Open strings in electric fields and time dependent backgrounds

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LPTHE, Paris

Utrecht

May 16, 2003

w/ B. Durin, hep-th/0302159
w/ M. Berkooz, to appear

transparencies available from
http://www.lpthe.jussieu.fr/~pioline/seminars.html

Introduction

- Much effort in string theory has been directed into searching for compactifications to flat Minkowski space which reproduce the Standard Model at low energies. Alas, $t_{LHC} > 2008$, and chances to observe strings directly are moderate.

- In contrast, observational cosmology is undergoing a fast revolution, from an order-of-magnitude Regime to a high-precision Era, posing a new challenge to string theory:

$$\omega_\Lambda = 71.0\% \ , \quad \omega_{baryon} = 4.7\% \ , \quad \omega_{dark} = 24.3\%$$

- While string-inspired cosmological scenarios have been much discussed in effective field theory, string theory in time-dependent backgrounds remains a mostly uncharted territory.

Strings in time-dependent backgrounds

Perturbative string theory is well-suited for S-matrix computations in asymptotically flat space.

Many questions arise in trying to generalize to (smooth) time-dependent backgrounds:

- No (unitary) analytic continuation to Euclidean signature, neither in target space nor on the worldsheet: amplitudes are superficially divergent, modular group acts ergodically...
- Many different choices of vacuum are possible, how can one implement Bogolioubov transformations from one to another ? Is worldsheet locality sacred ?
- Observables are unclear, especially in the case of closed universes, or with pathological asymptotic regions like such as the Cheshire's Cat Universe and its whiskers.

String Field Theory seems a crying need in order to address these issues.

Strings at cosmological singularities

More questions arise in relation with spacelike singularities, which a purported theory of quantum gravity had better address:

- Can perturbative string theory still hold, despite the infinite blueshift towards the singularity ?
- Can extra degrees of freedom of string theory resolve spacelike singularities, or rather prevent their appearance ? How can one evade the no-bounce theorem ?
- If instead spacelike singularies signify the End or Beginning of time, how can one specify boundary conditions there ?
- Is the BKL oscillatory behaviour generic also in string theory ? As different bits of the string fall outside of causal contact at the spacelike singularity, does the string reduce to a Matrix model ?

Cosmological Singularities: a Toy Story

Various toy models have been proposed recently to study time-dependence and cosmological singularities in string theory:

- The Lorentzian orbifold, quotient of $R^{1,1}$ by a boost J: this gives a free-field realization of the Milne Universe

  $$ds^2 = -dt^2 + t^2 dx^2 \ , \quad x \equiv x + 2\pi$$

  together with two whiskers with CTC,

  $$ds^2 = -r^2 dt^2 + dr^2 \ , \quad t \equiv t + 2\pi$$

  Horowitz Steif; Seiberg; Nekrasov

- The Parabolic orbifold, quotient of $R^{1,2}$ by the product of a boost J_{01} and a rotation R_{12},

  $$ds^2 = -2dy^+ dy^- + (y^+)^2 dy^2 \ , \quad y \equiv y + 2\pi$$

  which is better thought of as a singular gravitational wave.

  Simon; Liu Moore Seiberg

- Flux branes and null branes, where the boost is combined with a translation on an extra coordinate, hence lifting any fixed point; WZW models such as the Nappi Witten cosmology, which reduces to the Lorentzian orbifold at the singularity.

  Cornalba Costa; LMS; Craps Kutasov Rajesh
  Elitzur Giveon Kutasov Rabinovici

Toys are broken

- These models all seem to be plagued with perturbative divergences, related to a large backreaction at the singularity. Divergences may be avoided by fine-tuning initial conditions.

Liu Moore Seiberg
Berkooz Craps Kutasov Rajesh

- In addition, due to high blue-shift, the images of the particles on the covering space may non-perturbatively form a large black hole, that eats up the space. Combining with a translation does not cure this instability except in high dimension.

Horowitz Polchinski

- These models are also highly non-generic trajectories on the cosmological billiard: can one study more general Kasner singularities ? find the BKL behaviour ?

