MHV, CSW, BCFW: field theory techniques for string theory amplitudes

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A point of philosophy

last few years (re)learned new things about fields

- MHV: simple amplitudes
- CSW: simpler Feynman rules [Mason]
- BCFW: on-shell recursion [Skinner]

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applications to string theory?

[Stieberger]

Outline

1. MHV: α'
2. CSW: DBI
3. BCFW: recursion
4. Conclusions and outlook

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ordinary string theory amplitudes?

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ordinary open string theory tree amplitudes in (mainly) 4D flat background?

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Disk superstring amplitudes in a flat background

- insert vertex operators on edge of disk with definite ordering, fix 3, integrate over other insertion points, sum over orderings
- problem: sum over complicated integrals
- up to 2005: $\leq$ 5-gluon amplitude [Medina, Brandt, Machado, 02]

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Simple properties of string amplitudes

- pole properties: kinematic limits by conformal invariance
- color ordering: only poles in adjacent channels
- simplest limits: soft (massless particles only) and collinear (IR divergences)

(Effective) space-time supersymmetry

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- absence of fermion-helicity violating amplitudes (worldsheet parity), exact amplitudes:

$$A(+^n) = A(-^n) = A(+,-^n) = A(-,+^n) = 0 \quad \forall n > 2$$

- $\rightarrow$ amplitude with e.g., two $+$ and rest $-$ (Maximal Helicity Violating (MHV)): only collinear massless poles

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- superstring theory has same collinear limit as field theory

MHV amplitudes in string theory

$$A_{\text{sub}}(1^-, 2^+, \ldots j^- \ldots n^+) = \left(\frac{\langle 1j \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n1 \rangle} \right) Q(p_1 \ldots p_n)$$

- tree level YM: $Q = 1$ [Parke-Taylor, 88], ($\mathcal{N} = 4$ loop level: $Q = @$ weak and strong coupling)

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$$A_4(1^-, 2^-, 3^+, 4^+) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \frac{\Gamma(1-\alpha' s)\Gamma(1-\alpha' t)}{\Gamma(1-\alpha'(s+t))}$$

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  - known for $n = 4, 5, 6, 7$ [prev, Stieberger-Taylor, 06-07]
  - not obvious from calculation
  - but see [Berkovits-Maldacena, 08], appendix A, [Berkovits]
- field theory expansion: Q is polynomial in α'
- order by order in α'?

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- soft limits + 5 point amplitude determine [Stieberger-Taylor, 06]

$$Q = 1 - \alpha'^2 \frac{\pi^2}{6} \sum_{i<j<k<l} \langle ij \rangle [jk] \langle kl \rangle [li] + \mathcal{O}\left(\alpha'^3\right)$$

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- … much computer calculation … (and the 6 point function)

$$\alpha'^3 \frac{\zeta(3)}{24} \Big(42[s_{12}s_{34}s_{56}] + 18[s_{13}s_{24}s_{56}] - 9[s_{13}s_{23}s_{56}] + 9[s_{13}s_{25}s_{46}] - 3[s_{14}s_{25}s_{36}]$$
$$+ 36[s_{12}s_{15}s_{34}] - [s_{12}s_{12}s_{12}] + 96i[\epsilon_{1234}s_{56}] + 24i[\epsilon_{1234}s_{45}] - 24i[\epsilon_{1234}s_{35}] \Big)$$

- $s_{12} = (P_1 + P_2)^2 \quad \epsilon_{1234} = \epsilon_{\mu\nu\rho\sigma} P_1^\mu P_2^\nu P_3^\rho P_4^\sigma$, summed over cyclic permutations

How we got α'^3

Algorithm:

- construct polynomial, cyclic basis of momentum invariants of n particles
  - Schouten identities (solved)
  - momentum conservation
- restrict to polynomials which reduce well under soft limits (up to relations)
- establish dimension, find stabilizer, determine coefficients
- check collinearity
- $\alpha'^3 \rightarrow n = 6$, 13 polynomials, checked up to $n = 8$

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- effective field theory? Abelian limits?

