MHV and (non)-MHV amplitudes in N=4, N=2 and marginal deforms

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1. On SYM amplitudes at strong coupling -- beyond the MHV case

2. On marginal deformation of N=4

3. On amplitudes in N=2 SQCD

1. SYM amplitudes at strong coupling

- Alday and Maldacena (0705.0303) gave a string theory prescription for computing planar amplitudes N=4 SYM at strong coupling using AdS/CFT.

- Amplitudes are determined by a classical string solution and contain a universal exponential factor -- the action of the classical string.

$$A_n(p_i, h_i) = K\, e^{i\sqrt{\lambda} S_{cl}}$$

$$= K\, e^{-\frac{\sqrt{\lambda}}{2\pi}\text{Area}_{cl}}$$

- In gauge theory the only amplitudes which are (almost) under control at strong coupling are expressions for MHV amplitudes:

via the exponential ansatz of Bern-Dixon-Smirnov
augmented in the exponent by the
conformally-invarinat Reminder function
Drummond-Henn-Korchemsky-Sokatchev;
Bern-Dixon-Kosower-Roiban-Spradlin-Vergu-Volovich '08

- Still Open Question -- non-MHV: examine the amplitudes dependence on helicities and particle-types of external states.

(Abel-Forste-VVK 0705.2113):

- Argue/conjecture:

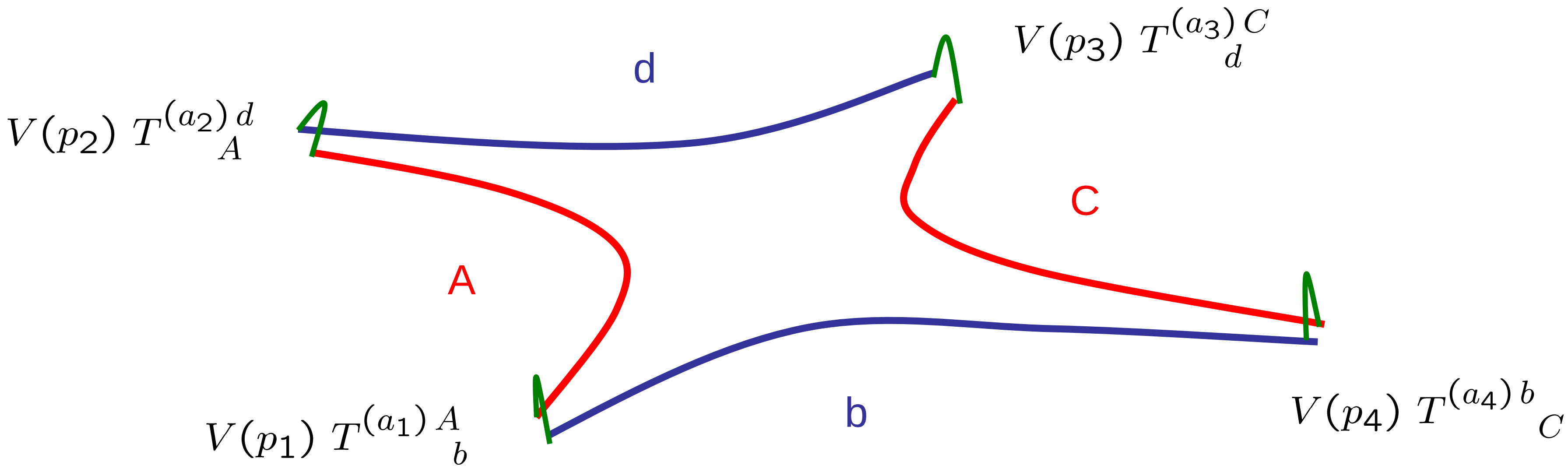
  the prefactor K at strong coupling should be ~ to tree-level SYM amplitude for the same process.

- => non-MHV scattering amplitudes in N=4 SYM simplify dramatically in the strong coupling limit.

  (at weak coupling one wouldn’t expect exponentiation of non-MHV’s)

- At strong coupling:

  all (MHV and non-MHV) n-point amplitudes are given by the (known) tree-level Yang-Mills result times the universal exponential.



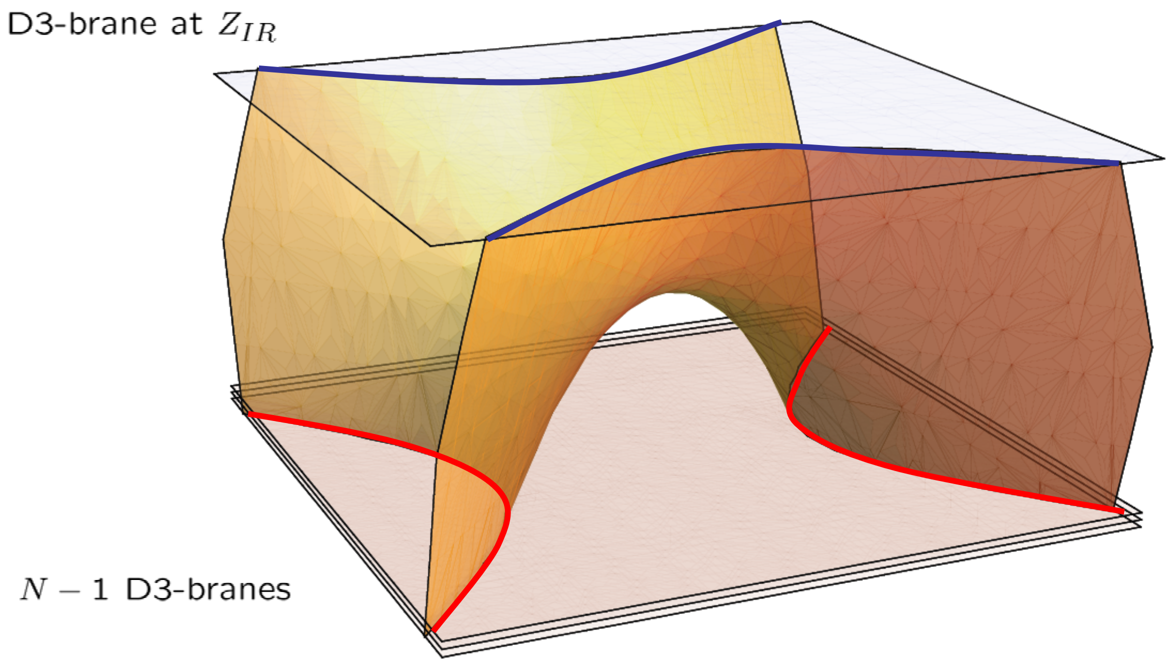
Scattering of 4 open strings ending on N coincident D3-branes.

A,b,C,d are the Chan-Paton indices labeling the branes on which strings end.

External states: strings with one end on the Nth brane, b=d=N,
and the other end on the remaining N-1 branes, A,C=1… N-1.

N^{th} D3-brane is separated from the stack of N -1 branes and placed at $Z=Z_{IR}$.