Damour Henneaux

More toys: open strings in electric fields

For the purpose of studying time-dependence in string theory, it may be simpler to consider time-dependent D-brane configurations, or equivalently open strings in electric fields:

- Backreaction in the closed string sector may be neglected as $g_s \to 0$. Yet production of open strings is retained. Backreaction in the open string sector is analogous to D-brane recoil.

- Powerful techniques are available: boundary states, string field theory . . . Classical configurations can often be found explicitly due to the fact that the worldsheet theory is free in the bulk.

- Analogues of spacelike singularities are D-brane collisions, or (in the simplest case) a constant electric field. Analogues of null singularities are null scissor configurations, or a constant null field.

Bachas Hull

- The analogy is very precise: charged open strings in an electric field have (half) the same mode structure as twisted closed strings in a Lorentzian orbifold. Issues of physical states can be dealt in the same way.

Outline

1. Introduction
2. Open strings in constant electromagnetic field
3. Open strings in electromagnetic waves
4. Open strings in a constant electric field, revisited
5. Remarks on Milne universe

Open strings in a constant electromag field

- Open strings couple to an electromagnetic field through their boundary only. The embedding coordinates are therefore free bosons in the bulk of the Minkowskian strip $0 < \sigma < \pi, \quad \tau \in R$,

$$X^\mu(\tau,\sigma) = f^\mu(\tau+\sigma) + g^\mu(\tau-\sigma)$$

- The electric field may be different on each of the two D-branes. The boundary conditions at $\sigma = \sigma_a \in \{0, \pi\}$

$$\partial_\sigma X^\mu + (2\pi\alpha') F^\mu{}_{\nu;a}(X) \partial_\tau X^\nu = 0$$

- For a constant F, this is a linear system of non-local ODEs,

$$\begin{aligned} \dot{f}^\mu - \dot{g}^\mu + (2\pi\alpha') F^\mu{}_{\nu;0} \left(\dot{f}^\nu + \dot{g}^\nu \right) &= 0 \\ T\dot{f}^\mu - T^{-1}\dot{g}^\mu + (2\pi\alpha') F^\mu{}_{\nu;1} \left(T\dot{f}^\nu + T^{-1}\dot{g}^\nu \right) &= 0 \end{aligned}$$

where $\cdot = d/d\tau$ and $Tf(\tau) = f(\tau+\pi)$.

- This can be solved in Fourier space, $T = e^{-i\pi\omega}, \partial/\partial\tau = -i\omega$. Eigenmodes satisfy, assuming $[F_0, F_1] = 0$,

$$e^{-2\pi i\omega_n} = \frac{1+F_0}{1-F_0} \cdot \frac{1-F_1}{1+F_1}$$

Open strings in a constant electromag field

The dispersion relation again:

$$T^2 = e^{-2\pi i \omega_n} = \frac{1+F_0}{1-F_0} \cdot \frac{1-F_1}{1+F_1}$$

- Magnetic field: $F = b \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \rightarrow \{ib, -ib\}$ hence $|T| = 1$ and frequencies are real:

  $$\omega_n = n \pm \nu \ , \quad \pi\nu = \text{ArcTan } b_1 - \text{ArcTan } b_0$$

  The string is stable and follows Landau orbits.

- Electric field: $F = e \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \rightarrow \{e, -e\}$ hence $|T| \neq 1$ and frequencies have an imaginary part:

  $$\omega_n = n \pm i\nu \ , \quad \pi\nu = \text{ArcTanh } e_1 - \text{ArcTanh } e_0$$

  This instability is due to Schwinger pair production:

  $$A = (e_0 + e_1) \int_0^\infty \frac{dt}{t^{13}} e^{-\pi\nu^2 t} \frac{1}{\eta^{21}(it)\theta_1(\nu t \mid it)}$$

  $$\Im(A) \sim \sum_{k=1}^{\infty} a(k) \exp\left(-\frac{\pi k}{\nu}\right)$$

Bachas Porrati

Born-Infeld critical electric field

- At the critical electric field $e_a = 1/\alpha'$, the electric force pulling the two ends of the string apart overwhelms the string tension, leading to the production of stretched macroscopic strings, that discharge the condensator at infinity.

