

Open strings in electric fields and the Milne universe

Boris Pioline

LPTHE, Paris

Strings 2003, Kyoto

w/ Micha Berkooz, to appear

Introduction

Despite obvious physical relevance, **time dependent backgrounds** in string theory have been hard to come by:

- String perturbation technology is well suited for **S-matrix** computations around a **fixed coherent** vacuum state
- Time dependence implies **production of squeezed states**, and **ambiguous choice of vacuum**: is string field theory mandatory ?
- First quantized string theory requires **analytic continuation** of target space and world sheet, often problematic
- String theory is not content on a finite time interval, but often forces us into **Big Bang / Big Crunch singularities, CTC, ...**

Milne Universe, the bright side

These issues may be easier to address in solvable CFT such as orbifolds of flat space, WZW, ...:

- The **Lorentzian orbifold**: $R^{1,1}/Z$, $X^\pm \equiv X^\pm e^{\pm\beta}$ describes the **Milne universe**, ie a circle linearly contracting and reexpanding in time:

$$ds^2 = -dT^2 + T^2 dy^2, \quad y \equiv y + \beta$$

together with Rindler whiskers with CTC:

$$ds^2 = -r^2 d\eta^2 + dr^2, \quad \eta \equiv \eta + \beta$$

and non-Hausdorff light cone, all attached at a **cosmological singularity**

Horowitz Steif; Seiberg; Nekrasov

- This same singularity arises **locally** in many other examples: $SU(2) \times Sl(2)/U(1) \times U(1)$, $Sl(2)/U(1)$, AdS/Γ , ... yet it is far from generic: Kasner singularity, BKL oscillatory behavior

*Nappi Witten, Elitzur Giveon Kutasov Rabinovici
Craps Kutasov Rajesh*

- Variants have been proposed which ought to simplify things even further: electric Melvin universe, null boost, ...

Costa Cornalba Kounnas; Liu Moore Seiberg; ...

Milne Universe, the dark side

Unfortunately, many difficulties lie in the shade:

- Tree level scattering amplitudes of **untwisted states** are badly divergent, due to large graviton exchange near the singularity, signaling **large backreaction**

Berkooz Craps Kutasov Rajesh, LMS

- **Physical states** do not seem to exist in **twisted sectors**, hence may not condense and resolve the singularity

Nekrasov

- Non perturbatively, interactions of an incoming particle with its orbifold images will lead to **black hole formation** and gravitational collapse

Horowitz, Polchinski

Moving D-branes, electric fields

Open strings offer a simpler arena to investigate time-dependence without dealing with gravitational instabilities:

- The closest open string analogue of the Milne Universe is a **head-on D-brane collision**. Analogues of twisted closed strings are open strings stretched between D-branes. *The cosmological singularity issue is replaced by bound state formation...*

Bachas; Douglas Kabat Pouliot Shenker

- Even simpler, the T-dual situation is open strings in a **constant electric field**:

$$F_{+-} = \begin{pmatrix} E & \\ & -E \end{pmatrix}$$

11 and 22 strings are like untwisted closed strings, 12 and 21 are like winding strings. *Space-time non-commutativity may cause us some trouble...*

Bachas Porrati; Bachas Hull

Particle production and backreaction

Physically, the fate of these backgrounds is rather clear:

- For open strings in a constant electric field, **Schwinger pair production** of **charged open strings** will screen the electric field, or **discharge** the condensator at infinity.
- For moving D-branes, **stretched strings** will be pair produced, and will accelerate/slow down the collision. The final outcome depends on the **production/recombination** rates.
- Q1: *Can backreaction be consistently incorporated in open string theory ?*
- Q2: *Can pair production resolve the Milne singularity ?*

Q1 and Q2 are still too hard, but we'll answer

- Q0: *Does there exist physical states in charged or twisted sectors which are liable to condense in pairs ? What are the rules to compute their scattering amplitudes ?*

