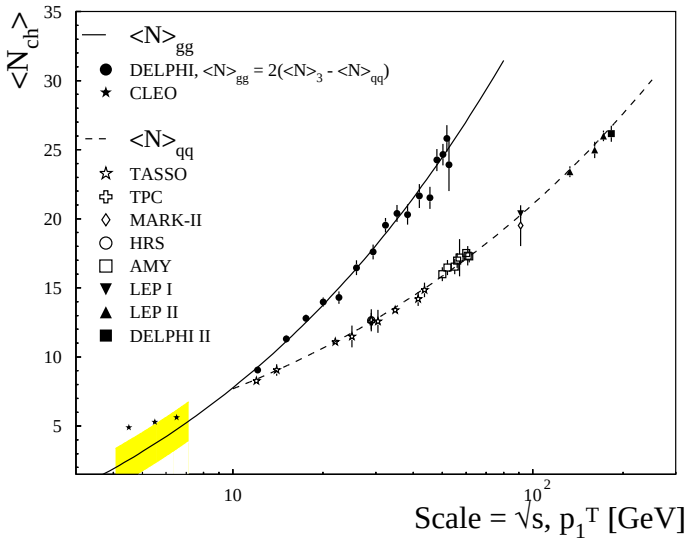


Fig. 6: DELPHI comparison of charged hadron multiplicities from tagged quark and gluon jets [32].

The ratio of the slopes, once again, provides an independent measure of the ratio of the colour charges, which is consistent with (5) [32]:

$$\frac{C_A}{C_F} = 2.246 \pm 0.062_{\text{stat.}} \pm 0.008_{\text{syst.}} \pm 0.095_{\text{theo.}}. \qquad (6)$$

Since the total numbers match, it is time to ask a more delicate question about energy-momentum distribution of final hadrons versus that of the underlying parton ensemble. One should not be too picky in addressing such a question. It is clear that hadron-hadron correlations, for example, will show resonant structures about which the quark-gluon speaking pQCD can say little, if anything, at the present state of the art. Inclusive single-particle distributions, however, have a better chance to be closely related. Triggering a single hadron in the detector, and a single parton on paper, one may compare the structure of the two distributions to learn about dynamics of hadronization.

It is important to stress that QCD coherence is crucial for treating particle multiplication **inside** jets, as well as for hadron flows **in-between** jets.

3.3 Multiplicity flows between jets (another global direct tricky evidence)

"QCD Radiophysics" deals with particle flows in the angular regions *between jets* in various multi-jet configurations. These particles do not belong to any particular jet, and their production, at the pQCD level, is governed by *coherent* soft gluon radiation off the multi-jet system as a whole as off a composite *antenna* (hence, "radiophysics").

The ratios of particle (gluon) flows in different inter-jet valleys are given by parameter-free PT$^{\{1\}}$ predictions and reveal the so-called "string" [21] or "drag" effects [22].

At the level of the PT accompanying gluon radiation (*QCD radiophysics*) such ratios are quite simple and straightforward to derive. They depend only on the number of colours (N_c) and on the geometry of the underlying ensemble of hard partons forming jets.

Lund string effect. For example, the classical *string effect* – the ratio of the multiplicity flow between a quark (antiquark) and a gluon to that in the $q\bar{q}$ valley in symmetric ("Mercedes") $q\bar{q}g$ three-jet e^+e^-

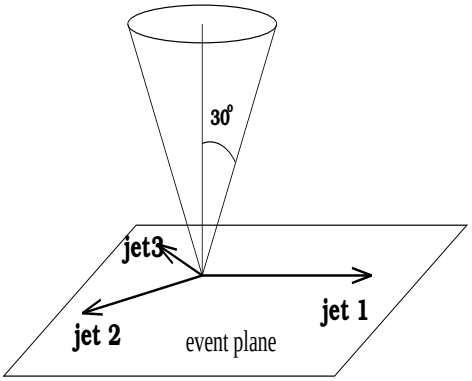
annihilation events reads

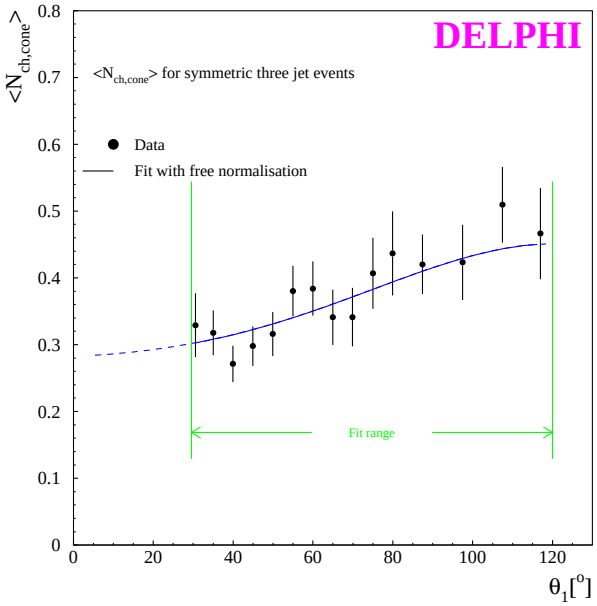
$$\frac{dN_{qg}^{(q\bar{q}g)}}{dN_{q\bar{q}}^{(q\bar{q}g)}} \simeq \frac{5N_c^2-1}{2N_c^2-4} = \frac{22}{7} \simeq \pi.$$

We see that emitting an energetic gluon off the initial quark pair depletes accompanying radiation in the *backward* direction: colour is *dragged out* of the $q\bar{q}$ valley. This destructive interference effect is so strong that the resulting multiplicity flow between quarks falls below that in the *least favourable* direction *transversal* to the 3-jet event plane:

$$\frac{dN_{\perp}^{(q\bar{q}\gamma)}}{dN_{q\bar{q}}^{(q\bar{q}g)}} \simeq \frac{N_C+2C_F}{2(4C_F-N_c)} = \frac{17}{14}\,.$$

The following pictures demonstrate the DELPHI study of the particle flow in the out-of-plane direction, as a function of the angle Θ_1 between the two softer jets (one of them the gluon) [33]. The particle flow increases with angle in a full accord with the theoretical expectation based on the coherent gluon radiation off the three-prong colour antenna.





Another Example. A comparison of the hadron flows in the $q\bar{q}$ valley in $q\bar{q}\gamma$ events with a gluon jet replaced by an energetic photon results in the ratio

$$\frac{dN_{q\bar{q}}^{(q\bar{q}\gamma)}}{dN_{q\bar{q}}^{(q\bar{q}g)}} \simeq \frac{2(N_c^2-1)}{N_c^2-2} = \frac{16}{7}, \quad \text{(experiment: } 2.3 \pm 0.2\text{)}.$$

It is not strange at all that with *gluons* one can get, e.g.,

$$q\bar{q}: \quad 1+1 \;=\; 2 \qquad \text{while} \qquad q\bar{q}+g: \quad 1+1 \;+\; \frac{9}{4} \;=\; \frac{7}{16}, \tag{7}$$

which is simply the radiophysics of composite antennas, or *quantum mechanics* of conserved colour charges. The first equation of these quantum arithmetics problems describes symbolically the density of soft gluon radiation between two quarks in a $q\bar{q}\gamma$ event, with 1 standing for the colour quark charge.

Replacing the colour-neutral photon by a gluon one gets an additional emitter with the relative strength $\frac{9}{4}$, as shown in the l.h.s. of the second equation in (7). In spite of having added an additional

emitter, the resulting soft gluon yield in the $q\bar{q}$ direction (r.h.s.) *decreases* substantially as a result of destructive interference between three elements of a composite colour antenna.

Nothing particularly strange, you might say. What *is* rather strange, though, is that this naive perturbative{1} wisdom is being impressed upon junky 100–300 MeV pions which dominate hadron flows between jets in the present-day experiments such as the OPAL study shown in Fig. 7.

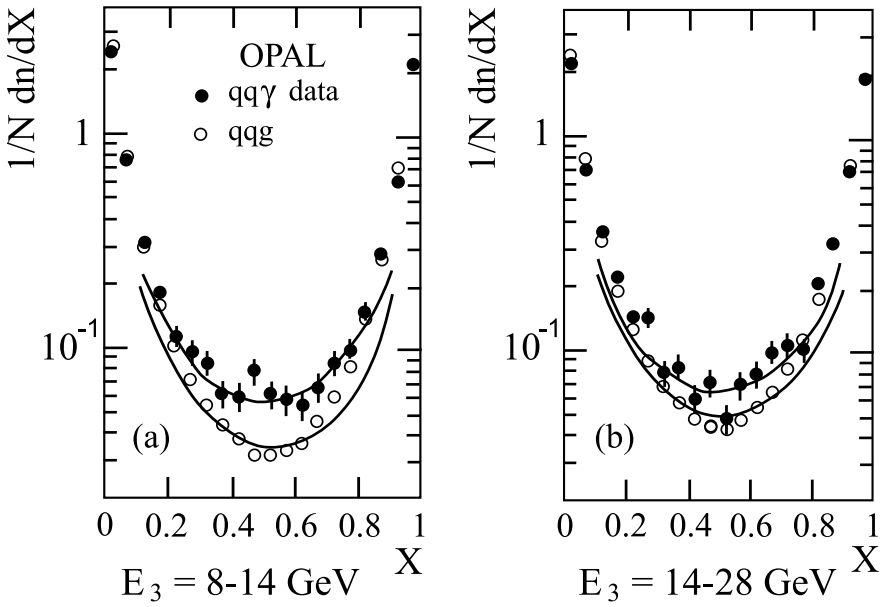


Fig.7: Particle flows in the $q\bar{q}$ valley in $q\bar{q}\gamma$ and $q\bar{q}g$ events [34] versus an analytic parameter-free prediction based on the soft gluon radiation pattern [35].

