Closed strings in the Misner Universe

a toy model of a Big Crunch ?

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Talk based on

hep-th/0307280 w/ M. Berkooz

hep-th/0405126 w/ M. Berkooz, and M. Rozali

hep-th/0407216 w/ M. Berkooz, B. Durin and D. Reichmann

hep-th/0501145 w/ B. Durin

slides available from

`http://www.lpthe.jussieu.fr/˜pioline/seminars.html`

String theory and time-dependent backgrounds

- Observational Cosmology is currently supplying us with abundant high-precision experimental data, yet our understanding of string theory in time-dependent backgrounds is mostly limited to effective field theory.
- While inflationay field theoretical models have so far been successful in describing these data, they predict their own demise at the initial singularity. Can string theory resolve the singularity, e.g. by turning the Big Bang into a Big Bounce ?
- (Closed) string field theory would appear to be the natural framework to address time dependent backgrounds, unfortunately it remains untractable to this day, and may not even exist in principle. Holography for time-dependent processes is perhaps the most promising option, but still in the making.
- To what extent can we use the first-quantized, on-shell perturbative formalism to describe time-dependent backgrounds, and in particular compute cosmological particle production ?

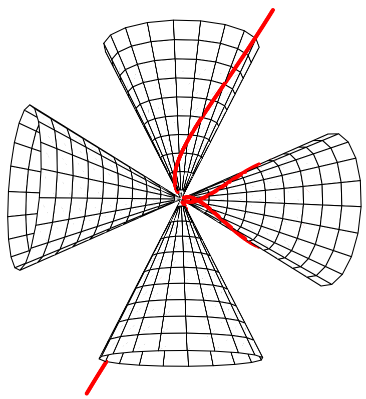
Cosmological backgrounds in perturbative string theory

- Even disregarding quantum (g_s) corrections, string theory backgrounds undergo classical (α') corrections compared to general relativity. Few examples of cosmological solutions of string theory are known where these effects are resummed: cosmological singularities typically remain.

Antoniadis Bachas Ellis Nanopoulos, Kounnas Lüst, Nappi Witten...

- In this talk, we will discuss an example of a classically exact cosmological background with a space-like singularity: Misner space, aka the "Lorentzian" orbifold.

  Our aim will be to understand classical aspects of string propagation in this cosmological background, and compute tree-level particle/string production rates.



- A much more ambitious task is to incorporate gravitational backreaction, and determine whether or not the cosmological singularity is resolved.

Outline of the talk

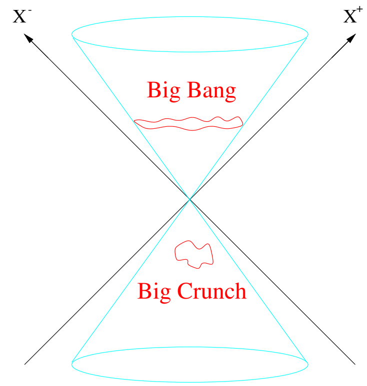
0. Misner, Milne, Rindler, and friends
1. Classical and quantum particles in Misner space
2. Winding strings in Misner space: the short and the long
3. First quantization, and an analogy to charged particles
4. Schwinger pair production of winding strings
5. Coherent condensation of winding strings
6. Discussion

0. The Lorentzian orbifold

- One of the (apparently) simplest time-dependent solution in string theory is the quotient of flat Minkowski space by a discrete boost, also known as Misner space (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^\pm \sim e^{\pm 2\pi\beta} X^\pm$$



- The future (past) regions $X^+X^- > 0$ describes a cosmological universe often known as the Milne universe (1932), linearly expanding away from a Big Bang singularity (or contracting into a Big Crunch singularity):

$$ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + (dX^i)^2 \ , \quad \theta \equiv \theta + 2\pi \ , \quad X^\pm = T e^{\pm\beta\theta}/\sqrt{2}$$

This is a special case of a Kasner geometry, with Kasner exponents $(1, 0, 0, \ldots)$.

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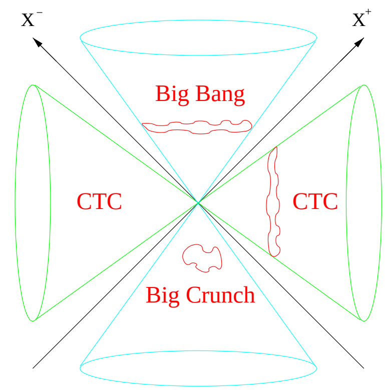
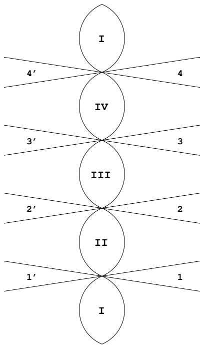


- In addition, the spacelike regions $X^+X^- < 0$ describe two static regions with compact time, sometimes called Rindler regions or whiskers, leading to closed time-like curves:

$$ds^2 = dr^2 - \beta^2 r^2 d\eta^2 + (dX^i)^2 \quad , \eta \equiv \eta + 2\pi \; , \quad X^\pm = \pm r e^{\pm\beta\eta}/\sqrt{2}$$

- Finally, the lightcone $X^+X^- = 0$ gives rise to a null, non-Hausdorff locus attached to the singularity.

Close relatives of the Misner Universe



- Misner space was first introduced as a local model of Lorentzian Taub-NUT space, which can be viewed as a cyclic universe, isomorphic to $R^{1,1}/boost \times S^2$ around each singularity.
- It also provides a local model for the Nappi-Witten cosmology, described by a gauged WZW model.

