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Spatial analysis of real estate price bubbles: Paris, 1984–1993

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Abstract

The Paris housing price bubble that started around 1984 came to a burst in 1990–1991. In order to get insight into the spatial mechanism of speculative bubbles, we examine this episode at the level of the twenty districts composing Paris intra-muros. The analysis is carried out in two steps. First, we empirically describe the price movements in the different districts. Prices in the best areas are seen to peak first and by and large to decrease in proportion to their former increase. Our second objective is to characterize each district in terms of the relative strength of speculative trading versus price–supply inelasticity. We show that the various price trajectories can fairly well be described in the framework of a partial equilibrium model which stylizes the behaviour of the different kinds of agents in the market. The model thus provides a classification of the districts in terms of speculative trading intensiveness versus supply inelasticity. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

1.1. Price waves in speculative bubbles

An important characteristic of real estate is its spatial dimension. Previous

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studies about speculative behaviour and price bubbles only rarely considered this dimension; one exception is Tirtiroglu (1992, see the discussion below) although he focused on a different topic than the present paper. In this paper[1] the emphasis is on relative price levels, i.e. prices in one district with respect to others. We do not try to explain why a bubble started. In other words no attempt is made to identify the sources that triggered speculation. A partial answer to that question may be found in Abraham and Hendershott (1996)[2]. Taking it as given that a bubble developed, we analyse where it started and how it spread throughout the city. Comparing its magnitude in different districts will enable us to learn something about the microeconomic behaviour of buyers and sellers. The behaviour of economic agents is different across the different districts (for a schematic map of the districts see Fig. 4). What makes the present study particularly interesting is the fact that a city such as Paris consists of a number of differentiated districts in which the behaviour of economic agents can be analysed in detail. There are twenty districts in Paris intra-muros. Roughly speaking, the most expensive districts are in the western part of the city (district numbers: 6, 7, 15, 16, part of 17); the medium price districts are in the center and in the south (1, 2, 3, 4, 5, 14); the lower-price districts are in the east and in the north (11, 12, 13, 18, 19, 20). One should note that even in the latter prices were on average higher than in other provincial French cities.

It is sometimes claimed (without much supporting evidence) that "the best areas peak first and hold on best". While the first part of this assertion is confirmed by our data the second is not: Prices in the best areas very much fall in proportion to their former increase. Abraham and Hendershott arrived to the same conclusion by considering city samples in different parts of the USA (see below Section 2.1). An interesting consequence can be drawn from such findings, namely that the shape of housing bubbles is roughly symmetric with respect to peak prices, a conclusion that is also supported by the curves in Fig. 1(a).

Our analysis will be rather empirical in the sense that we shall not try to connect the phenomenon to basic economic determinants and causes such as for instance the evolution of rentals, the variations in real interest rates or changes in the degree of risk aversion[3]. Nonetheless, the economic significance and consequences of the speculative process should be emphasized; it resulted in a capital gain representing a sizeable part of national saving and even of national income. In order to get a

[1]The author would like to thank two anonymous referees as well as the editor for their remarks and suggestions which helped to simplify and clarify the paper.

[2]These authors developed a lagged appreciation mechanism which generates such bubbles in a fairly satisfactory way. Yet, in the concluding remarks of their paper, they recognized that the way bubbles start largely remains a mystery.

[3]Let us point out that fundamentals of the housing market in a city such as Paris may take a great variety of forms; as an illustrative example one may mention the fact that the creation of a new opera house in the late eighties (next to Bastille) has had a significant effect on the prices in several neighbouring districts (3rd, 4th, 11th).