These and many similar phenomena are being seen experimentally. What the nature seems to be telling us, is that

- The *colour field* that an ensemble of hard primary **partons** (parton antenna) develops, determines, on the one-to-one basis, the structure of final flows of **hadrons**.
- The Poynting vector of the colour field gets translated into the hadron Poynting vector without any visible reshuffling of particle momenta at the hadronization stage.

When viewed *globally*, confinement is about *renaming* a flying-away quark into a flying-away pion rather than about forces *pulling* quarks together.

3.4 Inclusive hadron distribution inside jets (local direct evidence)

A similar message comes from the study of the energy distribution of particles *inside* jets.

An inclusive energy spectrum of soft bremsstrahlung partons in QCD jets has been derived in 1984 in the so-called MLLA – the Modified Leading Logarithmic Approximation [7, 36]. This approximation takes into account all essential ingredients of parton multiplication in the next-to-leading order. They are: parton splitting functions responsible for the energy balance in parton splitting, the running coupling $\alpha_s(k_\perp^2)$ depending on the relative transverse momentum of the two offspring and exact angular ordering. The latter is a consequence of soft gluon coherence and plays, as we shall discuss below, an essential rôle in parton dynamics. In particular, gluon coherence suppresses multiple production of very small momentum gluons. It is particles with intermediate energies that multiply most efficiently. As a result, the energy spectrum of relatively soft secondary partons in jets acquires a characteristic hump-backed shape. The position of the maximum in the logarithmic variable $\xi = -\ln x$, the width of the hump and its height increase with Q^2 in a predictable way.

The shape of the inclusive spectrum of all charged hadrons (dominated by $\pi^{\pm}$) exhibits the same features. This comparison, pioneered by Glen Cowan (ALEPH) and the OPAL collaboration, has since become a standard test of analytic QCD predictions. First scrutinized at LEP, the similarity of parton and hadron energy distributions has been verified at SLC and KEK e^+e^- machines, as well as at HERA and Tevatron where hadron jets originate not from bare quarks dug up from the vacuum by a highly virtual photon/Z^0 but from hard partons kicked out from initial hadron(s).

In Fig. 8 (DELPHI) the comparison is made of the spectra of all charged hadrons at various annihilation energies Q with the so-called "distorted Gaussian" fit [37] which employs the first four moments (the mean, width, skewness and kurtosis) of the MLLA distribution around its maximum.

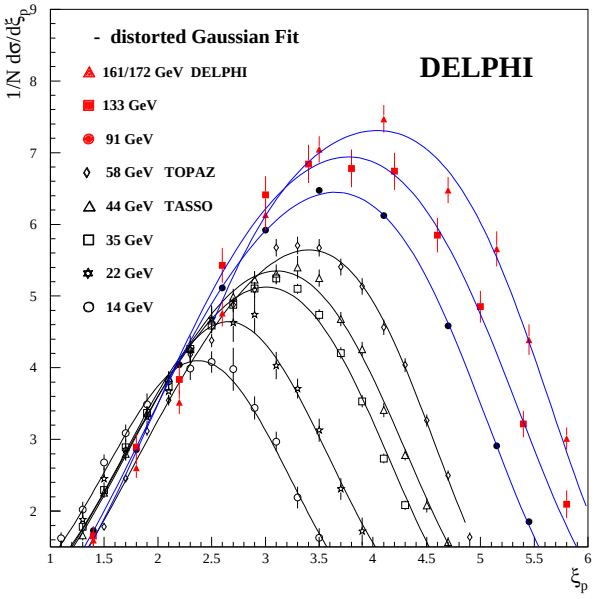


Fig. 8: Hadron energy spectra in $e^+e^- \to h^{\pm} + X$

Shall we say, a (routine, interesting, wonderful) check of yet another QCD prediction?

I would rather not. Such a close similarity offers a deep puzzle, even a worry, rather than a successful test.

Indeed, after a little exercise in translating the values of the logarithmic variable $\xi = \ln(E_{\rm jet}/p)$ in Fig. 8 into GeVs you will see that the actual hadron momenta at the maxima are, for example, p=$\frac{1}{2}Q \cdot e^{-\xi_{\max}} \simeq 0.42$, 0.85 and 1.0 GeV for Q=14, 35 GeV and at LEP-I, Q=91 GeV. Is it not surprising that the PT spectrum is mirrored by that of the pions (which constitute 90% of all charged hadrons produced in jets) with momenta well below 1 GeV?!

For this very reason the observation of the parton-hadron similarity was initially met with a serious and well grounded scepticism: it looked more natural (and was more comfortable) to blame the finite hadron mass effects for falloff of the spectrum at large ξ (small momenta) rather than seriously believe in applicability of the PT$^{\{1\}\{2\}}$ consideration down to such disturbingly small momentum scales.

This worry has been answered by CDF. Andrey Korytov and Alexei Safonov carried out meticulous studies of the energy distribution of hadrons produced inside a restricted cone of the opening half-angle Θ_c around the jet axis.

As we have already mentioned above discussing the Lund hadroproduction picture, theoretically it is not the energy of the jet but the maximal parton transverse momentum inside it, $k_{\perp\max} \simeq E_{\rm jet} \sin\Theta_c$, that determines the hardness scale and thus the yield and the distribution of the accompanying radiation [38]. This means that by choosing a small opening angle one can study relatively small hardness scales but in a cleaner environment: due to the Lorentz boost effect, eventually all particles that form a short small-Q^2 QCD "hump" are now relativistic and concentrated at the tip of the jet.

For example, selecting hadrons inside a cone $\Theta_c \simeq 0.14$ around an energetic quark jet with $E_{\rm jet} \simeq 100$ GeV (LEP-II) one would see that very "dubious" $Q = 14$ GeV curve in Fig. 8 but now with the maximum boosted from 0.45 GeV into a comfortable 6 GeV range.

The CDF Fig. 9 [39, 40] shows the change of the energy spectrum of charged hadrons with the opening angle for a given invariant mass of the system of two large transverse momentum jets, in comparison with the analytic MLLA expression for soft secondary gluons. Similar results for a broad range of dijet masses, $78\,{\rm GeV} \le M_{jj} \le 537\,{\rm GeV}$, will soon be made public.

A close similarity between the hadron yield and the full MLLA parton spectra can no longer be considered accidental and be attributed to non-relativistic kinematical effects.

3.5 Brave gluon counting

Modulo $\Lambda_{\rm QCD}$, there is only one unknown in this comparison, namely, the overall normalization of the spectrum of hadrons relative to that of partons (bremsstrahlung gluons).

Strictly speaking, there should/could have been another free parameter, that which quantifies one's bravery in applying the pQCD dynamics. It is the minimal transverse momentum cutoff in parton cascades, $k_\perp > Q_0$. The strength of successive $1 \to 2$ parton splittings is proportional to $\alpha_s(k_\perp^2)$ and grows with $k_\perp$ decreasing. The necessity to terminate the process at some low transverse momentum scale where the PT coupling becomes large (and eventually hits the formal "Landau pole" at $k_\perp = \Lambda_{\rm QCD}$) seems imminent. Surprisingly enough, it is not.