Elitzur Giveon Kutasov Rabinovici;Johnson Svendsen

- A close variant of Misner space, where the discrete boost is combined with a translation along a transverse direction,

$$ds^2 = -2dX^+dX^- + dX^2 + (dX^i)^2 \, , \quad (X^\pm, X) \sim (e^{\pm 2\pi\beta} X^\pm, X + 2\pi R)$$

  describes the geometry away from two moving cosmic strings. The cosmological singularity is smoothed out, but regions with CTC remain.

Gott 91, Grant 93; Cornalba, Costa, Kounnas

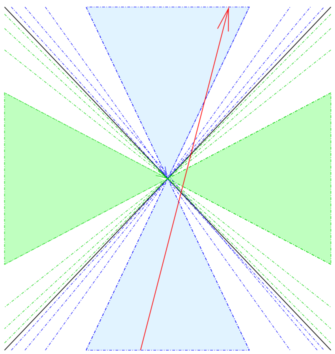
- KK reduction along the cosmological circle leads to a cosmological solution of Einstein-dilaton gravity with no potential, possibly relevant for ekpyrotic scenarios.

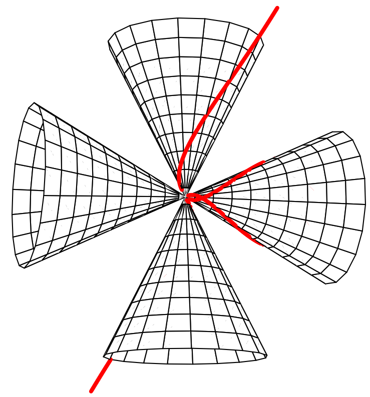
Khoury Ovrut Seiberg Steinhard Turok

1. Classical particles in the Misner Universe

- Classical particles propagate along straight lines on the covering space:

$$X^{\pm} = x_0^{\pm} + p^{\pm}\tau \ , \quad M^2 = 2p^+p^- \ , \quad j = p^+x_0^- - p^-x_0^+$$





- As the particle approaches the singularity from the past, it starts spinning faster and faster, $\theta \sim \log |T|$, implying copious gravitational emission and possibly large backreaction.

Polchinski Horowitz

- In terms of the covering space coordinates, the trajectory is regular. The trajectories in the past and future Milne regions can be related unambigously, by propagating for a finite proper time in the Rindler wedges.

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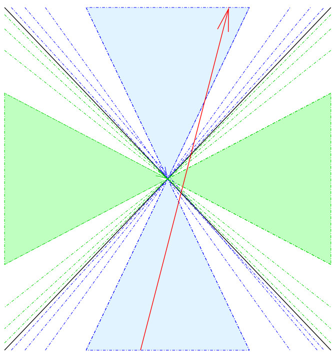
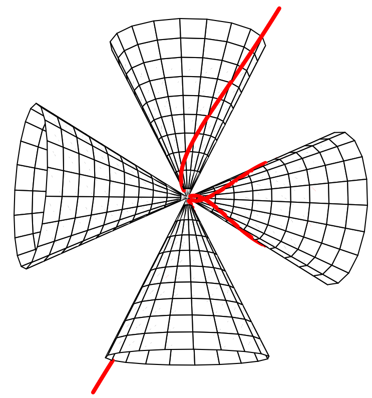




- In the Rindler wedges, the particle reaches a finite radius. As it approaches the horizon, it winds infinitely many times around the time direction.
- From the point of view of a static observer in the Rindler wedge, an infinite number of particles are periodically emitted from and reabsorbed by the singularity. The total Rindler energy is infinite.

Quantum particles in the Misner Universe

- Quantum mechanically, the particle wave function obeys the Klein-Gordon equation in this curved geometry. At fixed boost momentum j, this reduces to a Schrödinger equation in a 1-dim Liouville-type potential:

$$\frac{1}{r}\partial_r r \partial_r + \frac{j^2}{r^2} = M^2 , \qquad r = e^y , \qquad V(y) = -j^2 + M^2 e^{2y} \equiv 0$$

$$-\frac{1}{T}\partial_T \partial_T - \frac{j^2}{T^2} = M^2 , \qquad T = e^x , \qquad V(x) = -j^2 - M^2 e^{2x} \equiv 0$$

  The singularity is at infinite distance in the canonically normalized x or y coordinate.

- The wave functions can be defined globally by continuing across the horizons. The *in* and *out* states defined at $T = -\infty$ and $T = +\infty$ are identical, hence no overall particle production. However, there is copious particle production between $T = -\infty$ and $T = 0$.
- Wave functions of boost momentum j and spin s can be expressed as superpositions of plane waves on the covering space ($s =$spin)

$$f_{j,M^2,s}(x^+, x^-) = \int_{-\infty}^{\infty} dv \exp\left(ik^+ x^- e^{-2\pi\beta v} + ik^- x^+ e^{2\pi\beta v} + ivj + vs \right)$$

  For $|s| > 1$ the v integral can be defined by shifting the contour.

2. Closed strings in Misner space

As in standard orbifold constructions, closed strings fall into two classes:

- UNTWISTED SECTOR: closed strings in the covering space, which are invariant under the orbifold projection. These topologically trivial states behave at low energy just like the ordinary point particles described previously.
- TWISTED SECTORS: open strings in the covering space, which close up to the action of the orbifold group:

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau) \ , \quad \nu = 2\pi w \beta$$

  These correspond to "cosmic" strings which wind w-times around the compact direction.

In usual Euclidean orbifolds, twisted states are localized near the orbifold fixed point, and their condensation leads to a resolution of the singularity, e.g. $R^4/Z_k \to ALE_k$.

Can the condensation, or rather pair production, of twisted states resolve the cosmological singularity ?

