

Phenomenology

Gavin P. Salam

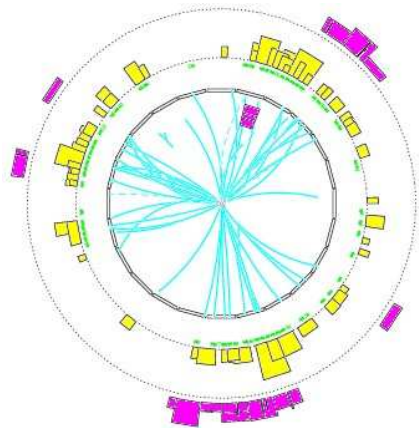
LPTHE, Universities of Paris VI and VII and CNRS

BUSSTEPP

Plymouth, August 2004

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Lecture 4
(QCD jets)

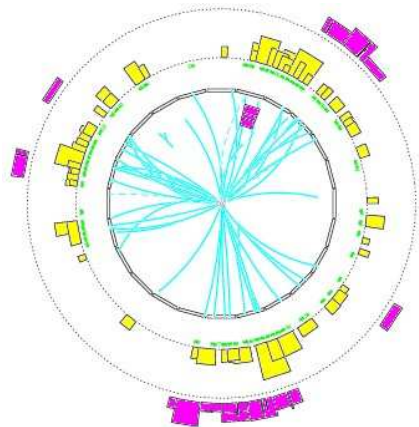


Previous lecture

- Divergent matrix element for emission of soft and collinear gluons.
- ‘Good’ observables are insensitive to this — infrared and collinear safe.
- But complex event structure is still present (and must be understood for many practical uses of QCD).

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- Try to see how event structure builds up.
- See when that information is relevant.



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- ↳ Perturbation theory too cumbersome, *e.g.* $n!$ growth of number of diags.
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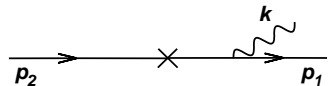
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To understand jet structure need to consider *multiple soft gluon emission*.

Before doing so, useful to examine some simple QED cases.

Soft photon radiated from e^+e^- pair



$$M_{e^+e^-\gamma} = -M_{e^+e^-} g_e \left(\frac{p_1 \cdot \epsilon}{p_1 \cdot k} - \frac{p_2 \cdot \epsilon}{p_2 \cdot k} \right)$$

Divergent denominator: near on-shell propagator

This is the source of the enhancement of soft-photon radiation.

Same mechanism in QCD

Squared amplitude for photon emission:

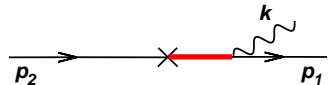
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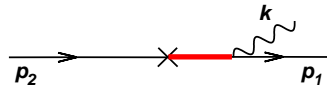
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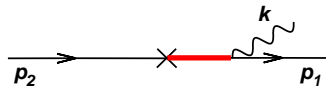
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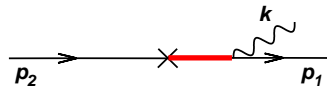
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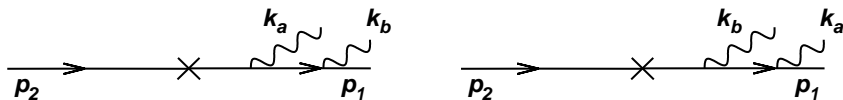
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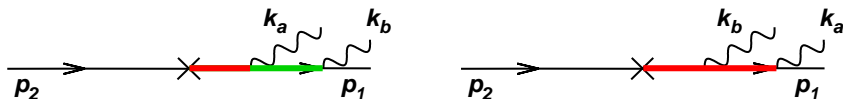
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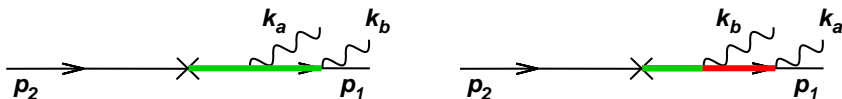
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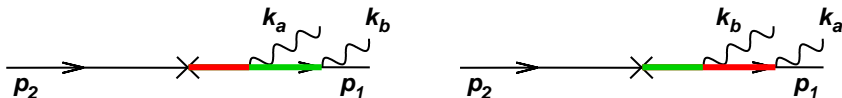
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Squared amplitude for double soft photon emission

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Independent emission, eq.(1), holds if both are soft.