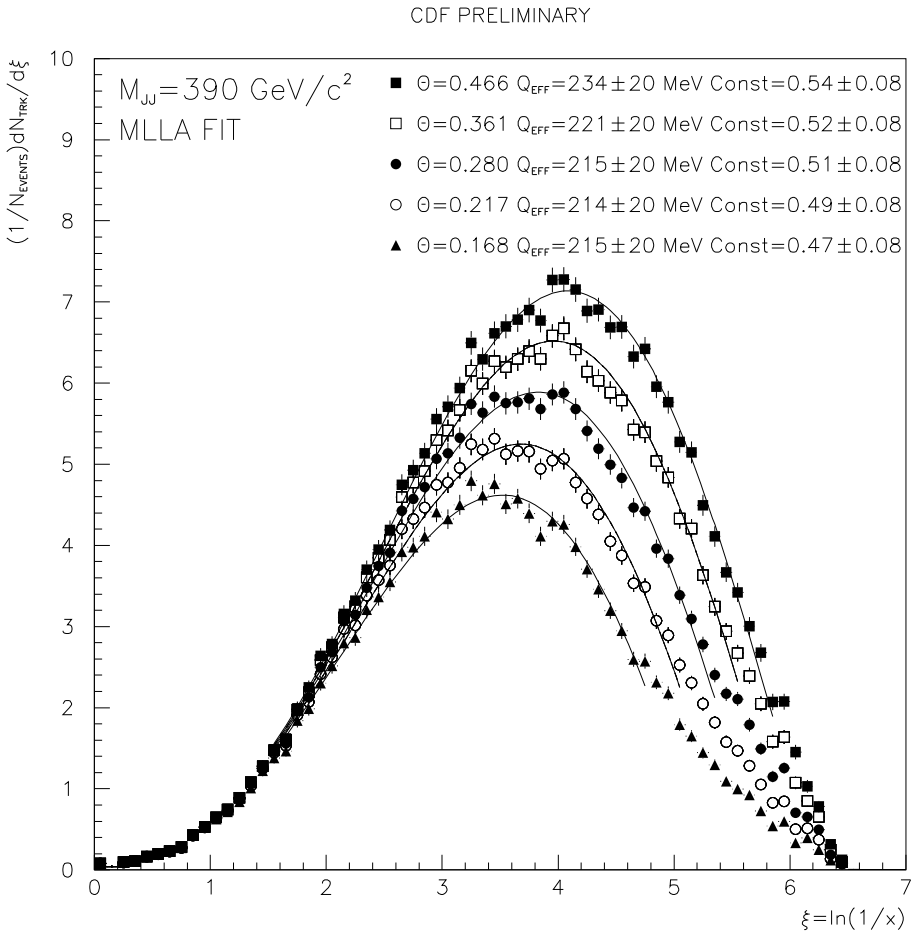


Fig. 9: Inclusive energy distributions of charged hadrons in large–$p_\perp$ jets [39].

As we shall see in the next Section, the inclusive parton energy distribution turns out to be a CIS QCD prediction, believe it or not. It is its crazy $Q_0 = \Lambda_{\rm QCD}$ limit (the so-called "limiting spectrum") which is shown by solid curves in Fig. 9.

Choosing the minimal value for the collinear parton cutoff Q_0 can be looked upon as shifting, as far as possible, responsibility for particle multiplication in jets to the PT dynamics. This brave choice can be said to be dictated by experiment, in a certain sense. Indeed, with increase of Q_0 the parton distributions *stiffen* (parton energies are limited from below by the kinematical inequality $xE_{\rm jet} \equiv k \geq k_\perp > Q_0$). The maxima would move to larger x (smaller ξ), departing from the data.

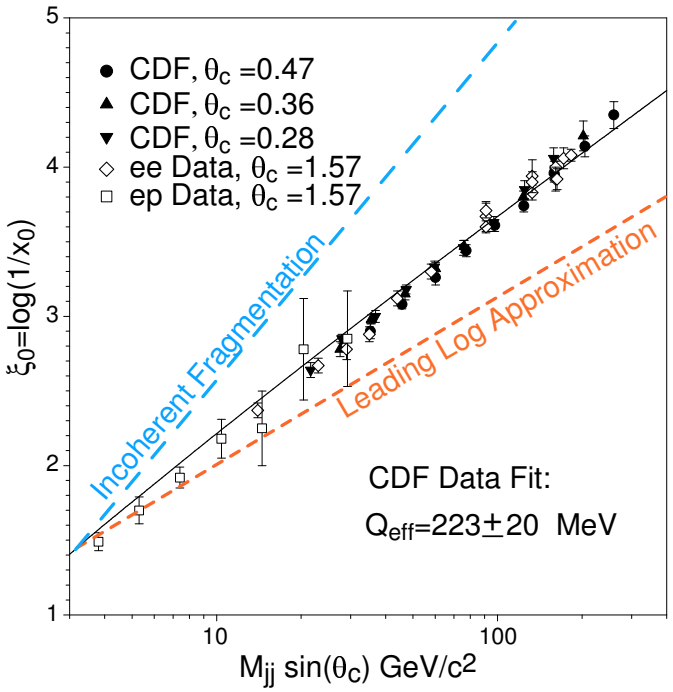


Fig. 10: Position of the maximum versus MLLA [41].

A clean test of "brave gluon counting" is provided by Fig. 10 where the position of the hump, which is insensitive to the overall normalization, is compared with the parameter-free analytic MLLA prediction [41]. An overlaid prediction of the incoherent hadronization model (long-dashed line) shows how the maximum of the energy distribution would have moved if the Field–Feynman fragmentation picture were applicable. Comparison with the DLA expectation (short dashes) demonstrates the rôle of the NLO effects parton cascades (MLLA).

Spectacular verification of the local duality hypothesis was recently reported by A. Safonov [41]. It showed remarkable stability of the only parameter of the game – $Q_{\rm eff}$ – wrt variations of the dijet mass M_{JJ} and the opening angle of the cone Θ_c (left template in Fig. 11).

This parameter plays a double rôle in the naive limiting spectrum: that of $\Lambda_{\rm QCD}$ and of the collinear cut Q_0. The evolution of the spectrum with the hardness of the process ($M_{jj}\sin\Theta_c$) obviously depends on $\Lambda_{\rm QCD}$, while the position of the maximum is sensitive to Q_0. The right template in Fig. 11 demonstrates an impressive correlation between the two independent determinations of $Q_{\rm eff}$ [41].

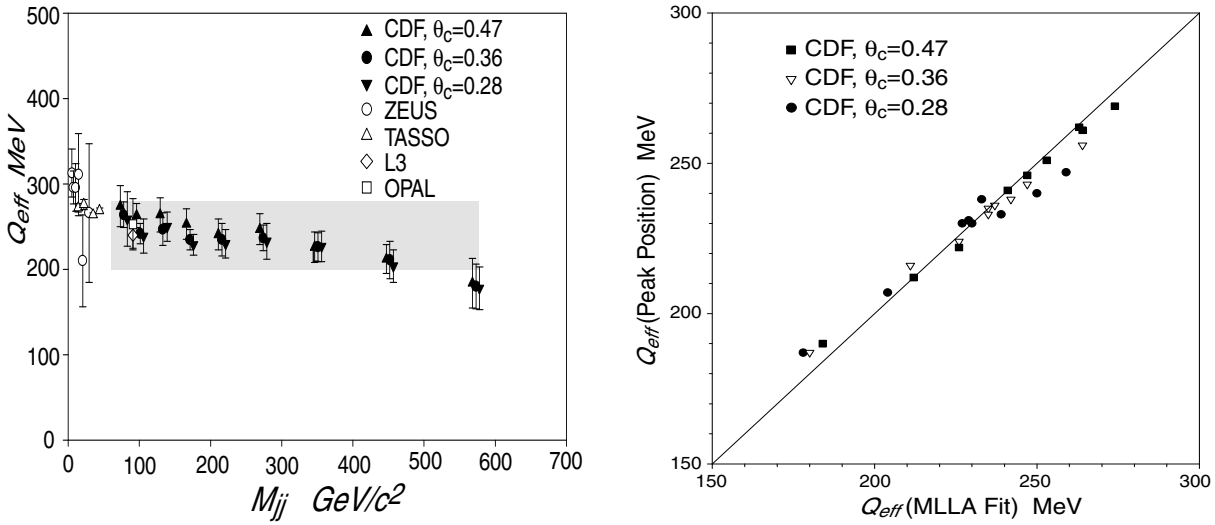


Fig. 11: Experimental results on the $Q_{\rm eff}$ parameter of the limiting MLLA spectrum [41].

4. HUMPBACKED PLATEAU AND THE ORIGIN OF LPHD

Here we are going to derive together the QCD "prediction" of the inclusive energy spectrum of relatively soft particles from QCD jets. I put the word *prediction* in quotation marks on purpose. This is a good example to illustrate the problem of filling the gap between the QCD formulae, talking quarks and gluons, and phenomena dealing, obviously, with hadrons.

Let me first make a statement:

It is QCD coherence that allows the prediction of the inclusive soft particle yield in jets

practically from "first principles".

You have all the reasons to feel suspicious about this. Indeed, we have stressed above the similarity between the dynamics of the evolution of space-like (DIS structure functions) and time-like systems (jets). On the other hand, you are definitely aware of the fact that the DIS structure functions cannot be calculated perturbatively.

In spite of the similarity between the space- and time-like evolution of *hard* partons, $x \sim 1$, there is an essential difference between *small–x* physics of DIS structure functions and the jet fragmentation. In the case of the *space-like evolution*, in the limit of small Bjorken–x the problem becomes essentially non-perturbative and pQCD loses control of the DIS cross sections [42]. In contrast, studying small-Feynman-x particles originating from the *time-like evolution* of jets offers a gift and a puzzle: all the richness of the confinement dynamics reduces to a mere overall normalization constant.