Electric field vs Milne Universe

- Eigenmodes of closed strings **twisted sector** of order w are free fields satisfying

$$X^\pm(\sigma + 2\pi, \tau) = e^{\pm\nu} X^\pm(\sigma, \tau) , \quad \nu = w\beta$$

hence the normal mode expansion:

$$X_R^\pm(\tau - \sigma) = \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1/2} \alpha_n^\pm e^{-i(n \pm i\nu)(\tau - \sigma)}$$

$$X_L^\pm(\tau + \sigma) = -\frac{i}{2} \sum_{n=-\infty}^{\infty} (-n \mp i\nu)^{-1/2} \tilde{\alpha}_n^\pm e^{-i(-n \mp i\nu)(\tau + \sigma)}$$

with canonical commutation relations

$$\begin{aligned} [\alpha_m^+, \alpha_n^-] &= -(m + i\nu) \delta_{m+n} , & [\tilde{\alpha}_m^+, \tilde{\alpha}_n^-] &= (m + i\nu) \delta_{m+n} \\ (\alpha_m^\pm)^* &= \alpha_{-m}^\pm , & (\tilde{\alpha}_m^\pm)^* &= \tilde{\alpha}_{-m}^\pm \end{aligned}$$

- **Charged open strings** are obtained by identifying

$$\alpha_n^\pm = \tilde{\alpha}_{-n}^\pm , \quad \nu = \frac{2}{\pi} \operatorname{arctanh}(E/\pi) ,$$

and adding a canonical pair of constant zero modes x_0^\pm with $[x_0^+, x_0^-] = i\pi/E$

- In particular, zero-modes are **isomorphic** (after rescaling), and involve two commuting pairs of **hermitian conjugate variables**,

$$[\alpha_0^+, \alpha_0^-] = -i\nu , \quad [\tilde{\alpha}_0^+, \tilde{\alpha}_0^-] = i\nu ,$$

Are there twisted physical states ?

- Representing these oscillators on a Fock space with vacuum $|0\rangle$ annihilated by all $\alpha_{n>0}^\pm$ and by α_0^- , the normal ordered worldsheet Hamiltonian reads

$$L_0 = -\sum_{n=1}^{\infty} (\alpha_n^+)^* \alpha_n^- - \sum_{n=1}^{\infty} (\alpha_n^-)^* \alpha_n^+ \\ -\alpha_0^+ \alpha_0^- + \frac{1}{2} i\nu(1 - i\nu) - 1 + L_{int}$$

with a similar answer for \tilde{L}_0 .

- Due to the $i\nu/2$ term in the ground state energy, all states obtained by acting on $|0\rangle$ by creation operators $\alpha_{n<0}^\pm$ and by α_0^+ will have imaginary energy, hence the physical state condition $L_0 = 0$ has no solutions.

Nekrasov

- Rk: this does not contradict L_0 being hermitian, since these states also have zero norm !

One-loop amplitude

- Irrespective of this, the **one-loop vacuum energy** for open strings in electric fields can be computed,

$$A_{open} = i\pi V_{26} E \int_0^\infty \frac{dt}{(4\pi^2 t)^{13}} \frac{e^{-\pi\nu^2 t/2}}{\eta^{21}(it/2) \theta_1(t\nu/2; it/2)}$$

$$\theta_1(\nu, \tau) = 2q^{1/8} \sin \pi\nu \prod_{n=1}^{\infty} (1 - e^{2\pi i\nu} q^n)(1 - q^n)(1 - e^{-2\pi i\nu} q^n)$$

Poles at $t = 2k/\nu, k \in \mathbb{Z}$ contribute to the **imaginary** part, which agrees with the sum of Schwinger production rates for each particle in the spectrum:

$$w = \frac{1}{2(2\pi)^{25}} \frac{2E}{\nu} \sum_{k=1}^{\infty} (-)^{k+1} \left(\frac{|\nu|}{k}\right)^{13} \sum_{N=-1}^{\infty} c_b(N) \exp\left(-2\pi k \frac{N}{|\nu|} - \pi k |\nu|\right)$$

where $\eta^{-24}(q) = \sum_{N=-1}^{\infty} c_b(N) q^N$.

Bachas Porrati

- Similarly, for closed strings on Milne space one obtains a **modular invariant** integral,

$$A_{closed} = \int_{\mathcal{F}} \sum_{l,w=0}^{\infty} \frac{d\tau d\bar{\tau}}{(2\pi^2 \tau_2)^{13}} \left| \frac{e^{-2\pi\lambda^2 w^2 \tau_2}}{(2\pi)^3 \eta^{21}(\tau) \theta_1(i\lambda(l + w\tau); \tau)} \right|^2$$

in agreement with the proposed treatment of zero modes.

