

Open and Closed Superstring Disk Amplitudes

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Workshop on the interface between N=4 SUSY and QCD
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Outline

I. Progress in higher point open superstring amplitudes (tree) (in particular multi-gluon scattering)

- SUSY Ward identities relating various amplitudes to all orders in α'
- Compact representation for amplitudes with many external massless string states

II. Open & closed vs. pure open string disk amplitudes

- Sort of generalized KLT on the disk

Review: perturbative QCD amplitudes (tree)

- Compact representation for amplitudes with many external particle legs, by use of spinor basis ($\lambda, \tilde{\lambda}$)

e.g.: MHV N -gluon amplitude

$$A(g_1^-, g_2^-, g_3^+, g_4^+, \dots, g_N^+) = (\sqrt{2}g_{YM})^{N-2} \text{Tr}(T^1 \dots T^N) \frac{\langle 12 \rangle^4}{\prod_{k=1}^N \langle k k+1 \rangle}$$

Parke, Taylor 1986
Berends, Giele 1989

$$\langle ij \rangle \sim \sqrt{k_i k_j}$$

- SUSY Ward identities relating various amplitudes

Grisaru, Pendleton, van Nieuwenhuizen (1977); Parke, Taylor, (1985); ...;
Bianchi, Elvang, Freedman (2008)

$$*e.g.:* A(g_1^-, g_2^-, g_3^+, g_4^+, \dots, g_N^+) = \frac{\langle 12 \rangle^2}{\langle 34 \rangle^2} A(\phi_1^-, \phi_2^-, \phi_3^+, \phi_4^+, g_5^+, \dots, g_N^+)$$

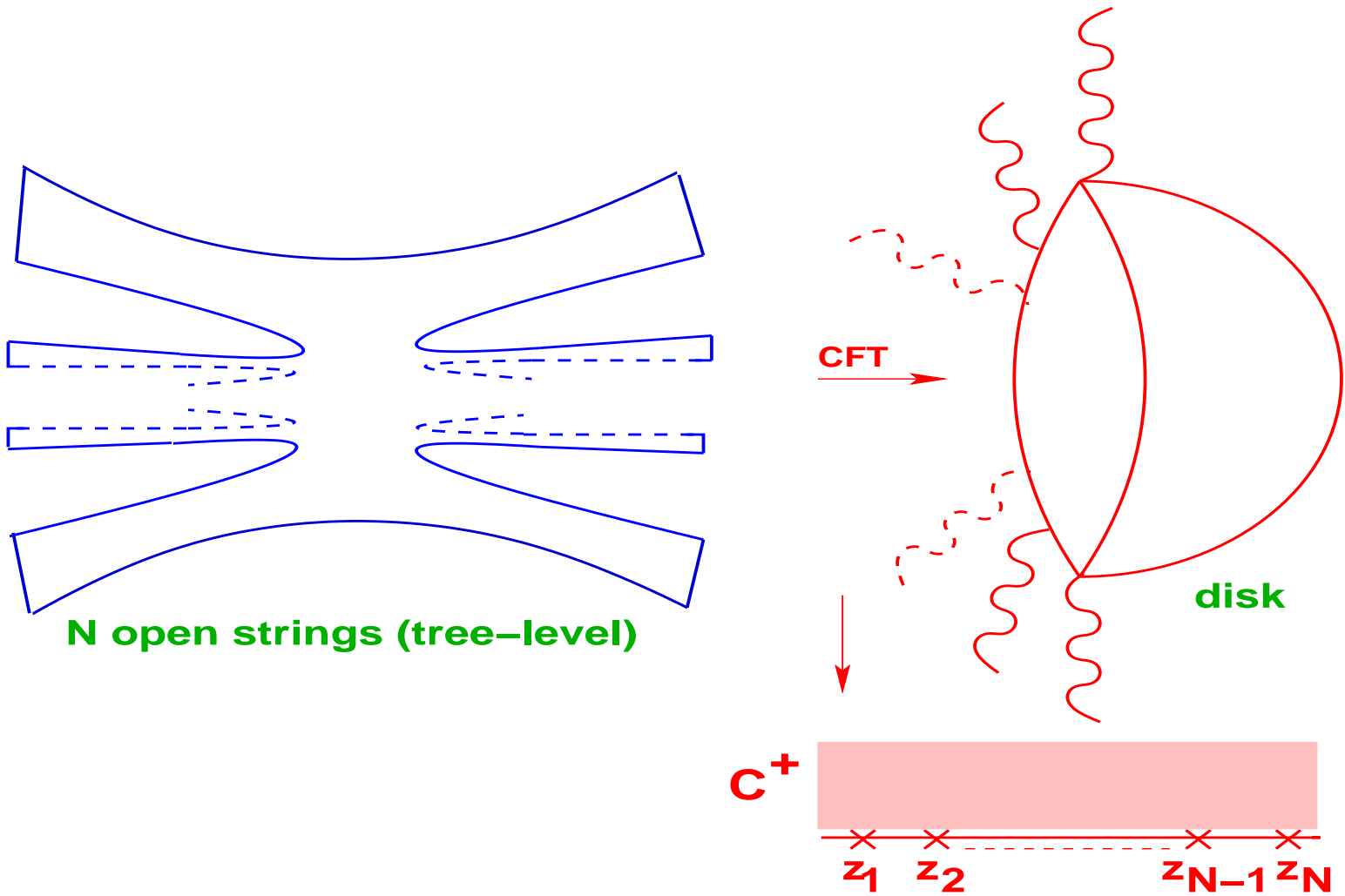
- More efficient techniques to compute QCD amplitudes; CSW, BCFW rules

Higher point amplitudes in superstring theory (tree)

Do similar simple properties also hold for superstring (tree)–amplitudes ?