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- k_b radiated 'after' k_a
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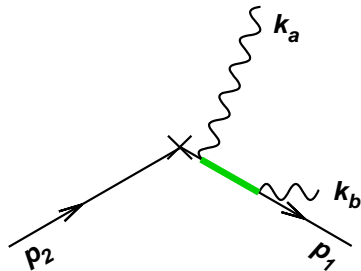
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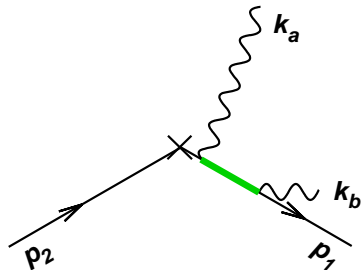
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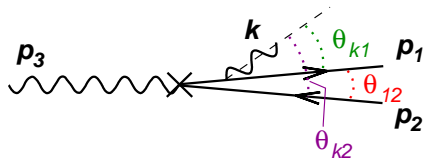
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Hard photon p_3 , & outgoing e^+e^- pair close in angle, $\theta_{12} \ll 1$.

What is radiation pattern of soft photon k ?



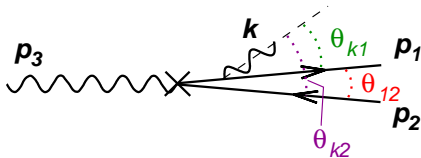
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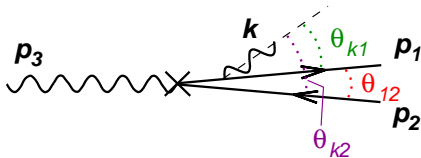
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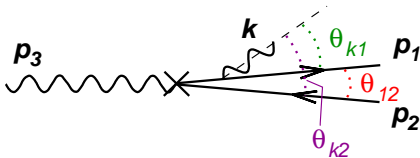
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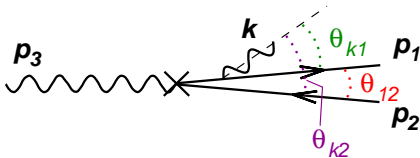
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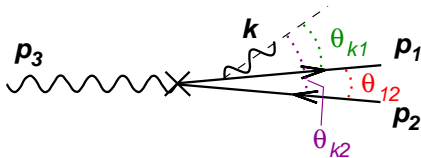
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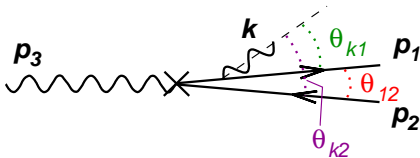
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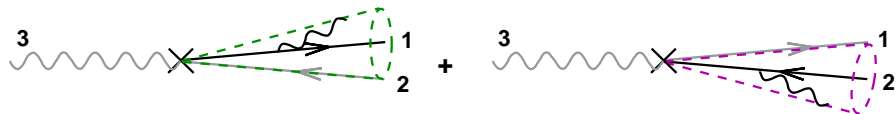
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Photon emission from e^+e^- pair behaves, approximately, as if it is restricted to two cones of radiation:



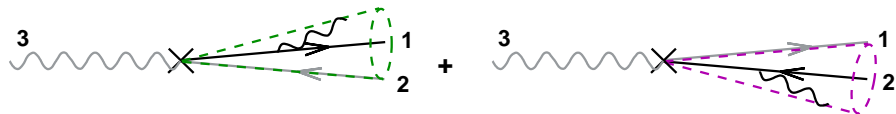
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Called *angular ordering*. Relation is exact after integration over angles.