Tree-level scattering of untwisted states

- Tree-level scattering amplitudes of untwisted sector states can be computed from those in flat space by the inheritance principle,

$$\langle V(j_1,k_1)\dots V(j_n,k_n)\rangle_{Misner} = \int dv_1\dots dv_n\, e^{i(j_1v_1+\dots+j_nv_n)}$$
$$\langle V(e^{\beta v_1}k_1^+, e^{-\beta v_1}k_1^-, k_1^i)\dots V(e^{\beta v_n}k_n^+, e^{-\beta v_n}k_n^-, k_n^i)\rangle_{Minkowski}$$

- String amplitudes are exponentially suppressed in the high energy regime (fixed s/t, s/u). However, in the deep inelastic regime, ($s\to\infty$, t fixed), they exhibit Regge behavior $A\sim s^t$, as if strings acquired a size $\sqrt{\ln s}$. The total amplitude

$$\langle V(j_1,k_1)\dots V(j_n,k_n)\rangle_{Misner} \propto \int dv\; v^{-\frac{1}{2}(k_1^i-k_3^i)^2+i(j_2-j_4)}$$

  diverges if $(k_1^i-k_3^i)^2\le 2$, as a result of large graviton exchange near the cosmological singularity.

Berkooz Craps Rajesh Kutasov

- *Could higher order corrections, e.g. resummation of ladder diagrams, lead to a finite amplitude ?*

Deser McCarthy Steif; Cornalba Costa

Winding strings in Misner space

- Truncating to zero-modes, ie viewing the string as rigid, the classical solutions of the equations of motion $(\partial_\tau^2 - \partial_\sigma^2)X^\pm = 0$ in the conformal gauge are

$$X^\pm(\tau,\sigma) = \frac{1}{2\nu} e^{\mp\nu\sigma}\left[\pm\alpha_0^\pm e^{\pm\nu\tau} \mp \tilde{\alpha}_0^\pm e^{\mp\nu\tau}\right] \;, \quad \alpha_0^\pm, \tilde{\alpha}_0^\pm \in R$$

  where the mass-shell and level-matching conditions require

$$2\alpha_0^+\alpha_0^- = M^2 \,, \quad 2\tilde{\alpha}_0^+\tilde{\alpha}_0^- = \tilde{M}^2$$

  Here $\mu^2 = M^2 + \tilde{M}^2$ and $j = (M^2 - \tilde{M}^2)/(2\nu)$ are the mass and boost momentum of the string, originating from internal oscillations / motion in the transverse directions.

- In particular, the world-sheet coordinate σ winds w times around the target-space compact coordinate θ or η, while τ generates motion along the Milne time, or Rindler radius:

$$2X^+X^- = \left\{ \begin{matrix} T^2 \\ -r^2 \end{matrix} \right\} = (\alpha_0^+\tilde{\alpha}_0^- e^{2\nu\tau} + \alpha_0^-\tilde{\alpha}_0^+ e^{-2\nu\tau} - \alpha_0^+\alpha_0^- - \tilde{\alpha}_0^+\tilde{\alpha}_0^-)/(4\nu^2)$$

Short and long strings

Up to shifts and translations of σ and τ, the physical state conditions are solved by $\alpha_0^{\pm} = \epsilon M/\sqrt{2}$, $\tilde{\alpha}_0^{\pm} = \tilde{\epsilon}\tilde{M}/\sqrt{2}$. Choosing $j = 0$ for simplicity, we thus have two very different types of solutions:

- SHORT STRINGS ($\epsilon\tilde{\epsilon} = 1$):

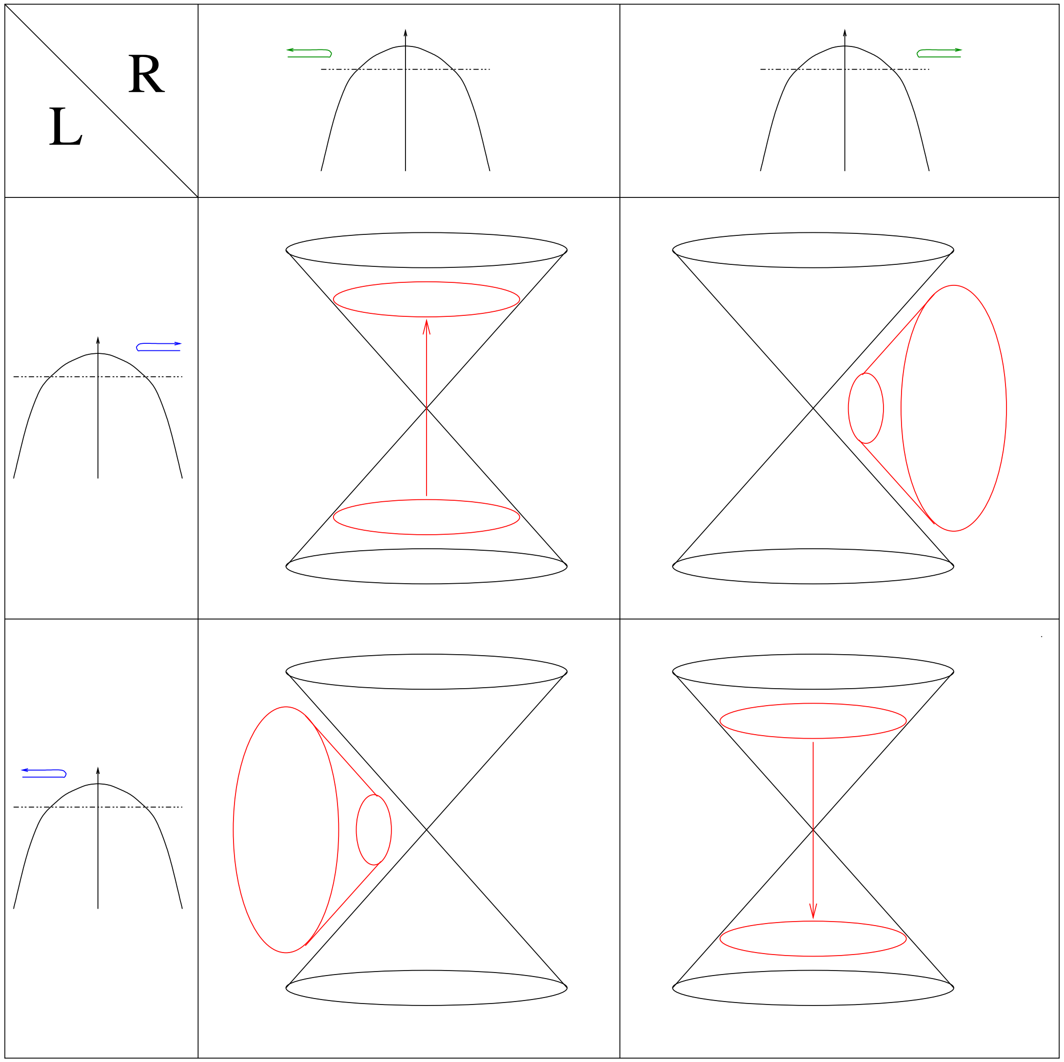
$$X^{\pm}(\sigma, \tau) = \frac{\epsilon M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma} \ , \quad T = \frac{\epsilon M}{\nu} \sinh(\nu\tau) \ , \quad \theta = \nu\sigma$$