- SUSY Ward identities relating various amplitudes to all orders in α'
St. St., T.R. Taylor, arXiv:0708.0574
- Compact representation for amplitudes with many external massless string states
St. St., T.R. Taylor, arXiv:0711.4354
- More efficient technique to compute superstring amplitudes; recursion relations
work in progress

Tree-level scattering of N open strings



Full N -point string S -matrix

$$A(1, 2, \dots, N; \alpha') = \sum_{\text{kinematics } \mathcal{K}_{\mathcal{I}}} \mathcal{K}_{\mathcal{I}} F \begin{bmatrix} n_a^I \\ n_{ab}^I \end{bmatrix}$$

generalized Euler integrals \iff
multiple Gaussian hypergeometric functions

$$F \begin{bmatrix} n_a^I \\ n_{ab}^I \end{bmatrix} = \int_0^1 dx_1 \dots \int_0^1 dx_{N-3} \prod_{a=1}^{N-3} x_a^{1+a-N+n_a} \prod_{b=a}^{N-3} x_a^{2\alpha' k_{b+3} \left(k_1 + \sum_{j=a+3}^{b+2} k_j \right)}$$

$$\times \left(1 - \prod_{j=a}^b x_j \right)^{2\alpha' k_{2+a} k_{3+b} + n_{ab}}, \quad \begin{array}{l} b \geq a = 1, 2, \dots, N-3, \\ n_a, n_{ab} = 0, \pm 1 \end{array}$$

All F 's can be expressed by $(N-3)!$ basis functions

α' -expansion \iff multiple Euler-Zagier sums

Oprisa, St. St., hep-th/0509042
St. St., Taylor, hep-th/0609175

N=4: Veneziano–amplitude

E.g.: $N = 4$

$$\int_0^1 dx x^{2\alpha'k_1k_2-1} (1-x)^{2\alpha'k_2k_3} = \frac{1}{s_1} \frac{\Gamma(1+s_1) \Gamma(1+s_2)}{\Gamma[1+s_1+s_2]}$$
$$= \frac{1}{s_1} - s_2 \zeta(2) + (s_1 + s_2) s_2 \zeta(3) + \dots \quad , \quad \begin{aligned} s_1 &= 2\alpha' k_1 k_2 \\ s_2 &= 2\alpha' k_2 k_3 \end{aligned}$$

Beta–function relevant for scattering of four open strings

Veneziano–amplitude

E.g. $N=7$:

$$\begin{aligned}
 & \int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 dw \frac{x^{s_2} (1-x)^{s_3} y^{t_2} (1-y)^{s_4} z^{t_6} (1-z)^{s_5} w^{s_7} (1-w)^{s_6}}{(1-xy)(1-wz)(1-yz)} (1-wxyz)^{s_1-t_1+t_4-t_7} \\
 & \times (1-xy)^{-s_3-s_4+t_3} (1-wz)^{-s_5-s_6+t_5} (1-yz)^{-s_4-s_5+t_4} (1-wyz)^{s_5+t_1-t_4-t_5} (1-xyz)^{s_4-t_3-t_4+t_7} \\
 & = \mathcal{I}_0 + \mathcal{I}_{1a} (s_1 + s_2 + s_3 + s_4 + s_5 + s_6 + s_7) + \mathcal{I}_{1b} (t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + t_7) + \mathcal{O}(\alpha'^2)
 \end{aligned}$$

with Multiple Euler–Zagier sums $\mathcal{I}_0, \mathcal{I}_{1a}, \mathcal{I}_{1b}$:

$$\begin{aligned}
 \mathcal{I}_0 &= \int \frac{1}{(1-xy)(1-wz)(1-yz)} = \sum_{\substack{n_1, n_2=0 \\ n_3=1}}^{\infty} \frac{1}{n_3 (1+n_1) (n_1+n_2+1) (n_2+n_3)} = \frac{27}{4} \zeta(4) \\
 \mathcal{I}_{1a} &= \int \frac{\ln w}{(1-xy)(1-wz)(1-yz)} = \sum_{\substack{n_1, n_3=1 \\ n_2=0}}^{\infty} \frac{1}{n_1 n_3^2 (n_1+n_2) (n_2+n_3)} = \frac{7}{2} \zeta(5) - 4 \zeta(2) \zeta(3) \\
 \mathcal{I}_{1b} &= \int \frac{\ln y}{(1-xy)(1-wz)(1-yz)} = \sum_{\substack{n_1, n_2=1 \\ n_3=0}}^{\infty} \frac{1}{n_1 n_2 (n_2+n_3) (n_1+n_3)^2} = -\frac{9}{2} \zeta(5) + \zeta(2) \zeta(3)
 \end{aligned}$$

