

# Phenomenology

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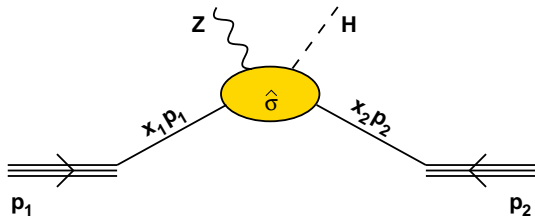
BUSSTEPP  
Ambleside, August 2005

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## Lecture 4

(Processes with incoming protons)

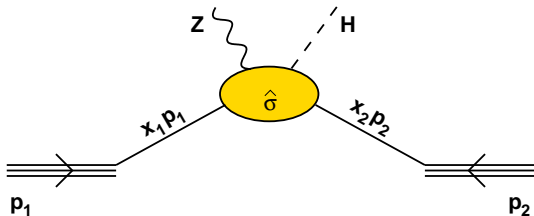
Recall Higgs production in hadron-hadron collisions:



$$\sigma = \int dx_1 f_{q/p}(x_1, \mu^2) \int dx_2 f_{\bar{q}/\bar{p}}(x_2, \mu^2) \hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2), \quad \hat{s} = x_1 x_2 s$$

- Total X-section is *factorized* into a 'hard part'  $\hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)$  and 'normalization' from parton distribution functions (PDF).
- Measure total cross section  $\leftrightarrow$  *need to know PDFs* to be able to test hard part (e.g. Higgs electroweak couplings).
- Picture seems intuitive, but
  - how can we determine the PDFs? NB: non-perturbative
  - does picture really stand up to QCD corrections?

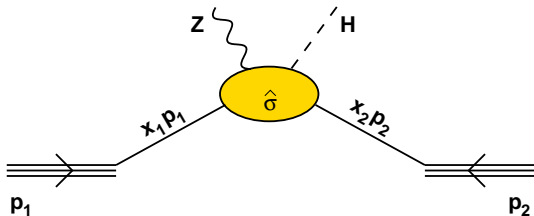
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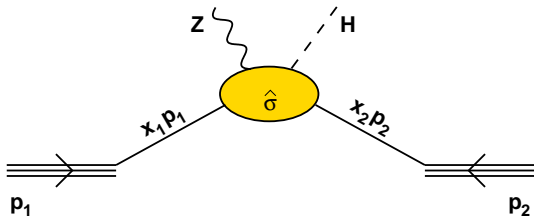
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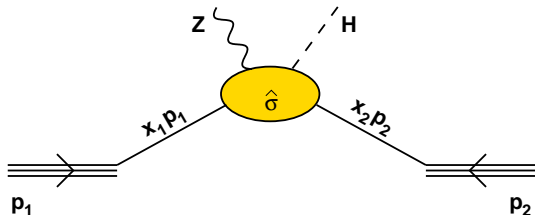
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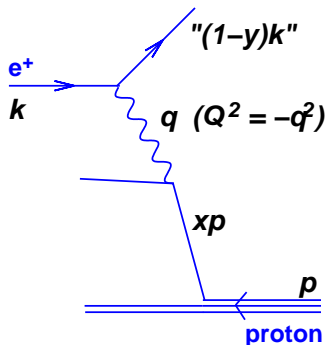


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Hadron-hadron is complex because of two incoming partons — so start with simpler Deep Inelastic Scattering (DIS).

### Kinematic relations:

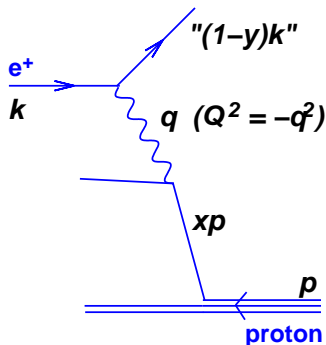


$$x = \frac{Q^2}{2p \cdot q}; \quad y = \frac{p \cdot q}{p \cdot k}; \quad Q^2 = xys$$

$\sqrt{s}$  = c.o.m. energy

- $Q^2$  = photon virtuality  $\leftrightarrow$  *transverse resolution* at which it probes proton structure
- $x$  = *longitudinal momentum fraction* of struck parton in proton
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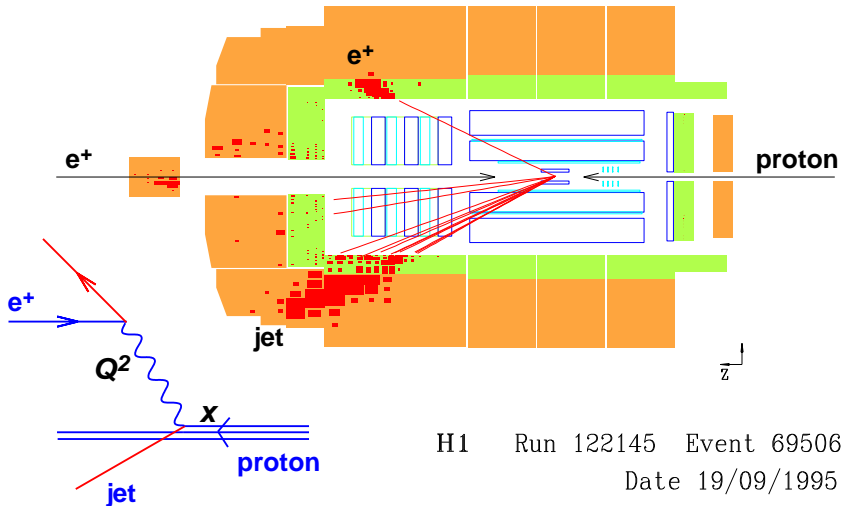
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## Deep Inelastic scattering (DIS): example



$$Q^2 = 25030 \text{ GeV}^2; \quad y = 0.56; \quad \mathbf{x=0.50}$$



Write DIS X-section to zeroth order in  $\alpha_s$  ('quark parton model'):

$$\frac{d^2\sigma^{em}}{dx dQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4} \left( \frac{1 + (1-y)^2}{2} F_2^{em} + \mathcal{O}(\alpha_s) \right)$$

$\propto F_2^{em}$                       [structure function]

$$F_2 = x(e_u^2 u(x) + e_d^2 d(x)) = x \left( \frac{4}{9} u(x) + \frac{1}{9} d(x) \right)$$

[ $u(x)$ ,  $d(x)$ : parton distribution functions (PDF)]

NB:

- use perturbative language for interactions of up and down quarks
- but distributions themselves have a *non-perturbative* origin.

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$F_2$  gives us *combination* of  $u$  and  $d$ .  
How can we extract them separately?

Assumption ( $SU(2)$  isospin): neutron is just proton with  $u \leftrightarrow d$ :  
 proton = uud; neutron = ddu

**Isospin:**  $u_n(x) = d_p(x), \quad d_n(x) = u_p(x)$

$$F_2^p = \frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)$$

$$F_2^n = \frac{4}{9}u_n(x) + \frac{1}{9}d_n(x) = \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

*Linear combinations* of  $F_2^p$  and  $F_2^n$  give separately  $u_p(x)$  and  $d_p(x)$ .

Experimentally, get  $F_2^n$  from deuterons:  $F_2^d = \frac{1}{2}(F_2^p + F_2^n)$

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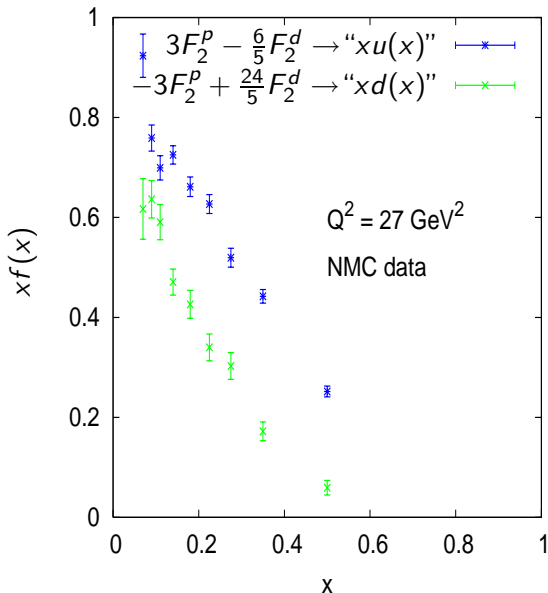
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Combine  $F_2^p$  &  $F_2^d$  data,  
deduce  $u(x)$ ,  $d(x)$ :

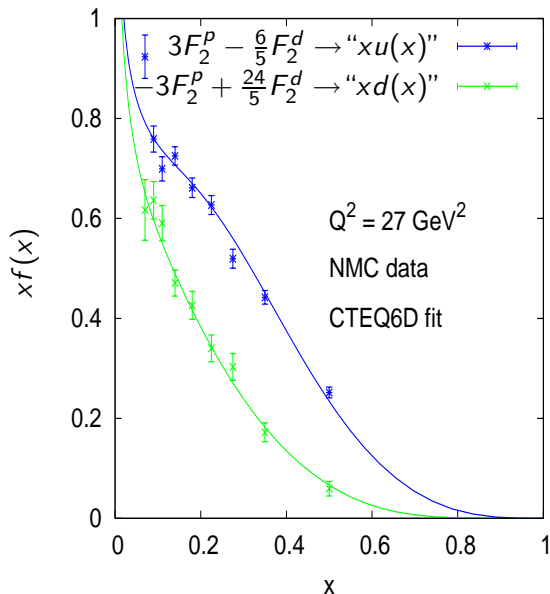
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How much  $u$  and  $d$ ?