  describes a "cosmic" string winding around the Milne circle, propagating from $T = -\infty$ to $T = +\infty$ (or reversely).

- LONG STRINGS ($\epsilon\tilde{\epsilon} = -1$):

$$X^{\pm}(\sigma, \tau) = \pm\frac{\epsilon M}{\nu\sqrt{2}} \cosh(\nu\tau) e^{\pm\nu\sigma} \ , \quad r = \frac{\epsilon M}{\nu} \cosh(\nu\tau) \ , \quad \eta = \nu\sigma$$

  describes an infinitely long string in the right Rindler patch, stretching from $r = \infty$ to $r = M/\nu$ and back to $r = \infty$; σ is now the proper time direction in the induced metric.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3. Quantizing the twisted sectors

- Restoring all oscillators, the twisted strings have a normal mode expansion:

$$X^{\pm} = \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha_n^{\pm} \, e^{-i(n \pm i\nu)(\tau - \sigma)} + \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \mp i\nu)^{-1} \tilde{\alpha}_n^{\pm} \, e^{-i(n \mp i\nu)(\tau + \sigma)}$$

with canonical commutation relations

$$[\alpha_m^+, \alpha_n^-] = -(m + i\nu)\delta_{m+n} \quad , \quad [\tilde{\alpha}_m^+, \tilde{\alpha}_n^-] = -(m - i\nu)\delta_{m+n}$$

$$(\alpha_m^{\pm})^* = \alpha_{-m}^{\pm} \quad , \quad (\tilde{\alpha}_m^{\pm})^* = \tilde{\alpha}_{-m}^{\pm}$$

- In particular, the quasi zero-mode sector, consists of two pairs of real (hermitian) canonically conjugate operators,

$$[\alpha_0^+, \alpha_0^-] = -i\nu \; , \quad [\tilde{\alpha}_0^+, \tilde{\alpha}_0^-] = i\nu$$

Fock space quantization

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated by half of them, say $\alpha^{\pm}_{n>0}\ ,\ \tilde{\alpha}^{\pm}_{n>0}\ ,\quad \alpha^{-}_{0}\ ,\ \tilde{\alpha}^{+}_{0}$
- The worldsheet Hamiltonian, normal-ordered wrt to this vacuum, reads

$$L_0 = -\sum_{n=0}^{\infty}(\alpha^{+}_{n})^{*}\alpha^{-}_{n} - \sum_{n=1}^{\infty}(\alpha^{-}_{n})^{*}\alpha^{+}_{n} + \frac{1}{2}i\nu(1-i\nu) - 1 + L_{int}$$

- The vacuum energy is recognized as the analytically continuation $\theta \longrightarrow i\nu$ of the twisted ground state energy $\frac{1}{2}\theta(1-\theta)$ of Euclidean rotation orbifolds.
- All states in the Fock space have imaginary energy, hence none satisfy the Virasoro condition $L_0 = \tilde{L}_0 = 0$. *In this scheme, there are no physical states in the twisted sectors*

Nekrasov

- However, this quantization scheme overlooks the fact that, unlike standard harmonic oscillators a and $a^{\dagger}$, α^{+}_{0} and α^{-}_{0} are self-hermitian operators ! Indeed, we shall argue that $\alpha^{\pm}_{0}$ need to be quantized like inverted harmonic oscillators.

Quantizing the quasi-zero-modes

- Consider instead the following unitary representation:

$$\alpha_0^{\pm} = i\partial_{\mp} \pm \frac{\nu}{2} x^{\pm}\,, \quad \tilde{\alpha}_0^{\pm} = i\partial_{\mp} \mp \frac{\nu}{2} x^{\pm}$$

acting on wave functions $f(x^+, x^-)$, with inner product

$$\langle f | g \rangle = \int dx^+ dx^- f^*(x^+, x^-)\, g(x^+, x^-)$$

- The zero modes $\alpha_0^{\pm}$ and $\tilde{\alpha}_0^{\pm}$ can be thought of as covariant derivatives in a constant electric field $F_{+-} = F_{01} = \pm\nu$.
- The zero-mode piece of L_0, including the bothersome $\frac{i\nu}{2}$, is the Klein-Gordon operator

$$L_0^{(zm)} = -\alpha_0^+ \alpha_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\nabla^+\nabla^- + \nabla^-\nabla^+)$$

for charged particle in a constant electric field ! This is well known to be equivalent to an inverted harmonic oscillator, just as the Landau problem is equivalent to the usual harmonic oscillator.