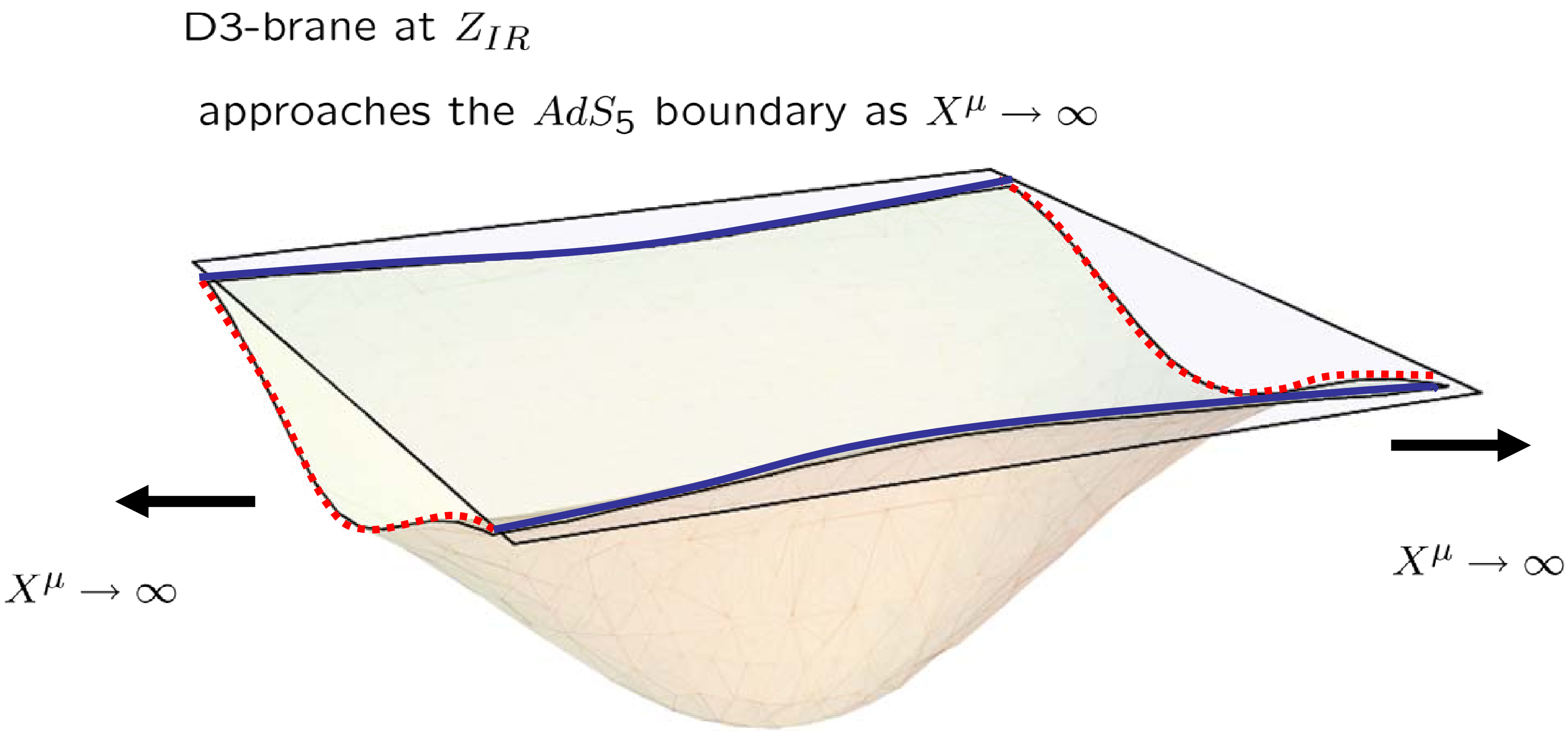
This implements an IR regularisation for the amplitudes where all the external states are in the (massive) bifundamental of SU(N-1) × U(1)



Scattering of open strings stretched between the separated IR brane and the stack of N-1 D3-branes.

In the Maldacena near-horizon limit the N -1 stack dissolves into the $AdS_5 \times S^5$ geometry and the IR brane is the only brane remaining.

The stretched strings worldsheet becomes the open string worldsheet curved into the AdS bulk.



String worldsheet bending into the AdS_5 bulk

- Vertex operators describing external states are located on the Dirichlet IR brane: the only brane remaining.

- External states, being the states of the boundary conformal SYM theory, should live on the boundary of the AdS_5 space, and this is where the boundary of the open string worldsheet must be.

- In terms of Poincare coordinates (X^μ,Z) the AdS_5 boundary is spanned by X^μ and is usually placed at the radial coordinate $Z \to 0$.

- But the boundary of AdS_5 is not only described by $Z \to 0$, but also by $Z = Z_{IR} \to \infty$ at large values of X^μ.

- Asymptotic external states live on the boundary of AdS_5, which (up to a constant rescaling by Z_{IR}) is the 4-dim Minkowski space.

- Use standard flat space definition of vertex operators V:

For a gluon state of momentum p_i and helicity $h_i=\pm$

$$V(p_i) = \int d\tau \, e^{ip_i^\mu X_\mu(\tau)} \varepsilon_\mu^\pm(p_i)(\partial_\tau X^\mu(\tau) + \ldots)$$

τ parameterises the boundary of the worldsheet (τ,σ)

$X^\mu = X^\mu(\tau, \sigma=0)$ is taken at the boundary

(with the radial coordinate $Z=Z_{IR}$)

and … indicate the supersymmetric completion

n-point open string amplitude:

$$A_n = \int DX\, V(p_1) \ldots V(p_n)\, e^{i\sqrt{\lambda}S[X]}$$

or

$$A_n = \prod_i \int d\tau_i \epsilon^{\pm}_{\mu}\left(p_i\right) \frac{\delta}{\delta J_\mu\left(\tau_i\right)} e^{iW[J]}$$

$$e^{iW[J]} = \int DX\, DZ \exp iS_{\text{eff}}$$

where

$$iS_{\text{eff}} := i\sqrt{\lambda}S\left[X, Z\right] + \sum_i ip_i^\mu X^\mu\left(\tau_i\right) + \int d\tau J_\mu\left(\tau\right)\partial_\tau X_\mu$$

Strong coupling limit amounts to a semicalssical regime where we extremise the effective action

$$iS_{\text{eff}} = i\sqrt{\lambda}S\left[X,Z\right] + \sum_i ip_i^{\mu}X^{\mu}\left(\tau_i\right) + \int d\tau J_{\mu}\left(\tau\right)\partial_{\tau}X_{\mu}$$

The bosonic action on the AdS_5 is

$$S\left[X,Z\right] = \int d\tau d\sigma \left(\partial_{\alpha}X^{\mu}\frac{1}{Z^2}\partial^{\alpha}X_{\mu} + \partial_{\alpha}Z\frac{1}{Z^2}\partial^{\alpha}Z\right)$$

The total effective action can be decomposed as

$$\sqrt{\lambda} S_{\text{cl}}^{\text{bulk}}(p_i) + \frac{1}{\sqrt{\lambda}} S_{\text{cl}}^{\text{boundary}}(\tau_i, p_i)$$

$\sqrt{\lambda} S_{\text{cl}}^{\text{bulk}}(p_i)$ is the Alday-Maldacena action. It does not depend on emission points τ_i and is a homogenous function of external momenta.

$\frac{1}{\sqrt{\lambda}} S_{\text{cl}}^{\text{boundary}}(\tau_i, p_i)$ is the generating functional for the tree-level amplitudes in flat 4D space at $Z = Z_{IR}$ =const.

In the limit

$\sqrt{\lambda} \to \infty$ keeping p_i^μ and Z_{IR} fixed

from $\frac{1}{\sqrt{\lambda}} S_{\text{cl}}^{\text{boundary}}(\tau_i, p_i)$ we pick up the poles contribution of the tree-level Veneziano amplitude in flat space, which is precisely the tree-level Yang-Mills amplitude.

$\sqrt{\lambda} S_{\text{cl}}^{\text{bulk}}(p_i)$ gives the universal exponent:

$$\lambda \to \infty : \qquad A_n = A_n^{\text{tree}} \, e^{i\sqrt{\lambda} S_{\text{cl}}^{\text{bulk}} - S_0}$$

$$\lambda \to \infty : \qquad A_n \propto A_n^{\text{tree}} e^{i\sqrt{\lambda} S_{\text{cl}}}$$

- If this factorised proposal is correct for general non-MHV amplitudes at strong coupling:

it is certainly not expected to hold at weak coupling where factorisation appears to be lost

Bern-Del Duca-Dixon-Kosower 0410224

Britto-Cachazo-Feng 0412103

But…note recent results of

Drummond-Henn-Korchemsky-Sokatchev 0807.1095

- Drummond-Henn-Korchemsky-Sokatchev 0807.1095

In perturbation theory NMHV amplitudes can be recast in the form:

$$A_n^{NMHV} = A_n^{MHV} \times [R_n^{NMHV} + \mathcal{O}(\varepsilon)]$$

- R_n^{NMHV} is a factor of Grassmann degree-four
- Its perturtbative expansion in λ starts from tree-level ~ λ^0 plus loop corrections
- It is a Lorentz scalar of vanishing helicity
- It is a dual-superconformal invariant.

$$A_n^{NMHV}(\lambda) \to A_n^{MHV\,tree} \underbrace{R_n^{NMHV}(\lambda)\, e^{i\sqrt{\lambda}S_{\text{cl}}}}$$

At strong coupling can it be $R_n^{NMHV}(0)$?