- By scaling $e_a \alpha' \to 1$ and $\alpha' \to 0$ while keeping the effective tension of charged strings fixed, one obtains NCOS, a theory of interacting non-commutative open strings, decoupled from closed strings, propagating in a fixed open string metric.

Gopakumar Maldacena Minwalla Strominger
Seiberg Sussking Toumbas

- This classical instability occurs even for neutral dipoles. In contrast, the non-perturbative Schwinger instability requires charged particles.

Open strings in a null electric field

- A generic $F_{\mu\nu}$ can always be brought to the electric or magnetic form depending on sgn $F_{\mu\nu}F^{\mu\nu}$. However there is a non-generic possibility,

$$F = \Phi dx \wedge dx^+ \ , \qquad x^{\pm} = (x^0 \pm x^1)/\sqrt{2} \ , \ x = x^2$$

  which satisfies $F_{\mu\nu}F^{\mu\nu} = 0$. In 4D, it amounts to a configuration with crossed fields $\vec{E} \perp \vec{B}$ of equal magnitude $|\vec{E}| = |\vec{B}|$.

- The matrix F^{μ}_{ν} now has a non-trivial Jordan form, (the only non-trivial from for $SO(1, d-1)$)

$$F = \Phi \begin{pmatrix} 0 & 1 & \\ & 0 & 1 \\ & & 0 \end{pmatrix} \rightarrow \{0, 0, 0\}$$

  hence the spectrum is unaffected, $\omega_n = n \in Z$. Precise eigenmodes do depend on Φ however.

- This agrees with the fact that there is no polarization in a configuration with null electric field. In fact, this configuration preserves half SUSY, namely the generators such that $\Gamma^+\epsilon = 0$.

- After T-duality on x, this describes a null scissor configuration, i.e. two intersecting straight D-branes whose intersection point moves with the speed of light.

Bachas Hull

Relativistic string in a pulse

- More generally, one may allow an arbitrary dependence in the light-cone time x^+:

$$A = \Phi(x^+) x dx^+ \ , \quad F = \Phi'(x^+) dx \wedge dx^+$$

All contractions of F_{x+} and ∂_+ vanish, hence this is an exact supersymmetric solution of the open strings eom to all orders in α'.

- In light-cone gauge $X^+ = x_0^+ + p^+\tau$, the boundary conditions receive a time-dependent source term,

$$\partial_\sigma X + (2\pi\alpha')p^+\Phi'_a(X^+) = 0 \ , \quad \sigma = \sigma^a \in \{0, \pi\}$$

while X is still free in the bulk, $(\partial_\tau^2 - \partial_\sigma^2)X = 0$. Classical solutions can be computed by linear response.

- Assuming that the electric field vanishes at $x^+ = \pm\infty$, it is now straightforward to compute the quantum mechanical transition amplitudes. An incoming string in its ground state will in general emerge in an excited state, depending on the profile $a(x^+)$.

- After T-duality along x, the bc becomes

$$X(\tau) = -(2\pi\alpha')\Phi^{(a)}(X^+(\tau)) + b^{(a)}$$

It describes open strings stretched between two D-branes with a null intersection: null scissors

Bachas

Exact travelling waves

- As a matter of fact, the class of electromagnetic waves which are exact solutions of open string theory is much larger:

$$A = \Phi(x^+, x^i) dx^+ \ , \quad F = \partial_i \Phi(x^+, x^i) dx^i \wedge dx^+$$

Maxwell's equations require that Φ be an harmonic function in transverse space.

- The corresponding open string metric is a gravitational wave in Brinkmann coordinates:

$$ds^2 = 2dx^+ dx^- + G_{ij} dx^i dx^j + (2\pi\alpha')^2 \left| \partial_i \Phi(x^+, x^i) \right|^2 (dx^+)^2$$

- In light-cone gauge $X^+ = x_0^+ + p^+ \tau$, the boundary conditions read

$$\partial_\sigma X^i + (2\pi\alpha') p^+ \partial_i \Phi(X) = 0$$

Just like closed strings in pp-waves, conformal invariance is broken in the light-cone gauge, but only through boundary effects.