(Space–time) SUSY transformation on world–sheet disk

Setup: World–sheets with boundaries and \mathcal{N} conserved SUSY charges Q_α^I
 $I = 1, \dots, \mathcal{N}$, with $Q_\alpha^I = \oint \frac{dz}{2\pi i} V_\alpha^I(z)$

$$V_\alpha^I(z) = \begin{cases} \mathcal{N} \text{ supercurrents,} \\ \text{extended into double cover} \end{cases}$$

Variation of
open string vertex operator $\mathcal{O}(z)$
under (inf.) SUSY transformation

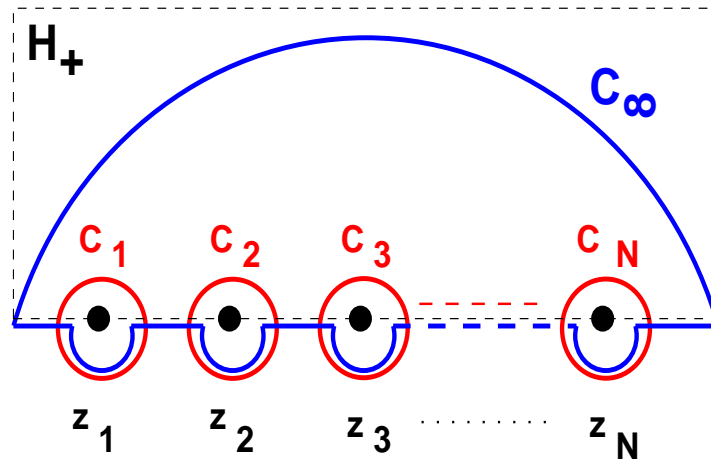
$$[Q^I(\eta_I) , \mathcal{O}(z)] := \oint_{C_z} \frac{dw}{2\pi i} \eta_I^\alpha V_\alpha(w) \mathcal{O}(z)$$

E.g.: $\mathcal{N} = 2$ gauge multiplet $(\phi, \lambda^1, \lambda^2, g)$: $Q^I(\eta_I, \bar{\eta}_I) := \eta_I^\alpha Q_\alpha^I + \bar{\eta}_{\dot{\alpha}I} \bar{Q}^{\dot{\alpha}I}$

$$\begin{aligned} [Q^I(\eta_I, \bar{\eta}_I) , \phi(z, k)] &= i \varepsilon^{IL} \langle \eta_I k \rangle \lambda^L(z, k) \\ [Q^I(\eta_I, \bar{\eta}_I) , \lambda^L(z, k)] &= -\delta^{IL} \langle \eta_I k \rangle g(z, k) - i \varepsilon^{IL} [\eta_I k] \phi(z, k) \\ [Q^I(\eta_I, \bar{\eta}_I) , g(z, k)] &= -[\eta_I k] \lambda^I(z, k) \end{aligned}$$

SUSY transformations of open string vertices

Consider:
$$\oint_{C_\infty} \frac{dw}{2\pi i} \eta_I^\alpha \langle V_\alpha^I(w) V_1(z_1) V_2(z_2) \dots V_N(z_N) \rangle \sim \oint_{C_\infty} \frac{dw}{2\pi i} w^{-2} = 0$$



$$\Rightarrow \sum_{l=1}^N \langle V_1(z_1) \dots V_{l-1}(z_{l-1}) [Q^I(\eta_I^\alpha), V_l(z_l)] V_{l+1}(z_{l+1}) \dots V_N(z_N) \rangle = 0$$

\Rightarrow SUSY Ward identity in superstring theory

N-gluon MHV amplitude in superstring theory

From supersymmetric Ward identities in string theory:

$$\begin{aligned} A(g_1^-, g_2^-, g_3^+, g_4^+, \dots, g_N^+) &= \\ &= \frac{\langle 12 \rangle^2}{\langle 34 \rangle^2} A(\phi_1^-, \phi_2^-, \phi_3^+, \phi_4^+, g_5^+, \dots, g_N^+) \end{aligned}$$

- valid to all orders in α'
- universal to all string compactifications
- any numbers of supersymmetries

This allows for very short expressions for the *N*-gluon MHV amplitude

E.g.: $A(g_1^-, g_2^-, g_3^+, g_4^+, g_5^+) = \text{Tr}(T^1 \dots T^5) (\sqrt{2} g_{YM})^3 \alpha'$ St. St., Taylor
arXiv:0708.0574

$$\times \frac{\langle 12 \rangle^2}{\langle 34 \rangle^2 \langle 45 \rangle} (\langle 41 \rangle [15] \mathbf{K}_1 + \langle 42 \rangle [25] \mathbf{K}_2)$$