- Total  $U = \int dx u(x)$
- $F_2 = x(\frac{4}{9}u + \frac{1}{9}d)$
- $u(x) \sim d(x) \sim x^{-1.25}$

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divergence

So why do we say  
proton =  $uud$ ?



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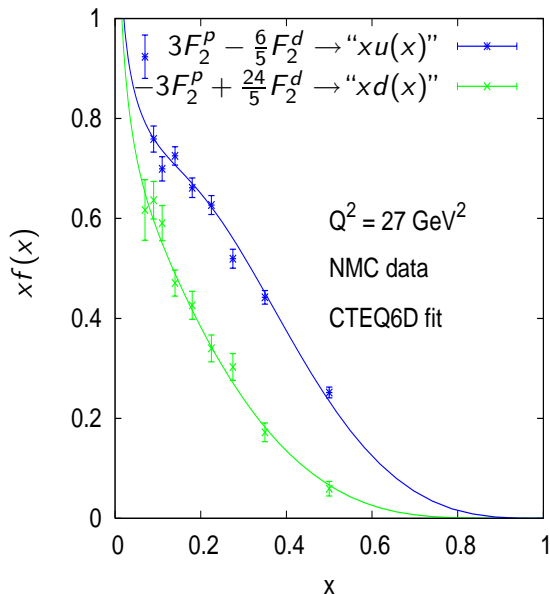
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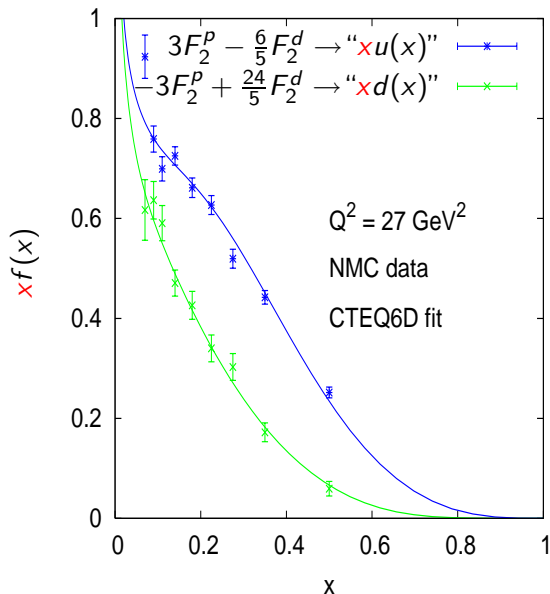
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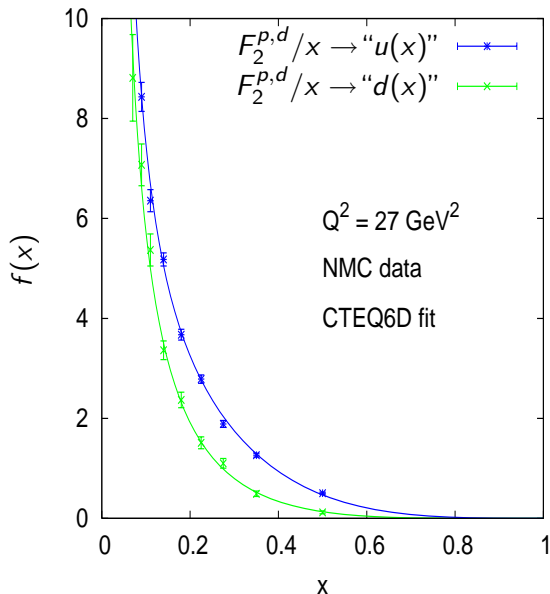
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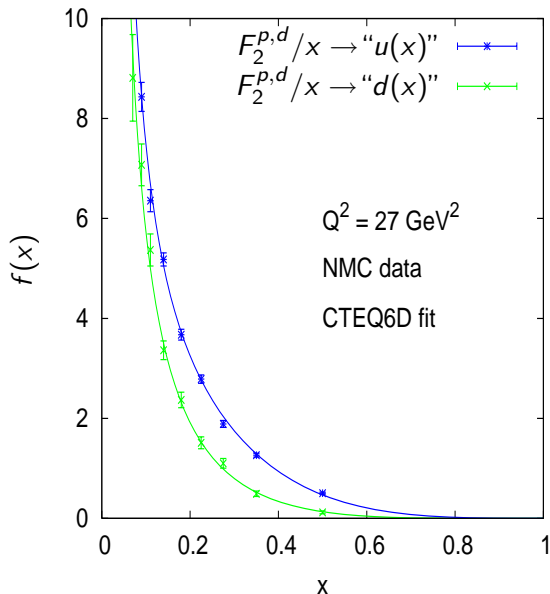
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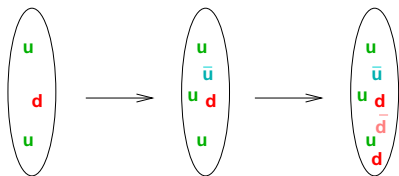
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Proton wavefunction *fluctuates* — extra  $u\bar{u}$ ,  $d\bar{d}$  pairs (*sea quarks*) can appear:

Antiquarks also have distributions,  $\bar{u}(x)$ ,  $\bar{d}(x)$

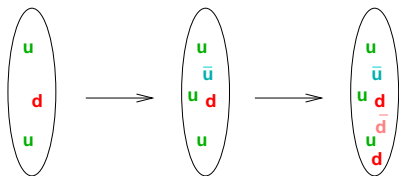
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NB: photon interaction  $\sim$  square of charge  $\rightarrow$  +ve

- Previous transparency: we were actually looking at  $\sim u + \bar{u}$ ,  $d + \bar{d}$
- Number of extra quark-antiquark pairs can be *infinite*, so

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as long as they carry little momentum (mostly at low  $x$ )



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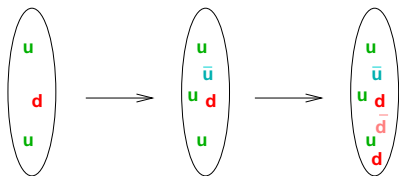
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When we say proton has 2 up quarks & 1 down quark we mean

$$\int dx (u(x) - \bar{u}(x)) = 2, \quad \int dx (d(x) - \bar{d}(x)) = 1$$

$u - \bar{u} = u_V$  is known as a *valence* distribution.

How do we measure *difference* between  $u$  and  $\bar{u}$ ? Photon interacts identically with both  $\rightarrow$  no good. . .

Question: what interacts differently with particle & antiparticle?

Answer:  $W^+$  or  $W^-$

See question sheet for more details. . .

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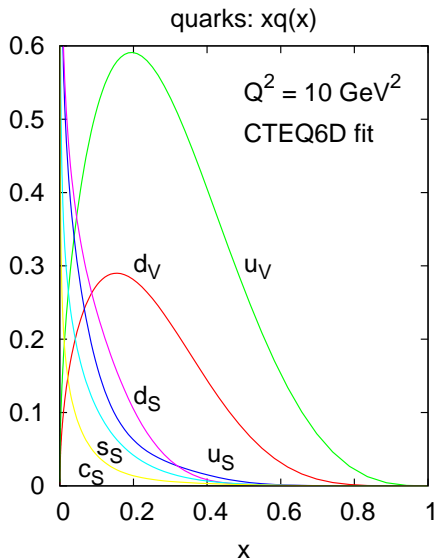
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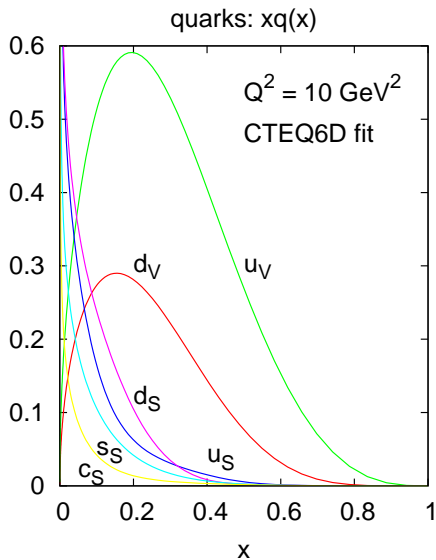
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These & other methods  $\rightarrow$  whole set of quarks & antiquarks