Charged particles vs. winding strings

- The classical trajectories of a charged particle in a constant electric field are hyperbolas centered at an arbitrary point,

$$X^{\pm} = x_0^{\pm} \pm \frac{1}{\nu} a_0^{\pm} e^{\pm\nu\tau}$$

Canonical quantization implies that the generators of translations do not commute:

$$[a_0^+, a_0^-] = -i\nu \ , \quad [x_0^+, x_0^-] = -\frac{i}{\nu} \ , \quad L_0 = -a_0^+ a_0^- + \frac{i\nu}{2} + \text{excited.}$$

- Similarly, the classical trajectories of winding strings are

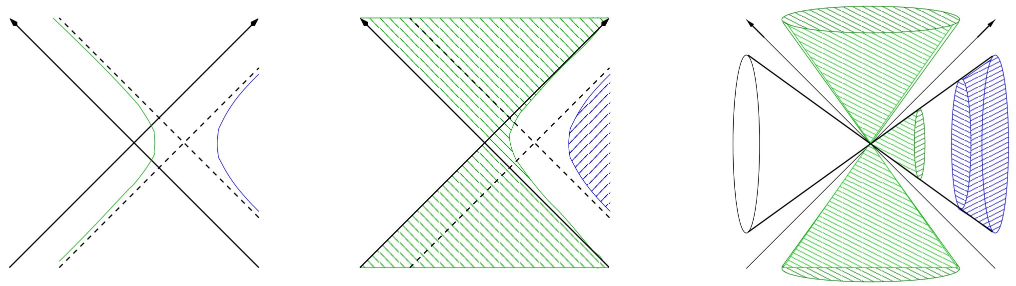
$$X^{\pm}(\tau, \sigma) = \frac{1}{2\nu} e^{\mp\nu\sigma} \left[\pm\alpha_0^{\pm} e^{\pm\nu\tau} \mp \tilde{\alpha}_0^{\pm} e^{\mp\nu\tau} \right] \ , \quad \alpha_0^{\pm}, \tilde{\alpha}_0^{\pm} \in R$$

with canonical commutation relations isomorphic to the above,

$$[\alpha_0^+, \alpha_0^-] = -i\nu \ , \quad [\tilde{\alpha}_0^+, \tilde{\alpha}_0^-] = i\nu$$

From open to closed strings

- The reason for the analogy is now clear: a point on the string worldsheet at fixed $\sigma + \tau$ experiences the same trajectory as a charged particle in a constant electric field.
- In particular, the closed string worldsheet can be obtained by smearing a charged particle trajectory under the action of the boost:



- SHORT STRINGS correspond to charged particle trajectories which begin and end in the Milne regions. LONG STRINGS correspond to charged particle trajectories which begin and end in the Rindler regions.

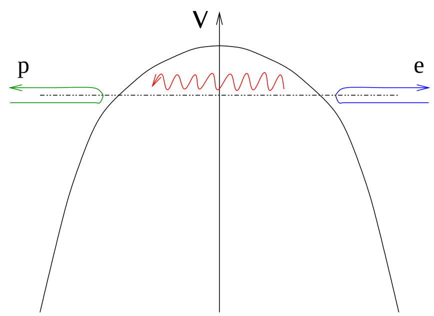
4. Tunneling and Schwinger pair production

- Changing basis to $\alpha_0^\pm = (P \pm Q)/\sqrt{2}$, the Klein-Gordon operator $M^2 = \alpha_0^+\alpha_0^- + \alpha_0^-\alpha_0^+$ can be rewritten as an inverted harmonic oscillator:

$$M^2 = -\frac{1}{2}(P^2 - Q^2)\ , \qquad [P, Q] = i$$

- The latter admits a respectable delta-normalizable spectrum of scattering states, in terms of parabolic cylinder functions.

- These correspond to non-compact trajectories of charged particles in the electric field. Tunnelling is just (stimulated) Schwinger pair creation,



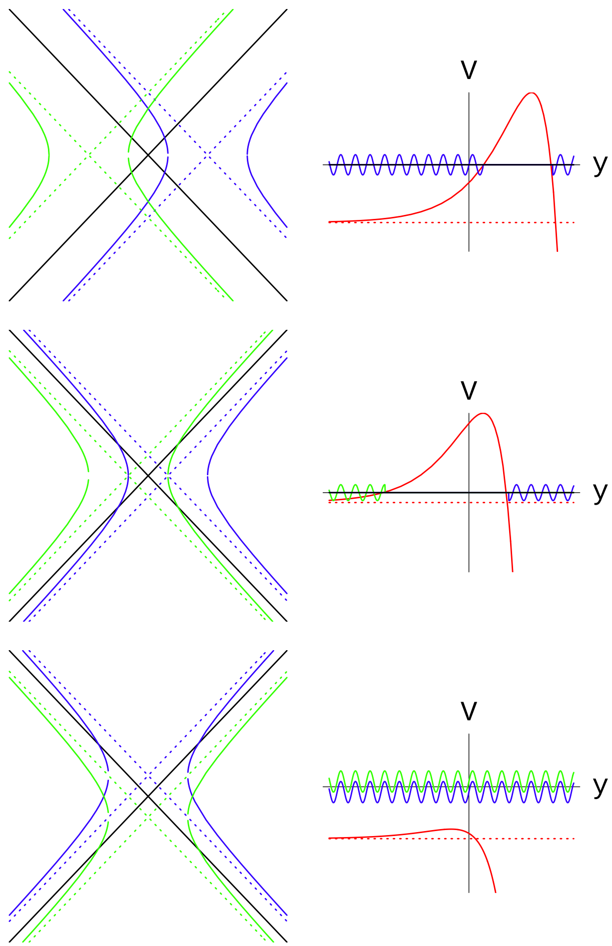
$$e^- \longrightarrow (1+\eta)\, e^- + \eta\, e^+$$

- The tunneling rate can be computed semiclassically, $\eta \sim \exp\left(-2 \oint P dQ\right) = e^{-\pi M^2/\nu}$ which reproduces the Schwinger pair creation rate.