- A constant magnetic field B_{ij} can be added, at the cost of using the open string metric in the harmonicity equation.

Solvable travelling waves

- The simplest harmonic solution is a linear potential $\Phi(x^+, x^i) = \phi_i(x^+)x^i$, leading to the uniform null field already discussed.

- The next simplest case is a quadratic potential

$$\Phi(x^+, x^i) = h_{ij}(x^+)x^i x^j/2$$

leading to a massive linear boundary condition:

$$\partial_\sigma X^i + p^+(h_a)_{ij}(x^+)X^j \ , \quad \sigma = \sigma^a$$

- This is very reminiscent of studies of open string tachyon condensation in BSFT. However,
(i) due to the tracelessness of h, the boundary deformation $p^+ \oint \Phi(x^+, x^i) dX^+$ is unbounded from below or above.
(ii) the worldsheet is a Lorentzian strip, instead of an Euclidean cylinder or annulus. *Can tachyon dynamics be derived from Born-Infeld ?*

Witten, Shatashvilii; Arutyunov Pankiewicz Stefanski, Bardakci Konechny

- As for gravitational waves, supersymmetric non-conformal boundary deformations, in particular integrable, can be used to construct on-shell exact backgrounds.

Maldacena Maoz

A word on T-duality

- In terms of the T-dual coordinate

$$\tilde{X}^i = f^i(\tau+\sigma) - g^i(\tau-\sigma)$$

the bc become, after differentiating once,

$$\partial_\tau^2 \tilde{X}^i + p^+ (h_a)_{ij} \partial_\sigma \tilde{X}^j = 0,$$

This is an open string with two beads of mass h_a^{-1}/p^+ at its ends.

- This corresponds to a boundary deformation $(h^{-1})_{ij}$ $\oint X^i \partial_\tau^2 X^j / p^+$ by an excited state. Deformations by more general excited states $X \partial^n X$ are also solvable.

- When $h = 0$, this is a Dirichlet bc. However, at finite coupling, D0-branes have finite mass $1/g_s$, hence $h \sim g_s$: D-brane recoil can be taken into account by going off-conformality.

- We will momentarily predict an instability of the T-dual system, at a critical line $m_0 - m_1 = \alpha' p^+$: fast elastic rotator ?

- A T-dual-like but inequivalent bc would be to take

$$\partial_\tau X^i + p^+ \partial_i \Phi(X) = 0$$

The ends of the open string follow the gradient lines of Φ: we are back to null scissors of arbitrary shape.

Point particles in electromagnetic waves

- The action for a charged particle is

$$S = \int \left[\frac{1}{2e} (\partial_\tau X^\mu)^2 - e\ m^2 \right] d\tau + A_\mu dX^\mu$$

- After choosing the gauge $e = 1$, the eom read

  1. $(d^2/d\tau^2) X^+ = 0$
  2. $(d^2/d\tau^2) X^i + \partial_i \Phi\ \partial_\tau X^+ + B_{ij} \partial_\tau X^j = 0$
  3. $(d^2/d\tau^2) X^- - \partial_i \Phi\ \partial_\tau X^i = 0$

- 1. can be integrated to $X^+(\tau) = x_0^+ + p^+ \tau$. 2. and 3. imply that

$$H = \frac{1}{2}(p_i)^2 + p^+ p^- + p^+ \Phi(X^+, X^i) + m^2$$

  is a constant, where $p_i = \partial_\tau X^i - B_{ij} X^j$ and $p^- = dX^-/d\tau - \Phi$ are the canonical momenta conjugate to X^i and to X^+.

- The motion in transverse coordinates is therefore that of a non-relativistic particle in an electrostatic potential $V = \Phi(X^+, X^i)$.