4.1 Solving the DIS evolution

So let us repeat that DIS structure functions at $x \sim 1$ cannot be calculated perturbatively[{1}] from first principles. Indeed there are input parton distributions for the target proton, which have to be plugged in as an initial condition for the evolution at some finite hardness scale $Q_0 = \mathcal{O}\,(1\,\mathrm{GeV})$. These initial distributions cannot be calculated "from first principles" nowadays but are subject to fitting. What pQCD controls then, is the scaling violation pattern. Namely, it tells us how the parton densities change with the changing scale of the transverse-momentum probe:

$$\frac{\partial}{\partial \ln k_\perp} D(x,k_\perp) = \frac{\alpha_{\mathrm{s}}(k_\perp)}{\pi} \int_x^1 \frac{dz}{z}\, P(z)\, D\left(\frac{x}{z}, k_\perp\right). \tag{8}$$

It is convenient to present our "wavefunction" D and "Hamiltonian" P in terms of the complex moment ω, which is Mellin conjugate to the momentum fraction x:

$$D_\omega = \int_0^1 dx\, x^\omega \cdot D(x)\,, \qquad D(x) = x^{-1} \int_{(\Gamma)} \frac{d\omega}{2\pi \mathrm{i}}\, x^{-\omega} \cdot D_\omega\,; \tag{9a}$$

$$P_\omega = \int_0^1 dz\, z^\omega \cdot P(z)\,, \qquad P(z) = z^{-1} \int_{(\Gamma)} \frac{d\omega}{2\pi \mathrm{i}}\, z^{-\omega} \cdot P_\omega\,, \tag{9b}$$

where the contour Γ runs parallel to the imaginary axis, to the right from singularities of D_ω (P_ω). It is like trading the coordinate ($\ln x$) for the momentum (ω) in a Schrödinger equation.

Substituting (9) into (8) we see that the evolution equation becomes algebraic and describes propagation in "time" $dt = \frac{\alpha_s}{\pi} d \ln k_\perp$ of a free quantum mechanical "particle" with momentum ω and the dispersion law $E(\omega) = P_\omega$:

$$\widehat{\mathbf{d}}\, D_\omega(k_\perp) \;=\; \frac{\alpha_{\mathrm{s}}(k_\perp)}{\pi} \cdot P_\omega\, D_\omega(k_\perp)\,; \qquad \widehat{\mathbf{d}} \equiv \frac{\partial}{\partial \ln k_\perp}\,. \tag{10}$$

To continue the analogy, our wavefunction D is in fact a multi-component object. It embodies the distributions of valence quarks, gluons and secondary sea quarks which evolve and mix according the 2×2 matrix "Hamiltonian" of the parton splitting functions $P[A \to B]$.

At small x, however, the picture simplifies. Here the valence distribution is negligible, $\mathcal{O}\,(x)$, while the gluon and sea quark components form a system of two coupled oscillators which is easy to diagonalize. What matters is one of the two energy eigenvalues (one of the two branches of the dispersion rule) that is *singular* at $\omega = 0$. The problem becomes essentially one-dimensional. Sea quarks are driven by the gluon distribution while the latter is dominated by gluon cascades. Correspondingly, the leading

energy branch is determined by gluon-gluon splitting $g \to gg$, with a subleading correction coming from the $g \to q(\bar{q}) \to g$ transitions,

$$P_\omega \;=\; \frac{2N_c}{\omega} - a \;+\; \mathcal{O}\left(\omega\right) \;, \quad a = \frac{11N_c}{6} + \frac{n_f}{3N_c^2} \,. \tag{11}$$

The solution of (10) is straightforward:

$$D_\omega(k_\perp) \;=\; D_\omega(Q_0) \cdot \exp\left\{ \int_{Q_0}^{k_\perp} \frac{dk}{k}\, \gamma_\omega(\alpha_{\rm s}(k)) \right\}, \tag{12a}$$

$$\gamma_\omega(\alpha_{\rm s}) \;=\; \frac{\alpha_{\rm s}}{\pi}\, P_\omega \,. \tag{12b}$$

The structure (12a) is of the most general nature. It follows from *renormalizability* of the theory, and does not rely on the LLA which we used to derive it. The function $\gamma(\alpha_{\rm s})$ is known as the "anomalous dimension"[12]. It can be perfected by including higher orders of the PT expansion. Actually, modern analyses of scaling violation are based on the improved next-to-LLA (two-loop) anomalous dimension, which includes $\alpha_{\rm s}^2$ corrections to the LLA expression (12b).

The structure (12a) of the x-moments of parton distributions (DIS structure functions) gives an example of a clever separation of PT and NP effects; in this particular case – in the form of two factors. It is the ω-dependence of the input function $D_\omega(Q_0)$ ("initial parton distributions") that limits predictability of the Bjorken-x dependence of DIS cross sections.

So, how comes then that in the time-like channel the PT answer turns out to be more robust?

4.2 Coherent hump in $e^+e^- \to h(x) + \ldots$

We are ready to discuss the time-like case, with $D_j^h(x,Q)$ now the inclusive distribution of particles h with the energy fraction (Feynman-x) $x \ll 1$ from a jet (parton j) produced at a large hardness scale Q.

Here the general structure (12a) still holds. We need, however, to revisit the expression (12b) for the anomalous dimension because, as we have learned, the proper evolution time is now different from the case of DIS.

In the time-like jet evolution, due to Angular Ordering, the evolution equation becomes non-local in $k_\perp$ space:

$$\frac{\partial}{\partial \ln k_\perp} D(x,k_\perp) = \frac{\alpha_{\rm s}(k_\perp)}{\pi} \int_x^1 \frac{dz}{z}\, P(z)\, D\left(\frac{x}{z},\, z\cdot k_\perp\right). \tag{13}$$

Indeed, successive parton splittings are ordered according to

$$\theta = \frac{k_\perp}{k_{||}} \;>\; \theta' = \frac{k'_\perp}{k'_{||}} \,.$$

Differentiating $D(k_\perp)$ over the scale of the "probe", $k_\perp$, results then in the substitution

$$k'_\perp = \frac{k'_{||}}{k_{||}} \cdot k_\perp \;\equiv\; z \cdot k_\perp$$

in the argument of the distribution of the next generation $D(k'_\perp)$.

The evolution equation (13) can be elegantly cracked using the Taylor-expansion trick,

$$D(z\cdot k_\perp) = \exp\left\{ \ln z \frac{\partial}{\partial \ln k_\perp} \right\} D(k_\perp) \;=\; z^{\frac{\partial}{\partial \ln k_\perp}} \cdot D(k_\perp). \tag{14}$$

[12] The name is a relict of those good old days when particle and solid state physicists used to have common theory seminars. If the coupling $\alpha_{\rm s}$ were constant (had a "fixed point"), then (12a) would produce the function with a non-integer (non-canonical) dimension $D(Q) \propto Q^\gamma$ (analogy – critical indices of thermodynamical functions near the phase transition point).

Turning as before to moment space (9), we observe that the solution comes out similar to that for DIS, (12a), but for one detail. The exponent $\widehat{\mathbf{d}}$ of the additional z-factor in (14) combines with the Mellin moment ω to make the argument of the splitting function P a *differential operator* rather than a complex number:

$$\widehat{\mathbf{d}} \cdot D_\omega = \frac{\alpha_s}{\pi} P_{\omega+\widehat{\mathbf{d}}} \cdot D_\omega \,. \tag{15}$$

This leads to the differential equation

$$\left(P^{-1}_{\omega+\widehat{\mathbf{d}}}\, \widehat{\mathbf{d}} \;-\; \frac{\alpha_s}{\pi} - \left[P^{-1}_{\omega+\widehat{\mathbf{d}}}, \frac{\alpha_s}{\pi} \right] P_{\omega+\widehat{\mathbf{d}}} \right) \cdot D = 0 \,. \tag{16}$$

Recall that, since we are interested in the small-x region, the essential moments are small, $\omega \ll 1$.

For the sake of illustration, let us keep only the most singular piece in the "dispersion law" (11) and neglect the commutator term in (16) generating a subleading correction $\propto \widehat{\mathbf{d}}\alpha_s \sim \alpha_s^2$. In this approximation (DLA),

$$P_\omega \simeq \frac{2N_c}{\omega} \,, \tag{17}$$

(16) immediately gives a quadratic equation for the anomalous dimension,[13]

$$(\omega + \gamma_\omega)\gamma_\omega \;-\; \frac{2N_c\alpha_s}{\pi} + \mathcal{O}\left(\frac{\alpha_s^2}{\omega}\right) \;=\; 0 \,. \tag{18}$$

The leading anomalous dimension following from (18) is

$$\gamma_\omega = \frac{\omega}{2}\left(-1 + \sqrt{1 + \frac{8N_c\alpha_s}{\pi\,\omega^2}} \right). \tag{19}$$

When expanded to first order in α_s, it coincides with that for the space-like evolution, $\gamma_\omega \simeq \alpha_s/\pi \cdot P_\omega$, with P given in (17). Such an expansion, however, fails when characteristic $\omega \sim 1/|\ln x|$ becomes as small as $\sqrt{\alpha_s}$, that is when

$$\frac{8N_c\alpha_s}{\pi}\, \ln^2 x \;\gtrsim\; 1 \,.$$

Now what remains to be done is to substitute our new weird anomalous dimension into (12a) and perform the inverse Mellin transform to find $D(x)$. If there were no QCD parton cascading, we would expect the particle *density* $xD(x)$ to be constant (Feynman plateau). It is straightforward to derive that plugging in the DLA anomalous dimension (19) results in the plateau density increasing with Q and with a maximum (hump) "midway" between the smallest and the highest parton energies, namely, at $x_{\max} \simeq \sqrt{Q_0/Q}$. The subleading MLLA effects shift the hump to smaller parton energies,

$$\ln\frac{1}{x_{\max}} = \ln\frac{Q}{Q_0}\left(\frac{1}{2} + c\cdot\sqrt{\alpha_s} + \ldots \right) \simeq 0.65\, \ln\frac{Q}{Q_0} \,,$$

with c a known analytically calculated number. Moreover, defying naive probabilistic intuition, the softest particles do not multiply at all. The density of particles (partons) with $x \sim Q_0/Q$ stays constant while that of their more energetic companions increases with the hardness of the process Q.