Conclusions for Part1:

$$\lambda \to \infty \; : \qquad A_n \propto A_n^{\text{tree}} \, e^{i\sqrt{\lambda} S_{\text{cl}}}$$

- If this factorised proposal is correct for general non-MHV amplitudes at strong coupling:
- non-MHV scattering amplitudes in N=4 SYM must simplify dramatically in the strong coupling limit
- There is much to learn about dual conformal invariance (esp in non-MHV context)
- …and about non-MHV amplitudes in general.

2. Amplitudes and marginal deformations of N=4 SYM

Following VVK hep-th/0512194

- Marginal deformations of N=4 SYM keep conformal invariance of the theory but reduce supersymmetry to N=1 (or even N=0).
- There is a continuous family of marginal deformations, and the original N=4 SYM is just a point on a moduli space of these theories.
- For simplicity concentrate here on the so-called real-beta deformations. Will show:

  => Planar amplitudes in these theories are identical to those in N=4 (up to an overall factor).

This is the deformation of the N=4 superpotential (preserves N=1 susy):

$$\mathrm{Tr}(\Phi_1 * \Phi_2 * \Phi_3 - \Phi_1 * \Phi_3 * \Phi_2) = \mathrm{Tr}(e^{i\pi\beta}\Phi_1\Phi_2\Phi_3 - e^{-i\pi\beta}\Phi_1\Phi_3\Phi_2)$$

Component Lagrangian:

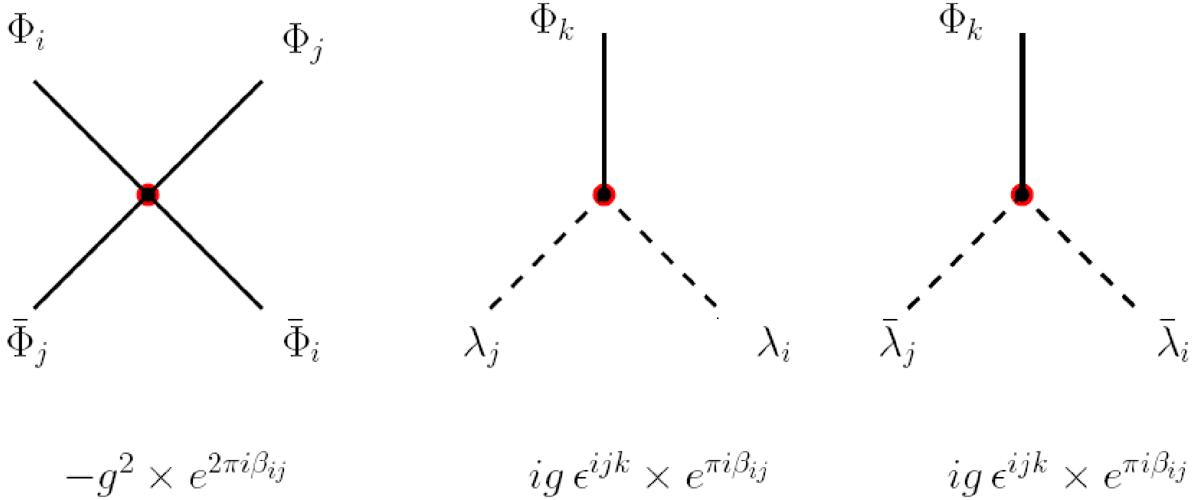
$$\mathcal{L} = \mathrm{Tr}\Big(\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu\bar{\Phi}^i)(D_\mu\Phi_i) - \frac{g^2}{2}[\Phi_i, \Phi_j]_{\beta_{ij}}[\bar{\Phi}^i, \bar{\Phi}^j]_{\beta_{ij}} + \frac{g^2}{4}[\Phi_i, \bar{\Phi}^i][\Phi_j, \bar{\Phi}^j]$$

$$+\lambda_A\sigma^\mu D_\mu\bar{\lambda}^A - ig([\lambda_4, \lambda_i]\bar{\Phi}^i + [\bar{\lambda}^4, \bar{\lambda}^i]\Phi_i) + \frac{ig}{2}(\epsilon^{ijk}[\lambda_i, \lambda_j]_{\beta_{ij}}\Phi_k + \epsilon_{ijk}[\bar{\lambda}^i, \bar{\lambda}^j]_{\beta_{ij}}\bar{\Phi}^k)\Big)$$

Where I introduced beta-deformed commutators:

$$[f_i, g_j]_{\beta_{ij}} := e^{i\pi\beta_{ij}} f_i g_j - e^{-i\pi\beta_{ij}} g_j f_i$$

$$\beta_{ij} = -\beta_{ji}\,, \quad \beta_{12} = -\beta_{13} = \beta_{23} := \beta$$



All β_R-dependent Feynman vertices in color-ordered perturbation theory.