N-gluon MHV amplitude in superstring theory

This allows to give very short expressions for the *N*-gluon amplitude:

$$A(g_1^-, g_2^-, g_3^+, g_4^+, g_5^+, g_6^+) = \alpha'^2 \frac{\langle 1 2 \rangle^2}{\langle 3 4 \rangle^2} [\tau(3, 2) K_1 + \tau(2, 3) K_2 + \tau(1, 2) K_3 \\ + \tau(1, 3) K_4 + \tau(2, 1) K_5 + \tau(3, 1) K_6]$$

$$\tau(a, b) = \frac{\langle 4 a \rangle \langle 4 b \rangle [a 5] [b 6]}{\langle 4 5 \rangle \langle 4 6 \rangle}$$

$$A(g_1^-, g_2^-, g_3^+, g_4^+, g_5^+, g_6^+, g_7^+) = \alpha'^3 \frac{\langle 1 2 \rangle^2}{\langle 3 4 \rangle^2} \sum_{I=1}^{24} \tau_I K_I ,$$

$$\tau(a, b, c) = \frac{\langle 4 a \rangle \langle 4 b \rangle \langle 4 c \rangle [a 5] [b 6] [c 7]}{\langle 4 5 \rangle \langle 4 6 \rangle \langle 4 7 \rangle}$$

6-gluon NMHV amplitude in superstring theory

From SUSY Ward identities:

Full 6-gluon NMHV amplitude can be constructed from
three partial subamplitudes (Mangano, Parke 1991)

$$\begin{aligned} A(g_1^-, g_2^-, g_3^+, g_4^+, g_5^-, g_6^+) &= \frac{\alpha'^4}{s_{12}^2 s_{34}^2} \left(y^2 A_g^Y - 2y \alpha_Y A_\lambda^Y + \alpha_Y^2 A_s^Y \right) \\ A(g_1^+, g_2^+, g_3^-, g_4^-, g_5^-, g_6^+) &= \frac{\alpha'^4}{s_{12}^2 s_{34}^2} \left(x^2 A_g^X - 2x \alpha_X A_\lambda^X + \alpha_X^2 A_s^X \right) \\ A(g_1^-, g_2^+, g_3^-, g_4^+, g_5^-, g_6^+) &= \frac{\alpha'^4}{s_{13}^2 s_{24}^2} \left(z^2 A_g^Z - 2z \alpha_Z A_\lambda^Z + \alpha_Z^2 A_s^Z \right) \end{aligned}$$

$$A_g^Y = A(\phi_1^-, \phi_2^-, \phi_3^+, \phi_4^+, g_5^-, g_6^+)$$

$$A_\lambda^Y = A(\phi_1^-, \phi_2^-, \phi_3^+, \phi_4^+, \lambda_5^-, \lambda_6^+)$$

$$A_s^Y = A(\phi_1^-, \phi_2^-, \phi_3^+, \phi_4^+, \phi_5^-, \phi_6^+)$$

St. St., Taylor, arXiv:0711.4354

Final result: 6-gluon NMHV superstring amplitude

$$\begin{aligned}
 A^Y &= A(g_1^-, g_2^-, g_3^+, g_4^+, g_5^-, g_6^+) = \text{Tr}(T^{a_1} T^{a_2} T^{a_3} T^{a_4} T^{a_5} T^{a_6}) \frac{(\sqrt{2} g_{YM})^4 \alpha'^5}{s_5} \\
 &\quad \times \left(N_1^Y \frac{\alpha_Y^2}{s_1^2 s_3^2} + N_2^Y \frac{\beta_Y^2}{s_1^2} + N_3^Y \frac{\gamma_Y^2}{s_3^2} + N_4^Y \frac{\alpha_Y \beta_Y}{s_1^2 s_3} + N_5^Y \frac{\alpha_Y \gamma_Y}{s_1 s_3^2} + N_6^Y \frac{\beta_Y \gamma_Y}{s_1 s_3} \right), \\
 A^X &= A(g_1^+, g_2^+, g_3^-, g_4^-, g_5^-, g_6^+) = \text{Tr}(T^{a_1} T^{a_2} T^{a_3} T^{a_4} T^{a_5} T^{a_6}) \frac{(\sqrt{2} g_{YM})^4 \alpha'^5}{s_5} \\
 &\quad \times \left(N_1^X \frac{\alpha_X^2}{s_1^2 s_3^2} + N_2^X \frac{\beta_X^2}{s_1^2} + N_3^X \frac{\gamma_X^2}{s_3^2} + N_4^X \frac{\alpha_X \beta_X}{s_1^2 s_3} + N_5^X \frac{\alpha_X \gamma_X}{s_1 s_3^2} + N_6^X \frac{\beta_X \gamma_X}{s_1 s_3} \right), \\
 A^Z &= A(g_1^-, g_2^+, g_3^-, g_4^+, g_5^-, g_6^+) = \text{Tr}(T^{a_1} T^{a_2} T^{a_3} T^{a_4} T^{a_5} T^{a_6}) \frac{(\sqrt{2} g_{YM})^4 \alpha'^5}{s_5} \\
 &\quad \times \left(N_1^Z \frac{\alpha_Z^2}{s_{13}^2 s_{24}^2} + N_2^Z \frac{\beta_Z^2}{s_{13}^2} + N_3^Z \frac{\gamma_Z^2}{s_{24}^2} + N_4^Z \frac{\alpha_Z \beta_Z}{s_{13}^2 s_{24}} + N_5^Z \frac{\alpha_Z \gamma_Z}{s_{13} s_{24}^2} + N_6^Z \frac{\beta_Z \gamma_Z}{s_{13} s_{24}} \right)
 \end{aligned}$$