Nekrasov, Cornalba Costa

Space-time representation of zero-modes

- Zero-modes can be **unitarily** represented as covariant derivatives acting on wave functions $f(x^+, x^-)$ of the **center of motion** of the charged string,

$$\alpha_0^\pm = i\partial_\mp \mp \frac{\nu}{2}x^\pm, \quad \tilde{\alpha}_0^\pm = i\partial_\mp \pm \frac{\nu}{2}x^\pm$$

- The zero-mode piece of L_0 , including the evil $\frac{i\nu}{2}$,

$$L_0^{(0)} = -\alpha_0^+ \alpha_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\alpha_0^+ \alpha_0^- + \alpha_0^- \alpha_0^+)$$

is just the **Klein-Gordon operator** of a particle of 2D mass $M^2 = -2L_0^{(0)}$ and charge ν .

- For open strings, only one pair (α_0^+, α_0^-) appears in the worldsheet Hamiltonian L_0 . The other one $(\tilde{\alpha}_0^+, \tilde{\alpha}_0^-)$ describes the position of **center of the hyperbolic trajectory**, subject to the uncertainty principle.
- For closed strings, the difference $L_0^{(0)} - \tilde{L}_0^{(0)}$ is the zero-mode **boost momentum**,

$$\mathcal{M}^2 - \tilde{\mathcal{M}}^2 = -i\nu (x^+ \partial_+ - x^- \partial_-) := J^{(0)}$$

The matching condition equates

$$\beta w J = N_L - N_R$$

In addition, the **orbifold** projection requires the total boost momentum J to be **integer**.

KG and the inverted harmonic oscillator

- Defining $\alpha_0^\pm = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator just becomes an **inverted harmonic oscillator**:

$$M^2 = \alpha_0^+ \alpha_0^- + \alpha_0^- \alpha_0^+ = \frac{1}{2}(P^2 - Q^2)$$

- Diagonalizing (\tilde{P}, M^2) in P representation:

$$f(x^+, x^-) = \int d\tilde{p} \psi_{\tilde{p}}(u) e^{-i(\tilde{p} + \frac{1}{2}\nu x)t}$$

where $u = (\tilde{p} + \nu x)\sqrt{2/\nu} \propto P$. Then $\psi_{\tilde{p}}(u)$ is a eigenmode of the **inverted harmonic oscillator**,

$$\left(-\partial_u^2 - \frac{1}{4}u^2 + \frac{M^2}{2\nu}\right) \psi_{\tilde{p}}(u) = 0$$

- The latter admits a respectable delta-normalizable spectrum of scattering states, in terms of **parabolic cylinder functions**.

Moore

- These correspond to **non-compact** trajectories of charged particles in the electric field. Waves on the right side of the potential describe electrons, waves on the left side describe positrons. **Tunnelling** is just (stimulated) **Schwinger pair creation**,

$$e^- \rightarrow (1 + \eta) e^- + \eta e^+$$

In/out vacua and particle production

- Combinations which create an **incoming** electron or positron are

$$\phi_{in}^+ = D_{-\frac{1}{2} + i\frac{M^2}{2\nu}}(e^{-\frac{3i\pi}{4}}u)e^{-i\tilde{p}t}e^{i\nu xt/2},$$

$$\phi_{in}^- = e^{-\frac{3i\pi}{4}}D_{-\frac{1}{2} + i\frac{M^2}{2\nu}}(e^{-i\pi/4}u)e^{i\tilde{p}t}e^{-i\nu xt/2},$$

while **outgoing** electron or positrons are created by

$$\phi_{out}^\pm(t, x) = [\phi_{in}^\pm(-t, x)]^*$$

- The in and out vacua are related by a non-trivial **Bogolioubov transformation**

$$|0, in\rangle = \mathcal{N} \exp\left[-\frac{1}{\sqrt{2\pi}}\Gamma\left(\frac{1}{2} + i\frac{M^2}{2\nu}\right)e^{-\frac{\pi M^2}{2\nu}}\right] a_{\tilde{p}}^* b_{\tilde{p}}^* |0, out\rangle$$

with overlap

$$|\langle 0, out | 0, in \rangle|^2 = \exp\left[-\int d\tilde{p} \ln(1 + e^{-\pi M^2/\nu})\right]$$

which reproduces the Schwinger pair creation rate, after properly interpreting $\int d\tilde{p}$.