Coherent sum of radiation reduces to *incoherent sum* (over cones)

Photon emission from e^+e^- pair behaves, approximately, as if it is restricted to two cones of radiation:



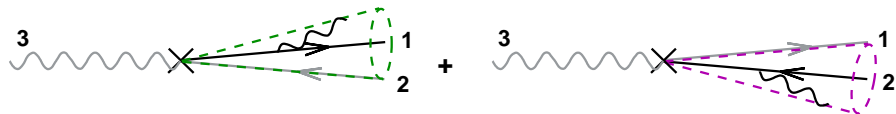
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Ideas of multiple soft emission and coherence apply also to QCD.

'Novel' aspects come from $SU(3)$ (colour), in particular gluon's charge.

$k_b \ll k_a \ll p_1, p_2$ (just maximally divergent diags; 'a' radiated off p_2)

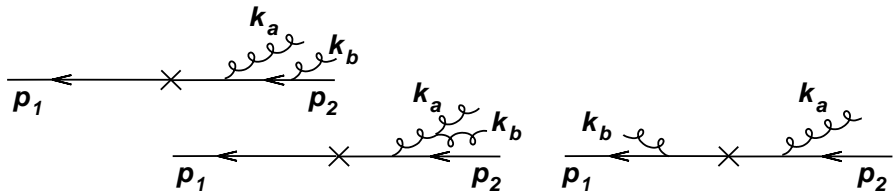
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$$M_{q\bar{q}ab} = -g_s^2 \frac{p_2 \cdot \epsilon_a}{p_2 \cdot k_a} \bar{u}(p_1) \times \left(-t^A t^B \frac{p_2 \cdot \epsilon_b}{p_2 \cdot k_b} + if^{ABC} t^C \frac{k_a \cdot \epsilon_b}{k_a \cdot k_b} + t^B t^A \frac{p_1 \cdot \epsilon_b}{p_1 \cdot k_b} \right) v(p_2)$$

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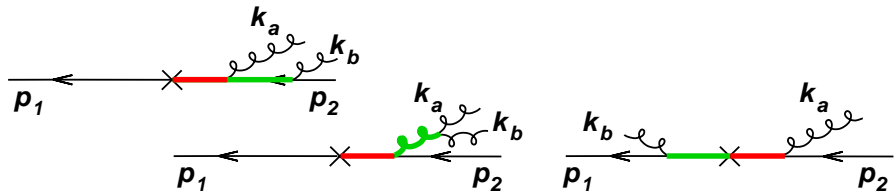
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Note structure as *incoherent sum over dipoles*

Consistency: off each quark we have

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Organizing squared amplitude

We have already assumed coupling is small and that gluon energies are *strongly ordered* ($\omega_b \ll \omega_a$).

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Need to organize it better — introduce an extra ‘small’ parameter:

- **EITHER:** Assume that their angles are also strongly ordered.

e.g. for $\omega_a \gg \omega_b$: $\theta_{a1} \gg \theta_{b1}$, or $\theta_{a1} \ll \theta_{b1}$

Then exploit *coherence* to simplify colour.

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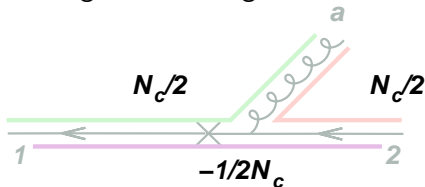
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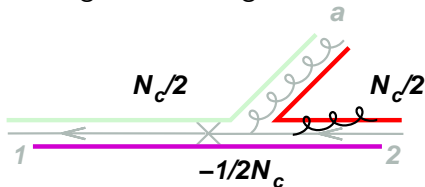
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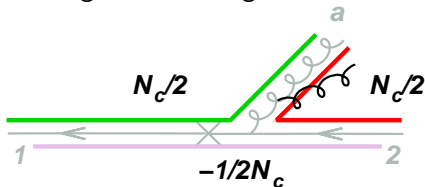
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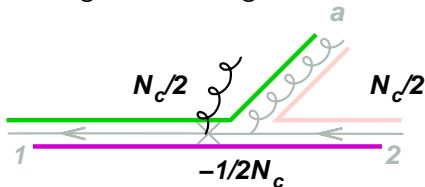