Notation

$$Y = k_3 + k_4 + k_6 , \quad y = \langle 12 \rangle [34] Y^2 ,$$

$$X = k_1 + k_2 + k_6 , \quad x = [12] \langle 34 \rangle X^2 ,$$

$$Z = k_2 + k_4 + k_6 , \quad z = \langle 13 \rangle [24] Z^2 .$$

$$\alpha_Y = - \langle 12 \rangle [34] [6|Y|5] , \quad \beta_Y = \langle 12 \rangle [46] [3|Y|5] , \quad \gamma_Y = \langle 51 \rangle [34] [6|Y|2] ,$$

$$\alpha_X = - [12] \langle 34 \rangle [6|X|5] , \quad \beta_X = [12] \langle 45 \rangle [6|X|3] , \quad \gamma_X = [61] \langle 34 \rangle [2|X|5] ,$$

$$\alpha_Z = - \langle 13 \rangle [24] [6|X|5] , \quad \beta_Z = \langle 13 \rangle [46] [2|Z|5] , \quad \gamma_Z = \langle 51 \rangle [24] [6|Z|3] ,$$

with: $[6|Y|5] = [63] \langle 35 \rangle + [64] \langle 45 \rangle ,$ etc.

$$s_{ij} = 2\alpha' k_i k_j ,$$

$$s_i = \alpha' (k_i + k_{i+1})^2 , \quad t_i = \alpha' (k_i + k_{i+1} + k_{i+2})^2 \quad (i + 6 \equiv i)$$

α' -Expansion

E.g.:

$$N_1^X = -\zeta(2) s_1 s_3 + \dots ,$$

$$N_2^X = \frac{s_1}{s_2 s_4 t_1} - \zeta(2) \left(\frac{s_1 s_6}{s_2 s_4} + \frac{s_1^2}{s_4 t_1} + \frac{s_1 s_5}{s_2 t_1} \right) + \dots ,$$

$$N_3^X = \frac{s_3}{s_2 s_6 t_2} - \zeta(2) \left(\frac{s_3 s_4}{s_2 s_6} + \frac{s_3 s_5}{s_2 t_2} + \frac{s_3^2}{s_6 t_2} \right) + \dots ,$$

$$N_4^X = \zeta(2) \left(\frac{s_1 t_2}{s_2} + \frac{s_1 t_3}{s_4} \right) + \dots ,$$

$$N_5^X = \zeta(2) \left(\frac{s_3 t_1}{s_2} + \frac{s_3 t_3}{s_6} \right) + \dots ,$$

$$N_6^X = \frac{t_3}{s_2 s_4 s_6} + \zeta(2) \left(\frac{s_1 + s_3 - s_5}{s_2} - \frac{t_1 t_3}{s_2 s_4} - \frac{t_2 t_3}{s_2 s_6} - \frac{t_3^2}{s_4 s_6} \right) + \dots$$

N-gluon MHV amplitude in superstring theory

$$A(g_1^-, g_2^-, g_3^+, g_4^+, \dots, g_N^+; \alpha') = \left(1 - \alpha'^2 \frac{\zeta(2)}{2} F^{(N)} \right) A(g_1^-, g_2^-, g_3^+, g_4^+, \dots, g_N^+) + \mathcal{O}(\alpha'^3)$$

$F^{(N)}$ polynomial in kinematic invariants:

$$F^{(4)} = s_1 s_2$$

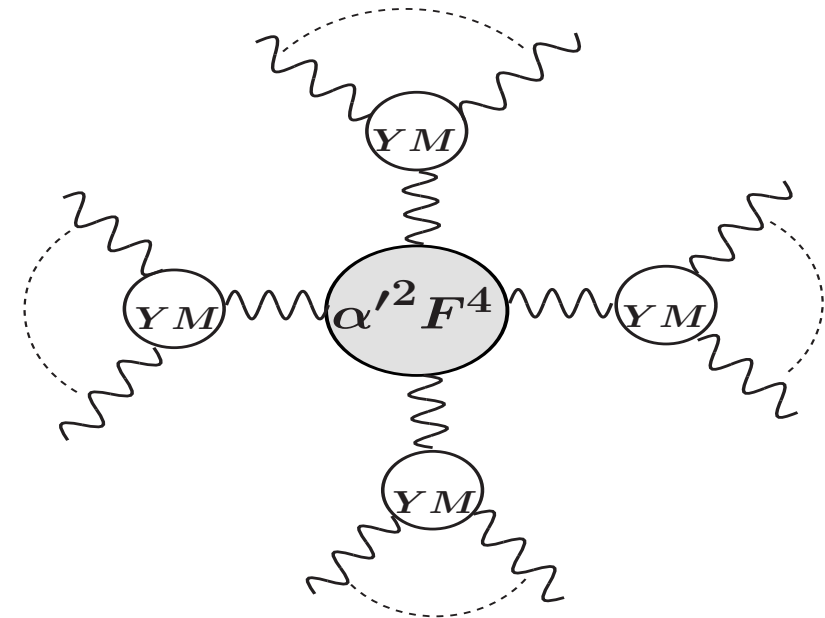
$$F^{(5)} = s_1 s_2 + s_2 s_3 + s_3 s_4 + s_4 s_5 + s_5 s_1 + 4i \epsilon_{1234}$$