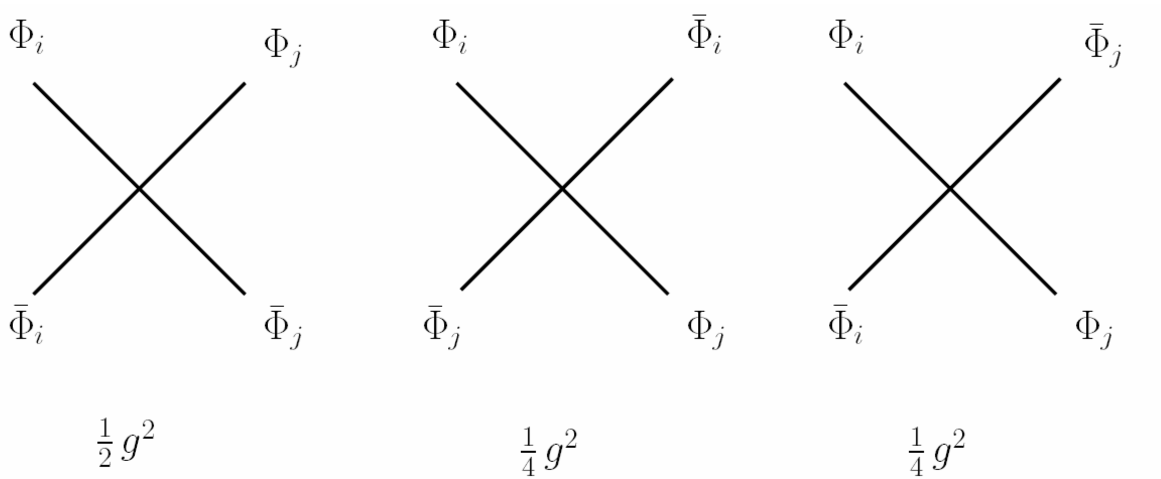
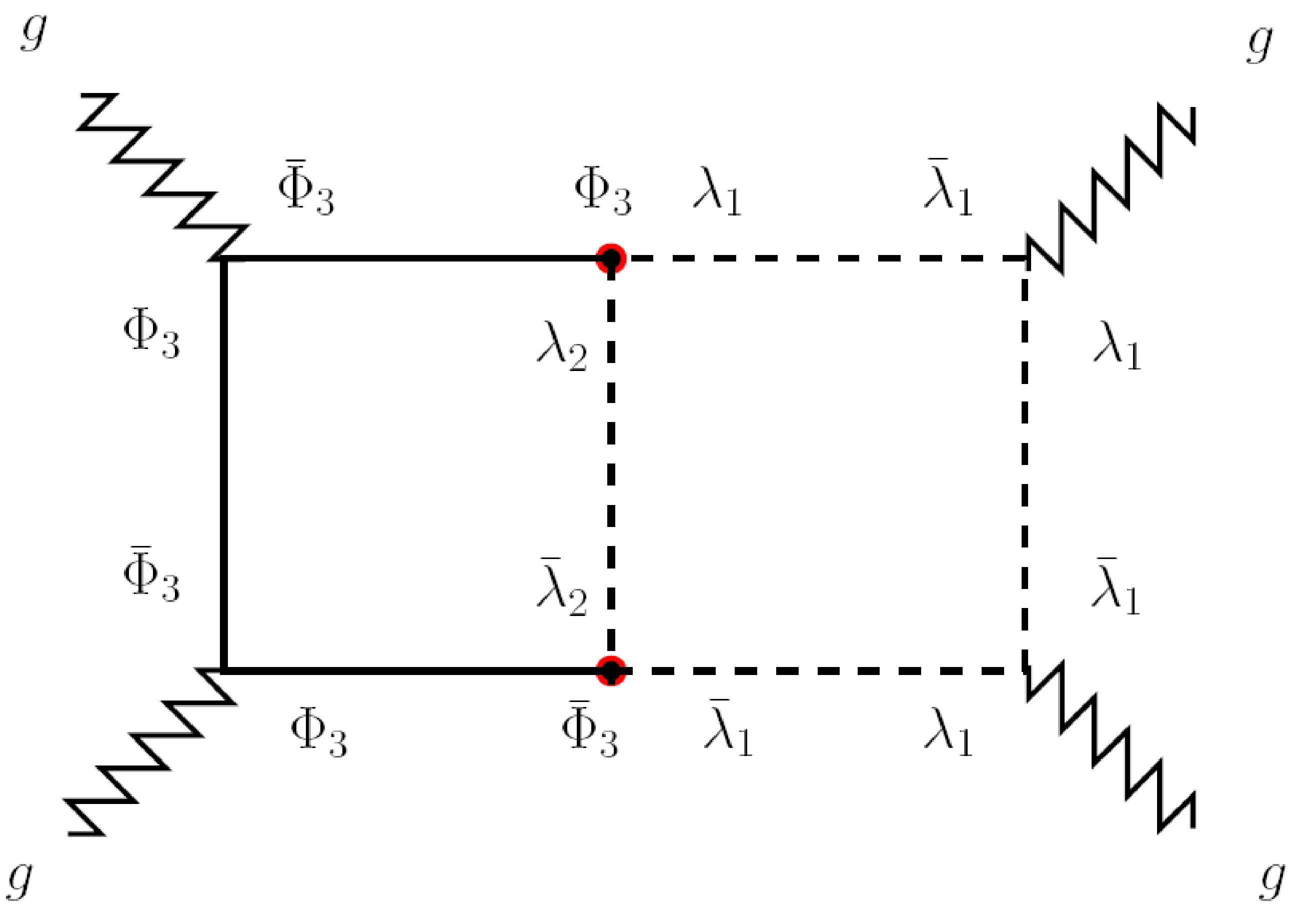


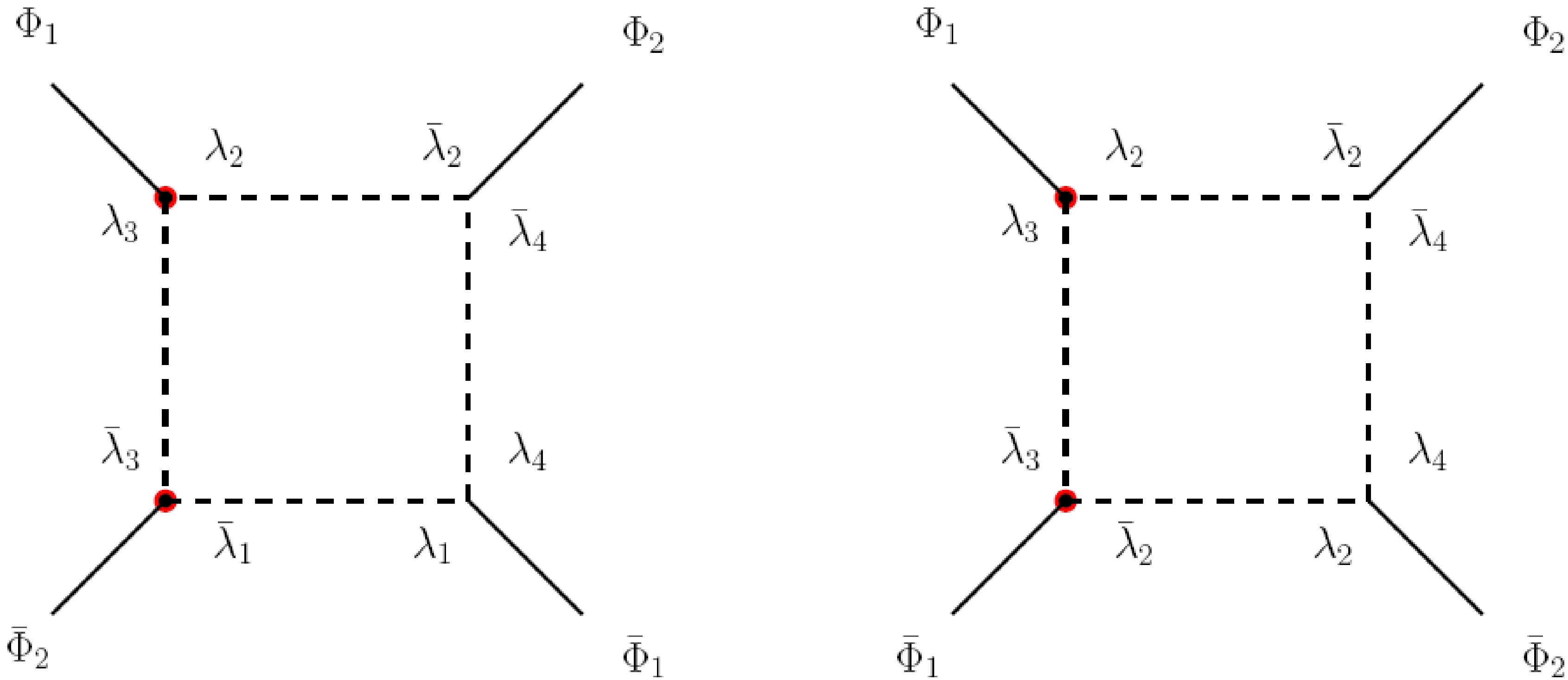
Figure 2: *β_R-independent ϕ^4 color-ordered vertices.*

- Total phase factor associated with any given planar amplitude is entirely determined by the external lines and does not depend on topologies and types of internal (loop) interactions.
- In other words, any planar loop amplitude in the beta-deformed theory is equal to the corresponding amplitude in the original N=4 SYM times an overall external phase factor.
- In particular, this universal phase factor can be read off the corresponding tree-level amplitude