Brezin Itzykson; Brout Massar Parentani Spindel

Schwinger pair production in Rindler space

- A similar reasoning applies for a charged particle in Rindler space:



- In the Rindler patch R, letting $f(r,\eta) = e^{-ij\eta} f_j(r)$ and $r = e^y$, one gets a Schrödinger equation for a particle in a potential

$$V(y) = M^2 e^{2y} - (j + \frac{1}{2}\nu \, e^{2y})^2$$

- If $j < 0$, the electron and positron branches are in the same Rindler quadrant. Tunneling corresponds to Schwinger particle production.
- If $0 < j < M^2/(2\nu)$, the two electron branches are in the same Rindler quadrant. Tunneling corresponds to Hawking radiation.
- *(If $j > M^2/(2\nu)$, the electron branches cross the horizons. regions. There is no tunneling, but partial reflection amounts to a combination of Schwinger and Hawking emission.)*

Rindler modes

- Incoming modes from Rindler infinity I_R^- read, in terms of parabolic cylinder functions:

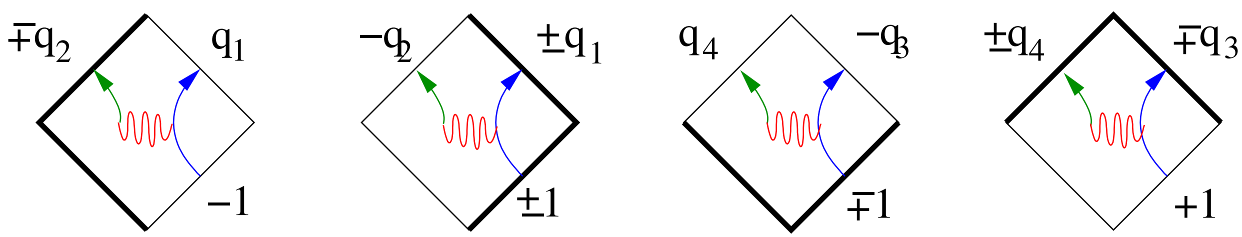
$$\mathcal{V}^j_{in,R} = e^{-ij\eta} r^{-1} M_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),-\frac{ij}{2}}(i\nu r^2/2)$$

Incoming modes from the Rindler horizon H_R^- read

$$\mathcal{U}^j_{in,R} = e^{-ij\eta} r^{-1} W_{i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}(-i\nu r^2/2)$$

- The reflection coefficients can be computed ($q_1 = 1 - q_2, q_3 = q_4 + 1$):

$$q_2 = e^{-\frac{\pi M^2}{2\nu}} \frac{|\sinh \pi j|}{\cosh\left[\pi\left(j - \frac{M^2}{2\nu}\right)\right]}\,, \quad q_4 = e^{-\frac{\pi M^2}{\nu}}/q_2$$



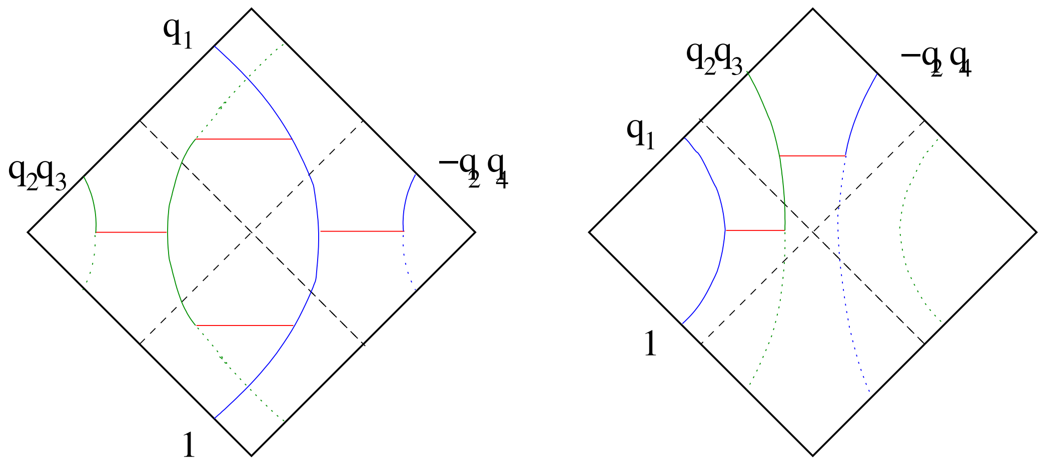
Unruh modes (charged particle)

- Global modes may be defined by patching together Rindler modes, ie by analytic continuation across the horizons. Unruh modes are those which are superposition of positive energy Minkowski modes,

$$\Omega^j_{in,+} = \mathcal{V}^j_{in,P} = (-i\nu X^+X^-)[X^+/X^-]^{-ij/2} W_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}(-i\nu X^+X^-)$$

$$\omega^j_{in,-} = \mathcal{U}^j_{in,P} = (i\nu X^+X^-)[X^+/X^-]^{-ij/2} M_{i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}(-i\nu X^+X^-)$$