$$F^{(6)} = s_1 s_2 + s_2 s_3 + s_3 s_4 + s_4 s_5 + s_5 s_6 + s_6 s_1$$

$$+ t_1 t_2 + t_2 t_3 + t_3 t_1 - s_1 s_4 - s_2 s_5 - s_3 s_6$$

$$+ 4i [\epsilon_{1234} + \epsilon_{1235} + \epsilon_{1245} + \epsilon_{1345} + \epsilon_{2345}]$$

⋮



St. St., Taylor, hep-th/0609175

Non-trivial duality: $\zeta(2)$ term agrees with one-loop field-theory result !

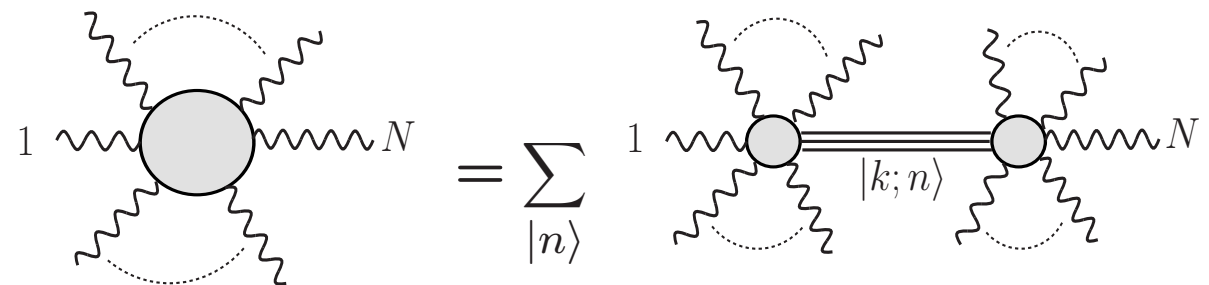
see also Dixon, Schabinger, to appear

Amplitudes from first principles

Analytic properties: Scattering amplitudes are computed *directly*

- use: $\left\{ \begin{array}{l} \bullet \text{ cyclic invariance} \\ \bullet \text{ soft-boson limit} \\ \bullet \text{ factorizing into collinear limits} \\ \bullet \dots \end{array} \right.$

String tree-level recursion relations:



\Rightarrow Construct amplitudes from first principles

work in progress

Effective D-brane action (α' -expansion)

Series of higher derivative terms (α' -corrections):

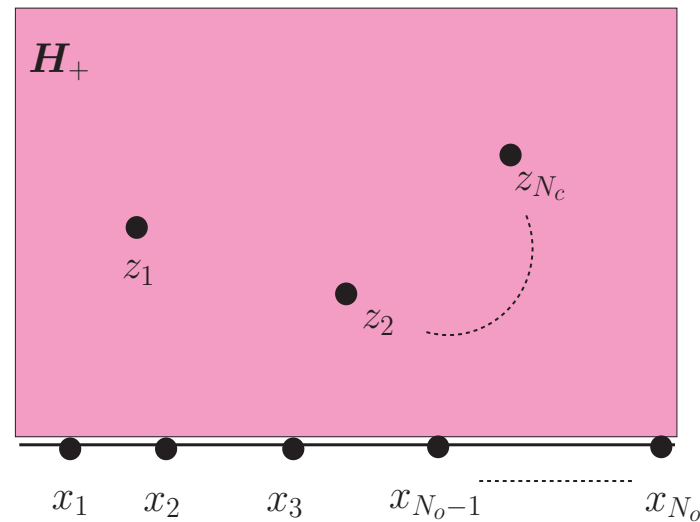
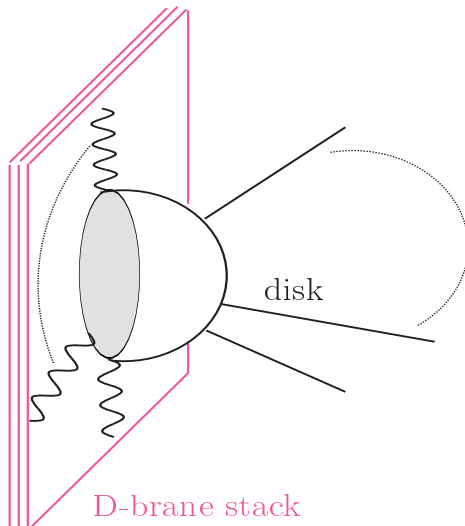
$$\mathcal{L}_{\text{effective}}^{Dp} = \text{Tr} \sum_{m \geq 4, n \geq 0} \alpha'^{\frac{1}{2}n+m-2} \zeta\left(\frac{1}{2}n + m - 2\right) D^n F^m$$