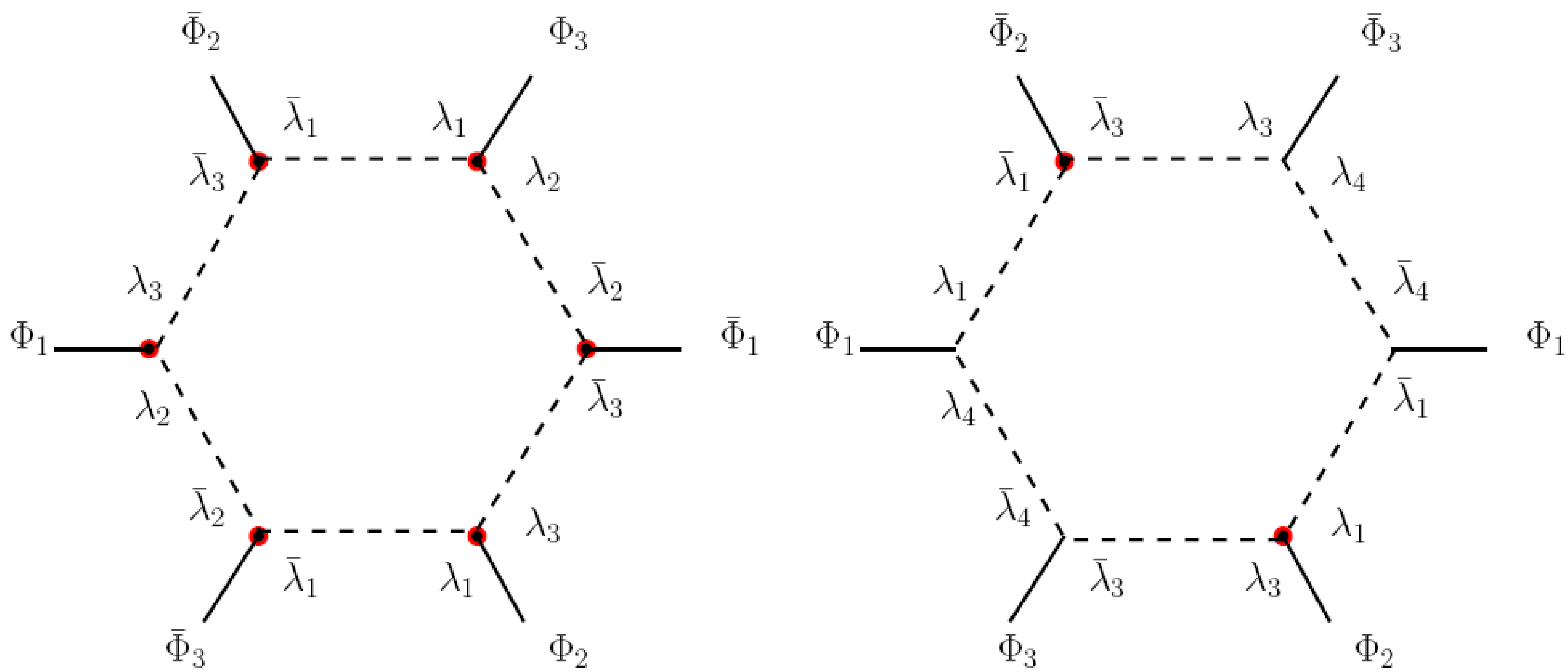
(or even better from the star-products…see below)



Example of a 2-loop diagram:
Red dots denote two beta-dependent Yukawa vertices. Their phases cancel and the total contribution is beta-independent.



Two examples of contributions to amplitudes with 4 external scalars. The first diagram is β-dependent, $A_4(\Phi_1, \Phi_2, \bar{\Phi}_1, \bar{\Phi}_2) \sim e^{2i\pi\beta}$. In the second amplitude, $A_4(\Phi_1, \Phi_2, \bar{\Phi}_2, \bar{\Phi}_1) \sim 1$, the phases cancel.



Two examples of Next-to-MHV amplitudes with six external scalars. The amplitude on the left, $A(\Phi_1, \bar{\Phi}_2, \Phi_3, \bar{\Phi}_1, \Phi_2, \bar{\Phi}_3,) \sim e^{-6i\pi\beta}$, and the amplitude on the right $A(\Phi_1, \bar{\Phi}_2, \bar{\Phi}_3, \Phi_1, \Phi_2, \Phi_3,) \sim e^{2i\pi\beta}$.

Real beta-deformations can be represented in terms of star-products (Lunin-Maldacena) introduced into the N=4 Lagrangian

$$\mathrm{Tr}(\Phi_1 * \Phi_2 * \Phi_3 - \Phi_1 * \Phi_3 * \Phi_2)$$

$$= \mathrm{Tr}(e^{i\pi\beta}\Phi_1\Phi_2\Phi_3 - e^{-i\pi\beta}\Phi_1\Phi_3\Phi_2)$$

$$f * g \equiv e^{i\pi\beta(Q_1^f Q_2^g - Q_2^f Q_1^g)} fg$$

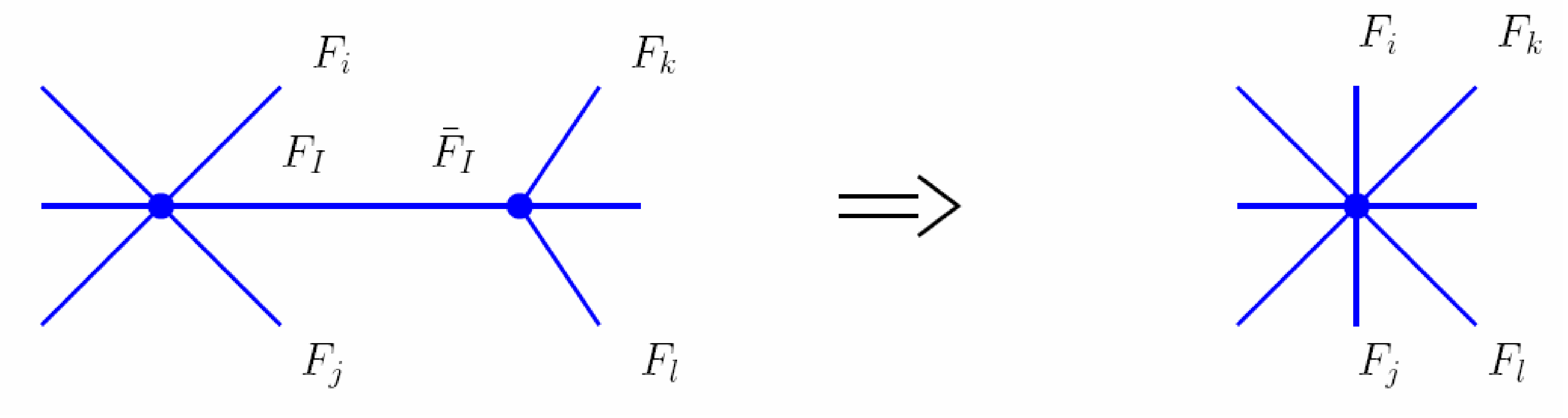
(Q_1, Q_2) are the $U(1)_1 \times U(1)_2$ charges of the fields

$$\Phi_1 : \quad (Q_1\,, Q_2) = (0\,, -1)$$

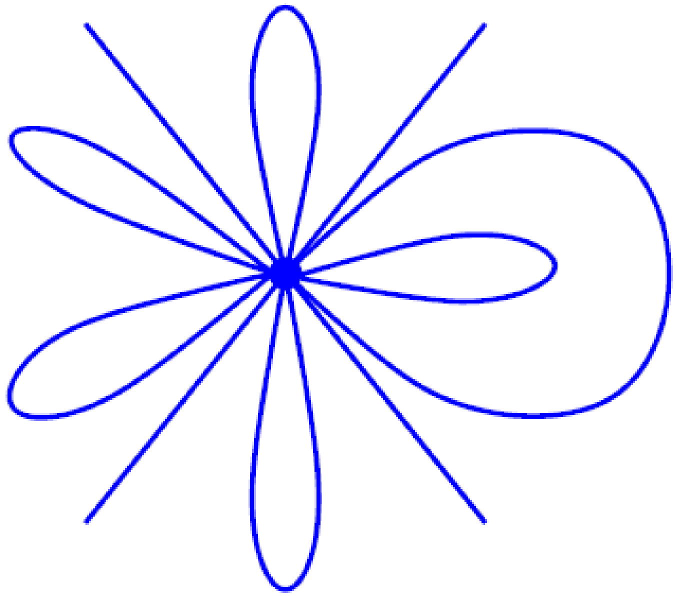
$$\Phi_2 : \quad (Q_1\,, Q_2) = (1\,, 1)$$

$$\Phi_3 : \quad (Q_1\,, Q_2) = (-1\,, 0)$$

$$V : \quad (Q_1\,, Q_2) = (0\,, 0)$$



At tree level all vertices can be joined together into an effective vertex without changing the total β-phase of the diagram.
$\bar{F}_I$ has opposite Q-charges to the F_I field on the other end of the internal propagator.



