The dipole picture in DIS: saturation and heavy quarks

Grégory Soyez[1] *

Brookhaven National Laboratory - Physics Department
Building 510, Upton, NY 11973 - USA

We discuss the description of the proton structure function within the dipole factorisation framework. We parametrise the forward dipole amplitude to account for saturation as predicted by the small-x QCD evolution equations. Contrarily to previous models, the saturation scale does not decrease when taking heavy quarks into account. We show that the same dipole amplitude also allows to reproduce diffractive data and exclusive vector meson production.

In these proceedings [1] we shall concentrate on Deep Inelastic Scattering (DIS) at small x. In this regime, the photon-proton cross-section can be factorised as a convolution between the wavefunction for a virtual photon to fluctuate into a quark-antiquark pair and the interaction T between this colourless dipole and the proton:

$$\sigma_{L,T}(x,Q^2) = 2\pi R_p^2 \sum_f \int d^2r\, dz\, \left|\Psi_{L,T}(r,z;Q^2)\right|^2 T(r,x), \tag{1}$$

where the factor $2\pi R_p^2$ arises from integration over the impact parameter. The photon wavefunction can be computed from perturbative QED and we are left with the parametrisation of the hadronic dipole amplitude. To that aim, we usually rely on the observation that the small-x DIS data satisfy *geometric scaling* [2], meaning that, instead of being a function of both Q^2 and x, they appear to be a function of $\tau = \log(Q^2/Q_s^2(x)) = \log(Q^2/Q_0^2) - \lambda\log(1/x)$ only, where $Q_s(x)$ is known as the *saturation scale*. Since $r \sim 1/Q$ in (1), this property suggests that the dipole amplitude is a function of $rQ_s(x)$ only.

Since the small-x domain extends down to small Q^2, the dipole amplitude is sensitive to the unitarity bound $T < 1$. There are two broad classes of models which differ by their way to implement that boundary. The first approach is to use an eikonal form as initially proposed in [3], followed by more precise analysis to incorporate DGLAP evolution and masses for the heavy quarks [4].

The second approach, that we follow through these proceedings, is to use predictions directly from the Balitsky-Kovchegov equation describing the QCD evolution to small x. It resums the BFKL logarithms of $1/x$ and satisfies unitarity by including saturation effects.

In contrast with the eikonal models which include it by hand, it has been proven [5] that the solutions of the BK equation satisfy the property of geometric scaling. More precisely,

$$T(r,x) \overset{rQ_s \lesssim 2}{\propto} \exp\left[-\gamma_c z - \frac{z^2}{\kappa\lambda\log(1/x)}\right] \quad \text{with } z = \log\left(\frac{4}{r^2Q_s^2}\right),\ Q_s^2 = \left(\frac{x}{x_0}\right)^{-\lambda}\ \text{GeV}^2. \tag{2}$$

In this expression, γ_c, λ and κ are obtained directly from the BFKL kernel. The first term in the exponential, surviving at asymptotically small x, satisfies geometric scaling, while the second term violates geometric scaling and describes how it is approached when x decreases. Geometric scaling is thus respected when the second term can be neglected *i.e.* in a window

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extending up to $z = \sqrt{\kappa\lambda \log(1/x)}$, which extends beyond the saturation momentum itself. This is an important message one learns from the BK equation: the saturation effects are not only important for $rQ_s > 1$ (or $Q^2 \le Q_s^2$); they do affect the physical amplitude at larger values of Q^2, in a window growing like $\sqrt{\log(1/x)}$ and where the dipole amplitude T is significantly smaller than 1.

We thus can use (2) to parametrise T at small dipole sizes and match it continuously with an expression of the form $1 - \exp(-(az+b)^2)$, describing the solutions of the BK equation in the deep saturation domain. The parameters λ, x_0 and R_p are then fitted[a] to reproduce the latest HERA measurements of the inclusive proton structure function for $x \le 0.01$. This method has been successfully applied by Iancu, Itakura and Munier (IIM) [7], as shown in the first line of Table 1, where the sum over quark flavours in (1) only account for three light quarks.

model	γ_c	v_c	x_0	R_p	χ^2/n
IIM	0.6275	0.253	$2.67\ 10^{-5}$	3.250	$\approx$0.9
IIM+c,b	0.6275	0.195	$6.42\ 10^{-7}$	3.654	1.109
new fit[6]	0.7065	0.222	$1.19\ 10^{-5}$	3.299	0.963

Table 1: Values of the parameters and χ^2 per data point for (i) the original IIM model, the IIM model with heavy quarks and fixed γ_c and (iii) the new, adapted, model.

One of the general issues of these models is that, in both classes of models, once the mass of the heavy quarks is taken into account in (1), the saturation scale drops down by a factor ≈ 2 (≈ 500 MeV instead of ≈ 1 GeV). This is illustrated by the second line of Table 1, where we see that, once the contribution from heavy quarks is included, the quality of the fit becomes poor and x_0 decreases severely.