**NB: also strange and charm quarks**

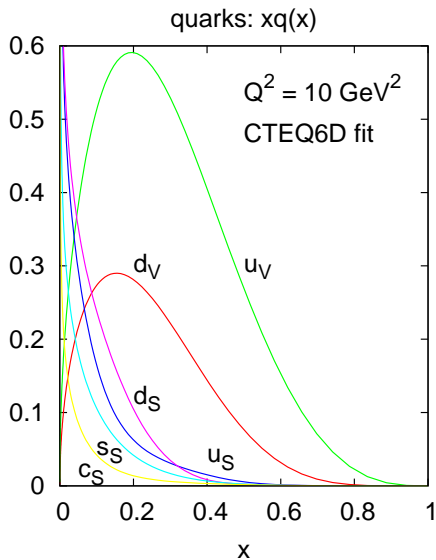
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  - $x \rightarrow 1 : xq_V(x) \sim (1-x)^3$   
quark counting rules
  - $x \rightarrow 0 : xq_V(x) \sim x^{0.5}$   
Regge theory
- sea quarks ( $u_S = 2\bar{u}, \dots$ ) fairly *soft* (low-momentum)
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Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx x q_i(x) = 1$$

$q_i$	momentum
$d_V$	0.111
$u_V$	0.267
$d_S$	0.066
$u_S$	0.053
$s_S$	0.033
$c_S$	0.016
total	0.546

*Where is missing momentum?*

Only parton type we've neglected so far is the

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Not directly probed by photon or  $W^\pm$ .

NB: need to know it for  $gg \rightarrow H$

To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD splitting.

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<b>total</b>	<b>0.546</b>

*Where is missing momentum?*

Only parton type we've neglected so far is the

**gluon**

Not directly probed by photon or  $W^\pm$ .

NB: need to know it for  $gg \rightarrow H$

To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD splitting...

Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx x q_i(x) = 1$$

$q_i$	momentum
$d_V$	0.111
$u_V$	0.267
$d_S$	0.066
$u_S$	0.053
$s_S$	0.033
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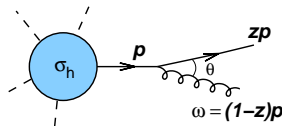
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To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD splitting...

Previous lecture: calculated  $q \rightarrow qg$  ( $\theta \ll 1$ ,  $\omega \ll p$ ) for final state of arbitrary hard process ( $\sigma_h$ ):

$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta^2}{\theta^2}$$



*Rewrite* with different kinematic variables

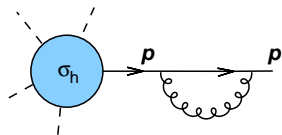
$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

$$\omega = (1-z)p$$

$$k_t = \omega \sin \theta \simeq \omega \theta$$

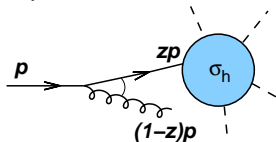
If we avoid distinguishing  $q + g$  final state from  $q$  (infrared-collinear safety), then divergent real and virtual corrections *cancel*

$$\sigma_{h+V} \simeq -\sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$



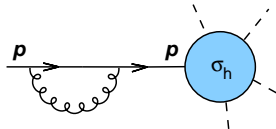
For initial state splitting, hard process occurs *after splitting*, and momentum entering hard process is modified:  $p \rightarrow zp$ .

$$\sigma_{g+h}(p) \simeq \sigma_h(zp) \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$



For virtual terms, momentum entering hard process is unchanged

$$\sigma_{V+h}(p) \simeq -\sigma_h(p) \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$



Total cross section gets contribution with *two different hard X-sections*

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \int \frac{dk_t^2}{k_t^2} \frac{dz}{1-z} [\sigma_h(zp) - \sigma_h(p)]$$

NB: We assume  $\sigma_h$  involves momentum transfers  $\sim Q \gg k_t$ , so ignore extra transverse momentum in  $\sigma_h$

## Initial-state collinear divergence

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \int_0^{Q^2} \frac{dk_t^2}{k_t^2} \int \frac{dz}{1-z} [\sigma_h(zp) - \sigma_h(p)]$$

- In soft limit ( $z \rightarrow 1$ ),  $\sigma_h(zp) - \sigma_h(p) \rightarrow 0$ : *soft divergence cancels*.
- For  $1 - z \neq 0$ ,  $\sigma_h(zp) - \sigma_h(p) \neq 0$ , so  $z$  integral is non-zero but finite.

**BUT:**  $k_t$  integral is just a factor, and is *infinite*

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So how do we do QCD calculations in such cases?

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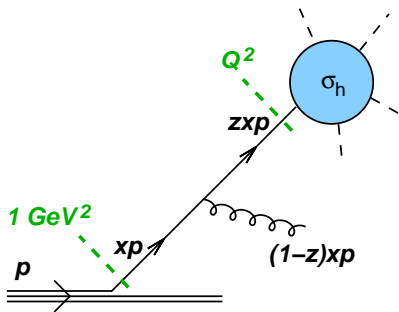
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By what right did we go to  $k_t = 0$ ?

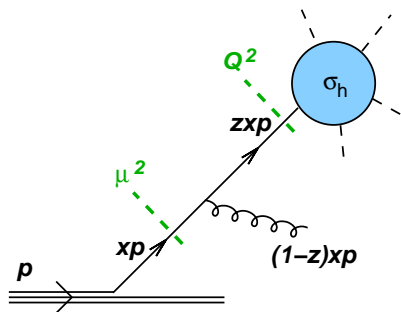
We assumed pert. QCD to be valid for all scales, but *below 1 GeV it becomes non-perturbative.*

Cut out this divergent region, & instead put non-perturbative quark distribution in proton.

$$\sigma_0 = \int dx \sigma_h(xp) q(x, 1 \text{ GeV}^2)$$

$$\sigma_1 \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_{1 \text{ GeV}^2}^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{finite (large)}} \underbrace{\int \frac{dx dz}{1-z} [\sigma_h(zxp) - \sigma_h(xp)]}_{\text{finite}} q(x, 1 \text{ GeV}^2)$$

In general: replace  $1 \text{ GeV}^2$  cutoff with arbitrary *factorization scale*  $\mu^2$ .



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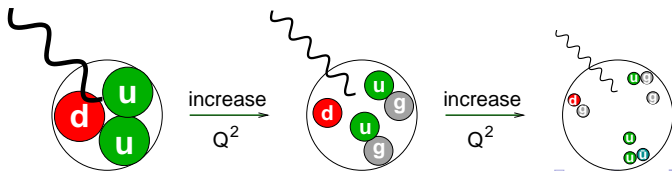
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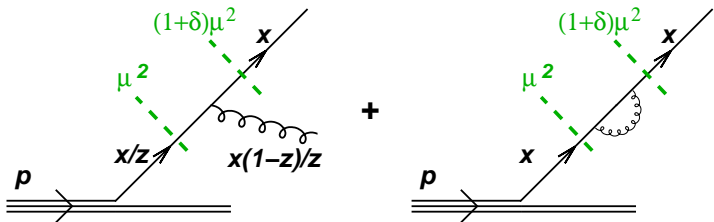
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Real and virtual have different longitudinal momenta
- Situation analogous to renormalization: need to *regularize* (but in IR instead of UV).  
Technically, often done with dimensional regularization
- Physical sense of regularization is to separate (*factorize*) proton non-perturbative dynamics from perturbative hard cross section.  
Choice of factorization scale,  $\mu^2$ , is arbitrary between  $1 \text{ GeV}^2$  and  $Q^2$
- In analogy with running coupling, we can *vary factorization scale* and get a *renormalization group equation* for parton distribution functions.  
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Change convention: (a) now *fix outgoing* longitudinal momentum  $x$ ; (b) *take derivative* wrt factorization scale  $\mu^2$



$$\frac{dq(x, \mu^2)}{d \ln \mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 dz p_{qq}(z) \frac{q(x/z, \mu^2)}{z} - \frac{\alpha_s}{2\pi} \int_0^1 dz p_{qq}(z) q(x, \mu^2)$$

$p_{qq}$  is real  $q \leftarrow q$  **splitting kernel**:  $p_{qq}(z) = C_F \frac{1+z^2}{1-z}$

Until now we approximated it in soft ( $z \rightarrow 1$ ) limit,  $p_{qq} \simeq \frac{2C_F}{1-z}$

Awkward to write real and virtual parts separately. Use more compact notation:

$$\frac{dq(x, \mu^2)}{d \ln \mu^2} = \frac{\alpha_s}{2\pi} \underbrace{\int_x^1 dz P_{qq}(z) \frac{q(x/z, \mu^2)}{z}}_{P_{qq} \otimes q}, \quad P_{qq} = C_F \left( \frac{1+z^2}{1-z} \right)_+$$