Reduced planar loop diagram. Planarity implies that none of the lines can intersect. Contractions can be removed without affecting the β-phase. Reduced diagram has the same β-phase as the original Feynman diagram.

$\Rightarrow$ Planar amplitudes in real-beta-deformed theories are identical to those in N=4 (up to an overall factor).

3. MHV amplitudes in N=2 SQCD

Following Glover-VVK-Williams 0805.4190

- One cannot hope for miracles, but what about the N=2 SQCD? Conformal invariance can be switched on and off – does it play a role? If so, is it sufficient?

- Calculated at 1-loop in N=2 SQCD:
- In N=2 SQCD already MHV amplitudes differ from N=4 for general values of N_f and N_c.
- However, for $N_f=2N_c$ where the N=2 SQCD is conformal, all 1-loop amplitudes (with all external particles in the adjoint representation) are identical to the N=4 results.

The full set of n-point MHV amplitudes in N=4 SYM is formed by all N=4 superpartners of $A_n(g^-, g^-)$

$$A_n(g^-, g^-)\ , \quad A_n(g^-, \lambda_A^-, \lambda^{A+})\ , \quad A_n(\lambda_A^-, \lambda_B^-, \lambda^{A+}, \lambda^{B+})\ ,$$

$$A_n(g^-, \lambda^{1+}, \lambda^{2+}, \lambda^{3+}, \lambda^{4+})\ , \quad A_n(\lambda_A^-, \lambda^{A+}, \lambda^{1+}, \lambda^{2+}, \lambda^{3+}, \lambda^{4+})\ ,$$

$$A_n(\lambda^{1+}, \lambda^{2+}, \lambda^{3+}, \lambda^{4+}, \lambda^{1+}, \lambda^{2+}, \lambda^{3+}, \lambda^{4+})\ ,$$

$$A_n(\overline{\phi}_{AB}, \lambda^{A+}, \lambda^{B+}, \lambda^{1+}, \lambda^{2+}, \lambda^{3+}, \lambda^{4+})\ ,$$

$$A_n(g^-, \overline{\phi}_{AB}, \phi^{AB})\ , \quad A_n(g^-, \overline{\phi}_{AB}, \lambda^{A+}, \lambda^{B+})\ , \quad A_n(\lambda_A^-, \lambda_B^-, \phi^{AB})\ ,$$

$$A_n(\lambda_A^-, \phi^{BC}, \overline{\phi}_{BC}, \lambda^{A+})\ , \quad A_n(\lambda_A^-, \phi^{AB}, \overline{\phi}_{BC}, \lambda^{C+})\ , \quad A_n(\lambda_A^-, \overline{\phi}_{BC}, \lambda^{A+}, \lambda^{B+}, \lambda^{C+})\ ,$$

$$A_n(\overline{\phi}, \phi, \overline{\phi}, \phi)\ , \quad A_n(\overline{\phi}, \phi, \overline{\phi}, \lambda^+, \lambda^+)\ , \quad A_n(\overline{\phi}, \overline{\phi}, \lambda^+, \lambda^+, \lambda^+, \lambda^+)$$

where we used the $SU(4)_R$ labelling conventions for scalars,

$$\overline{\phi}_{AB} = 1/2\, \epsilon_{ABCD}\, \phi^{CD} = (\phi^{AB})^\dagger$$

To switch from N=4 SYM:

$$\mathcal{W}_{\mathcal{N}=4} = igTr(\Phi_1\Phi_2\Phi_3 - \Phi_1\Phi_3\Phi_2)$$

to N=2 SQCD with N_f fundamental flavours:

$$\mathcal{W}_{\mathcal{N}=2} = g\sum_{f=1}^{N_f} \tilde{Q}_f \Phi Q_f$$

make the substitution

$$V \leftrightarrow V = (g^{\pm}, \lambda_1^{\pm}) , \quad \Phi \leftrightarrow \Phi_1 = (\phi^{12}, \lambda_2^{\pm}) ,$$

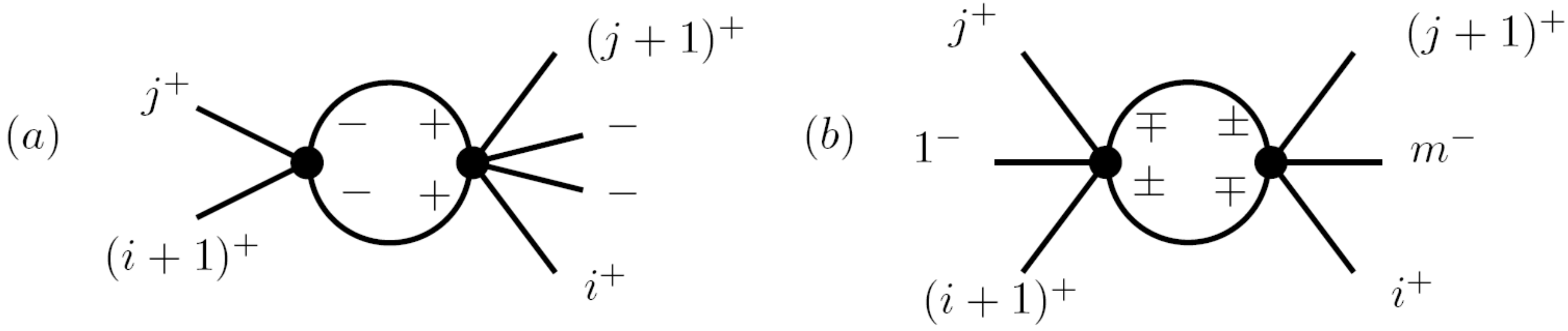
$$Q_f \leftrightarrow \Phi_2 = (\phi^{31}, \lambda_3^{\pm}) , \quad \tilde{Q}_f \leftrightarrow \Phi_3 = (\phi^{23}, \lambda_4^{\pm})$$

With these substitutions:

- The list of N=2 MHV amplitudes is the same
- The tree-level expressions for MHV’s are the same
- This is no longer the case at one-loop level (N=4 is reduced to N=2).
- But when $N_f=2N_c$ 1-loop MHV results in N=2 are the same as in N=4