Recently [6], I have shown that it was possible to accommodate, for the first time, the IIM model to include heavy quark contributions without having the inconvenient that the saturation scale goes down. The underlying idea is to allow the slope γ_c to become a free parameter of the fit. As shown on the third line of Table 1, this does not only brings the parameters closer to the original IIM model, especially x_0 as we will comment further later on, but it also results in a much better χ^2. To obtain the parameters mentioned in Table 1, one has restricted the Q^2 range to $Q^2 \le 150\ \text{GeV}^2$, though the parameters are stable when we vary this limit. Note however that we do expect corrections from resummation of the DGLAP logarithms at larger Q^2.

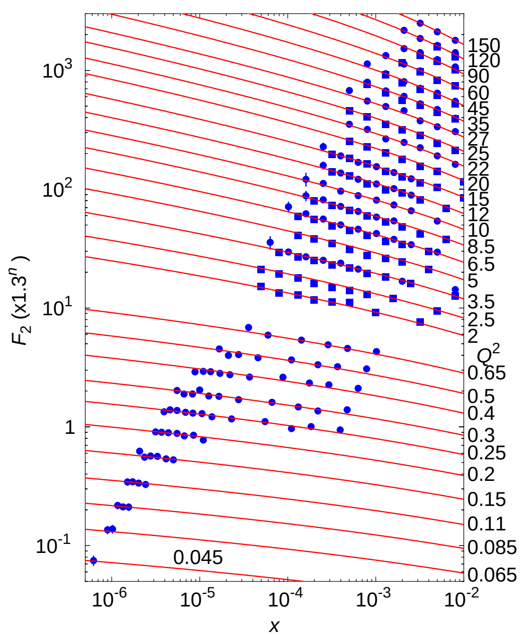

Figure 1: Description of the F_2^p HERA data.

[a] γ_c and κ are fixed to the value predicted from the leading-order BFKL kernel.

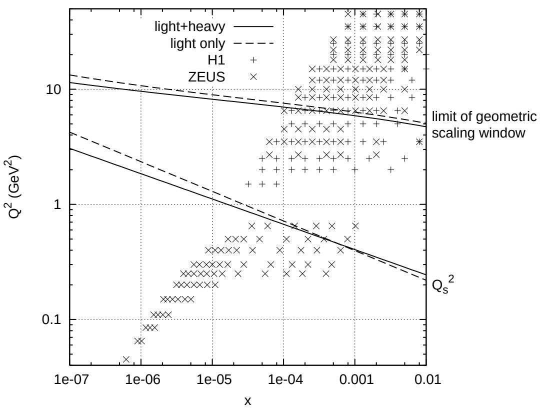

Figure 2: Saturation line and extension of the geometric scaling window with HERA data.

Figure 1 shows how well the F_2^p data [8] are reproduced. More interestingly, we have compared on Fig. 2 the saturation scale obtained in this new parametrisation with the one of the IIM model (bottom lines). They are clearly of the same order, showing that it is possible to include the heavy quarks in the dipole picture and at the same time keep a dipole amplitude with a saturation scale around 1 GeV at HERA. We also observe on Fig. 2 (upper lines) that the data up to $Q^2 = 5 - 7$ GeV^2 lie inside the geometric-scaling window and are thus sensitive to the physics of saturation.

Now that heavy quarks are properly integrated into the picture, we can have predictions for the charm and bottom structure functions. One sees from Figs. 3 that we achieve a good description of those data. This figure also shows the predictions for the longitudinal structure function where the model once again agrees with the data.

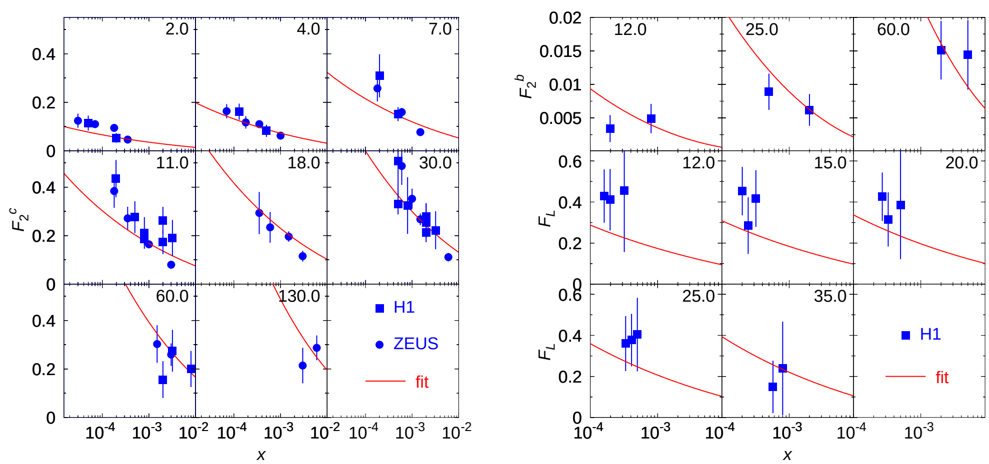

Figure 3: The charm structure function (left plot), bottom structure function (top right) and longitudinal structure function (bottom right).