This involves the *plus prescription*:

$$\int_0^1 dz [g(z)]_+ f(z) = \int_0^1 dz g(z) f(z) - \int_0^1 dz g(z) f(1)$$

$z = 1$  divergences of  $g(z)$  cancelled if  $f(z)$  sufficiently smooth at  $z = 1$

Proton contains both quarks and gluons — so DGLAP is a *matrix in flavour space*:

$$\frac{d}{d \ln Q^2} \begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q \leftarrow q} & P_{q \leftarrow g} \\ P_{g \leftarrow q} & P_{g \leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

[In general, matrix spanning all flavors, anti-flavors,  $P_{qq'} = 0$  (LO),  $P_{\bar{q}g} = P_{qg}$ ]

Splitting functions are:

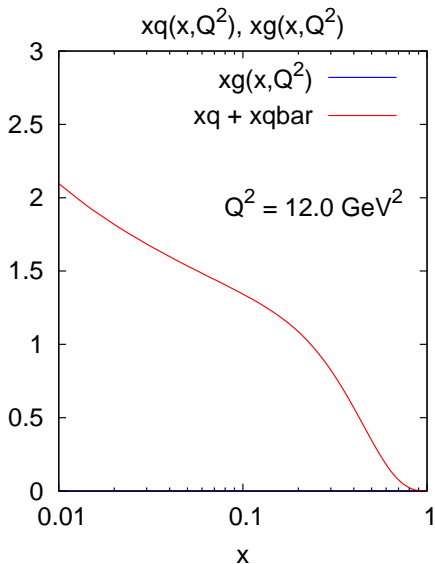
$$P_{qg}(z) = T_R [z^2 + (1-z)^2], \quad P_{gq}(z) = C_F \left[ \frac{1 + (1-z)^2}{z} \right],$$

$$P_{gg}(z) = 2C_A \left[ \frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \right] + \delta(1-z) \frac{(11C_A - 4n_f T_R)}{6}.$$

Have various symmetries / significant properties, e.g.

- $P_{qg}, P_{gg}$ : *symmetric*  $z \leftrightarrow 1-z$  (except virtuals)
- $P_{qq}, P_{gg}$ : *diverge* for  $z \rightarrow 1$  soft gluon emission
- $P_{gg}, P_{gq}$ : *diverge* for  $z \rightarrow 0$  Implies PDFs grow for  $x \rightarrow 0$

## Effect of DGLAP (initial quarks)



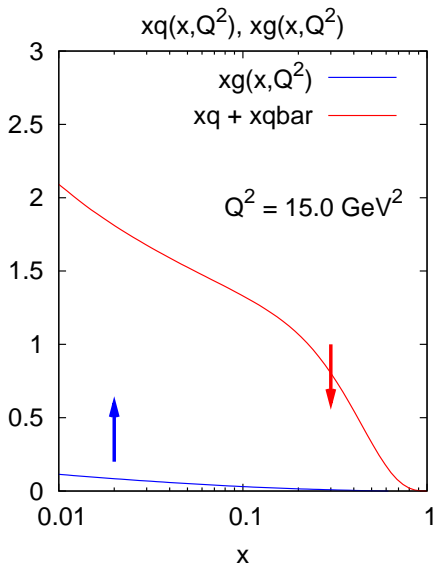
Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q \leftarrow q} \otimes q$$