This is a powerful legitimate consequence of pQCD coherence. We turn now to another, no less powerful though less legitimate, consequence.

[13] It suffices to use the next-to-leading approximation to the splitting function (11) and to keep the subleading correction coming from differentiation of the running coupling in (16) to get the more accurate MLLA anomalous dimension γ_ω.

4.3 Coherent damping of the Landau singularity

The time-like DLA anomalous dimension (19), as well as its MLLA improved version, has a curious property. Namely, in sharp contrast with DIS, it allows the momentum integral in (12a) to be extended to very small scales. Even integrating down to $Q_0 = \Lambda$, the position of the "Landau pole" in the coupling, one gets a finite answer for the distribution (the so-called *limiting spectrum*), simply because the $\sqrt{\alpha_{\rm s}(k)}$ singularity happens to be integrable!

It would have been poor taste to trust this formal integrability, since the very PT approach to the problem (selection of dominant contributions, parton evolution picture, etc.) relied on $\alpha_{\rm s}$ being a numerically small parameter. However, the important thing is that, due to time-like coherence effects, the (still perturbative but "smallish") scales, where $\alpha_{\rm s}(k) \gg \omega^2$, contribute to γ basically in a ω-*independent* way, $\gamma + \omega/2 \propto \sqrt{\alpha_{\rm s}(k)} \neq f(\omega)$. This means that "smallish" momentum scales k affect only an overall *normalization* without affecting the *shape* of the x-distribution.

Since such is the rôle of the "smallish" scales, it is natural to expect the same for the truly small – non-perturbative – scales where the partons transform into the final hadrons. This hypothesis (LPHD) reduces, mathematically, to the statement (guess) that the NP factor in (12a) has a finite $\omega \to 0$ limit:

$$D_\omega^{(h)}(Q_0) \;\to\; K^h = \text{const}, \quad \omega \to 0\,.$$

In other words, decreasing Q_0 we start to lose control of the interaction intensity of a parton with a given x and $k_\perp \sim Q_0$ (and thus may err in the overall production rate). However, such partons do not branch any further, do not produce any soft offspring, so that the *shape* of the resulting energy distribution remains undamaged. We see that colour coherence plays here a crucial rôle.

Thus, according to LPHD, the x-shape of the so-called "limiting" parton spectrum which is obtained by formally setting $Q_0 = \Lambda$ in the evolution equations, should be mathematically similar to that of the inclusive distribution of (light) hadrons h. Another essential property is that the "conversion coefficient" K^h should be a true constant independent of the hardness of the process producing the jet under consideration.

4.4 Another world

It is important to realize that knowing the spectrum of *partons*, even knowing it to be a CIS quantity in certain sense, does not guarantee on its own the predictability of the *hadron* spectrum. It is easy to imagine a world in which each quark and gluon with energy k produced at the small-distance stage of the process would have dragged behind its personal "string" giving birth to $\ln k$ hadrons in the final state (the Feynman plateau). The hadron yield then would be given by a convolution of the parton distribution with a logarithmic energy distribution of hadrons from the parton fragmentation.

If it were the case, each parton would have contributed to the yield of non-relativistic hadrons and the hadron spectra would peak at much smaller energies, $\xi_{\rm max} \simeq \ln Q$, in a spectacular difference with experiment.

Physically, it could be possible if the non-perturbative (NP) hadronization physics did not respect the basic rule of the perturbative[{1}] dynamics, namely, that of colour coherence.

There is nothing wrong with the idea of convoluting time-like parton production in jets with the inclusive NP parton→hadron fragmentation function, the procedure which is similar to convoluting space-like parton cascades with the NP initial parton distributions in a target proton to describe DIS structure functions.

What the nature is telling us, however, is that this NP fragmentation has a finite multiplicity and is *local* in the momentum space. Similar to its PT counterpart, the NP dynamics has a short memory: the NP conversion of partons into hadrons occurs locally in the configuration space.

The fact that even a legitimate finite smearing due to hadronization effects does not look mandatory

makes one think of a deep duality between the hadron and quark-gluon languages applied to such a global characteristic of multi-hadron production as an inclusive energy spectrum.

The message is, that "brave gluon counting", that is applying the PT$^{\{1\}}$ language all the way down to very small transverse momentum scales, indeed reproduces the x- and Q-dependence of the observed inclusive energy spectra of charged hadrons (pions) in jets.

Experimental evidence in favour of LPHD is mounting, and so is the list of challenging questions to be answered by the future quantitative theory of colour confinement.

5. PROBING THE NON-PERTURBATIVE$^{\{1\}}$ DOMAIN WITH PERTURBATIVE$^{\{1\}}$ TOOLS

> *There is no heresy in it, and if not manifestly defined in Scripture, yet it is an opinion of good and wholesome use in the cours and actions of a man's life, and would serve as an hypothesis to solve many doubts whereof common philosophy affordeth no solution.*
>
> Sir Thomas Browne, ca 1635 [43]

Let us discuss the test case of the total cross section of e^+e^- annihilation into hadrons as an example.

To predict $\sigma^{\rm tot}_{\rm hadr}$ one calculates instead the cross sections of quark and gluon production, $(e^+e^- \to q\bar{q})$ + $(e^+e^- \to q\bar{q}+g)$ + etc., where quarks and gluons are being treated *perturbatively*$^{\{1\}}$ as real (unconfined, flying) objects. The *completeness* argument provides an apology for such a brave substitution:

> Once instantaneously produced by the electromagnetic (electroweak) current, the quarks (and secondary gluons) have nowhere else to go but to convert, *with unit probability*, into hadrons in the end of the day.

This *guess* looks rather solid and sounds convincing, but relies on two hidden assumptions:

1. The allowed hadron states should be numerous as to provide the quark-gluon system the means for "regrouping", "blanching", "fitting" into hadrons.
2. It implies that the "production" and "hadronization" stages of the process can be separated and treated independently.

1. To comply with the first assumption the annihilation energy has to be taken large enough, $s \equiv Q^2 \gg s_0$. In particular, it fails miserably in the resonance region $Q^2 \lesssim s_0 \sim 2M^2_{\rm res}$. Thus, the point-by-point correspondence between hadron and quark cross sections,

$$\sigma^{\rm tot}_{\rm hadr}(Q^2) \stackrel{?}{=} \sigma^{\rm tot}_{q\bar{q}+X}(Q^2),$$

cannot be sustained except at very high energies. It can be traded, however, for something more manageable.

Invoking the dispersion relation for the photon propagator (causality $\Longrightarrow$ analyticity) one can relate the *energy integrals* of $\sigma_{\rm tot}(s)$ with the correlator of electromagnetic currents in a deeply Euclidean region of large *negative* Q^2. The latter corresponds to small space-like distances between interaction points, where the perturbative$^{\{1\}}$ approach is definitely valid.

Expanding the answer in a formal series of local operators, one arrives at the structure in which the corrections to the trivial unit operator generate the usual perturbative $^{\{1\}}$ series in powers of α_s (logarithmic corrections), whereas the vacuum expectation values of dimension-full (Lorentz- and colour-invariant) QCD operators provide non-perturbative $^{\{1\}}$ corrections suppressed as powers of Q.

This is the realm of the famous ITEP sum rules [44] which proved to be successful in linking the parameters of the low-lying resonances in the Minkowsky space with expectation values characterising a non-trivial structure of the QCD vacuum in the Euclidean space. The leaders among them are the gluon

condensate $\alpha_s G^{\mu\nu} G_{\mu\nu}$ and the quark condensate $\langle\psi\bar{\psi}\rangle \langle\psi\bar{\psi}\rangle$ which contribute to the total annihilation cross section, symbolically, as

$$\sigma^{\rm tot}_{\rm hadr}(Q^2) - \sigma^{\rm tot}_{q\bar{q}+X}(Q^2) \;=\; c_1 \frac{\alpha_s G^2}{Q^4} + c_2 \frac{\langle\psi\bar{\psi}\rangle^2}{Q^6} + \dots \,. \tag{20}$$

2. Validating the second assumption also calls for large Q^2. To be able to separate the two stages of the process, it is *necessary* to have the production time of the quark pair $\tau \sim Q^{-1}$ to be much smaller than the time $t_1 \sim \mu^{-1} \sim 1\,{\rm fm}/c$ when the first hadron appears in the system. Whether this condition is *sufficient*, is another valid question. And a tricky one.