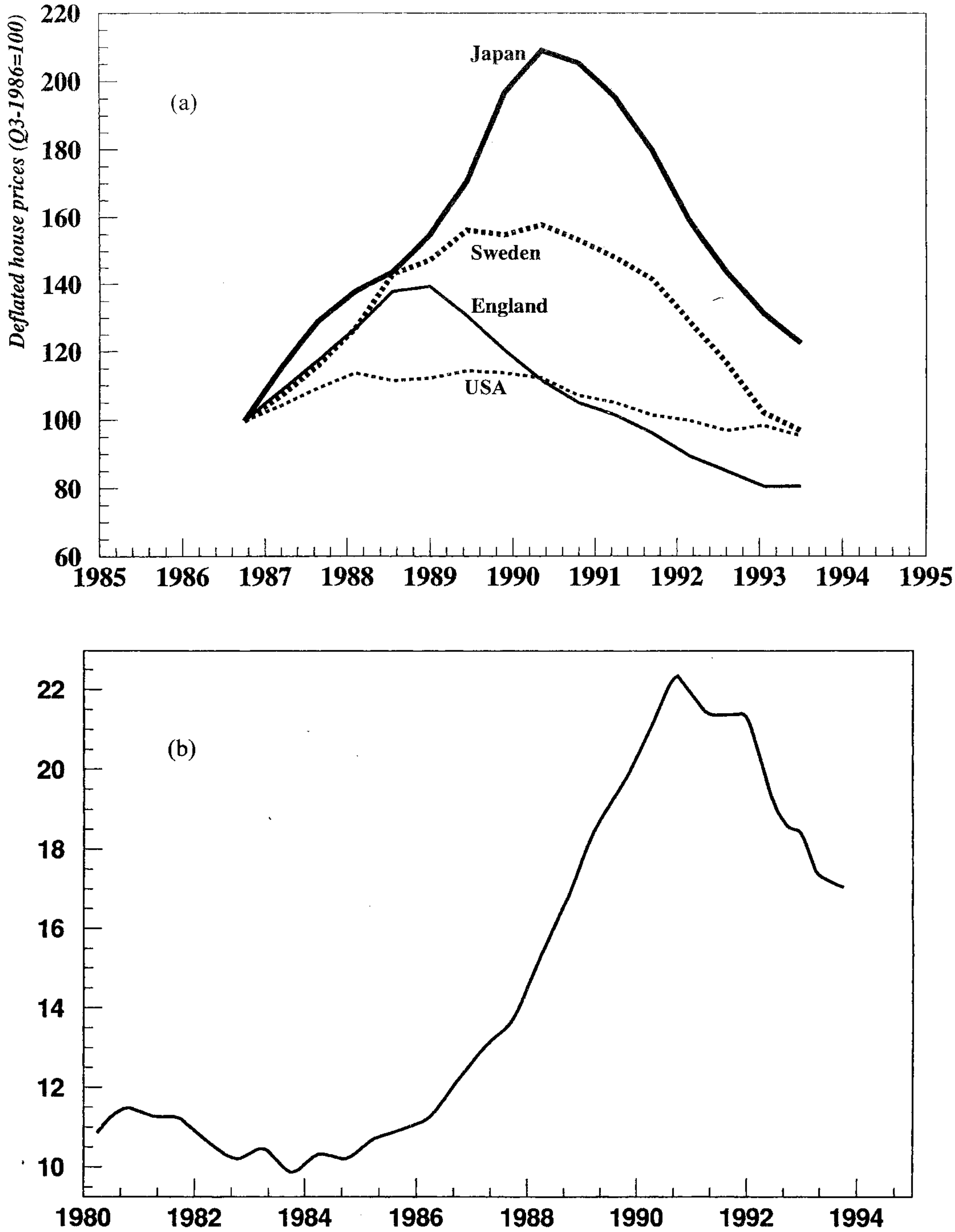

Fig. 1. (a) Evolution of deflated house prices (1986–1994). England: London; Japan: six major cities; Sweden and USA: national indexes. Source: Ramses 1994, Dunod, Paris. (b) Average price of real estate in Paris. The figure corresponds to deflated prices per square meter for 'old' apartments averaged over the twenty districts of Paris intra-muros. The prices have been deflated using the cost of living price index (including price of tobacco, i.e. the 'old' INSEE series). Source: Chambre des Notaires de Paris.

rough idea of its magnitude, let us consider that Paris intra-muros is a 10 km × 10 km square. Let us assume an average price of about 20 000 F/m^2 (which is rather a conservative estimate); if we suppose five storied buildings, this leads to a total estimate of about 10 000 billions francs[4]. This represents about five times the market value of the securities listed on the Paris stock market (2091 billions francs by June 1992)[5]. Since the price increase is roughly of the order of 100% in real terms (Fig. 4) we arrive for the capital gain over a period of 5 to 6 years to a figure of about 5000 billions francs; for the sake of comparison the French GNP amounted to 6766 billions francs in 1991. These orders of magnitude show that real estate bubbles can have a significant impact on the economy. In fact, the collapse of the housing market brought several large banks and insurance companies to the brink of bankruptcy (e.g. the Crédit Lyonnais, Crédit Foncier and GAN). In Japan too, the burst of the speculative bubble considerably weakened several leading banking institutions, e.g. Fuji Bank, Daiwa Bank and Nippon Credit Bank (Far Eastern Economic Review Dec. 4, 1997, in this connection see also Stone and Ziemba, 1993, p. 154). The 1997 financial crisis in Thailand and Malaysia was, at least in part, triggered by the burst of real estate bubbles.

The paper is organized as follows. In Section 2 we give an empirical overview; our purpose in Section 3 is to assess and to compare the magnitude of the following two market forces. (i) In Paris intra-muros supply in terms of new constructions is limited, a situation which is somewhat similar to that in Hong Kong, Singapore or Honolulu. This is likely to produce an upward trend; this effect will be referred to as the inelasticity effect. (ii) On the other hand, in the period under consideration there was clearly an incentive to take advantage of the booming market by anticipating future price increases. It is a common observation that the euphoric atmosphere prevailing in boom periods may lead traders to overshoot the mark[6]. In cobweb price models this is translated in the fact that large price fluctuations appear whenever the system comes near to its stability boundary. This dynamic effect may be called the speculative component. It is the analog of the dynamic adjustment process in Abraham and Hendershott's (1997) paper (see below).

It turns out that the second effect predominates in the best areas while the opposite holds in lower price districts.

A last remark is in order; in this paper, the expression "price bubble" refers to the general, nontechnical meaning with which it has been used since the 19th

[4]In this calculation we do not take into account the area occupied by streets and by public buildings; but on the other hand, Paris intra-muros represents but a small fraction (may be of the order of 20% to 30%) of the whole metropolitan area.