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- quark is depleted at large  $x$
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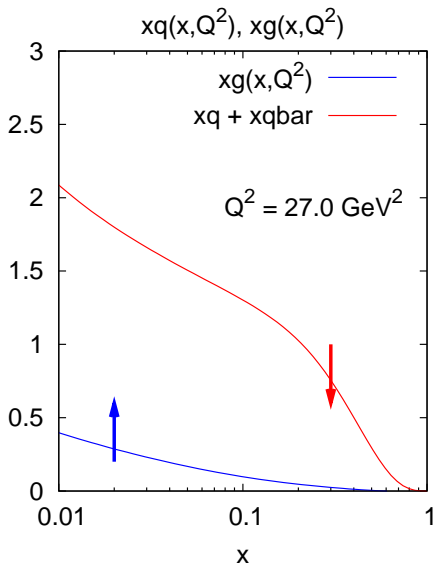
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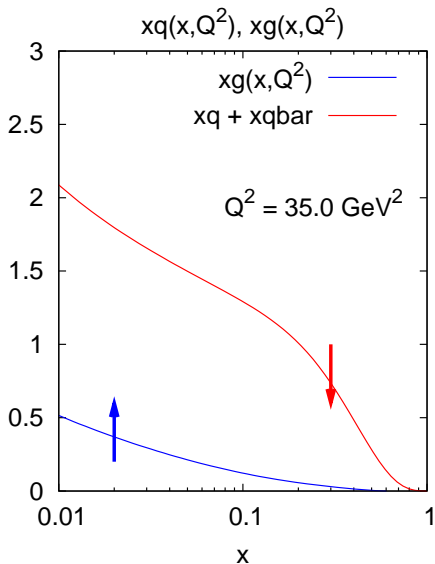
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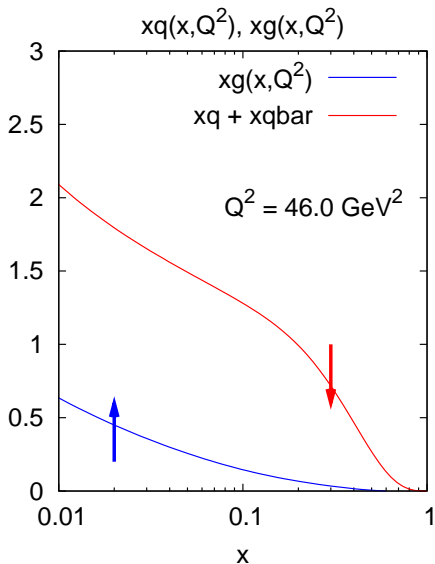
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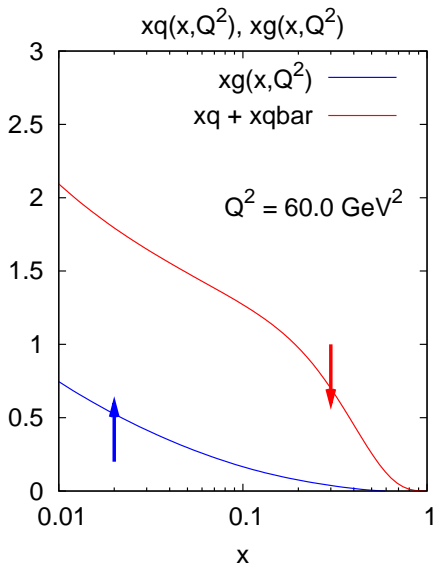
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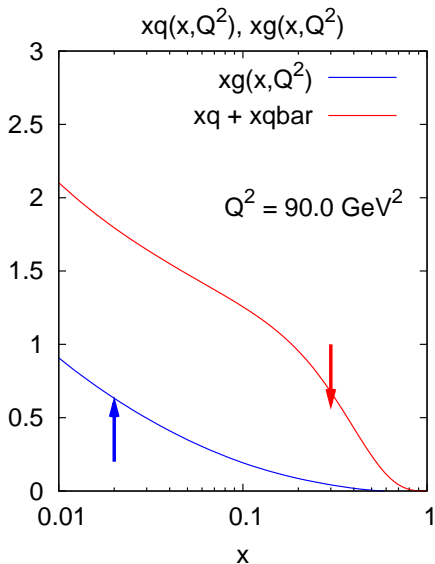
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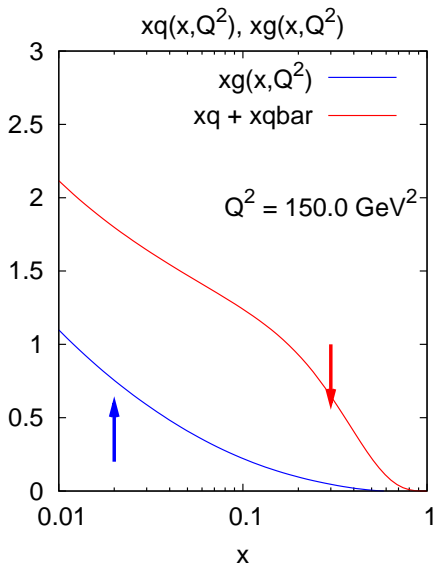
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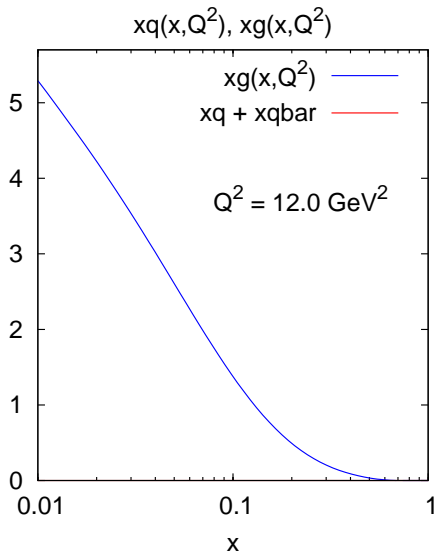
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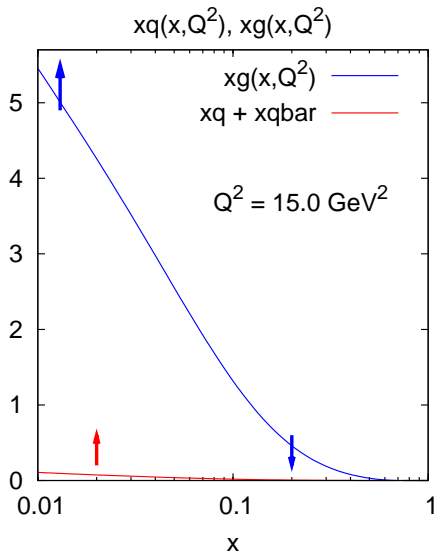
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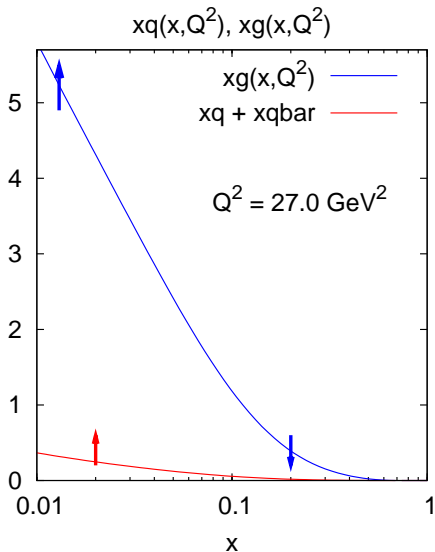
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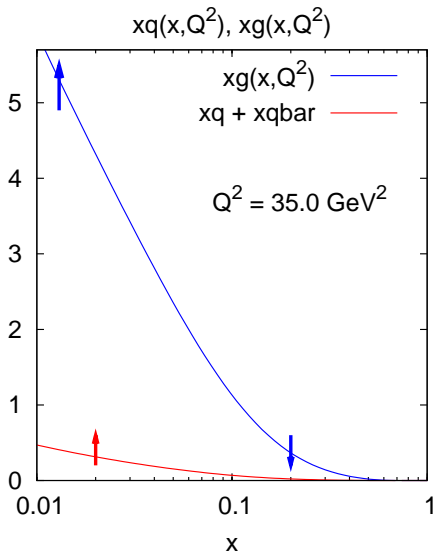
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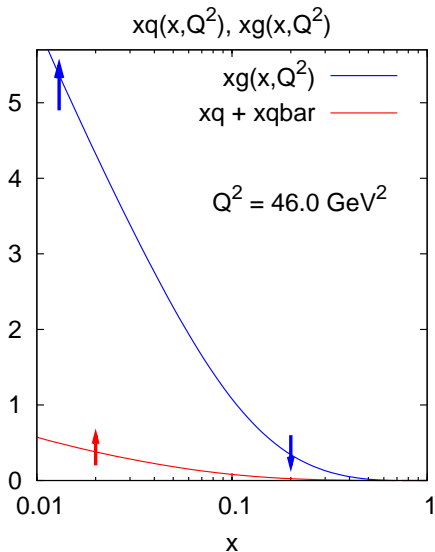
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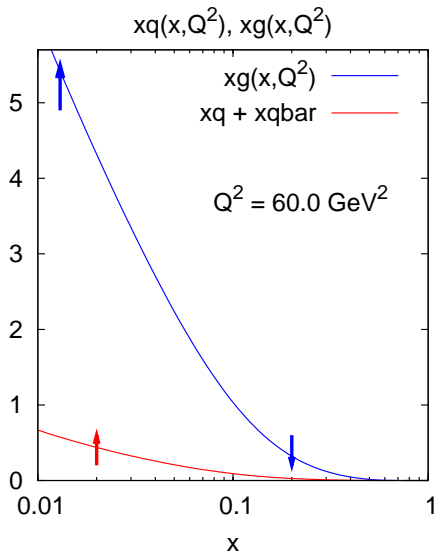
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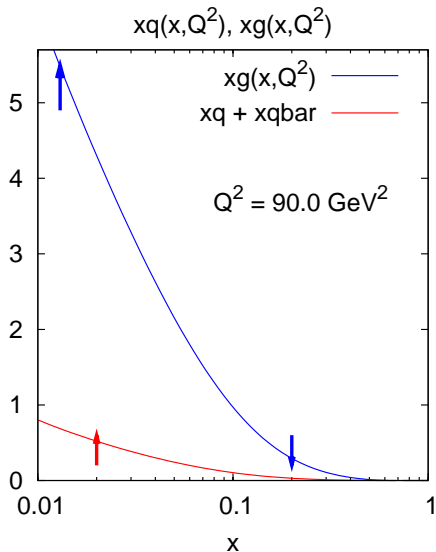
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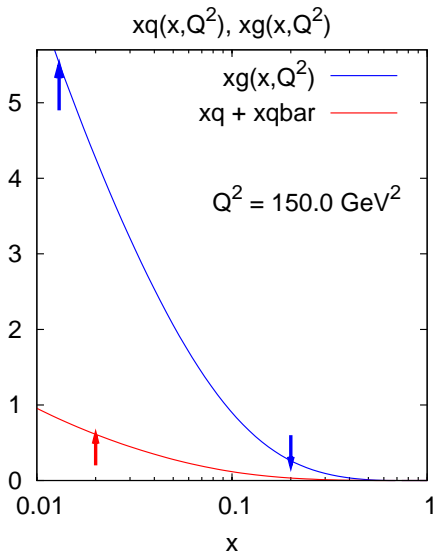
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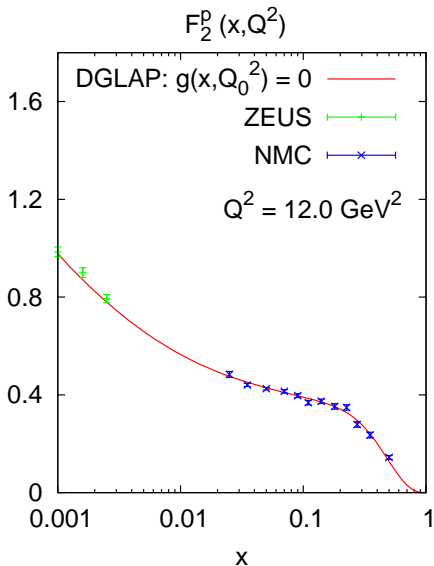
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- As  $Q^2$  increases, partons lose longitudinal momentum; distributions all shift to lower  $x$ .
- gluons can be seen because they help drive the quark evolution.

Now consider data

## DGLAP with initial gluon = 0



Fit quark distributions to  $F_2(x, Q_0^2)$ ,  
at *initial scale*  $Q_0^2 = 12 \text{ GeV}^2$ .

NB:  $Q_0$  often chosen lower

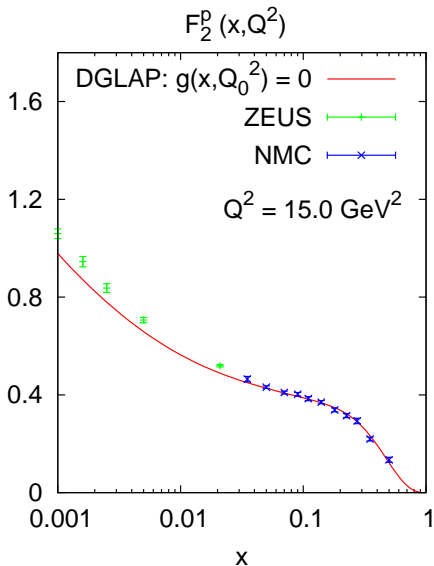
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$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to  
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Complete failure!