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Abelian effective action: Dirac-Born-Infeld

- Abelian case has derivative expansion
- leading terms: DBI action

$$S_{\mathrm{DBI}} = -1 + \frac{1}{\pi^2 \alpha'^2} \int d^4x \sqrt{-\det\left(\eta_{\mu\nu} + \pi\alpha' F_{\mu\nu}\right)}$$

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- infinite series of vertices
- scattering amplitudes? (besides 4-point, [Rosly-Selivanov, 02])

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- helpful fact: on-shell helicity states are BPS DBI solutions

$$F^{+}_{\dot{\alpha}\dot{\beta}}[A^{+}] = i\sqrt{2}p_{\dot{\alpha}}p_{\dot{\beta}} \qquad F^{+}_{\dot{\alpha}\dot{\beta}}[A^{-}] = 0$$
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- DBI in terms of F_+ and F_-: [Tseytlin, 99]

$$S_{\mathrm{DBI}} = \frac{1}{\pi^2\alpha'^2}\int d^4x\sqrt{\left(1+\frac{\pi^2\alpha'^2}{8}(F_+^2+F_-^2)\right)^2 - \frac{\pi^4\alpha'^4}{16}F_+^2F_-^2}$$
$$= \frac{1}{4}F_+^2 - \frac{\pi^2\alpha'^2}{32}\left(F_-^2F_+^2\right) + \frac{\pi^4\alpha'^4}{256}\left(F_-^4F_+^2 + F_+^4F_-^2\right) + \mathcal{O}(\alpha'^6)$$

- all vertices proportional to $F_+^2F_-^2 \rightarrow \ldots$

Deriving scattering amplitudes from DBI

- in accordance with the SUSY Ward identity:

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- see [Rosly-Selivanov, 02]: helicity conservation in DBI by $U(1)$ S-duality

Helicity conservation in DBI: diagrammatically

- S-duality obvious in reformulation [Rocek-Tseytlin, 97]

$$\mathcal{L}_{\mathrm{DBI}} = \frac{i}{2\pi\alpha'}\left(-ia\bar{a} + \lambda a - \bar{\lambda}\bar{a} + \frac{\sqrt{\pi\alpha'}}{2} a\bar{a}(\lambda - \bar{\lambda})\right) - \frac{1}{4}F^2 - i\frac{\sqrt{\pi\alpha'}\lambda}{8}F_+^2 + i\frac{\sqrt{\pi\alpha'}\bar{\lambda}}{8}F_-^2$$

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- "effective Higgs-gluon couplings" [RB-Schwinn, 08]:

$$-i\frac{\sqrt{\pi\alpha'}\lambda}{8}F_+^2 \rightarrow -\frac{1}{4}\frac{i\sqrt{\pi\alpha'}\lambda}{2 + i\sqrt{\pi\alpha'}\lambda} : F_+^2 : \quad + \mathrm{c.c.}$$

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- derivative corrections? higher dimensions? non-Abelian case? string field theory?

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- $z \rightarrow \infty$ behavior related to UV properties [Arkani-Hamed-Cachazo-] [-Kaplan, 08]

Example

Veneziano amplitude

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$$\hat{s} = s \quad \hat{t} = t - z' \quad \hat{u} = u + z'$$

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- with $z' = 2\alpha'(p_3^{\mu} n_{\mu})z$
- do finite z' sums: residues @∞ vanish

BCFW recursion

$$A_4^{\text{part}}(s,t) = \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+1)} \frac{\Gamma(\alpha' s - 1)}{\Gamma(\alpha' s - 1 - n)} \left(\frac{1}{\alpha' t - 1 + n} \right)$$

$$A_4^{\text{part}}(u,t) = \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+1)} \frac{\Gamma(1 - \alpha' s + n)}{\Gamma(1 - \alpha' s)} \left(\frac{1}{\alpha' t - 1 + n} + \frac{1}{\alpha' u - 1 + n} \right)$$

Four point functions in any string theory

simple observation

Any four point function in open string theory in flat background consists of some rational function of the momenta times a 'Veneziano' factor.

- $z' \to \infty$ is closely related to Regge poles:

$$\frac{1}{z'} \frac{\Gamma(\alpha' t + z' - 1)}{\Gamma(1 - \alpha' u + z')} \to (z')^{-\alpha' s} \left(1 + \mathcal{O}\left(\frac{1}{z'}\right)\right)$$

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- issue about reality conditions on kinematic invariants
- practical value? closure on gluons only?
- if true, 'three' vertices only:
- string field theory? CFT? topological field theory?
- other backgrounds?

Outline

Conclusions and outlook

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- join the fun!
  - more corrections? (α', # fields, # dimensions, # derivatives, g_s)
  - closed string? (KLT?)
  - other backgrounds?
  - $\mathcal{N} = 2$ strings? $\rightarrow$ [Berkovits,0x]
  - twistors / pure spinors?
  - your question?