[5]True, the proportion of the total asset that changes hands every year is probably smaller for housing than for securities.

[6]In Spiers (1994) words: "Beantown househunters could have held off buying at the peak if they had weighted the overexcited buzz of neighbors and colleagues against contrasting signs of economic slowdown."

century, that is to say a long time before the emergence of the concept of rational expectations.

1.2. Previous studies

Only few studies focused on the spatial aspect of the question of price bubbles. As we already mentioned Tirtiroglu (1992) is one exception. In line with former studies, particularly Case and Shiller (1989), he examined the issue of market efficiency in the light of spatial diffusion of house price changes. The data set concerns a large number of Connecticut towns in the period October 1981–September 1988. By using a control group in addition to his study group, Tirtiroglu was able to sharpen the conclusions of previous authors. On the whole, his findings added to the growing evidence of inefficiency in housing markets. There are at least two major differences between Tirtiroglu's paper and the present paper. (i) Our objective is not to test market efficiency, but to analyse the spatial diffusion of price changes per se. (ii) Our data set comprises distinct districts within the same city rather than a sample of different cities.

The general question of bubbles in housing prices has been taken up particularly by Shiller (1990), by Case and Shiller (1990), by Meese and Wallace (1994) and more recently by Abraham and Hendershott (1996). In this last paper the determinants of real house price appreciation are divided into two groups; the first group (which includes construction cost and income growth) determines the equilibrium prices, while the second group consists of factors providing a dynamic adjustment, such as lagged appreciation and differences between actual and equilibrium prices. Such a model succeeds in explaining 3/5 of the variance of price movements during the whole period 1977–1992.

One should also mention the paper by Mankiw and Weil (1989) which provides a stimulating overview of real estate price evolution, in spite of the fact that their conclusions have been questioned by different authors, e.g. Hendershott (1991). Finally, from a statistical point of view one should mention a paper by Gatzlaff and Ling (1994) which examines different methods (for instance the hedonic method) for correcting possible heterogeneity in real estate data sets. The data set used in the present paper is for apartments not for houses. In addition, these apartments belong to a fairly homogeneous type (apartments in standard five or six storied buildings); therefore no significant correction for homogeneity will be required here.

2. Summarizing statistical evidence

First of all let us briefly describe the data base of the "Chambre des Notaires" (Lawyer Chamber) on which our investigation is based. It provides average transaction prices for each of the twenty districts composing Paris intra-muros (the map in the upper right corner of Fig. 4 shows the location of these districts). The transactions only concern "old" apartments (i.e. apartments that have been sold at least twice). The frequency is half-yearly before 1991; subsequently prices are recorded every quarter. Prices are given not by apartments, but directly per square meters, thus providing a fairly homogeneous data set.

2.1. The speculative bubble

That the bubble in the Paris housing market was not an isolated phenomenon is shown by Fig. 1(a). In spite of some heterogeneity in the data (some are for major cities while others are national indexes) there are clearly two subgroups. In England and in the USA prices peaked in 1988–1989, while in Japan, Sweden and France (as shown in Fig. 1(b)) they peaked about two years later. In the case of the USA one should note that, due to the country's geographic extension, the national average summarizes different local scenarios of which Abraham and Hendershott (1997, Table 1) provides the following overview: in Northeast cities, real prices rose at a rate of 14 percent (per year) between 1983 and 1988, and subsequently fell at a rate of 6 percent in 1988–1992; in Western cities, the respective price changes were 7.2 percent for 1984–1990 and − 4 percent for 1990–1992; in the Midwest, these figures were 3.2 for 1984–1988 and − 0.3 percent for 1988–1992. In addition let us point out that price changes in different districts of major American cities display more diversity than in Europe. For instance between March 1990 and March 1993, prices in West Los Angeles fell by 17 percent (per year) while they rose by 21 percent in the poorer East Los Angeles district (New York Times, 5 September 1993).

Fig. 1(b) shows the evolution of the average (deflated) price per square meter in Paris from 1980 to the end of 1993. It can be observed that the bubble began to build up by 1986 after a period characterized by rather flat prices. The data depicted in Fig. 1(b) can be complemented by saying that the price decrease continued beyond 1994; thus between March 1995 and March 1996 prices in Paris intra-muros fell by about 16 percent; the decrease was by contrast much slower for the whole metropolitan area (Ile de France) with a decrease of only 2.7 percent.

2.2. The diffusion of the speculative wave

In Fig. 2 we consider a West–East cross section of Paris; the origin is taken at Notre-Dame (the historical center of Paris); negative figures refer to districts west of the 5th district (in which Notre-Dame is located), positive figures refer to eastern districts. In Fig. 2(a) the dates corresponding to the maxima of the price curves are represented according to distances to Notre-Dame. With the single exception of the 7th district there is a direct relationship between peak times and distances. A least-square fit yields a velocity of 7.8 ($\pm$2.5) km/year (confidence level of 0.95) for the speculative wave.