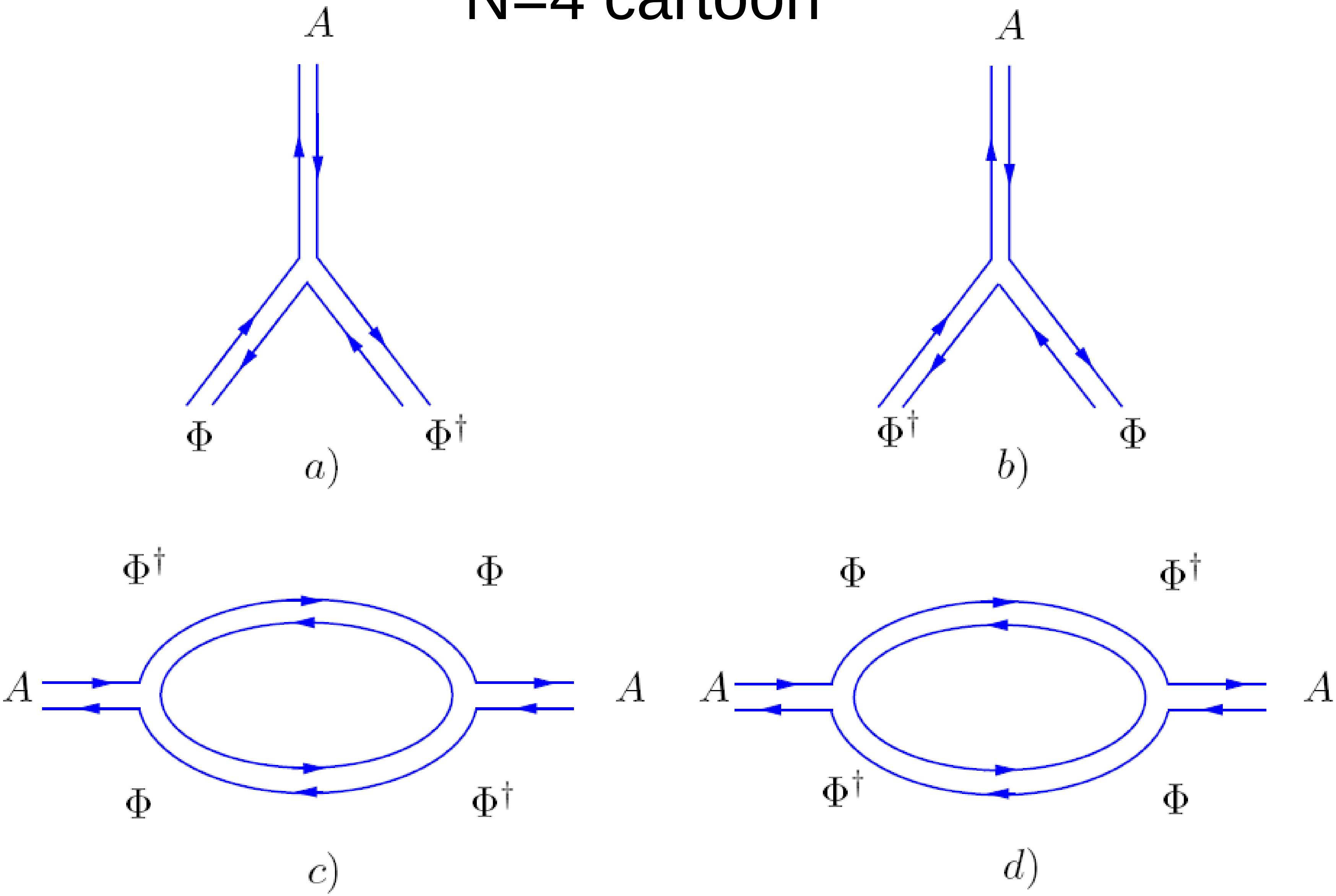
[when all external particles are in the adjoint]

MHV diagrams at 1-loop

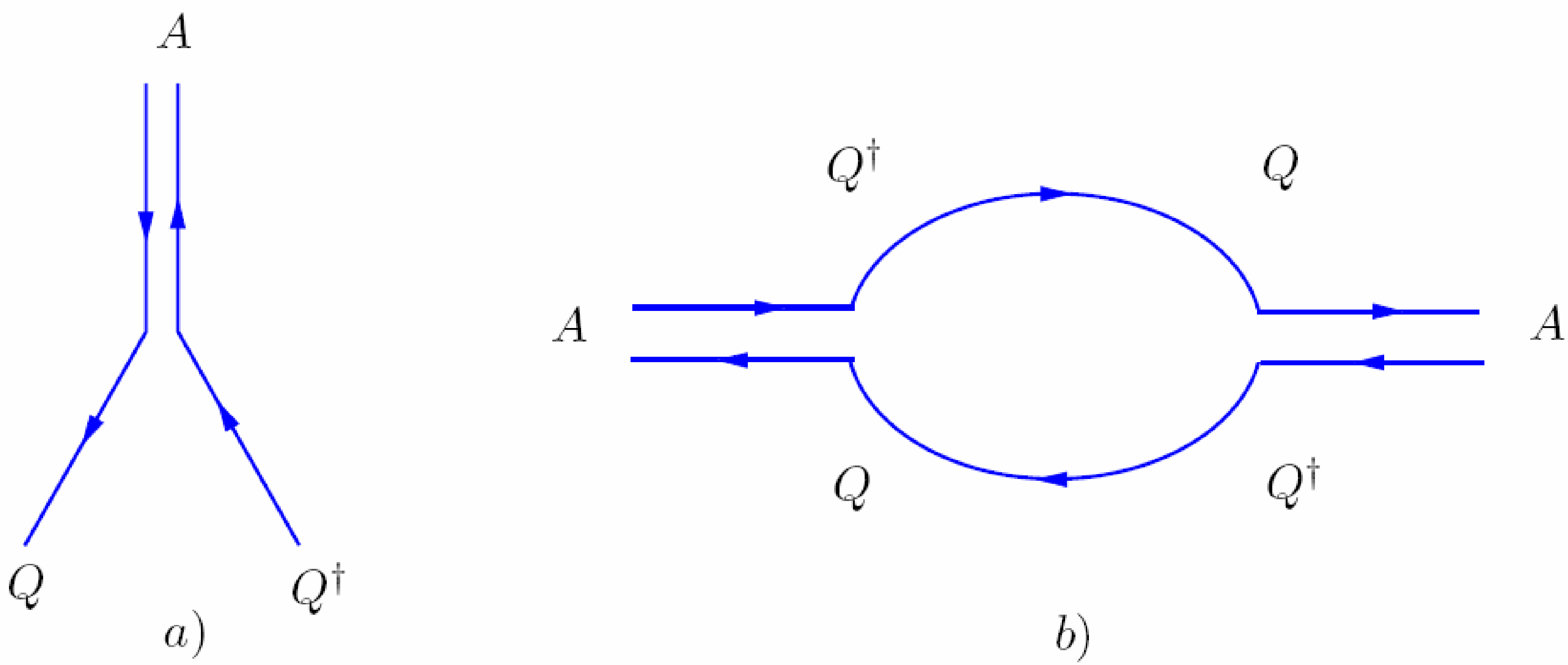


In (a) only gluons circulate in the loop
in (b) there are loop contributions from gluons, fermions and scalars.