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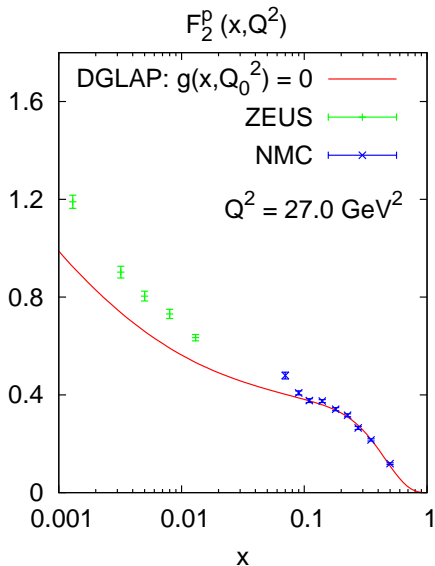
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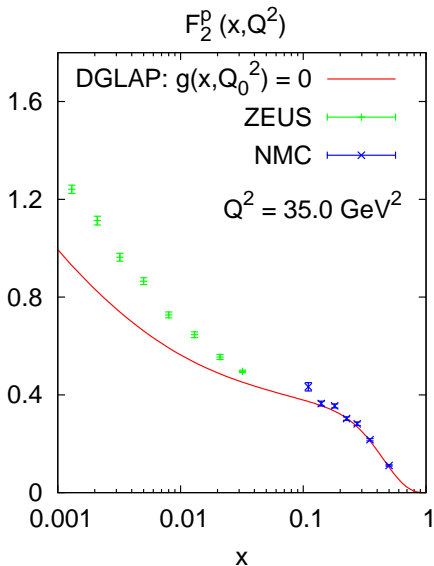
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NB:  $Q_0$  often chosen lower

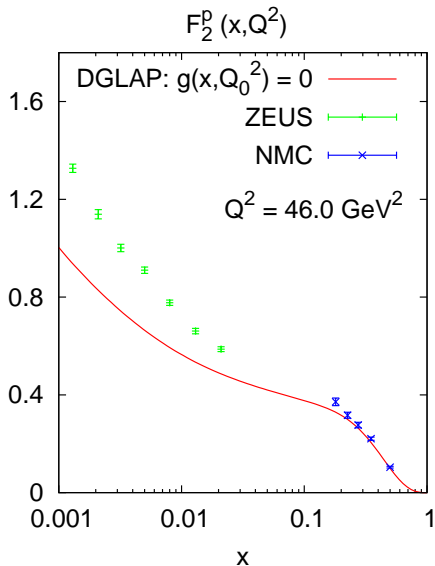
Assume there is no gluon at  $Q_0^2$ :

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Complete failure!

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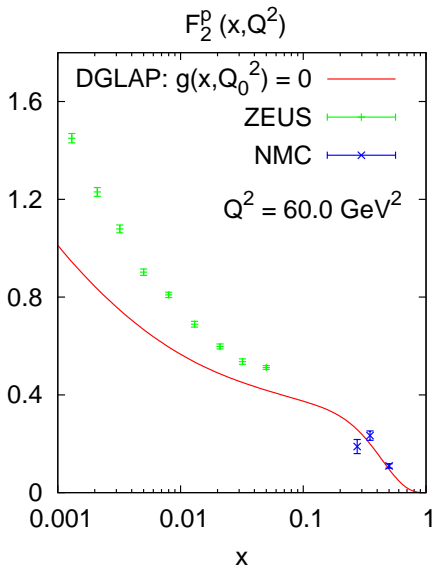
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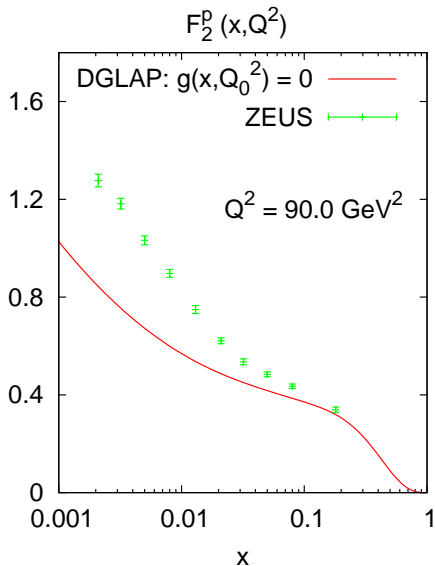
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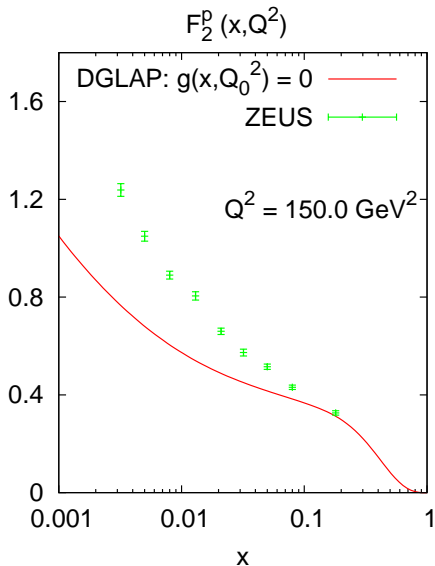
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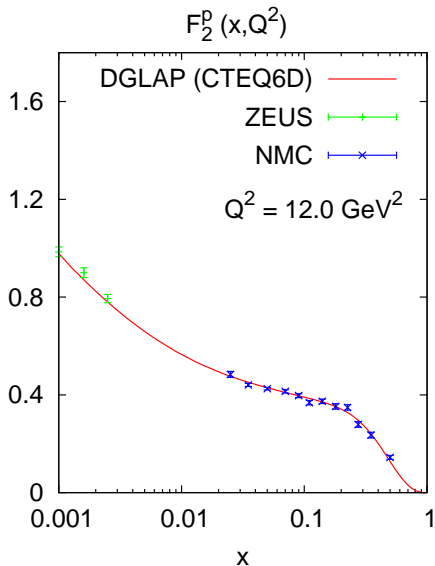
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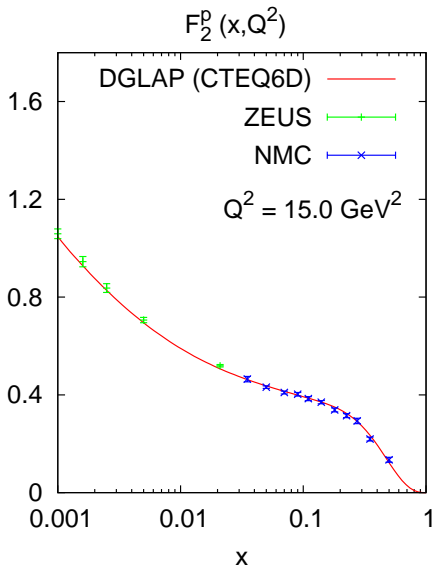
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Done for us by CTEQ, MRST, ...  
PDF fitting collaborations.

Success!