With this new model for the forward dipole amplitude that includes heavy quarks and remains compatible with small-x QCD evolution, we can also look at exclusive processes. As we show now, one of the major power of the dipole picture is that it allows, with the same parametrisation of the dipole amplitude, to describe both inclusive and diffractive processes.

The first of those measurements we shall consider is the diffractive structure function. Generally speaking, this quantity probes correlations inside the proton and is sensitive to the square of the dipole amplitude. However, considering only fluctuations of the photon into a colourless $q\bar{q}$ state only allows to describe the limit of large β (or the limit small diffractive mass $M_X \ll Q$). To go to smaller values of β (keeping $x_{\rm pom} = x/\beta \ll 1$), we also have to consider the radiation of one additional gluon from the initial quark-antiquark pair. We can then show that the interaction between the resulting $q\bar{q}g$ pair and the proton can also be expressed in terms of the dipole amplitude T, leading again to a contribution proportional to T^2. Based on the new parametrisation [6], it has been shown [9] that the HERA measurements of the diffractive structure function are well reproduced.

The final observable we shall consider is the production of exclusive vector mesons or on-shell photons (DVCS). The differential cross-section $d\sigma^{\gamma^* p \to V p}/dt$ can also be factorised, this time as a convolution between a vertex for the virtual photon to fluctuate into a $q\bar{q}$ pair, the interaction between that dipole and the proton, and the vector-meson wavefunction to account for the final state. The main difference with (1), beside the presence of the vector-meson wavefunction, is that one has to account for the momentum transfer dependence of the dipole scattering amplitude. An intuitive method is to make a Fourier transform and go to impact-parameter space. We shall rather use the result of studies of the full BK equation including its momentum-transfer dependence. It is predicted [10] that the saturation scale is constant at small t and increases like $|t|$ at large t. Introducing one parameter to implement that dependence and a second one to describe the proton form factor (taken of the form $\exp(-b|t|)$), we have reached [11] a successful description of the differential and total cross-sections for exclusive productions of ρ, ϕ and J/Ψ mesons, as well as the for DVCS measurements. The description of the ρ-meson differential cross-section is given as an example on Fig. 4.

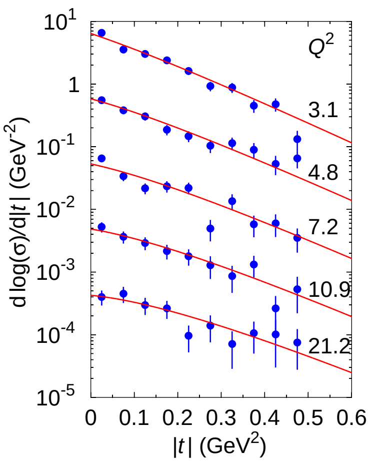

Figure 4: Description of the ρ-meson differential cross-section.

References

[1] Slides: `http://indico.cern.ch/contributionDisplay.py?contribId=249&sessionId=30&confId=24657`

[2] A. M. Staśto, K. Golec-Biernat, and J. Kwiecinski, Phys. Rev. Lett. **86**, 596 (2001) [arXiv:hep-ph/0007192].

[3] K. Golec-Biernat and M. Wusthoff, Phys. Rev. D **59** (1999) 014017 [arXiv:hep-ph/9807513]; Phys. Rev. D **60** (1999) 114023 [arXiv:hep-ph/9903358].

[4] K. Golec-Biernat and S. Sapeta, Phys. Rev. D **74** (2006) 054032 [arXiv:hep-ph/0607276].

[5] E. Iancu, K. Itakura and L. McLerran, Nucl. Phys. A **708** (2002) 327 [arXiv:hep-ph/0203137]; S. Munier and R. Peschanski, Phys. Rev. Lett. **91**, 232001 (2003) [arXiv:hep-ph/0309177].

[6] G. Soyez, Phys. Lett. B **655** (2007) 32 [arXiv:0705.3672 [hep-ph]].

[7] E. Iancu, K. Itakura and S. Munier, Phys. Lett. B **590** (2004) 199 [arXiv:hep-ph/0310338].

[8] J. Breitweg *et al.* [ZEUS Collaboration], Phys. Lett. B **487** (2000) 273 [arXiv:hep-ex/0006013]; S. Chekanov *et al.* [ZEUS Collaboration], Eur. Phys. J. C **21** (2001) 443 [arXiv:hep-ex/0105090]; C. Adloff *et al.* [H1 Collaboration], Eur. Phys. J. C **21** (2001) 33 [arXiv:hep-ex/0012053].

[9] C. Marquet, Phys. Rev. D **76** (2007) 094017 [arXiv:0706.2682 [hep-ph]].

[10] C. Marquet, R. B. Peschanski and G. Soyez, Nucl. Phys. A **756** (2005) 399 [arXiv:hep-ph/0502020]. C. Marquet and G. Soyez, Nucl. Phys. A **760** (2005) 208 [arXiv:hep-ph/0504080].

[11] C. Marquet, R. B. Peschanski and G. Soyez, Phys. Rev. D **76** (2007) 034011 [arXiv:hep-ph/0702171].