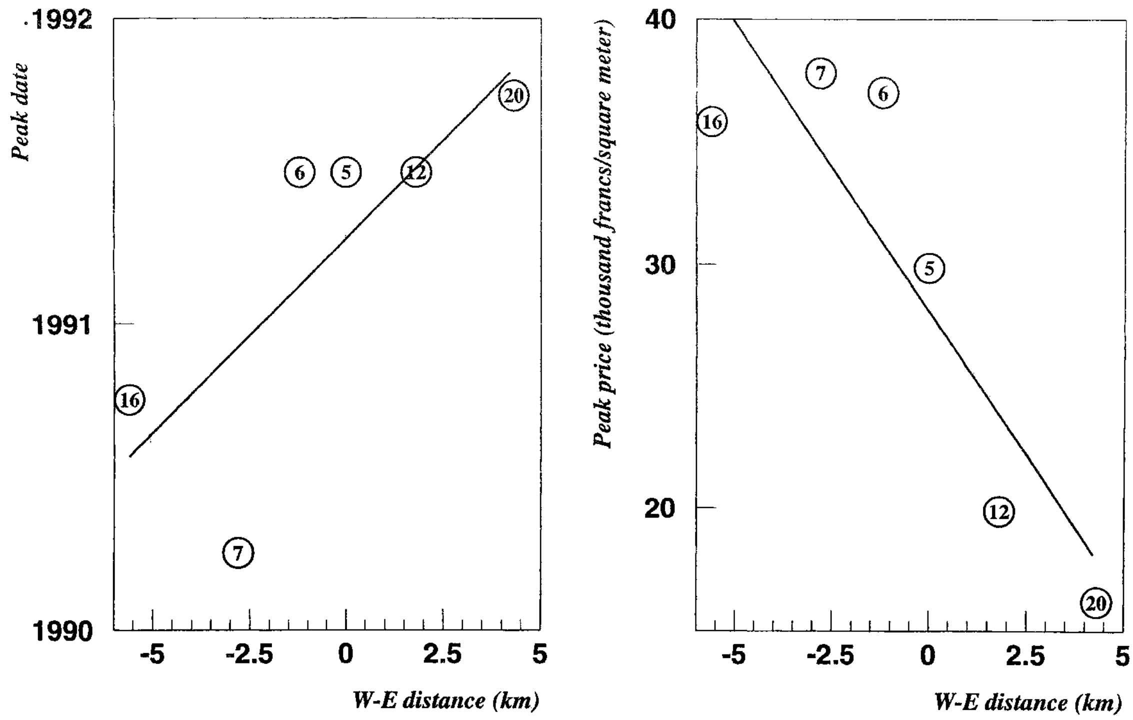

Fig. 2. (a,b) Price peaks in a West–East cross section. Horizontal scale: distance of districts to the center of Paris (West: negative distances; East: positive distances). The left chart shows peak dates in 6 districts. The right charts shows peak prices in the same districts. The straight lines are least square fits.

By extending the results obtained for speculative contagion in commodity markets (Roehner, 1989, 1995), it might be guessed that the interdependence between western and eastern districts increased as the bubble developed. This can hardly be tested here however; in order to estimate a multivariate autoregressive model higher frequency data (monthly data, or even better, weekly data) would be required.

Fig. 2(b) provides a characterization of western versus eastern districts in terms of peak price levels. The price pattern in different districts was almost the same before and during the price bubble; however, the price gap between the most expensive districts and the cheapest ones was substantially amplified by the speculative bubble. For instance, the price ratio between the 7th and the 20th district was equal to 1.68 in the first semester of 1984; for peak prices the same ratio amounts to 2.33.

3. Inelasticity versus speculative behaviour

First of all let us examine in more detail the connection that we mentioned in Section 1 between price volatility and demand (or supply) elasticity. Mankiw and Weil (1989, p. 246) observed that shifts in housing demand have great impact on housing prices; this suggests that both the supply and demand are highly inelastic as can easily be inferred from the following (quasi-static) model (Mankiw and Weil, 1989, p. 247):

$$\ln S = a + b \ln p, \quad b > 0, \quad \ln D = c - d \ln p + \ln \delta, \quad d > 0,$$

where δ denotes an exogenous shift in demand. The equilibrium condition $S=D$ yields:

$$\ln p = \frac{c-a}{b+d} + \frac{\ln \delta}{b+d},$$

Thus:

$$\frac{d \ln p}{d \ln \delta} = \frac{1}{b+d},$$

which means that, in order for the left hand side to be large, both b and d have to be small, i.e. supply and demand for housing have to be highly inelastic. In the subsequent model demand and supply inelasticities will be one of the mechanisms by which large fluctuations may be generated.

3.1. The model

We assume that the operators in the real estate market can be divided into two subgroups[7]:

1. Residents who buy and sell apartments for personal use.
2. Speculators and property developers who make money by selling and buying property.