Strictly speaking, due to gluon bremsstrahlung off the primary quarks, the perturbative production of secondary gluons and $q\bar{q}$ pairs spans an immense interval of time, ranging from a very short time, $t_{\rm form} \sim Q^{-1} \ll t_1$, all the way up to a macroscopically large time $t_{\rm form} \lesssim Q/\mu^2 \gg t_1$.

This accompanying radiation is responsible for formation of hadron jets. It does not, however, affect the total cross section. It is the rare hard gluons with large energies and transverse momenta, $\omega \sim k_\perp \sim Q$, that only matter. This follows from the celebrated Bloch-Nordsieck theorem which states that the logarithmically enhanced (divergent) contributions due to real production of *collinear* ($k_\perp \ll Q$) and *soft* ($\omega \ll Q$) quanta cancel against the corresponding virtual corrections:

$$\sigma^{\rm tot}_{q\bar{q}+X} = \sigma_{Born}\left(1 + \frac{\alpha_s}{\pi}\left[\infty_{\rm real} - \infty_{\rm virtual}\right] + \dots\right) = \sigma_{Born}\left(1 + \frac{3}{4}\frac{C_F\alpha_s(Q^2)}{\pi} + \dots\right).$$

The nature of the argument is purely perturbative{1}. Can the Bloch-Nordsieck result hold beyond pQCD?

Looking into this problem produced an extremely interesting result that has laid a foundation for the development of perturbative{2} techniques aimed at analysing non-perturbative{1} effects.

V. Braun, M. Beneke and V. Zakharov have demonstrated that the real-virtual cancellation actually proceeds much deeper than was originally expected [45].

Let me briefly sketch the idea.

- First one introduces an infrared cutoff (non-zero gluon mass m) into the calculation of the radiative correction.
- Then, one studies the dependence of the answer on m. A CIS quantity, by definition, remains finite in the limit $m \to 0$. This does not mean, however, that it is insensitive to the modification of gluon propagation. In fact, the m-dependence provides a handle for analysing the *small transverse momenta* inside Feynman integrals. It is this region of integration over parton momenta where the QCD coupling gets out of perturbative{1} control and the genuine non-perturbative physics comes onto the stage.
- Infrared sensitivity of a given CIS observable is determined then by the first non-vanishing term which is *non-analytic* in m^2 at $m = 0$.

In the case of one-loop analysis of $\sigma_{\rm tot}$ that we are discussing, one finds that in the sum of real and virtual contributions not only the terms singular as $m \to 0$,

$$\ln^2 m^2\,, \qquad \ln m^2\,,$$

cancel, as required by the Bloch-Nordsieck theorem, but that the cancellation extends [45, 46] also to the whole tower of *finite* terms

$$m^2 \ln^2 m^2\,, \quad m^2 \ln m^2\,, \quad m^2\,, \quad m^4 \ln^2 m^2\,, \quad m^4 \ln m^2\,.$$

In our case the first *non-analytic* term appears at the level of m^6:

$$\frac{3}{4}\frac{C_F\alpha_s}{\pi}\left(1 + 2\frac{m^6}{Q^6}\ln\frac{m^2}{Q^2} + \mathcal{O}\left(m^8\right)\right).$$

It signals the presence of the non-perturbative$^{\{1\}}$ Q^{-6} correction to $\sigma_{\rm tot}$, which is equivalent to that of the ITEP quark condensate in (20). (The gluon condensate contribution emerges in the next order in α_s.)

A similar program can be carried out for other CIS quantities as well, including intrinsically Minkowskian observables which address the properties of the final state systems and, unlike the total cross sections, do not have a Euclidean image.

5.1 Event shapes in e^+e^- annihilation

The most spectacular non-perturbative$^{\{1\}}$ results were obtained for a broad class of *event shape variables* (like Thrust T, C-parameter, Broadening B, and alike). As has long been expected [47], these observables possess relatively large $1/Q$ confinement correction effects.

Employing the "gluon mass" as a large-distance trigger was formalised by the so-called dispersive method [48]. There it was also suggested to relate new non-perturbative$^{\{1\}}$ dimensional parameters with the momentum integrals of the effective QCD coupling α_s in the infrared domain. Though it remains unclear how such a coupling can be rigorously defined from the first principles, the *universality* of the coupling makes this guess verifiable and therefore legitimate. All the observables belonging to the same class $1/Q^p$ with respect to the nature of the leading non-perturbative$^{\{1\}}$ behaviour, should be described by the same parameter.

***Whose* coupling is it?** Approaching the borderline where PT$^{\{1\}}$ gluons are about to disappear, one may talk about *gluers* as carriers of the NP$^{\{1\}}$ PT$^{\{2\}}$ colour field. A formal definition of gluers is as follows.

A *gluer* is a miserable *gluon* which hasn't got enough time to truly behave like one because its hadronization time is comparable with its formation time, $t_{\rm form.} \simeq \omega/k_\perp^2 \sim t_{\rm hadr.} \simeq \omega R_{\rm conf.}^2$. Contrary to respectful PT gluons with small transverse size, $k_\perp \gg R_{\rm conf.}^{-1}$, gluers are not "partons": they do not participate in PT$^{\{2\}}$ cascading (don't multiply). According to the above definition, gluers have *finite transverse momenta* (though may have *arbitrarily large energies*).

The rôle of gluers consists in providing comfortable conditions for *blanching* colour parton ensembles (jets) produced in hard interactions. By examining the space-time picture of the parton formation [7] one can convince oneself that formation of a gluer is a signal of hadronization process taking place in a given space-time region, *locally* in the configuration space (recall the problem of soft confinement!)

Having transverse momenta of the order of the inverse confinement scale makes their interaction strength potentially large, $\alpha_{\rm s}(R_{\rm conf.}^{-1}) \sim 1$. A uniform distribution in (pseudo)rapidity, together with finite transverse momenta with respect to the direction of the charge (jet, subjet) makes the gluers representatives of the Lund string [21].

The basic idea (see [48] and references therein) was to relate (uncalculable) NP$^{\{1\}}$ corrections to (calculable) PT$^{\{1\}}$ cross sections/observables through the intensity of *gluer* emission – $\alpha_{\rm s}$ in the infrared domain. In particular, an extended family of event shapes (not to forget energy-energy correlations [24], out-of-plane transverse momentum flows [49] etc.) can be said to measure the first moment of the perturbative$^{\{2\}}$ non-perturbative$^{\{1\}}$ coupling,

$$\alpha_0 \;\equiv\; \frac{1}{\mu_I}\int_0^{\mu_I} dk\,\alpha_s(k^2), \qquad \mu_I = 2\,{\rm GeV}, \tag{21}$$

where the choice of the infrared boundary value μ_I is a matter of convention.

The Broadening story: a mistake but not an error. A wonderful example of a mistake, in a sense of the introductory Section, was provided by the recent turbulent story of the Broadening measure.

B is defined as the sum of the moduli of transverse momenta of particles wrt the Thrust axis of the e^+e^- annihilation event. Originally the NP contribution to B was naturally thought to accumulate

gluers with rapidities up to $\log Q$. As a result theorists expected the PT$^{\{1\}}$ distribution in B to acquire a $\ln Q$-enhanced NP shift.

The data however refused to go along [50, 51]. Fits based on the $\log Q$-enhanced shift were bad and produced too small a value of $\alpha_s(M_Z)$, and the NP parameter α_0 inconsistent with that extracted from analyses of the Thrust and C-parameter means and distributions.

Universality of α_0 and thus viability of the very notion of the infrared-finite coupling was seriously questioned. Pedro Movilla Fernández who reported the findings of the resurrected JADE collaboration at the QCD-1998 conference in Montpellier did not stop at that [51]. He came up with the study of "what is wrong" with the Broadening measure as such. A comparison of MC-generated B distributions at parton and hadron levels produced an unexpected result. While the T and C cases showed the expected shift patterns, the bump of the *hadronic* B distribution turned out to be not only shifted but also squeezed as compared with the *partonic* one. This looked pretty anti-intuitive: how can one get a *narrower* distribution after a smearing due to hadron production?

A solution came with recognition of the fact that the B measure is more sensitive to quasi-collinear emissions than other event shapes, and is therefore strongly affected by an interplay between PT and NP radiation effects. With account of the omnipresent PT gluon radiation, the *direction of the quark* that forms the jet under consideration can no longer be equated with the direction of the Thrust axis (employed in the definition of B). As a result, the range of the pseudorapidity of gluers contributing to the NP shift decreases from $\ln Q$ downto $\ln(1/B)$. The shift becomes B-dependent giving rise to a narrower distribution all right [52].