N=4 cartoon

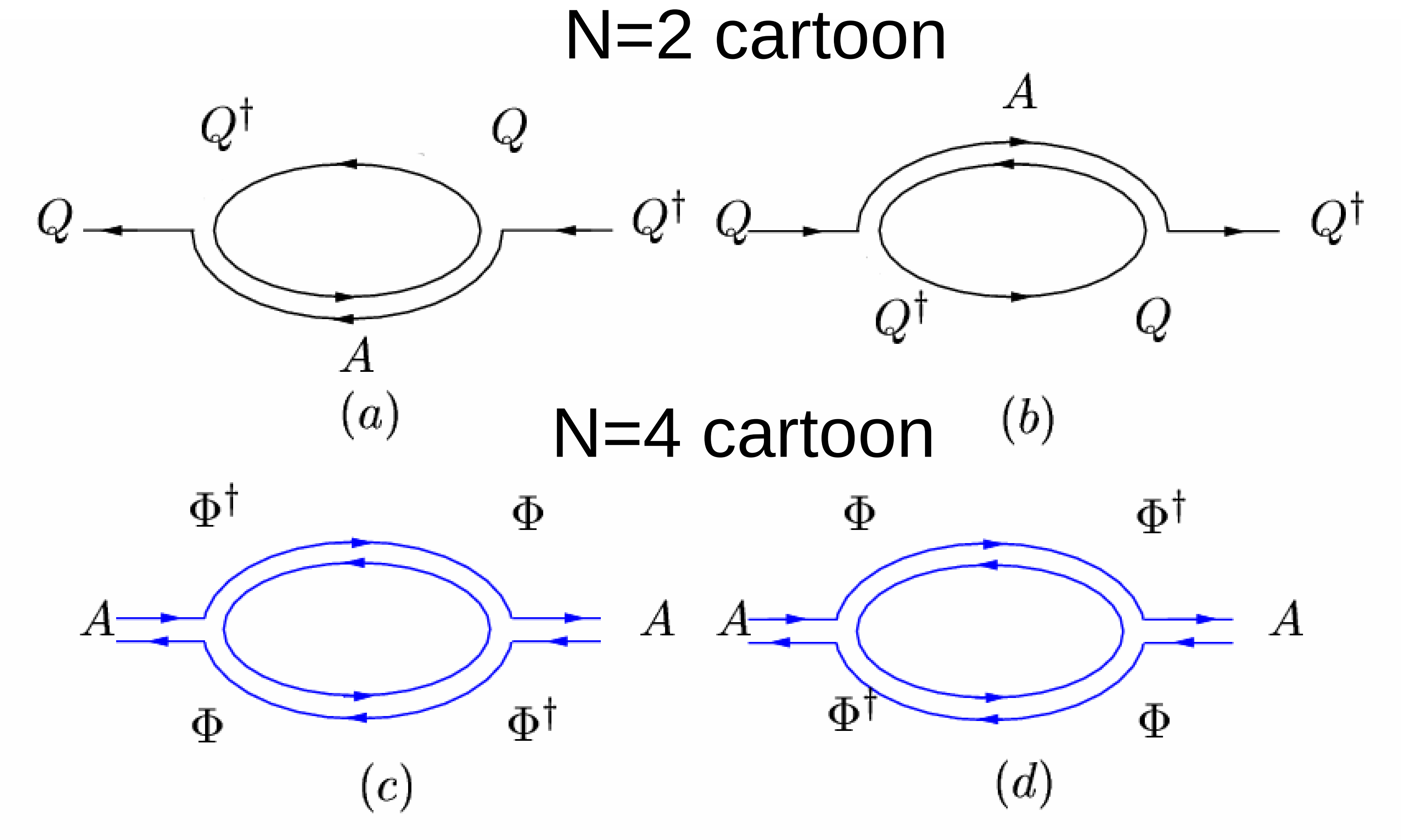


N=2 cartoon

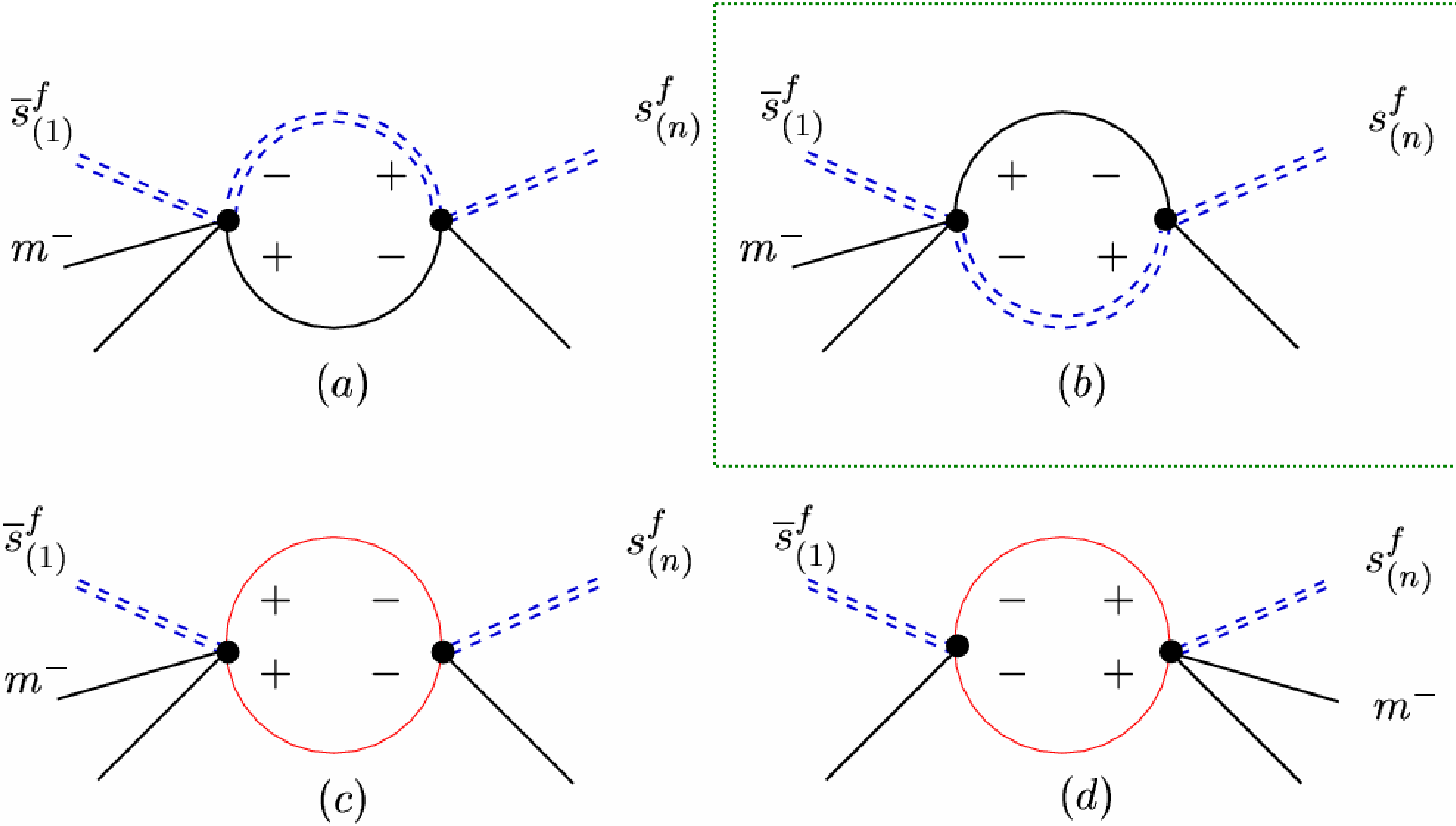


But when N_f=2N_c the sum over N_f
in the loop gives the same result as in N=4 !

However the agreement is lost for fundamental external legs…even in the superconformal case



This diagram is needed to match to N=, but it is non-planar !



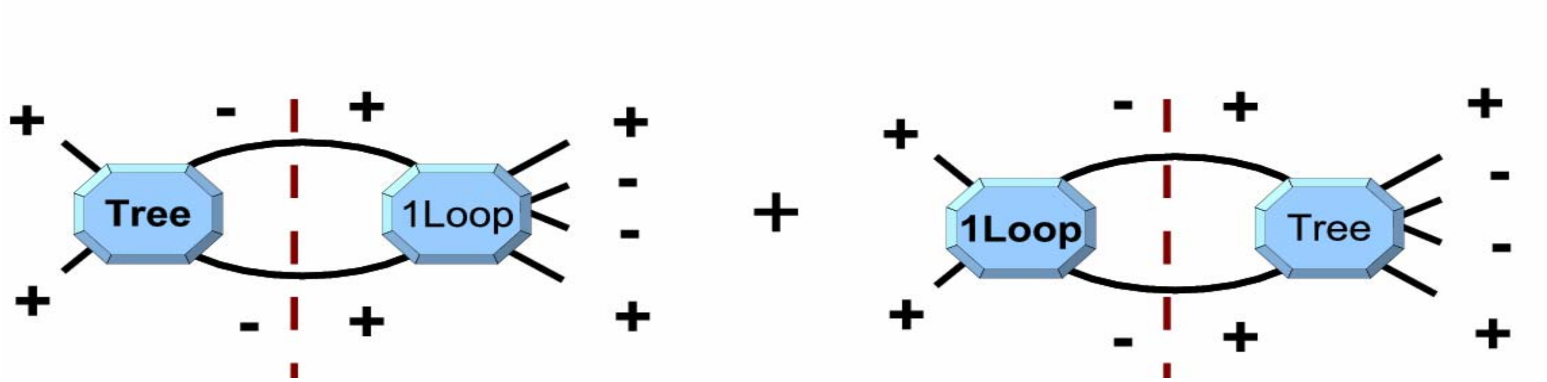
also thanks to Dixon, Kosower, Vergu: private communication

What if it did work at 1-loop level?

MHV amplitudes form a closed class: to construct higher loop MHV amplitudes using MHV rules one does not need non-MHVs.

Thus if MHV's did match at tree-level and at 1-loop level one could hope that all higher-loop MHV amplitudes will also mathc between Superconformal N=2 and N=4 theories.

Alas….it doesn't look good.



However note that at 2-loops the lightlike Wilson loops are clearly identical in N=2 and N=4 (and the highest transcendentality is satisfied in both cases…)

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2. On marginal deformation of N=4

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