Going back to Nerlove (1958), there is a long tradition in developing models which involve a mix of agents with different dynamic responses. While in Nerlove paper the aim was a combined description of short- and long-term phenomena, we shall here use that technique in order to model different levels of responsiveness to speculative contagion. In the following, we shall ignore the impact of many exogenous factors, the most important of which is the influence of real interest rates. In connection with this point, two remarks are in order:

- Real interest rates certainly have an influence on the housing market (since they determine the real price to be paid by those buyers who are unable to pay cash) especially in the long-run. This impact has for instance been estimated in the

[7]Building contractors who build and sell new buildings will be omitted since the number of new units built is negligible in Paris intra-muros.

case of the United States by Hendershott (1991) in his extension of Mankiw and Weil's model. For the period 1970–1987, he obtained the result: $\Delta \ln p = -0.11 + 1.08\Delta \ln S - 0.049\Delta r - 0.0022r$, $R^2 = 0.38$ r denotes real (after-tax) interest rates measured from 10-year Treasury bond rates. Thus the impact of variations in real interest rates (Δr) appears to be about 20 times smaller than the impact of variations in supply ($\Delta \ln S$).

- Once a speculative bubble has been initiated, its momentum and evolution appears to be rather robust with respect to a number of exogenous factors (in spite of the substantial role these factors may play in more "normal" conditions).

We denote the supply and demand functions of subgroup number i at time t by $S_t^{(i)}$ and $D_t^{(i)}$. We shall assume log-linear demand and supply functions, that is to say, $S_t^{(i)}$ and $D_t^{(i)}$ are supposed to be linear functions of:

$$p'_t = \log_c p_t,$$

where p_t denotes the price at time t and log c is the logarithm of base c, i.e.:

$$\log_c x = \frac{\ln x}{\ln c}. \tag{1}$$

The parameter c is related to the elasticity of supply and demand with respect to prices, see below Eq. (7).

Remark. *The introduction of the $\log_c$ could seem unusual; in fact, it is just a matter of notational convenience; it would be equivalent to consider ordinary logarithms and to introduce the parameter ln c as a multiplicative constant.*

Regarding the two different subgroups, we shall assume the following net-supply functions:

1. *Net supply (or excess-supply) function of residents.* Very often residents are buying and selling simultaneously, sometimes down or up sizing and sometimes just relocating. We do not treat the buying and selling components separately since the data at our disposal do not allow us to identify selling and buying elasticities separately anyway. Since residents are able to know the actual market prices at the time they decide to buy or to sell one has:

$$s_t^{(1)} = S_t^{(1)} - D_t^{(1)} = -c_1 + \gamma_1 p'_t. \tag{2}$$

2. *Net supply function of speculators.* We assume the market clearing condition to hold at any time; in other words, we ignore inventories. Given that inventories are not taken into account, again we lump the demand and supply side together. Speculators are assumed to withhold their property (and even to buy additional

ones) as long as prices are climbing; once prices have reached a plateau ($p'_{t-1} = p'_{t-2}$), their propensity to sell (resp. buy) is assumed to be in direct (resp. opposite) relation to the price level of the plateau. These assumptions are embodied in the following equation:

$$s^{(2)} = S_t^{(2)} - D_t^{(2)} = -c_2 + \gamma_2 p'_t - g_2(p'_{t-1} - p'_{t-2}) \quad g, h > 0. \tag{3}$$

3.2. Analytical solution

The equilibrium condition reads:

$$s_t = (1-k)s_t^{(1)} + ks_t^{(2)} = 0 \quad 0 < k < 1, \tag{4}$$

where k represents the proportion of the speculators. Substituting (2) and (3) we obtain the following second order equation (see Appendix A):

$$p'_t - a(k)p'_{t-1} + a(k)p'_{t-2} = d(k), \tag{5a}$$

where:

$$a = \frac{g_2}{\gamma_2}, \quad \gamma = \frac{\gamma_1}{\gamma_2}, \quad a(k) = \frac{a}{1 + \gamma(1-k)/k},$$

$$d(k) = \frac{c_2/\gamma_2 + [(1-k)/k](c_1/\gamma_2)}{1 + \gamma[(1-k)/k]}, \tag{5b}$$

The stability condition for the equilibrium solution of Eq. (5a) reads (Priestley, 1981, p. 127):

$$a(k) < 1.$$

As $a(k)$ becomes closer to 1, the stability of the solution decreases; now, $a(k)$ is a monotonous function of k and since $a(0) = 0$ and $a(1) = a$, we see that $a(k)$ increases from 0 to a as k increases from 0 to 1. Therefore there are two different cases:

- if $a < 1$ stability is ensured whatever the value of k may be.
- if $a > 1$, the solution becomes unstable for k larger than a critical value k_c.

The solution of the inhomogeneous Eq. (5a) is given by the sum of the solution of the homogeneous equation and of the constant $d(k)$. The latter does not play a great role here since we are interested in transient behaviour rather than in the long-term trend. The constants of integration will be determined from the condition that the bubble approximately starts at the beginning of the period of interest. One of these constants, namely φ, will be left indeterminate to account for contagion and delay effects between the different districts. Thus, the solution of (5a) takes the form (see Appendix A):

$$p_t = c^{p_t'}, \quad p_t' = a(k)^{t/2} \sin(\omega t - \varphi) + d(k), \quad \omega = \arccos(\sqrt{a(k)}/2), \tag{6}$$

where φ is an angle with an expected value of the order of $\pi/2$.