- Similarly, a relativistic string in an electromag wave behaves as a non-relativistic elastic dipole (possibly with overall charge)

Non-relativistic dipole and critical gradient

- We have seen that on the light-cone, a relativistic string behaves like a non-relativistic dipole. This implies that its tensive energy is proportional to the square of its length:

$$V_t = \frac{1}{\alpha' p^+}(x_L - x_R)^2$$

- For a quadrupolar wave, the electrostatic energy scales also like the square of the distance,

$$V_e = p^+ \left(h^0_{ij} x^i_L x^j_L - h^1_{ij} x^i_R x^j_R \right)$$

- There exists therefore a line of critical electric gradients

$$h_1 - h_0 - \pi (p^+)^2 \alpha' h_0 h_1 = 0$$

  at which the two forces balance against each other, leading to stretched macroscopic strings. This is the non-relativistic analogue of the Born-Infeld critical field. *What is the analogue of the open string metric ?*

- One may wonder if a decoupled theory of non-relativistic interacting open strings exists at that point, analogue to NCOS. This theory would exhibit light-like non-commutativity.

Aharony Gomis Mehem

First quantization

- Since the bulk theory is still free, one may separate X into left and right movers,

$$X^i = f^i(\tau+\sigma) + g^i(\tau-\sigma)$$

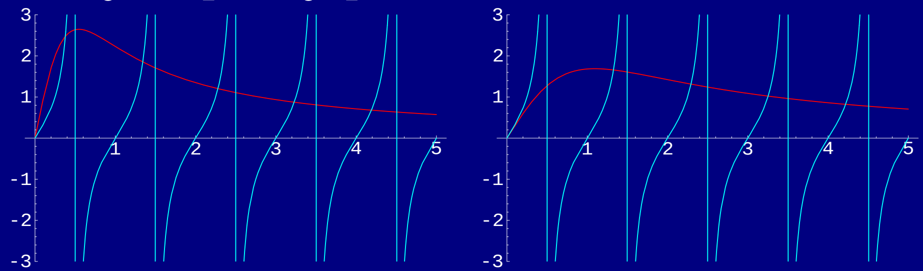
which satisfy boundary conditions:

$$\begin{aligned} \dot f(\tau) - \dot g(\tau) &+ p^+ h_0\big(f(\tau)+g(\tau)\big) \\ T^2\dot f(\tau) - \dot g(\tau) &+ p^+ h_1\big(T^2 f(\tau)+g(\tau)\big) = 0 \end{aligned}$$

- Again, we can work in Fourier space, and find the dispersion relation ($e_i = \pi p^+ h_i$)

$$\tan(\pi\omega) = \frac{(e_1-e_0)\pi\omega}{(\pi\omega)^2 + e_0 e_1}$$

Indeed, a pair of real roots disappear at the critical line $e_0 - e_1 - e_0 e_1 = 0$:



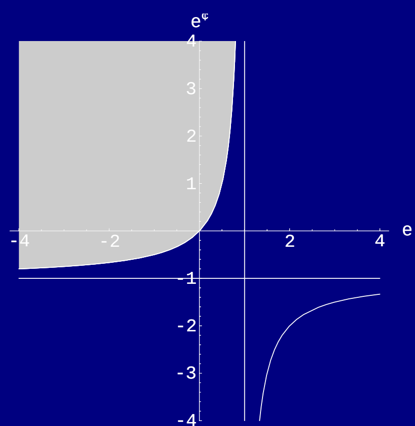
- The partition function for open strings in a quadrupolar field is then simply

$$Z_{op}(t,e_0,e_1) = q^{E_X}\prod_{n=1}^{\infty}(1-q^{\omega_n})^{-1}\times\begin{cases}(1-q^{\omega_0})^{-1} & \text{if } D>0\\ \big(1-q^{ik_0}\big)^{-1} & \text{if } D<0\end{cases}$$

with $E_X = \sum_{n=0}^{\infty}\omega_n$ the zero-point energy.