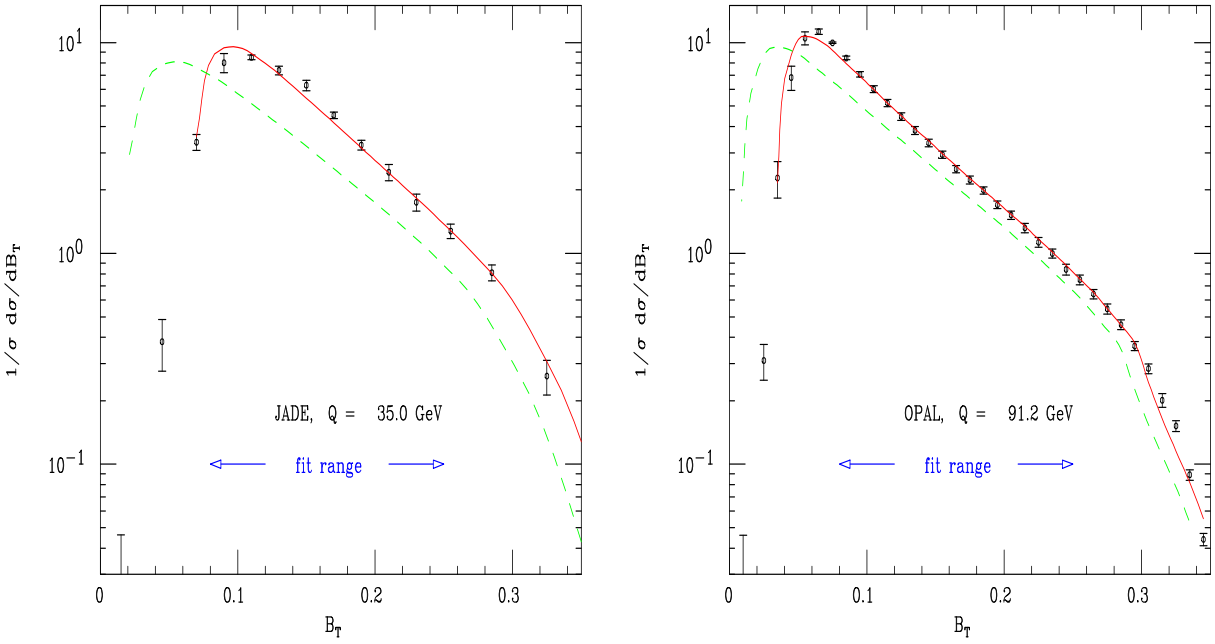


Fig.12: Perturbative (dashed) and NP-shifted/squeezed (solid) total Broadening distributions [52].

Three lessons were drawn from the Broadening drama.

- A pedagogical lesson the Broadenings taught, was that of the importance of keeping an eye on PT gluons when discussing effects of NP gluers. An example of a powerful interplay between the two sectors was recently given by the study of the energy-energy correlation in e^+e^- in the back-to-back kinematics [24]. The leading $1/Q$ NP contribution was shown to be promoted by PT$^{\{1\}}$ radiation effects to a much slower falling correction, $Q^{-0.32\text{-}0.36}$.
- The physical output of the proper theoretical treatment was the restoration of the universality picture: within a reasonable 20% margin, the NP parameters extracted from T, C and B means and distributions were found to be the same.

- A gnostic output was also encouraging. Phenomenology of NP contributions to event shapes has shown that it is a robust field with a high discriminative power: it does not allow one to be misled by theorists.

Looking forward to the Conclusions Section please keep in mind the key words “resurrected collaboration” and “error-free LEP data”.

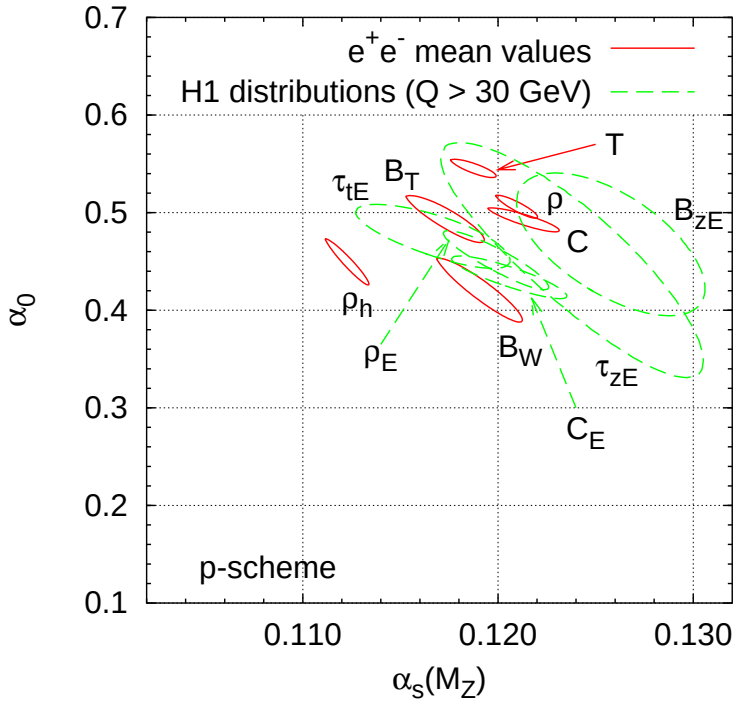
5.2 DIS jet shapes and non-global logs

Theoretical study of jet shapes in DIS was pioneered by Vito Antonelli, Mrinal Dasgupta and Gavin Salam by the derivation of resummed PT prediction for the Thrust distribution of the current fragmentation jet [53]. Two years later the Broadening measure followed [54].

On the way from T to B, Dasgupta and Salam stumbled upon a new source of significant purely perturbative$^{\{1\}}$ next-to-leading (SL) corrections which was previously overlooked in the literature. They dubbed these corrections “non-global logs” [55][14].

The final wisdom about DIS jet shapes can be found in [56].

5.3 On the universality of the infrared coupling – 2003



One standard deviation ellipses in the $\alpha_s(M_Z) - \alpha_0$ plane [57].

6. Conclusions

Dr. John Dee, a British scholar, mathematician, an alchemist, hermeticist, cabalist and adept in esoteric and occult lore, was Queen Elizabeth’s philosopher and astrologer.

[14] To put a long story short, the origin of these non-global effects has to do with the fact that in DIS one is forced to deal with characteristics of a single jet rather than shapes of the hard event as a whole. In other words, one resticts the measurements to a part of the available phase space; see [55] for details.

A visionary of the Empire, he coined the word Brittannia, was the first to apply Euclidean geometry to navigation, trained the great navigators, developed a plan for the British Navy and established the legal foundation for colonizing North America. Dee wrote a famous Mathematical Preface to his translation of Euclid in which he systematised future development of the sciences based on mathematics. He also extensively practiced as an angel conjuror (with Edward Kelley for many years his *skryer*) and, some say [58], was the one who in 1588 "*put a hex on the Spanish Armada which is why there was bad weather and England won*"[a].

John Dee (1527 – 1608)

[a] Speaking of practicality of communicating with angels: "[John Dee] *also speaks of seeing the sea, covered with many ships. Uriel warns them* [Dee & Kelley] *that foreign Powers are providing ships 'against the welfare of England, which shall shortly be put in practice.' . . . The defeat of the Spanish Armada took place . . . four years after this vision.*" [2]

The volume of this writeup prevents us from going deeper into the fascinating story of John Dee's life and endeavours. Dee's story is relevant to the present lectures: there is an important message to take home.

The "crystal egg" [15] John Dee used to communicate with spirits (and the cherub he identified as Archangel Uriel, in particular) rests, reportedly, in the British Museum along with his conjuring table [58, 59]. These were John Dee's *detector gadgets* if you please. More importantly, the *experimental data* that Dee collected in 1580's, his manuscripts and diaries are also being carefully preserved in the British Library [4, 60].

One might question the value of Dee's *De Heptarchiæ Mysticæ* (i.e. Detailed instructions for communicating with angels and employing their aid for practical purposes) as a source of inspiration and knowledge acquisition for the future. What cannot be questioned, however, is that the *experimental data* collected by LEP exactly 400 years after Dr. Dee was communicating with Uriel & Co, will remain, for many a year to come, an unmatched source of knowledge about the physics of hadrons.

Will there be a caring "*British Library*" to preserve LEP "diaries" for theorists who will sooner or later come close to deciphering the "Enochian Alphabet" of QCD confinement?

> *An angel tells him* [Dr. Dee] *they are to be "rocks in faith." "While sorrow be meansured thou shalt bind up thy fardell." He is not to seek to know the mysteries till the very hour he is called.* "**Can you bow to Nature and not honour the workman?**" [2]

ACKNOWLEDGEMENTS

I would like to thank Vladimir Dokchitser, Alexei Safonov and Gavin Salam for their help in the long course of preparing this writeup and to Nick Ellis for his patience in receiving it, as well as for the perfect organisation of the School. I am grateful to Jean Iliopoulos for Greek lessons and to Natalia Afanassieva for the enlightenment in regards of the rôle of Arch. Uriel in the development of fundamental science.

References

[1] Yu.L. Dokshitzer, *Perturbative QCD for Beginners*, CERN–Dubna School, Dubna, 1995.

[2] Charlotte Fell-Smith, *John Dee*, London, Constable & Co Ltd, 1909.

[3] "The highest initiatory teachings belonging to the New Age . . . are under the direction of Uriel.", Corinne Heline, *The Blessed Virgin Mary*, New Age Press, 1971, p. 110.