Brout Massar Parentani Spindel

Lorentzian vs Euclidean states

- Analytic continuation $X^0 \rightarrow e^{-i\pi/2}X^0$, $\nu \rightarrow e^{i\pi/2}\nu$ takes us from an electric field in $R^{1,1}$ to a **magnetic field in R^2** . At the same time, one should Wick rotate the worldsheet time.
- Under Wick rotation, **discrete states** with imaginary energy $i\nu(n + 1/2)$ become **normalizable** states of the \cup harmonic oscillator, ie **Landau states** in a magnetic field.
- Instead, the **continuous** spectrum of the \cap harmonic oscillator turns into **non-normalizable** states of the \cup harmonic oscillator.
- The heat kernel of \cup can be expressed as a sum over discrete eigenmodes

$$\sum_{n=0}^{\infty} e^{i(n+1/2)s} \psi_n^*(u_1) \psi_n(u_2) = \frac{1}{\sqrt{4\pi \sin s}} \exp \left[-\frac{1}{4} \left(\frac{u_1^2 + u_2^2}{\tan s} - \frac{2u_1 u_2}{\sin s} \right) \right]$$

but as the Schwinger parameter gets continued, becomes an integral over the continuous spectrum:

$$\begin{aligned} & \sum_{\epsilon=\pm} \int_{-\infty}^{\infty} dM^2 e^{iM^2 s} \psi^\epsilon(M^2, u_1) \psi^\epsilon(M^2, u_2) \\ &= \frac{1}{\sqrt{4\pi i \sinh s}} \exp \left[\frac{i}{4} \left(\frac{u_1^2 + u_2^2}{\tanh s} - \frac{2u_1 u_2}{\sinh s} \right) \right] \end{aligned}$$

Moore

Physical spectrum at low level

- The ground state **tachyon**

$$|T\rangle = \phi(x^+, x^-) |0_{ex}, k\rangle$$

should satisfy the Virasoro constraint

$$L_0 |T\rangle = \left[-\frac{1}{2} (\alpha_0^+ \alpha_0^- + \alpha_0^- \alpha_0^+) + \frac{1}{2} \nu^2 - 1 + \frac{1}{2} k_i^2 \right] |T\rangle$$

which is the two-dimensional KG equation, for a particle with mass $k_i^2 + \nu^2 - 2$ and charge ν .

- Level 1 states consist of

$$|A\rangle = \left(-f^+ \alpha_{-1}^- - f^- \alpha_{-1}^+ + f^i \alpha_{-1}^i \right) |0_{ex}, k\rangle$$

with the mass shell conditions

$$[M^2 - k_i^2 - \nu^2] f^i = 0, \quad [M^2 - k_i^2 - \nu^2 \mp 2i\nu] f^\pm = 0$$

The L_1 **Virasoro constraint** eliminates one polarization

- Despite the non-vanishing two-dimensional mass $k_i^2 - \nu^2$, the **spurious state** $L_{-1}\phi|0\rangle$ is still physical, eliminating an extra polarization. One thus has **$D - 2$ transverse** degrees of freedom, ie a **massless gauge boson** in D dimensions.
- For closed strings, the same goes through, with a further projection to $J = 0$: there is a **zero-momentum transverse graviton** in each twisted sector.

Quantizing charged KG - 2) light-cone

- In α^- representation, then $\alpha^+ = i\nu\partial/\partial\alpha^-$, hence $M^2 = i\nu\alpha^-\partial/\partial\alpha^-$ is just a **rescaling** operator. Eigenmodes are power law

$$f_{k_+}(x^+, x^-) = (2\nu)^{-\frac{1}{4} + \frac{iM^2}{4\nu}} \Gamma\left(-\frac{1}{4} - \frac{iM^2}{4\nu}\right) (k_+ + \nu x^-)^{-\frac{1}{2} + \frac{iM^2}{2\nu}} e^{ix^+(k_+ + \frac{1}{2}\nu x^-)}$$

This basis of functions is most appropriate to expand the modes at a fixed (early) x^- time, i.e. for **incoming electrons**.

- Alternatively, one may diagonalize $\tilde{\alpha}^+$: this is appropriate to relate **outgoing electrons**. The S-matrix relating the two basis is just **Fourier transform**.
- Canonical commutation relations read

$$\begin{aligned} [a_{k_+}, a_{k'_+}^*] &= \text{sgn}(k_+ + \nu x^-) (2\nu)^{-1/2} \left| \Gamma\left(-\frac{1}{4} - \frac{iM^2}{4\nu}\right) \right|^2 \delta(k_+ - k'_+) \\ [a_{k_-}, a_{k'_-}^*] &= \text{sgn}(k_- - \nu x^+) (2\nu)^{-1/2} \left| \Gamma\left(-\frac{1}{4} + \frac{iM^2}{4\nu}\right) \right|^2 \delta(k_- - k'_-) \end{aligned}$$

hence at time $k_+ + \nu x^- = 0$, **creation operator becomes annihilation**: this is pair creation.