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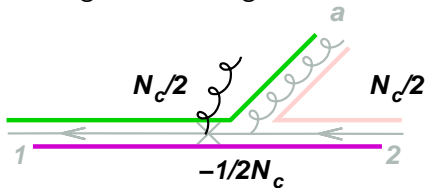


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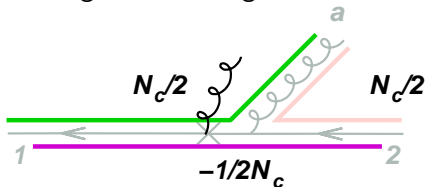


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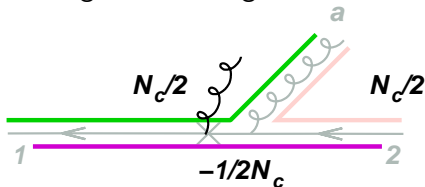


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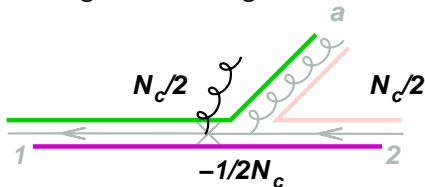


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- large-angle radiation doesn't resolve what happens at small angles.
- small-angle radiation doesn't care what is going on at larger angles.
- gluons on widely disparate angular scales do not 'talk to each other.'

Matrix element for n -gluon emission (strong θ and ω ordering)

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This is simplified version of a classic QCD result.

[Can be cast in many ways]

C_S : find the 'harder' (q , \bar{q} or $j < i$) parton j that gives the minimum angle in the denominator. Find the set S consisting of j and of all other partons k (q , \bar{q} or $k < i$), that satisfy $\theta_{kj} < \theta_{ij}$. C_S is the overall colour charge (C_F , C_A) of that set.

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This is simplified version of a classic QCD result.

[Can be cast in many ways]

C_S : find the 'harder' (q , \bar{q} or $j < i$) parton j that gives the minimum angle in the denominator. Find the set S consisting of j and of all other partons k (q , \bar{q} or $k < i$), that satisfy $\theta_{kj} < \theta_{ij}$. C_S is the overall colour charge (C_F , C_A) of that set.

Coherence:

- large-angle radiation doesn't resolve what happens at small angles.
- small-angle radiation doesn't care what is going on at larger angles.
- gluons on widely disparate angular scales do not 'talk to each other.'

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This is basis for *simulations* of QCD multi-gluon emission.

1. Start with a quark as the 'emitter'.

2. 'Scan' towards small angles.