- There are two types of Unruh modes, involving 2 or 4 tunneling events:



Unruh Modes (winding string)

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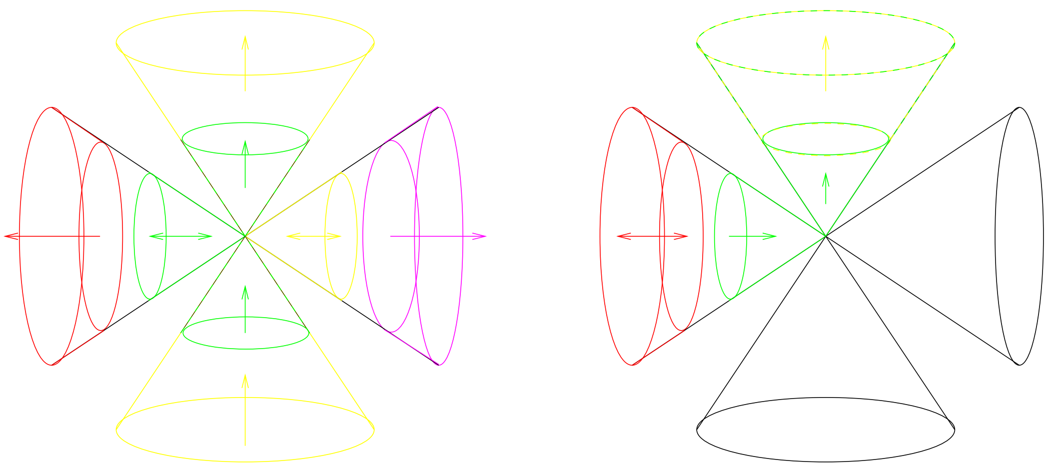
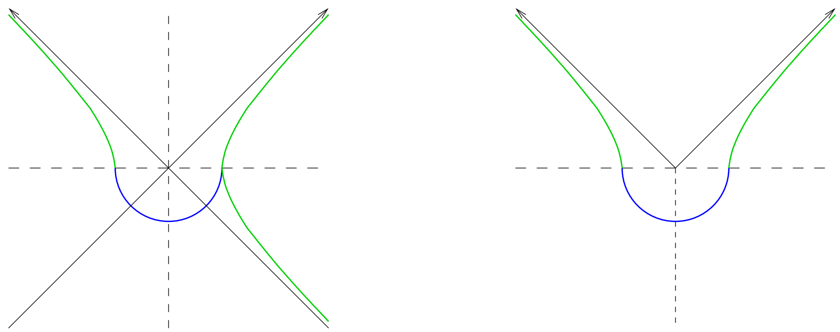
$$\omega^j_{in,-} = \mathcal{U}^j_{in,P} = (i\nu X^+X^-)[X^+/X^-]^{-ij/2} M_{i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}(-i\nu X^+X^-)$$

- The latter can be used as wave functions for winding strings:



Induced vs. spontaneous Schwinger pair production

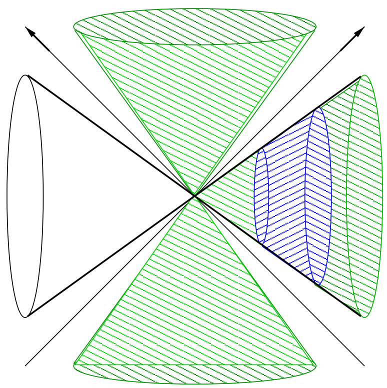
- In the charged particle problem, tunneling under the barrier corresponds to induced pair production. It can be described semi-classically by evolving in imaginary proper time, or equivalently in a magnetic field.

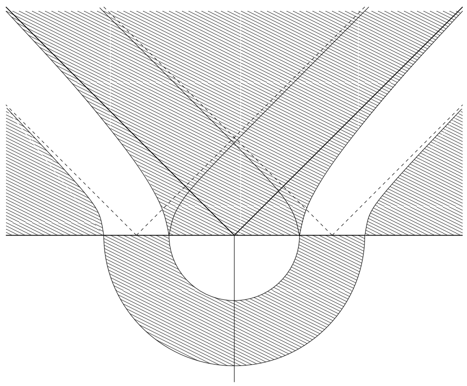


- Spontaneous pair production on the other hand involves no incoming state. It can be described semi-classically by cutting open an Euclidean periodic trajectory at the turning point, and evolving in real time henceforth.

Schwinger pair production of winding strings

- Similarly, in the closed string problem, tunneling under the barrier corresponds to induced pair production of winding strings.
- Spontaneous pair production of winding strings can be described by cutting open a periodic trajectory, either in imaginary proper time, or in the Euclidean rotation orbifold:





- In order to be more precise, one should specify a second quantized vacuum for winding strings. While obvious for the short strings, this is unclear for the long strings. Boundary conditions need to be imposed in the whiskers.
- Perhaps the best approach is to describe states in the cosmological regions as states in the tensor product of the left and right Rindler regions. Does that allow for an “holographic” description of a cosmology in terms of a static theory (with CTC) ?

Winding string production in more general geometries

- Using the covariant derivative representation for the zero-modes $\alpha_0^\pm$, $\tilde{\alpha}_0^\pm$, and going to adapted coordinates, we found the equation for the wave function $\psi(T)$ of a string with winding number w and momentum j in Milne space reads

$$-\frac{1}{T}\partial_T\, T\, \partial_T - \frac{j^2}{\beta^2 T^2} = m^2 + w^2\beta^2T^2$$

  where m^2 originates from excited modes and momentum in transverse directions.
- For $w = 0$, this is the ordinary Laplace equation for a particle of mass m in the Milne geometry $ds^2 = -dT^2 + \beta^2T^2d\theta^2$. The $\beta^2w^2T^2$ term is recognized as tensive energy.
- In more general geometries $ds^2 = -dT^2 + a^2(T)d\theta^2$, this generalizes straightforwardly to

$$-\frac{1}{a(T)}\,\partial_T\, a(T)\, \partial_T - \frac{j^2}{a(T)^2} = m^2 + w^2a^2(T)$$

  provided one can neglect the effect on internal oscillations. For a bouncing geometry, pair production corresponds to "back-scattering over a potential barrier".