Tomaras Tsamis Woodard

- In order to maintain charge conservation, one should also include the **incoming and out-going positron modes** on the other side of the wedge.

Charged KG in Rindler space

- For applications to the Milne universe, one should rather diagonalize the **boost momentum** J , ie consider an **accelerated observer**.

Gabriel Spindel; Mottola Cooper

- In the **Rindler** patch R , letting $f(r, \eta) = e^{-iJ\eta} f_J(r)$ and $r = e^y$, one gets a Schrodinger equation for a particle in a potential

$$\left[-r\partial_r r\partial_r + M^2 r^2 - \left(J + \frac{1}{2}\nu r^2 \right)^2 \right] f_J(r) = 0$$

- For $\nu = 0$, this reduces to a **Liouville wall**. For $\nu \neq 0$ the wall has finite height, and the potential is unbounded from below at large r .
- For $\nu J/M^2 > 1/2$, the energy is bigger than the barrier, the electron comes from infinity I_R^- into the horizon H_R^+ . For $\nu J/M^2 < 1/2$, the electron bounces off the barrier back to I_R^+ . Again, **tunnelling** corresponds to **stimulated pair emission from or into the horizon**.
- Incoming and outgoing Rindler/Unruh modes can be defined as usual. The Bogolioubov transformation shows **Schwinger pair creation in the bulk**, as well as **thermal particle production from the horizon** at $T_R = a/(2\pi) = \nu/(2\pi M)$.

Vertex operators and correlation functions

- Tree level correlators with at most 2 twisted sector states can be obtained on the Minkovskian strip, otherwise analytic continuation is mandatory.
- Upon Wick rotation to the Euclidean magnetic problem, physical states in the charged/twisted sectors become **non-normalizable modes** of the harmonic oscillator. **Don't panic**, this eliminates the **localized tachyon** !
- Vertex operators for the usual magnetic problem are constructed out of (boundary) **twist fields**, which generate cuts in (Z, \bar{Z}) ,

$$\partial Z(w)\sigma_\theta(0) = w^\theta \sigma_\theta^{(1)} + \dots, \quad \Delta_{\sigma_\theta} = \theta(1 - \theta)/2$$

Dixon Friedan Martinec Shenker; Hamidi Vafa

- These standard twist fields correspond to the harmonic oscillator ground state and its excitations. Instead, we need a new **continuous family** of twist fields with **irrational angle**.
- For $\beta \in \mathbb{Q}$, there should still be an infinite number of twisted sectors: **spectral flow / long strings**.

Twisted state production in Milne

- By analogy with electric field, one expects Schwinger creation of **correlated pairs** of twisted states on the Milne orbifold. Due to orbifold projection, lightest states will be produced in $J = 0$ states, hence **breaking spatial homogeneity**.
- For a given state of mass M , the density of pair creation can be evaluated semi-classically by computing the Jacobian

$$w(X^+, X^-) = \int dx_0^+ dx_0^- \delta[(X^+ - x_0^+)(X^- - x_0^-) + \frac{M^2}{2\epsilon^2}]$$
$$\delta[\epsilon x_0^+ x_0^- + \frac{M^2}{2\epsilon^2} - J] = \frac{1}{\sqrt{X^+ X^- (\epsilon^2 + 2M^2 X^+ X^-)}}$$

hence pair production is **peaked on the light-cone**.

- Unfortunately, the total pair creation number **diverges**, due to summing over winding sectors. Electric Melvin universe seems better off. . .
- A more tractable situation perhaps is to send in **squeezed** twisted states from $-\infty$, along with states one is interested to scatter. Can one use non-local worldsheet deformation ?

Aharony Berkooz Silverstein

Conclusions - speculations

- Open strings in an electric field are a simple arena to address conceptual issues of **Time and string theory**.
- We have laid some of the foundations to study this, but the hard work is ahead of us: S-matrix elements, recombination rate, backreaction...
- Despite strong formal analogies, there is **no duality** between open strings in electric fields and the Milne universe.
- Time-space non-commutativity however may still be important on black hole horizons, as 't Hooft taught us.
- Analogies with **$c = 1$ strings** are tantalizing, although open strings do not obey Fermi statistics. *Can one define some double scaling limit towards the top of the potential ? can squeezed states be dealt with by some bosonization technique ?*

Gose cho domo arigato gozaimashita
(Thank you very much for your attention)

Pari ni mata rainen
(See you in Paris next year)