Dynamical instability

- For a quadratic potential depending on a single direction, the motion is stable in the shaded region, extending slightly outside the domain of stability of a dipole with vanishing tension:



- For a traceless quadratic potential h_{ij}, the motion is always unstable, due to the convexity of the stability domain. However, this is a kinematical instability of the string probes, not of the background itself: much like the divergence of geodesics in purely gravitational plane waves,

$$ds^2 = 2dx^+dx^- + dx^2 + dy^2 - \mu^2(x^2 - y^2)(dx^+)^2$$

Marolf Zayas; Brecher Gregory Saffin

- (Former) atomic physicists know how to deal with these instabilities...

Strings in quadrupolar ion traps

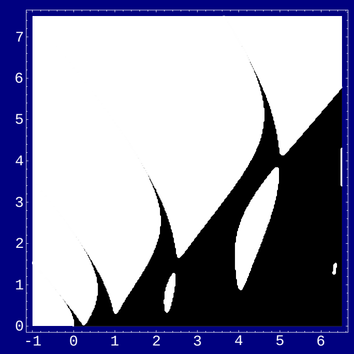
Several ways to make a stable electromagnetic trap:

a. The Penning trap: use a static magnetic field to confine charged particles in the transverse unstable plane:

$$V(x) = -\frac{e}{2}(x^2 + y^2 - 2z^2) \ , \qquad B = b dx \wedge dy$$

is stable if $b^2 > e$ and $e > 0$.

b. The RF or Paul trap: no magnetic field, but modulate the electric field at a frequency such that the particle experiences a restoring force on average: parametric resonance



$$V = (\omega^2 + \alpha^2 \cos t)(x^2 - y^2)$$

c. The quadrupolar trap: a static quadrupolar potential confines neutral particles with a negative polarizability, by drawing them to regions of low electric field strength: $W = -\alpha E^2$. *Degenerate excited states usually have negative polarizability*

Mechanisms a. and b. carry over to the string case easily.

Closed string channel and boundary state

- In the closed string channel the boundary states satisfies

$$\partial_\tau X + \frac{\hat{e}}{\pi} X \; |B(\hat{e})\rangle = 0$$

This is solved by the usual coherent state techniques,

$$|B(\hat{e})\rangle = \mathcal{N}(\hat{e})\, e^{i\frac{\pi p_0^2}{2\hat{e}}} \exp\left(\sum_{n=1}^{\infty} -\frac{1}{n}\frac{i\pi n + \hat{e}}{i\pi n - \hat{e}} \alpha_{-n}\tilde{\alpha}_{-n}\right) |0,\tilde{0}\rangle$$

- The partition function is therefore given by the overlap of the two boundary states,

$$Z_{cl}(\hat{t}, \hat{e}_0, \hat{e}_1) = \mathcal{N}(\hat{e}_0)\mathcal{N}(\hat{e}_1) \sqrt{\frac{2}{\hat{t} + i\left(\frac{1}{\hat{e}_1} - \frac{1}{\hat{e}_0}\right)}} e^{\pi\hat{t}/12}$$

$$\prod_{n=1}^{\infty} \left(1 - \frac{i\pi n + \hat{e}_0}{i\pi n - \hat{e}_0}\frac{i\pi n - \hat{e}_1}{i\pi n + \hat{e}_1} e^{-2\pi n\hat{t}}\right)^{-1}$$

Arutyunov Pankiewicz Stefanski, Bardakci Konechny

Open-closed duality

- Equality with the open string channel can be formally seen by representing the sum by a residue integral

$$\log Z_{op} = \frac{1}{2\pi} \int_C (\log \Phi_{cl}) \frac{d \log \Phi_{op}}{dz} dz$$

  with

$$\Phi_{op}(z) = 1 - e^{-2\pi i z} \frac{i\pi z + e_0}{i\pi z - e_0} \frac{i\pi z - e_1}{i\pi z + e_1}\,, \qquad \Phi_{cl}(z) = 1 - e^{-2\pi t z}$$

  Integrating by parts shows that

$$Z_{op}(t, e_0, e_1) = Z_{cl}(\hat{t}, \hat{e}_0, \hat{e}_1)$$

  where the deformation parameters are related by

$$\hat{e}_a = e_a t$$

  in full agreement with open/closed duality of the one-loop amplitude (after compactification of the light-cone).