[15] another source tells of "an Aztec/Mayan obsidian show stone"

"Uriel personifies the Divine Fire that comes down from the 3rd Aspect of Deity – Universal Mind – penetrating each plane until it reaches the physical."
Alice A. Bailey, *Treatise on Cosmic Fire*, Lucis Trust, p. 605;
For further details see www.uriel.org/Uriel/Archangels.htm.

[4] John Dee, *Compendium Heptarchiæ Mysticæ*, British Library catalogue number Additional MS. 36674; Edited by Joseph H. Peterson, 1999; www.esotericarchives.com/dee/chm.htm.

[5] M. Beneke, Phys. Rep. **317** (1999) 1.

[6] V.A. Khoze and W. Ochs, Int. J. Mod. Phys. **A 12** (1997) 2949.

[7] Yu.L. Dokshitzer and S.I. Troyan, "*Asymptotic Freedom and Local Parton-Hadron Duality in QCD Jet Physics*". 19^{th} Leningrad Winter School (in Russian), vol. 1, p. 144, Leningrad 1984;
Ya.I. Azimov et al., Zeit. Phys. **C 27** (1985) 65;
Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller and S.I. Troyan, *Basics of Perturbative QCD*, ed. J. Tran Thanh Van, Gif-sur-Yvette, Editions Frontiéres, 1991.

[8] G. Altarelli and G. Parisi, Nucl. Phys. **B 126** (1977) 298.

[9] Yu.L. Dokshitzer, Sov. Phys. JETP **46** (1977) 641.

[10] A.H. Mueller, Phys. Lett. **104 B** (1981) 161 ;
V.S. Fadin, Yad. Fiz. **37** (1983) 408.

[11] I.M. Dremin and J.W. Gary, Phys. Rep. **349** (2001) 301.

[12] G. Sterman and S. Weinberg, Phys. Rev. Lett. **39** (1977) 1436.

[13] I.T. Dyatlov, private communication.

[14] L.D. Landau, A. Abrikosov and I. Khalatnikov, Nuovo Cimento, Supplement **3** (1956) 80;
also in *Collected Papers of L.D. Landau*, ed. D ter Haar, Pergamon Press, 1965, No. 89, p. 701.

[15] L.D. Landau, *Theoretical Physics in the Twentieth Century, a Memorial Volume to Wolfgang Pauli*, ed. M. Fierz and V.F. Weisskopf, Interscience, New York, 1960, p. 245;
also in *Collected Papers*, ibid., No. 100, p. 800.

[16] F.J. Dyson, Sci. Amer. **199** (1958) 74.

[17] I. Pomeranchuk, Akad. Nauk SSSR, **103** (1955) 1005; **104** (1955) 51; **105** (1955) 461;
A.A. Abrikosov et. al, Phys. Rev. **111** (1958) 321; also in *Collected Papers*, ibid., No. 96, p. 761.

[18] G. Källén and W. Pauli, Math.–fys. Medd. **30** (1955) No. 7.

[19] I.B. Khriplovich, Yad. Fiz. **10** (1969) 409 [Sov. J. Nucl. Phys. **10** (70) 235].

[20] V.N. Gribov, Nucl. Phys. **B 139** (1978) 1.

[21] Bo Andersson, G. Gustafson and C. Peterson, Phys. Scripta **19** (1979) 184;
for a detailed review see Bo Andersson, "*The Lund Model*", Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol. **7** (1997) pp. 1–471.

[22] Ya.I. Azimov et al., Phys. Lett. **165 B** (1985) 147;
Yu.L. Dokshitzer, S.I. Troyan and V.A. Khoze, Sov. J. Nucl. Phys. **47**(1988) 881

[23] C.L. Basham, L.S. Brown, S.D. Ellis and S.T. Love Phys. Rev. Lett. **41** (1978) 1585; Phys. Rev. **D 17** (1978) 2298.

[24] Yu.L. Dokshitzer, G. Marchesini and B.R. Webber, JHEP 9907:012, 1999.

[25] S. Bethke, "α_s 2002," High-Energy Physics Int. Conference in Quantum Chromodynamics, Montpellier, France, July 2002, hep-ex/0211012.

[26] V. Matveev, R. Muradyan and A. Tavkhelidze, Lett. Nuovo Cim. **7** (1973) 719;
S.J. Brodsky and G.R. Farrar, Phys. Rev. **D 11** (1975) 1309.

[27] P. Hoyer, "*Soft perturbative QCD*," 6th JLAB Workshop on Exclusive and Semiexclusive Processes at High Momentum Transfer, Newport News, VA, May 1999, hep-ph/9908501.

[28] C. Bochna et al., Phys. Rev. Lett. **81** (1998) 4576.

[29] D. Haidt, "*The transition from $\sigma(\gamma^* p)$ to $\sigma(\gamma p)$*," DIS Workshop, Bologna, Italy, April 2001, p. 287.

[30] V.N. Gribov, "*QCD at Large and Short Distances*," Eur. Phys. J. **C 10** (1999) 71–90;
"*The Theory of Quark Confinement*", ibid., 91–105.

[31] I.Ya. Pomeranchuk and Ya.A. Smorodinsky, Journ. Fiz. USSR **9** (1945) 97.

[32] P. Abreu, et al., DELPHI Collab., Phys. Lett. **449 B** (1999) 383; Eur. Phys. J. **C 13** (2000) 573.

[33] By DELPHI Collaboration (K. Hamacher et al., DELPHI Collar., Int. Europhysics Conference on High-Energy Physics (EPS-HEP 99), Tampere, Finland, July 1999.

[34] R. Akers et al., OPAL Collab., Zeit. Phys. **C 68** (1995) 531.

[35] M.A. Buican, V.A. Khoze and W. Ochs, hep-ph/0212189.

[36] A.H. Mueller, Nucl. Phys. **B 213** (1983) 85. Erratum quoted, ibid., **B 241** (1984) 141.

[37] C.P. Fong and B.R. Webber, Nucl. Phys. **B 355** (1991) 54.

[38] Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller and S.I. Troyan, Rev. Mod. Phys. **60** (1988) 373.

[39] D. Goulianos, CDF Collab., 32nd Recontres de Moriond, Les Arcs, France, March 22, 1997.

[40] A.N. Safonov, CDF Collab., Int. Euroconference in Quantum Chromodynamics, Montpellier, France, July 7, 1999.

[41] A. Safonov, CDF Collab., *Recent results on jet fragmentation from CDF*", FERMILAB-CONF-02-082-E, Jun 2002. 8pp. Contributed to Lake Louise Winter Institute on Fundamental Interactions (LLWI 02), Lake Louise, Alberta, Canada, 17-23 Feb 2002.

[42] G. Camici and M. Ciafaloni, Phys. Lett. **395 B** (1997) 118;
A.H. Mueller, Phys. Lett. **396 B** (1997) 251.

[43] Sir Thomas Browne, *Religio Medici*, New York: P.F. Collier & son Company, 1909-14.

[44] *Vacuum Structure and QCD Sum Rules: Reprints*, ed. M.A. Shifman
(North-Holland, 1992: Current Physics, Sources and Comments, v. 10).

[45] M. Beneke, V.M. Braun and V.I. Zakharov, Phys. Rev. Lett. **73** (1994) 3058.

[46] M. Beneke and V.M. Braun, Nucl. Phys. **B 454** (1995) 253.

[47] B.R. Webber, Phys. Lett. **339 B** (1994) 148; see also Proc. Summer School on Hadronic Aspects of Collider Physics, Zuoz, Switzerland, August 1994, ed. M.P. Locher (PSI, Villigen, 1994);
Yu.L. Dokshitzer and B.R. Webber, Phys. Lett. **352 B** (1995) 451;
A.V. Manohar and M.B. Wise, Phys. Lett. **344 B** (1995) 407;
R. Akhoury and V.I. Zakharov, Phys. Lett. **357 B** (1995) 646.

[48] Yu.L. Dokshitzer, G. Marchesini and B.R. Webber, Nucl. Phys. **B 469** (1996) 93.

[49] A. Banfi, et al., Phys.Lett. **508 B** (2001) 269.

[50] C. Adloff et al., H1 Collab., Phys. Lett. **406 B** (1997) 256.

[51] P.A. Movilla Fernández, JADE Collab., QCD Euroconference, Montpellier, France, July 1998, Nucl. Phys. Proc. Suppl. **74** (1999) 384.

[52] Yu.L. Dokshitzer, G. Marchesini and G.P. Salam, [hep-ph/9812487]. Eur. Phys. J. **C 1** (1999) 3.

[53] V. Antonelli, M. Dasgupta and G.P. Salam, JHEP **0002** (2000) 001.

[54] M. Dasgupta and G.P. Salam, Eur. Phys. J. **C 24** (2002) 213.

[55] M. Dasgupta and G.P. Salam, Phys. Lett. **512 B** (2001) 323.

[56] M. Dasgupta and G.P. Salam, JHEP **0208** (2002) 032.

[57] G.P. Salam, private communication across the corridor.

[58] Paul Lee, *About Dr. John Dee*, www.johndee.org/DEE.html.

[59] Colin Wilson, *The Occult: A History*, Random House, 1971, p. 273.

[60] John Dee, *De Heptarchiæ Mysticæ*, British Library catalogue number Sloane 3191.