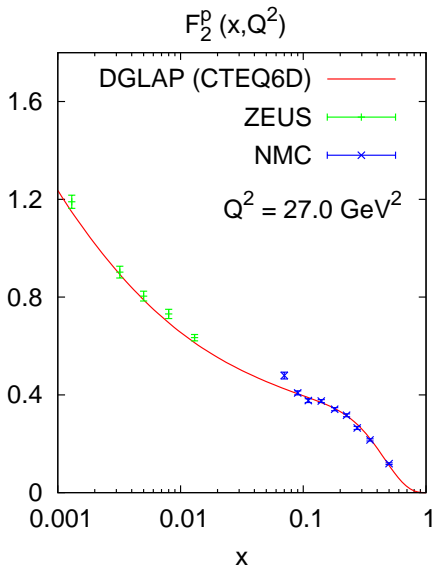
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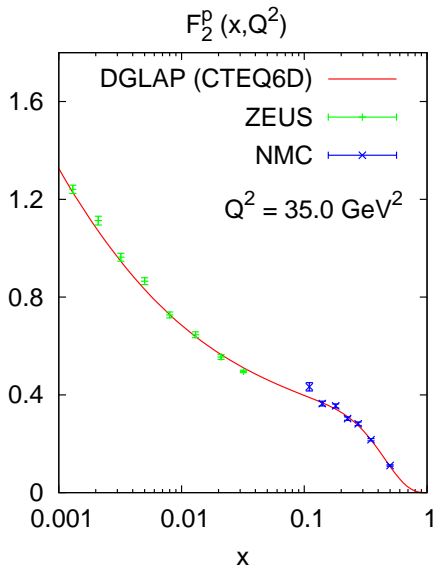
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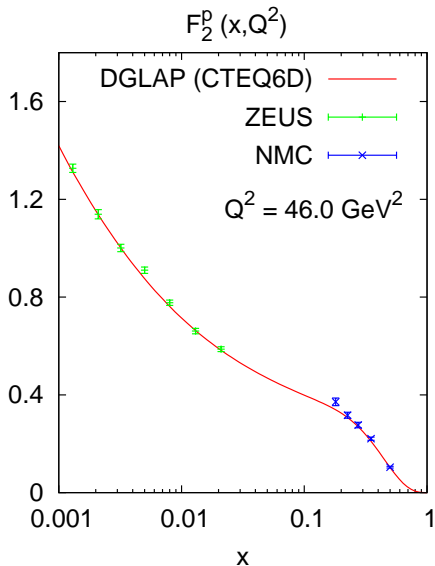
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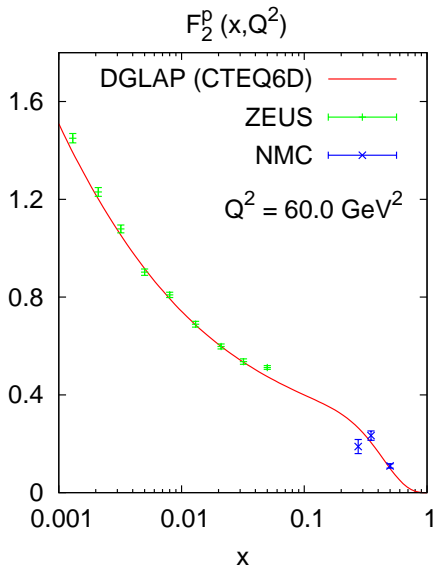
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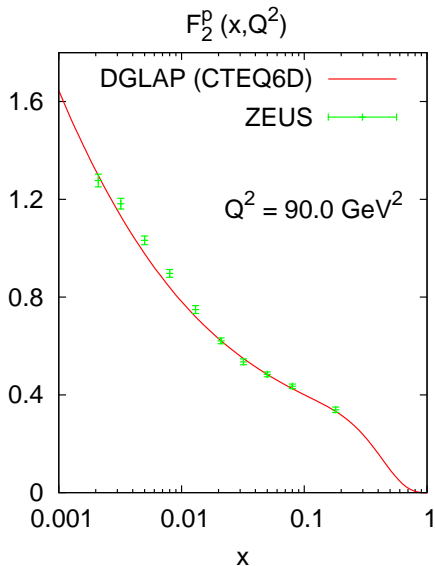
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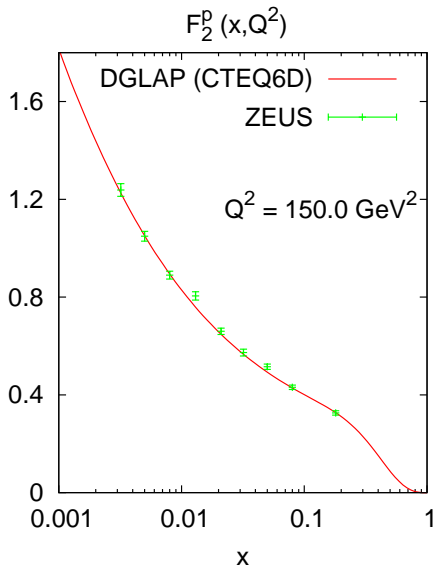
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DGLAP with initial gluon  $\neq 0$ 

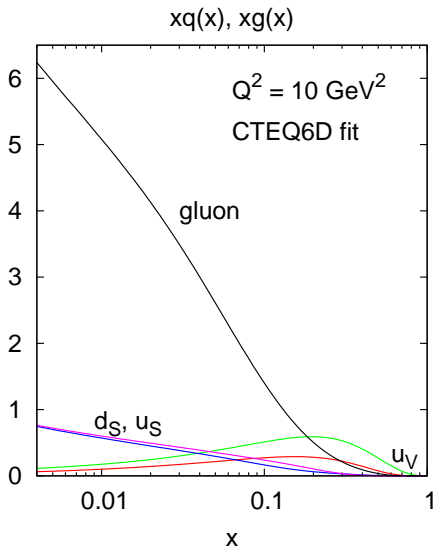
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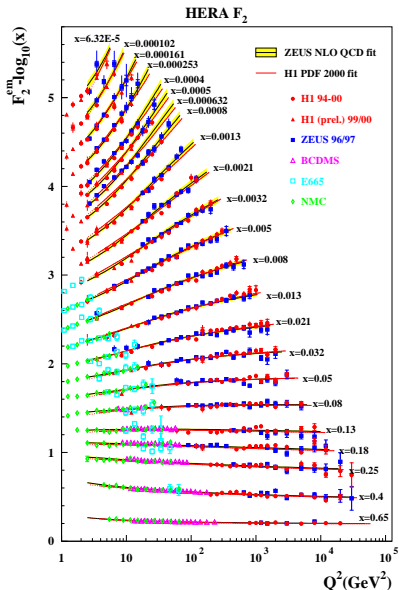
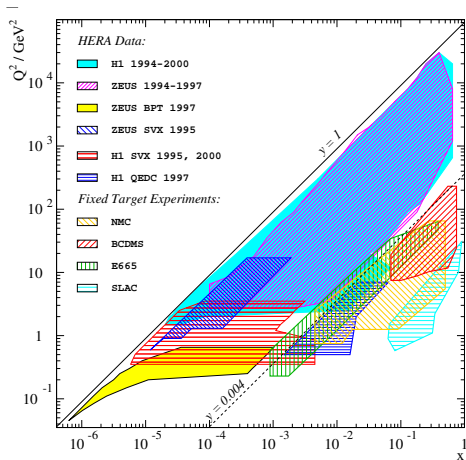
Success!

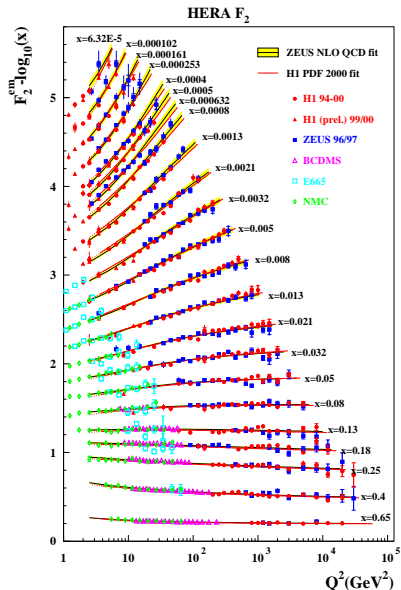
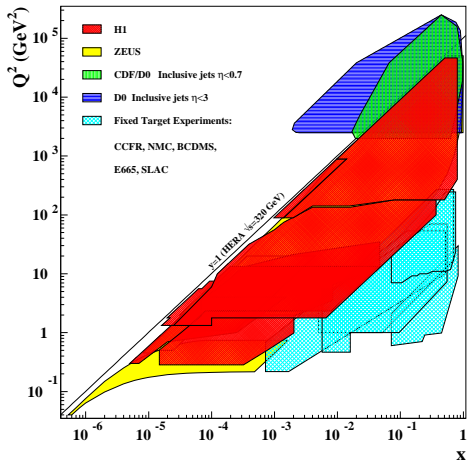


Gluon distribution is **HUGE!**

Can we really trust it?

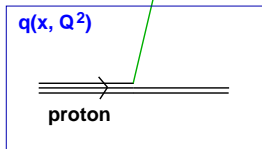
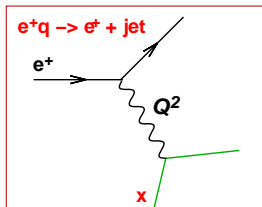
- Consistency: momentum sum-rule is now *satisfied*.  
NB: gluon mostly at small  $x$
- Agrees with vast range of data



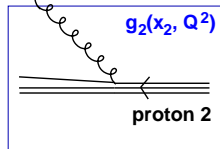
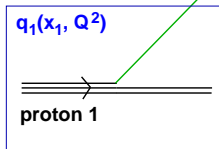
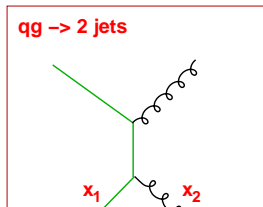


**Factorization** of QCD cross-sections into convolution of:

- hard (perturbative) process-dependent **partonic subprocess**
- non-perturbative, process-independent **parton distribution functions**



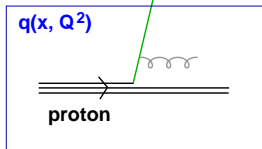
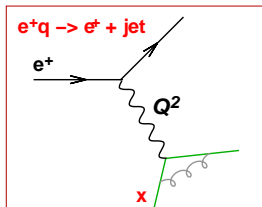
$$\sigma_{ep} = \sigma_{eq} \otimes q$$