- A careful proof takes much more effort, but can be made along the lines of a similar computation in the context of open strings in gravitational waves.

  Bergman Gaberdiel Green

- We now take $h_a(x^+)$ with finite support in x^+. At $\tau \to \pm\infty$ we have free field mode expansions,

$$X = x_0 + p_0\tau + +i \sum_{n\neq 0} \frac{2}{n} a_n \cos(n\sigma) e^{-in\tau}$$

  and a similar expansion with primes at $\tau \to \infty$.

- The two sets of modes are related by a symplectic matrix, the Bogolioubov transformation:

$$\begin{pmatrix} x'_0 \\ p'_0 \\ a'_m \end{pmatrix} = \begin{pmatrix} \alpha & \beta & A_n \\ \gamma & \delta & B_n \\ \tilde{A}_m & \tilde{B}_m & B_{mn} \end{pmatrix} \begin{pmatrix} x_0 \\ p_0 \\ a_n \end{pmatrix}$$

- In the Born approximation ($h \ll 1$), the incoming state is a source for the outcoming perturbation, and one finds easily e.g.

$$B_{mn} = \delta_{mn} + \frac{i}{\pi^2 n} \int_{-\infty}^{\infty} (e_0 - (Te_1))(p^+\tau) e^{-i(n-m)\tau} d\tau$$

- The final excitation number of the mode n is

$$\langle 0_{in} | a'_{-m} a_m | 0_{in} \rangle = \sum_{n\neq 0} |B_{m,-n}|^2 + \ldots$$

  hence the total energy diverges if $h_a(x^+)$ has a delta function singularity.

Two zero-modes in one

- In fact, the open string zero-mode has an ambiguity which corresponds to the splitting between left- and right-movers:

$$x_0 = f_0 + g_0 \ , \quad a = f_0 - g_0$$

a is the position of the T-dual D-brane, hence the value of the worldvolume $U(1)$ gauge field A_x on the original D-brane.

- In flat space, a can be changed by a gauge transformation hence has no physical meaning.

- In a time-dependent situation, this is no more the case: the difference $a(x^+ = +\infty) - a(x^+ = -\infty)$ is the electric field F_{+x}. In the Born approximation,

$$\delta f_0 - \delta g_0 = -\frac{1}{\pi} \int_{-\infty}^{\infty} e_0(p^+ \tau) X(\sigma = 0, \tau) d\tau$$

This is possibly the simplest computation of the backreaction of an open string on an electric background.

- Similar computations can be made in the adiabatic approximation, but keeping $h^a(\pm\infty)$ finite, as the limit $h \to 0$ is non adiabatic.

Electric field and Lorentzian orbifold

- Closed strings in the w-th twisted sector of the Lorentzian orbifold satisfy

$$X^{\pm}(\sigma+2\pi,\tau)=e^{\pm\nu}X^{\pm}(\sigma,\tau)\ ,\qquad \nu=w\beta$$

Expanding in left and right movers, we have the normal mode expansion:

$$\begin{aligned} X^{\pm}_R(\tau-\sigma) &= \frac{i}{2}\sum_{n=-\infty}^{\infty}(n\pm i\nu)^{-1/2}\alpha_n^{\pm}e^{-i(n\pm i\nu)(\tau-\sigma)} \\ X^{\pm}_L(\tau+\sigma) &= -\frac{i}{2}\sum_{n=-\infty}^{\infty}(-n\mp i\nu)^{-1/2}\tilde{\alpha}_n^{\pm}e^{-i(-n\mp i\nu)(\tau+\sigma)} \end{aligned}$$