5. Conformal perturbation theory

- While backreaction from quantum pair production cannot be treated in a first quantized formalism, one may consider deforming the orbifold CFT with a marginal twist field, i.e. adding a coherent superposition of winding strings:

$$S_\lambda = \int d^2\sigma \, \partial X^+ \bar{\partial} X^- + \lambda_{-w} V_{+w} + \lambda_{+w} V_{-w}$$

- While this deformation is marginal at leading order, it generates a one-point function for untwisted fields

$$\langle e^{ikX} \rangle_\lambda \sim \lambda_w \lambda_{-w} \, \langle w | e^{ikX} | -w \rangle \; ,$$

  which needs to be cancelled by deforming S at order λ^2 by an untwisted field: this is the untwisted field classically sourced by the winding string $V_{\pm w}$.
- In addition, the winding string sources twisted states whose winding number is a multiple of w:

$$\langle V_{-2w} \rangle_\lambda \sim \lambda_w \lambda_w \, \langle w | V_{-2w} | w \rangle \; ,$$

The size of twisted states

- The tree-level scattering of N untwisted states against one winding string can be computed by Hamiltonian quantization on the cylinder with twisted boundary conditions.
- As in flat space, the off-shell form factor is formally zero due to infinite zero-point fluctuations,

$$\langle -w|e^{ikX}(z,\bar{z})|w\rangle = \exp\left(-k^+k^-\Delta\right) \ , \quad \Delta \equiv \infty$$

- The characteristic size Δ may be made finite by a field redefinition of the untwisted vertex, e.g. normal ordering with respect to the untwisted vacuum:

$$\Delta(\nu) = \frac{1}{4}\sum_{n=1}^{\infty}\left(\frac{1}{n+i\nu}+\frac{1}{n+i\nu}-\frac{2}{n}\right)$$

This form factor is reminiscent of Moyal-type non-commutativity.

- At large ν, $\Delta(\nu) \sim 2\log\nu$, which indicates that the winding string grows to a size $\sqrt{\log w}$, T-dual to the Regge growth a high energy.

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Classical backreaction

- The untwisted fields sourced by a twisted state with wave function $f_(x^+, x^-)$ are then given by the zero-mode overlap

$$\langle -w| : e^{ikX}(z,\bar{z}) : |w\rangle = \exp\left(-k^+k^-\Delta\right) \int dx^+ dx^- e^{ikX} |f(x^+, x^-)|^2$$

- Three-point functions of twisted fields cannot be computed in Hamiltonian formalism, but can be obtained by analytic continuation of the amplitude for $p^+ \neq 0$ strings in the Nappi-Witten pp-wave:

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$$\int dx_1^\pm dx_2^\pm \, [f_1(x_1^\pm) f_2(x_2^\pm)]^* \, \exp\left[(x_1^+ - x_2^+)(x_1^- - x_2^-)\Xi(\nu_1,\nu_2)\right] \, f_3\left(\frac{\nu_1 x_1^\pm + \nu_2 x_2^\pm}{\nu_1+\nu_2}\right)$$

where the size of the non-locality is given by the (real) ratio

$$\Xi(\nu_1,\nu_2) = -i \frac{1 - \frac{i\nu_3}{\nu_1\nu_2}\frac{\gamma(i\nu_3)}{\gamma(i\nu_1)\gamma(i\nu_2)}}{1 + \frac{i\nu_3}{\nu_1\nu_2}\frac{\gamma(i\nu_3)}{\gamma(i\nu_1)\gamma(i\nu_2)}} \,, \quad \gamma(x) = \frac{\Gamma(x)}{\Gamma(1-x)}$$

Note that the non-locality scale $1/\sqrt{\Xi}$ diverges when $\nu_1\nu_2\gamma(i\nu_1)\gamma(i\nu_2) = i\nu_3\gamma(i\nu_3)$.

6. Discussion

- Winding string production can be understood semi-classically as scattering over the barrier in the (Milne) cosmology, or tunneling under the barrier in the doubly Wick-rotated (Rindler) region.
- Just as in the Schwinger problem, the pair production rate may be extracted from the tree-level two-point function in an appropriate basis of in/out states depending on the choice of vacuum. A great simplification of Misner space is that "oscillators go along for the ride".
- In particular, winding strings are copiously produced at the singularity (the production rate is infinite as $j = 0$). This suggests that the tree-level divergences found in the "bare" geometry may be resolved by including the back-reaction from twisted states.
- A proper handling of this issue may require "non-local string theories", tailored to described the production of correlated pairs, i.e. of a "squeezed state". Unfortunately this is little operative right now.

Discussion (cont)

- Using conformal perturbation theory, one may show that a coherent condensate of winding states does lead to Moyal-type non-locality. However, there are higher order corrections which need to be resummed. Can one solve the deformed theory exactly ?
- While particles can be quantized unambiguously in Misner space, using the “covering space vacuum”, boundary conditions in the whiskers are required for long strings. This hints at a “holographic” description of the cosmology in terms of a static theory, albeit with compact time.
- Finally, Misner space is very finely tuned wrt to initial conditions. Do whiskers appear at general Kasner singularities ? How about string theory on the (BKL) MIXMASTER Misner Universe ?