α'^0 1	F^2			
α'^1 0	F^3	$D^2 F^2$		
α'^2 $\zeta(2)$	F^4	$D^2 F^3$	$D^4 F^2$	
α'^3 $\zeta(3)$	F^5	$D^2 F^4$	$D^6 F^2$	
α'^4 $\zeta(4)$	F^6	$D^4 F^4$	$D^2 F^5$	
α'^5 $\zeta(2)\zeta(3), \zeta(5)$	F^7	$D^6 F^4$	$D^4 F^5$	$D^2 F^6$
\vdots	\dots	\dots	\dots	\dots

Degree of transcendentality \iff order in α' -expansion

Disk scattering of open and closed strings

$$\mathcal{A} = \sum_{\pi \in S_{N_o}/\mathbf{Z}_2} V_{\text{CKG}}^{-1} \left(\prod_{j=1}^{N_o} \int_{\mathcal{I}_\pi} dx_j \prod_{i=1}^{N_c} \int_{\mathcal{H}_+} d^2 z_i \right) \langle \prod_{j=1}^{N_o} : V_o(x_j) : \prod_{i=1}^{N_c} : V_c(\bar{z}_i, z_i) : \rangle$$



$V_o(x_i)$ = open string vertex operators inserted at x_i on the boundary of the disk

$V_c(\bar{z}_i, z_i)$ = closed string vertex operators inserted at z_i inside the disk

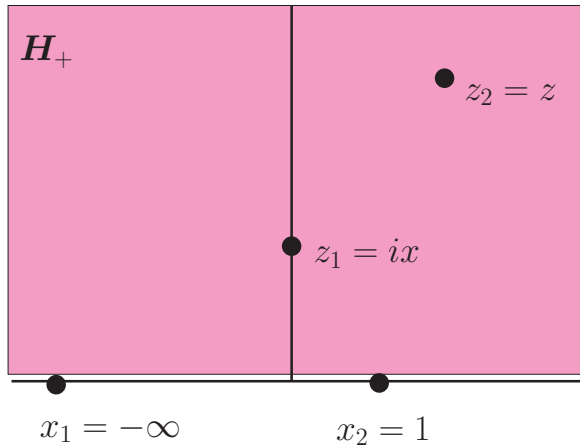
E.g.:

$N_o = 2, N_c = 1$	\implies	four open strings
$N_o = 3, N_c = 1$	\implies	five open strings
$N_o = 2, N_c = 2$	\implies	six open strings

Two open and two closed strings on the disk

With $PSL(2, \mathbf{R})$ transformation three arbitrary points $w_1, w_2 \in \mathbf{R}$ and $w_3 \in \mathbf{C}$ may be mapped to the points x_1, x_2 and z_1 :

Choice: $x_1 = -\infty$, $x_2 = 1$, $\bar{z}_1 = -ix$, $z_1 = ix$, $\bar{z}_2 = \bar{z}$, $z_2 = z$



with $z \in H_+$ and $x \in \mathbf{R}^+$

$$\begin{aligned}
 \mathcal{A}(1, 2, 3, 4) &= \int_{-\infty}^{\infty} dx \langle c(-\infty)c(1)c(ix) \rangle \\
 &\times \int_{\mathbf{C}} d^2z \langle : V_o(-\infty) : : V_o(1) : : V_c(-ix, ix) : : V_c(\bar{z}, z) : \rangle
 \end{aligned}$$

Two open & two closed strings versus six open strings on the disk

- generic structure of world-sheet disk amplitude of **two open & two closed strings**:

$$\begin{aligned}
 W(\alpha, \beta) \begin{bmatrix} \alpha_1, \lambda_1, \gamma_1, \beta_1 \\ \alpha_2, \lambda_2, \gamma_2, \beta_2 \end{bmatrix} &= \int_{-\infty}^{\infty} dx x^\beta (1+x)^{\alpha_1} (1-x)^{\alpha_2} \int_{\mathbf{C}} d^2z (1-z)^{\lambda_1} (1-\bar{z})^{\lambda_2} \\
 &\times (z-\bar{z})^\alpha (z-x)^{\gamma_1} (\bar{z}-x)^{\gamma_2} (z+x)^{\beta_1} (\bar{z}+x)^{\beta_2}
 \end{aligned}$$

- generic structure of world-sheet disk amplitude of **six open strings**:

$$\begin{aligned}
 F \begin{bmatrix} n_1, n_2, n_3 \\ n_4, n_5, n_6, n_7, n_8, n_9 \end{bmatrix} &= \int_0^1 dx \int_0^1 dy \int_0^1 dz x^{p_{23}+n_1} y^{p_{23}+k_{24}+p_{34}+n_2} z^{p_{16}+n_3} \\
 &\times (1-x)^{p_{34}+n_4} (1-y)^{p_{45}+n_5} (1-z)^{p_{56}+n_6} (1-xy)^{p_{35}+n_7} \\
 &\times (1-yz)^{p_{46}+n_8} (1-xyz)^{p_{36}+n_9} \quad , \quad n_i \in \mathbf{Z}
 \end{aligned}$$