There are three main parameters in our model, namely c, $a(k)$, φ; at this point it is in order to summarize their interpretation:

1. ln c is inversely proportional to the elasticity of supply (or demand) for the residents; in Appendix A it is shown that:

$$\ln c = \left(\frac{\gamma}{S^{(1)}}\right)\frac{1}{e_S} \quad e_S = (dS/dp)/(S/p) \tag{7}$$

   where e_S denotes the elasticity of supply.
   Thus e_S and c vary in opposite ways: The larger c, the smaller the elasticity of supply. In other words small values of c correspond to a fairly inelastic residents' supply.
2. $a(k)$ is related to the proportion of speculators by relation (5b) which implies:

$$k = 0 \Rightarrow a(k = 0) = 0; \; k = 1 \Rightarrow a(k = 1) = a.$$

   Thus $a(k)$ can be used as an alternative measure for the proportion k of speculators; $a(k)=0$ means that there are no speculators, while $a(k)=a$ means that there are only speculators.
3. The phase difference φ can be seen as a delay; the value $\varphi=\pi/2$ corresponds to a bubble starting in 1984, a date determined by the evolution of the average price (Fig. 1(b)). A value of φ smaller than $\pi/2$ indicates a district where the bubble started before 1984, and which therefore acted as a precursor in the building up of the bubble. A value of φ larger than $\pi/2$ indicates a district where the bubble started after 1982, and which therefore reacted with some delay to the speculation wave that was progressively sweeping the city.

Before we turn to the discussion of the numerical results, let us have a brief look at the empirical evidence. Fig. 3 shows three typical trajectories. The upper curve corresponds to the 7th district where the average price reached a record peak (for all Paris) of 37 787 F/square meter in the first semester of 1990 (i.e. about $6000 per square meter). The lower curve corresponds to the 18th district (located in the northern part of Paris) which experienced one of the most moderate price increases: between 1984 and 1990 current prices have been multiplied by a factor of "only" 2.6. The curve in between shows an intermediary case, namely the 5th district (the so-called Quartier Latin). Qualitatively, it can be seen that the delay between the 18th and the 7th district decreased as the bubble developed. In other

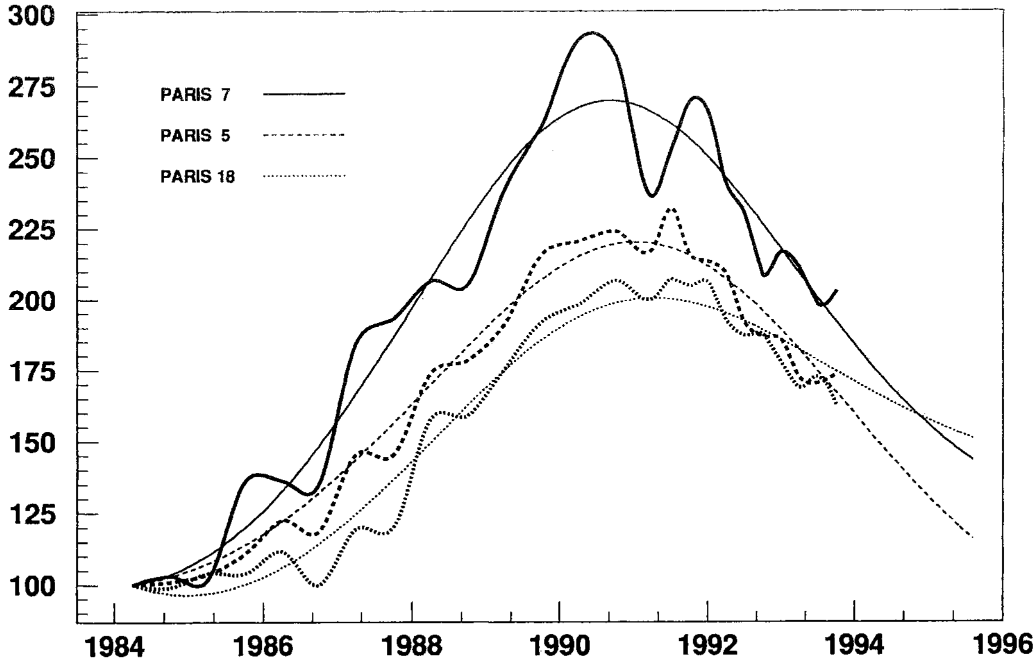

Fig. 3. Normalized (1984 = 100) deflated prices of real estate in Paris. The thick curves correspond to empirical data. The thin curves correspond to the theoretical function (5a,b) with estimated parameters as given in Table 1; the theoretical curves have been extrapolated to the period 1994–1996. Source: Chambre des Notaires de Paris.

words, once the market has reached fairly high price levels, it becomes more integrated and this has the effect of speeding up the propagation of price shocks.

3.3. Numerical results

Table 1 provides least squares estimates for the three parameters. Fig. 4 gives an overall view of the results in the (c, $a(k)$) plane which features inelasticity versus proportion of speculative trading.

The findings are fairly consistent with our qualitative knowledge of the housing market in Paris. The most speculative districts are (in increasing order) the 5th (Quartier Latin), the 16th, the 8th (Champs Elysées), the 6th (Saint Germain des Prés) and, above all, the 15th (opposite the 16th on the left-hand bank of the Seine); the fact that this last district was by far the most speculative area came to some extent as a surprise[8].