3. Stop when you find a gluon that is more collinear than the previous one.

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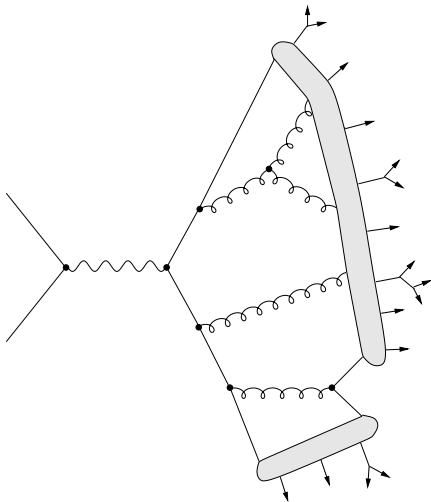
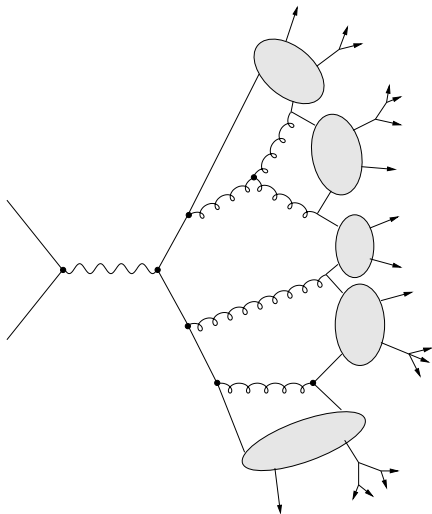
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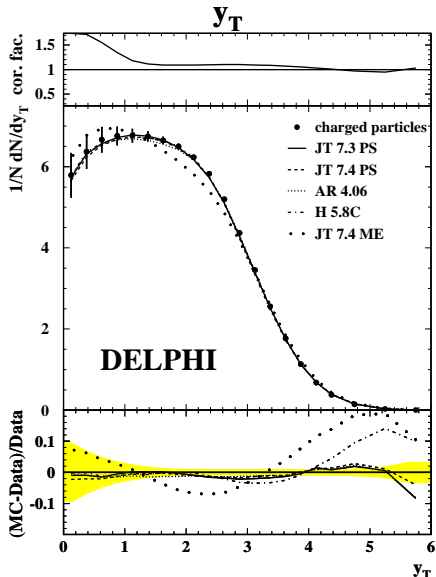
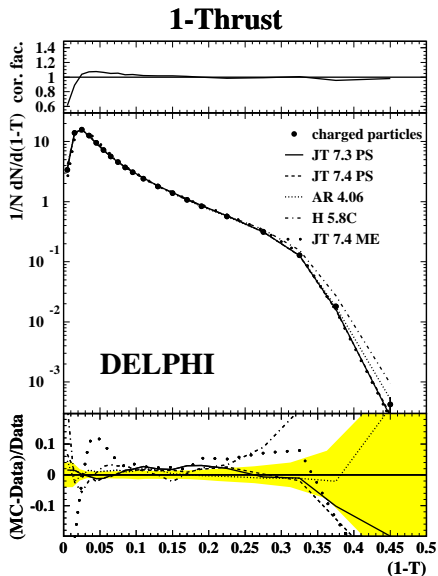


[fig. from B.R. Webber]

3rd 'problem' (not on problem sheet)

Lattice QCD can today calculate non-perturbative effects for the Υ system (a $b\bar{b}$ bound state), where the ratio of hard to soft scales is about 10 (the typical momentum scales are not m_b but $\alpha_s m_b$).

Make a guess as to how the difficulty of lattice calculations depends on the ratio of hard to soft scales. Assuming the continued validity of Moore's law (computing power doubles every 18 months), how long will it be before lattice can give a direct calculation of hadronisation effects in high-energy (100 GeV) collisions?



Why strongly ordered angles and energies?

It's easier, but is it justified?!

Answer: it depends on what you look at.

Use 'bootstrap' arguments. Calculate with 1-gluon soft-collinear approx:

✓ = dominated by soft-collinear region

✗ = dominated by hard region (→ NLO)

Results

✗ number of events with 3 hard jets

$$\sigma_{3\text{-jet}} \sim \alpha_s \int \frac{d\omega}{\omega} \frac{d\theta}{\theta} \Theta\left(\frac{\omega}{Q} - \epsilon\right) \Theta(\theta - \delta)$$

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Thrust — a QCD ‘guinea pig’

First discussion goes back to 1964. Serious work got going in late '70s.

Thrust is one of many continuous measures of the *event 'shape'*:

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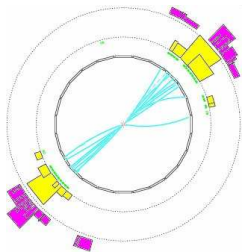
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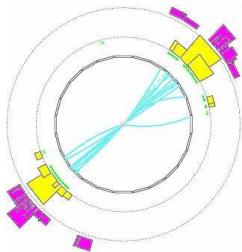
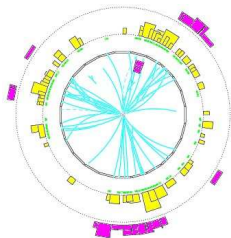
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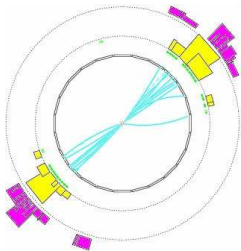
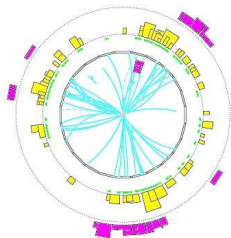
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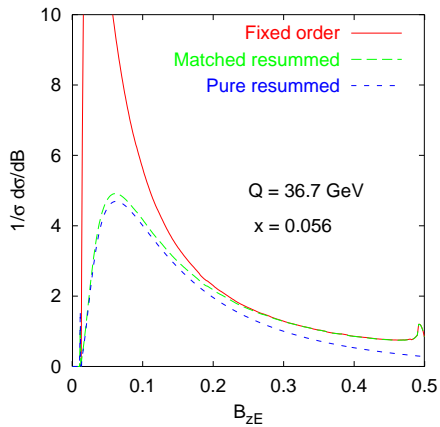
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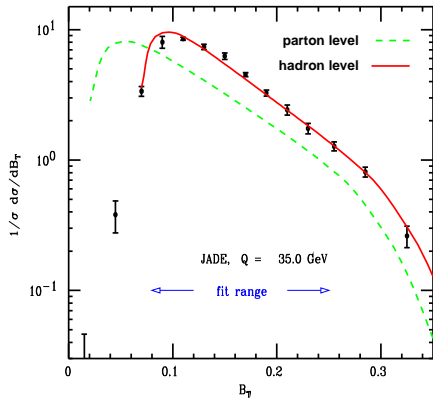
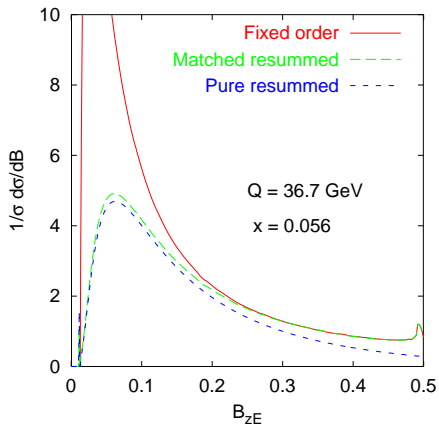
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This is called *resummation*





Range of QCD tools:

- **Fixed-order calculations (LO, NLO, ...)**
- Calculations based on multiple soft-collinear radiation
 - Monte Carlo 'cascade' event generators
 - analytical resummations $\sum (\alpha_s \ln^2 \tau)^n$
- Well-defined non-perturbative inputs (structure functions, fragmentation functions).
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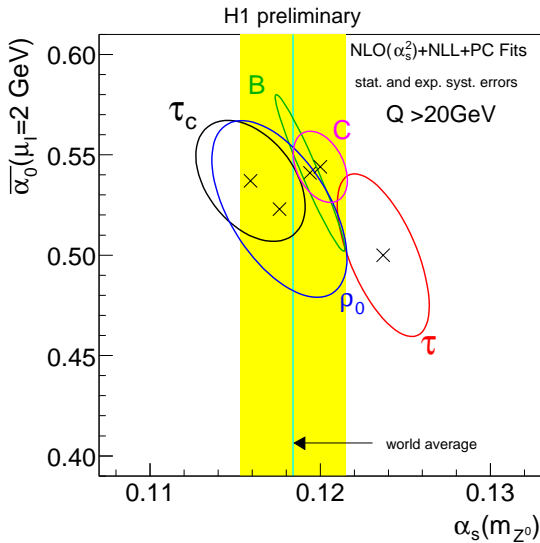
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EXTRA SLIDES

Fitting hadronisation with a single parameter



Jet clustering algorithms (for reference)

1. Define a distance measure for every pair of (pseudo)particles i and j

$$y_{ij} = \min(E_i^2, E_j^2) (1 - \cos \theta) \quad \text{'Durham' or 'k}_t\text{' measure}$$

2. Find the pair of (pseudo)particles with the smallest y_{ij} . If this y_{ij} is larger than some threshold y_{cut} , then stop.
3. Otherwise recombine i and j into a single 'pseudo-particle' and go to step 1.