- Upon identifying the oscillators

$$\alpha_n^{\pm}=a_n^{\pm}\ ,\qquad \tilde{\alpha}_{-n}^{\pm}=(a_n^{\pm})^*$$

and setting

$$X^{\pm}_{open}=x^{\pm}+X^{\pm}_R(\tau-\sigma)+X^{\pm}_L(-\tau-\sigma)\ ,$$

this reduces to the open string mode expansion

$$X^{\pm}=x^{\pm}+ia_0^{\pm}\phi_0^{\pm}(\sigma,\tau)+i\sum_{n=1}^{\infty}\left[a_n^{\pm}\phi_n^{\pm}(\sigma,\tau)-h.c.\right]$$

where $\phi_n^{\pm}(\sigma,\tau)=(n\pm i\nu)^{-\frac{1}{2}}\ e^{-i(n\pm i\nu)\tau}\ cos[(n\pm i\nu)\sigma]$

- Canonical commutation relations are

$$[\alpha_n^{+},(\alpha_n^{-})^*]=[\alpha_n^{-},(\alpha_n^{+})^*]=[\tilde{\alpha}_n^{-},(\alpha_n^{+})^*]=[\tilde{\alpha}_n^{-},(\tilde{\alpha}_n^{+})^*]=-1$$

No-physical state no-theorem

- The worldsheet Hamiltonian for open strings reads, after normal ordering ($a^\pm := a_0^\pm$)

$$L_0 = -\sum_{n=1}^{\infty}(n-i\nu)(a_n^+)^* a_n^- - \sum_{n=0}^{\infty}(n+i\nu)(a_n^-)^* a_n^+ + \frac{1}{2}i\nu(1-i\nu) + L_{int}$$

- Representing in a Fock space with vacuum annihilated by all $a^-_{n\geq 0}$ and $a^+_{n>0}$, eigenstates have imaginary energy. This does not contradict hermiticity, since they also have zero norm ! Hence the physical state condition L_0 has no solutions.

- For Milne Universe it is the same story with tildas. The no-physical state statement is warranted by modular invariance of the one-loop amplitude.

Nekrasov

Scattering states and tunnelling

- Alas, this vacuum is the one obtained by analytic continuation from Euclidean, i.e. for strings in a magnetic field (disregarding time direction). There physical states are Landau states, corresponding to discrete normalizable eigenmodes of the harmonic oscillator (times a continuous degeneracy label).

- In the Minkowskian (electric) case, the harmonic oscillator becomes inverted, and the continued Landau states now have imaginary energy. However there is now a continuum of delta-normalizable physical scattering states with real energy.

Das Jevicki; Moore; Alexandrov Kazakov Kostov

- Physically, these represent electrons and positrons being deflected by the electric field. Tunneling through the barrier is just induced Schwinger emission, $e^- \to \mu e^- + (1+\mu)e^+$.

Brezin Itzykson

- Scattering amplitudes may be computed in the Euclidean (magnetic) theory, after expanding the scattering states on the basis of (analytically continued) "Landau states" – and proper regularization.

Berkooz P., in progress

Rindler, Liouville, . . .

- A constant electric field $F = edx^+ \wedge dx^-$ preserves symmetries under boost. One may consider states of fixed boost momentum J, and use adapted Rindler coordinates $x^\pm = \pm e^{y \pm \eta}$ in the R region.

- Neutral massive particles have a rectilinear motion in Minkowski, but in Rindler coordinates experience a Liouville wall:

$$(dy/d\eta)^2 + 4m^2 e^{2y} = 0$$

Charged massive particles have an hyperbolic motion, but in Rindler coordinates experience a Liouville type potential barrier

$$(dy/d\eta)^2 + 4m^2 e^{2y} - (J + e\ e^{2y})^2 = 0$$

Tunnelling corresponds again to induced pair production or teleportation.

Narozhny Mur Fedotov

. . . Unruh and Milne . . .

- Winding closed strings in the Lorentzian orbifold behave exactly as massive charged particles in Rindler space, with boost momentum fixed by the matching condition $wJ = N_L - N_R$.

- Fixing the boundary conditions in the whiskers region should lead to Unruh-type vacua, with thermal particle production. *A stringy Hawking radiation computation ?*

- Winding strings will be pair-produced and should backreact on the geometry so as to “discharge the condensator”: is there enough time for the cosmological singularity to take place ?

- Electric fields are full of promises for the study of Time and String Theory...