[8] At this point, one should emphasize that the classification with respect to the speculative index k does not merely reflect the height of the price peak; indeed a huge price peak can obtain quite as well through a large value of the parameter c (i.e. high inelasticity) as through a value of k close to one (i.e. $a(k)$ close to a).

Table 1
Parameter estimates

District	Parameter estimates			Goodness of fit	
	Inelasticity c	Speculation $a(k)$	Delay φ	Coef. of curv.cor	RNRMS
2	1.76 (0.03)	0.43 (0.01)	2.22 (0.04)	0.86	0.081
3	1.75 (0.05)	0.68 (0.08)	1.73 (0.09)	0.96	0.047
4	1.53 (0.03)	0.98 (0.08)	1.32 (0.06)	0.93	0.059
5	1.45 (0.01)	1.12 (0.04)	1.26 (0.05)	0.97	0.037
6	1.39 (0.02)	1.36 (0.06)	0.98 (0.05)	0.93	0.063
7	1.78 (0.01)	0.79 (0.01)	1.52 (0.01)	0.95	0.060
8	1.50 (0.02)	1.33 (0.09)	1.01 (0.09)	0.92	0.072
9	1.81 (0.01)	0.53 (0.02)	1.93 (0.02)	0.96	0.046
10	1.70 (0.05)	0.63 (0.05)	1.96 (0.02)	0.96	0.047
11	1.48 (0.05)	0.87 (0.13)	1.73 (0.18)	0.97	0.038
12	1.43 (0.08)	0.79 (0.36)	1.86 (0.48)	0.92	0.057
13	1.46 (0.01)	0.95 (0.02)	1.64 (0.04)	0.97	0.039
14	1.43 (0.05)	0.84 (0.20)	1.67 (0.25)	0.95	0.045
15	1.29 (0.02)	1.57 (0.06)	0.70 (0.01)	0.94	0.049
16	1.49 (0.21)	1.20 (0.43)	0.96 (0.49)	0.96	0.048
17	1.53 (0.02)	0.89 (0.04)	1.45 (0.05)	0.97	0.042
18	1.75 (0.14)	0.56 (0.15)	2.17 (0.21)	0.96	0.046
19	1.76 (0.21)	0.48 (0.22)	2.32 (0.42)	0.92	0.062
20	1.55 (0.06)	0.70 (0.10)	1.97 (0.13)	0.96	0.042

The figures given in the table are (nonlinear) least squares estimates of the model's parameters. For each district there are 25 (quarterly) observations. Standard errors are given in parenthesis. The goodness of fit is estimated through the coefficient of curvilinear correlation and the standard residual normalized root mean square (RNRMS) defined as the square root of the ratio of squared residuals to squared observed values. The serial correlation of residual is positive, the Durbin Watson coefficient being on average equal to 1.01

The relationship between the delay $\varphi - \pi/2$ and the average price (prior to the occurrence of the bubble) in different districts is shown in the following table.

District	15	16	6	8	5	4	17	7	13	14
Delay	−0.87	−0.61	−0.59	−0.56	−0.31	−0.25	−0.12	−0.05	0.07	0.10
Price	9.6	11.0	11.6	10.1	9.7	9.4	7.9	10.3	7.2	8.8
District	3	11	12	9	10	20	18	2	19	
Delay	0.16	0.16	0.29	0.36	0.39	0.40	0.60	0.65	0.75	
Price	7.0	6.4	7.8	6.3	5.5	6.1	5.9	7.4	6.5	

In the above table the districts have been ranked according to the magnitude of the delay; at the beginning of the list are the south-west districts (15, 16, 8) where the bubble started. The correlation between delay and average price is equal to $R = -0.84$ ($R^2 = 0.70$). In other words the bubble originated in the wealthy