Two open & two closed strings versus six open strings on the disk

After splitting the complex integral into holomorphic and anti-holomorphic pieces

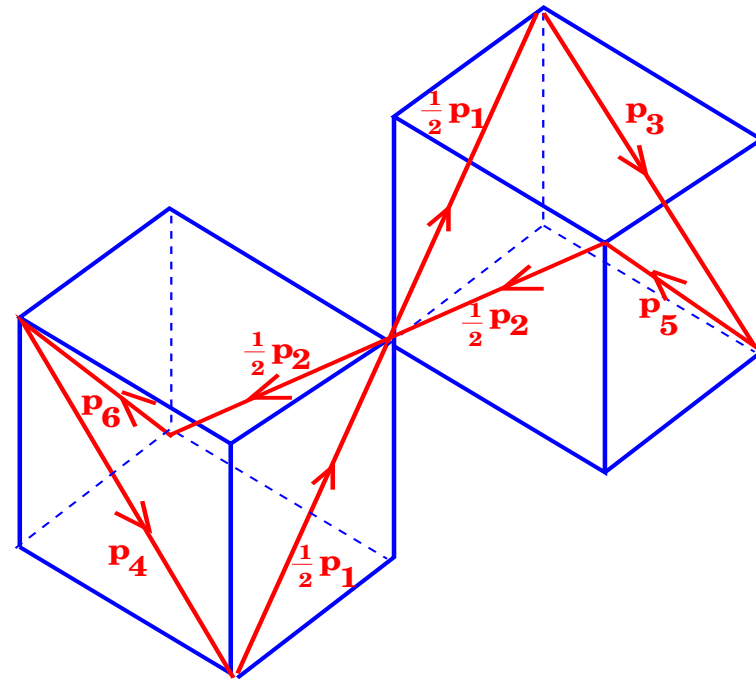
Answer: Six open strings, with:

$$z_1 = -\infty, \quad z_2 = 1, \quad z_3 = -\tau,$$

$$z_4 = \tau, \quad z_5 = \xi, \quad z_6 = \eta$$

$$p_1 = k_1, \quad p_2 = k_2,$$

$$p_3 = p_4 = \frac{1}{2}k_3, \quad p_5 = p_6 = \frac{1}{2}k_4$$



Four contributions, with open string ordering

$$\left\{ \begin{array}{l} I_1 : z_1 < z_6 < z_3 < z_5 < z_4 < z_2 \\ I_2 : z_1 < z_6 < z_3 < z_5 < z_2 < z_4 \\ I_3 : z_1 < z_3 < z_4 < z_5 < z_2 < z_6 \\ I_4 : z_1 < z_3 < z_2 < z_5 < z_4 < z_6 \end{array} \right.$$

Two open & two closed strings versus six open strings on the disk

$$W(\alpha, \beta) \begin{bmatrix} \alpha_1, \lambda_1, \gamma_1, \beta_1 \\ \alpha_2, \lambda_2, \gamma_2, \beta_2 \end{bmatrix} = \sigma_\gamma \sin(\pi\beta_2) (I_1 + I_2) + \sin(\pi\lambda_2) I_3 + \sigma_\lambda \sigma_\gamma \sin(\pi\gamma_2) I_4 + R$$

I_1, I_2, I_3 and I_4 six open string amplitudes $F[...]$

involves transformations:

$$\begin{aligned} I_1 : \quad \tau &\rightarrow -1 + \frac{2}{1 + yz}, & \xi &\rightarrow 1 - \frac{2y}{1 + yz}, & \eta &\rightarrow 1 - \frac{2}{x(1 + yz)} \\ I_2 : \quad \tau &\rightarrow \frac{1}{1 - 2yz}, & \xi &\rightarrow \frac{1 - 2y}{1 - 2yz}, & \eta &\rightarrow -\frac{2 - x}{x(1 - 2yz)} \\ I_3 : \quad \tau &\rightarrow \frac{xy}{2 - xy}, & \xi &\rightarrow \frac{(2 - x)y}{2 - xy}, & \eta &\rightarrow \frac{2 - xyz}{z(2 - xy)} \\ I_4 : \quad \tau &\rightarrow -\frac{1}{1 - 2xy}, & \xi &\rightarrow \frac{1 - 2y}{1 - 2xy}, & \eta &\rightarrow -\frac{2 - z}{z(1 - 2xy)} \end{aligned}$$

Open & closed vs. pure open string disk amplitudes

This map reveals
important relations between
open & closed string disk amplitudes
and pure open string disk amplitudes !

E.g.:

$$\langle A_{\mu_1}(x_1) A_{\mu_2}(x_2) G_{\mu_3\mu_4}(\bar{z}_1, z_1) G_{\mu_5\mu_6}(\bar{z}_2, z_2) \rangle \\ \sim \langle A_{\mu_1}(x_1) A_{\mu_2}(x_2) A_{\mu_3}(x_3) A_{\mu_4}(x_4) A_{\mu_5}(x_5) A_{\mu_6}(x_6) \rangle$$

Sort of generalized KLT on the disk

St. St. arXiv:0812.xxxx, to appear