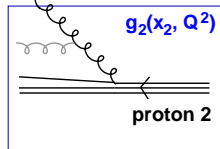
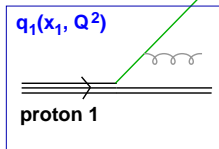
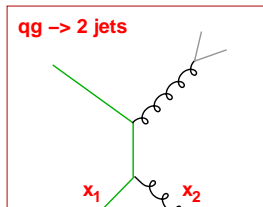
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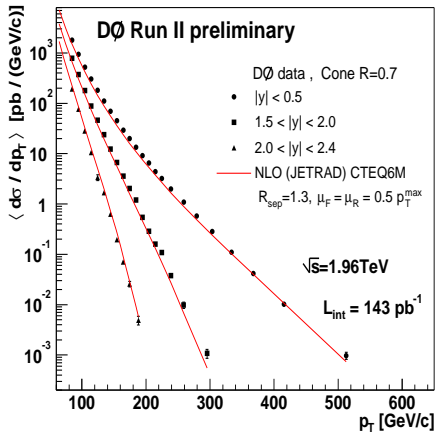
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Jet production in proton-antiproton collisions is *good test of large gluon distribution*, since there are large direct contributions from

$$gg \rightarrow gg, \quad qg \rightarrow qg$$

NB: more complicated to interpret than DIS, since many channels, and  $x_1, x_2$  dependence.

$$p_T \sim \sqrt{x_1 x_2 s} \text{ jet transverse mom.}$$

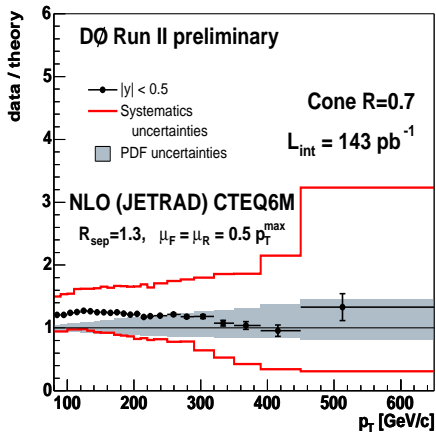
$$\sim Q$$

$$y \sim \frac{1}{2} \log \frac{x_1}{x_2}$$

$$y = \log \tan \frac{\theta}{2}$$

jet angle wrt  $p\bar{p}$  beams

Good agreement confirms factorization



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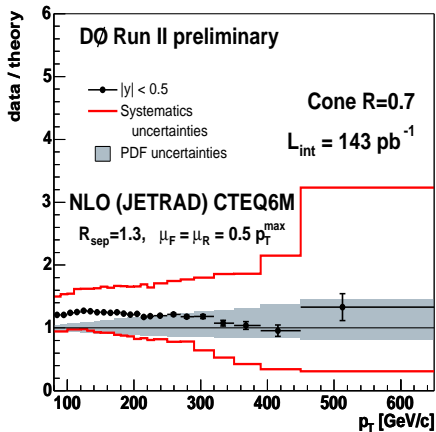
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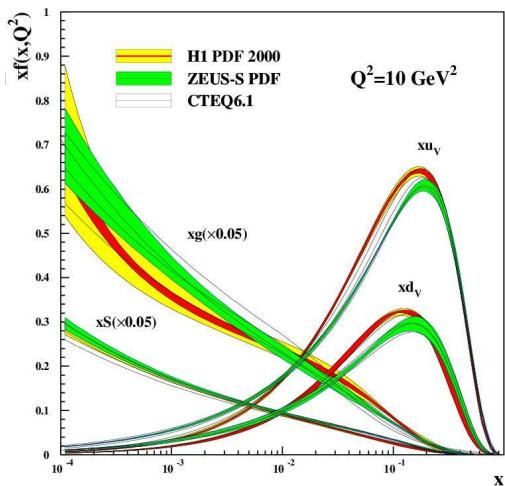
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Major recent activity is translation of experimental errors (and theory uncertainties) into *uncertainty bands on extracted PDFs*.

PDFs with uncertainties allow one to estimate *degree of reliability* of future predictions

Earlier, we saw leading order (LO) DGLAP splitting functions,  $P_{ab} = \frac{\alpha_s}{2\pi} P_{ab}^{(0)}$ :

$$P_{qq}^{(0)}(x) = C_F \left[ \frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right],$$

$$P_{qg}^{(0)}(x) = T_R [x^2 + (1-x)^2],$$

$$P_{gq}^{(0)}(x) = C_F \left[ \frac{1+(1-x)^2}{x} \right],$$

$$P_{gg}^{(0)}(x) = 2C_A \left[ \frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] \\ + \delta(1-x) \frac{(11C_A - 4n_f T_R)}{6}.$$

NLO:

$$P_{\text{ps}}^{(1)}(x) = 4 C_F \eta \left( \frac{20}{9} \frac{1}{x} - 2 + 6x - 4H_0 + x^2 \left[ \frac{8}{3} H_0 - \frac{56}{9} \right] + (1+x) \left[ 5H_0 - 2H_{0,0} \right] \right)$$

$$P_{\text{qg}}^{(1)}(x) = 4 C_A \eta \left( \frac{20}{9} \frac{1}{x} - 2 + 25x - 2\rho_{\text{qg}}(-x)H_{-1,0} - 2\rho_{\text{qg}}(x)H_{1,1} + x^2 \left[ \frac{44}{3} H_0 - \frac{218}{9} \right] \right. \\ \left. + 4(1-x) \left[ H_{0,0} - 2H_0 + xH_1 \right] - 4\zeta_2 x - 6H_{0,0} + 9H_0 \right) + 4 C_F \eta \left( 2\rho_{\text{qg}}(x) \left[ H_{1,0} + H_{1,1} + H_2 \right. \right. \\ \left. \left. - \zeta_2 \right] + 4x^2 \left[ H_0 + H_{0,0} + \frac{5}{2} \right] + 2(1-x) \left[ H_0 + H_{0,0} - 2xH_1 + \frac{29}{4} \right] - \frac{15}{2} - H_{0,0} - \frac{1}{2} H_0 \right)$$

$$P_{\text{gq}}^{(1)}(x) = 4 C_A C_F \left( \frac{1}{x} + 2\rho_{\text{gq}}(x) \left[ H_{1,0} + H_{1,1} + H_2 - \frac{11}{6} H_1 \right] - x^2 \left[ \frac{8}{3} H_0 - \frac{44}{9} \right] + 4\zeta_2 - 2 \right. \\ \left. - 7H_0 + 2H_{0,0} - 2H_1 x + (1+x) \left[ 2H_{0,0} - 5H_0 + \frac{37}{9} \right] - 2\rho_{\text{gq}}(-x)H_{-1,0} \right) - 4 C_F \eta \left( \frac{2}{3} x \right. \\ \left. - \rho_{\text{gq}}(x) \left[ \frac{2}{3} H_1 - \frac{10}{9} \right] \right) + 4 C_F^2 \left( \rho_{\text{gq}}(x) \left[ 3H_1 - 2H_{1,1} \right] + (1+x) \left[ H_{0,0} - \frac{7}{2} + \frac{7}{2} H_0 \right] - 3H_{0,0} \right. \\ \left. + 1 - \frac{3}{2} H_0 + 2H_1 x \right)$$

$$P_{\text{gg}}^{(1)}(x) = 4 C_A \eta \left( 1 - x - \frac{10}{9} \rho_{\text{gg}}(x) - \frac{13}{9} \left( \frac{1}{x} - x^2 \right) - \frac{2}{3} (1+x) H_0 - \frac{2}{3} \delta(1-x) \right) + 4 C_A^2 \left( 27 \right. \\ \left. + (1+x) \left[ \frac{11}{3} H_0 + 8H_{0,0} - \frac{27}{2} \right] + 2\rho_{\text{gg}}(-x) \left[ H_{0,0} - 2H_{-1,0} - \zeta_2 \right] - \frac{67}{9} \left( \frac{1}{x} - x^2 \right) - 12H_0 \right. \\ \left. - \frac{44}{3} x^2 H_0 + 2\rho_{\text{gg}}(x) \left[ \frac{67}{18} - \zeta_2 + H_{0,0} + 2H_{1,0} + 2H_2 \right] + \delta(1-x) \left[ \frac{8}{3} + 3\zeta_3 \right] \right) + 4 C_F \eta \left( 2H_0 \right. \\ \left. + \frac{2}{3} \frac{1}{x} + \frac{10}{3} x^2 - 12 + (1+x) \left[ 4 - 5H_0 - 2H_{0,0} \right] - \frac{1}{2} \delta(1-x) \right) .$$

$$P_{ab} = \frac{\alpha_s}{2\pi} P^{(0)} + \frac{\alpha_s^2}{16\pi^2} P^{(1)}$$

Curci, Furmanski  
& Petronzio '80



- Experiments tell us that proton really is what we expected ( $uud$ )
- Plus lots more: large number of 'sea quarks' ( $q\bar{q}$ ), gluons (50% of momentum)
- *Factorization* is key to usefulness of PDFs
  - Non-trivial beyond lowest order
  - PDFs depend on factorization scale, evolve with *DGLAP equation*
  - Pattern of *evolution gives us info on gluon* (otherwise hard to measure)
  - PDFs really are universal!
- *Precision* of data & QCD calculations steadily increasing.
- Crucial for understanding future signals of *new particles*, e.g. Higgs Boson production at LHC.