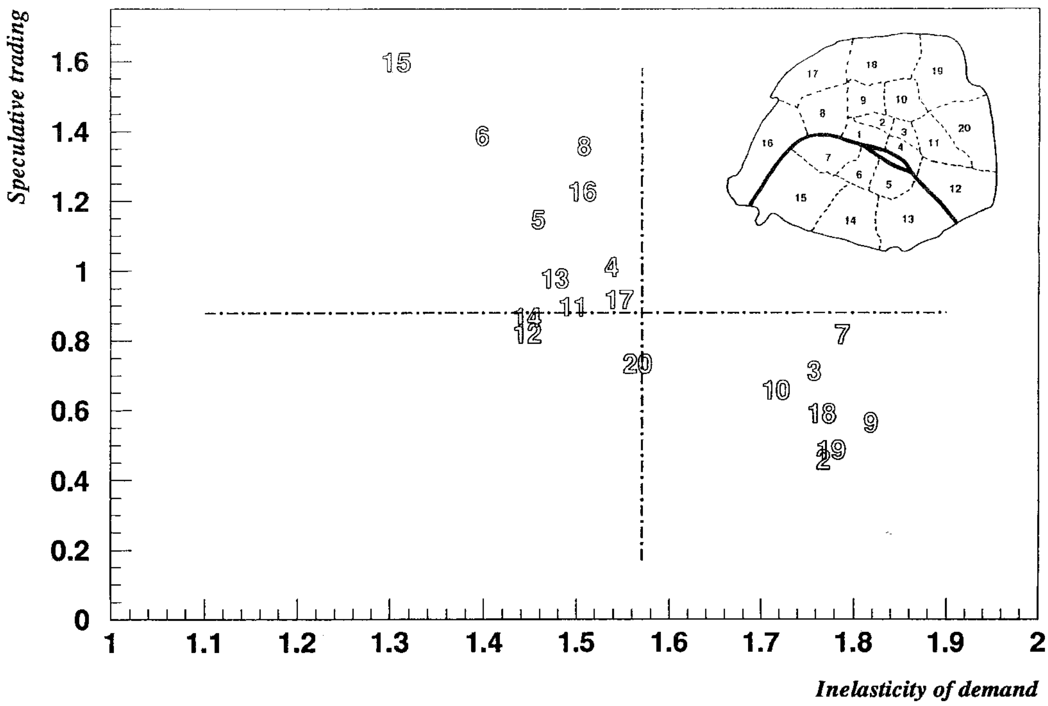

Fig. 4. Representation of estimated parameters for 19 districts in the $(c, a(k))$ plane. The map in the upper right corner shows the location of the districts. There is a negative correlation between inelasticity of demand (or supply) and speculative trading. Moreover the districts with high $a(k)$ values are those where the speculation started.

districts of the south-west, then swept northwards and eastwards to medium-priced districts and finally reached the cheapest districts.

It would be interesting to examine whether other housing booms in Paris displayed similar spatial transmission patterns; however the "Chambre des Notaires" data base came into operation only in the early 1980s and cannot be used for that purpose. While the leading role played by the 15th district may be somewhat novel, the fact that the 16th and the 8th districts are acting as forerunners might be true in previous price bubbles as well.

4. Conclusion

To our best knowledge spatial diffusion of speculative processes have rarely been studied for their own sake (see however a figure from Lösch, 1940, 1954 which is reproduced in Roehner, 1995, p. 4). In this paper we were able to show that the transmission of speculative attitudes plays a major economic role because it triggers price increases even in areas where they would not be expected on account of low income levels and of comparatively poor housing standards.

There is one major idea we would like to convey in this paper (and which may even be more important than the modeling procedure per se), namely the fact that the investigation of different realizations of a speculative bubble is likely to be much more informative than the mere analysis of the single series of average prices. One should of course also mention the fact that the price evolution in Paris was part of a world-wide real estate price bubble. It would be of considerable interest to compare the respective evolutions in Frankfurt, Geneva, London, Stockholm, Stuttgart, Tokyo or Zurich. Regarding Tokyo, Taniguchi (1992) reports that by the beginning of 1992, real estate prices in the Tokyo metropolitan area had dropped by 40 percent from their peak, something that happened only once before (in 1975) since the end of World War II. If this paper could encourage the publication of price data for some other large cities, then it may not have been completely fruitless.

The price data used in this study can be obtained upon request from the author (email address: roehner@lpthe.jussieu.fr).

Appendix A

Solution to Eq. (5a)

The condition: $s_t = (1-k)s_t^{(1)} + ks_t^{(2)} = 0$ reads:

$$(1-k)(-c_r + \gamma_r p_t') + k[-c_s + \gamma_s p_t' - g_s(p_{t-1}' - p_{t-2}')] = 0.$$

That is to say:

$$[(1-k)\gamma_r + k\gamma_s]p_t' - kg_s p_{t-1}' + kg_s p_{t-2}' = (1-k)c_r + kg_s.$$

Dividing by the coefficient of p_t' and arranging the different terms leads to Eq. 5(a,b). With r_1, r_2 denoting the roots of the characteristic equation:

$$r^2 - a(k)r + a(k) = 0,$$

the solution of Eq. 5(a,b) reads (Priestley, 1981, p. 130):

$$p_t' = Ar_1^t + Br_2^t + d(k).$$

Since the roots r_1, r_2 are complex it is convenient to write them in the form: $r_{1,2} = |r_{1,2}|\ e^{i\theta}$. Thus one obtains p_t' in the form:

$$p_t' = |r_{1,2}|(A' \cos \theta t + B' \sin \theta t) + d(k).$$

A simple calculation leads to: $|r_{1,2}| = \sqrt{a(k)}$ and $\cos \theta = \sqrt{a(k)}/2$.

By adjusting the constants A' and B' in order to fit the initial condition one is led to Eq. (6).

Relationship between the parameter c and the elasticity of supply

Remembering that $p'_t = \ln p_t / \ln c$, the supply part in Eq. (2) can be written:

$$S^{(1)} = -c + (\gamma/\ln c)\ln p_t,$$

which leads to:

$$\frac{dS^{(1)}}{dp_t} = (\gamma/\ln c)(1/p_t).$$

Thus, ln c and the elasticity of supply e_S are related in the following way:

$$e_S = (dS^{(1)}/dp)/(S^{(1)}/p) = \left(\frac{\gamma}{S^{(1)}}\right)\frac{1}{\ln c}.$$

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