

Holography in homogeneous wave backgrounds

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Intro 1: a new stringy regime for SYM

String theory on pp-wave backgrounds and Penrose limits have recently been the focus of much interest:

*Blau, Figueroa O'Farrill, Hull, Papadopoulos
Metsaev, Tseytlin
Berenstein Maldacena Nastase*

One motivation is to deeper understand AdS/CFT:

- Despite RR fluxes, string theory on the light-cone is (in principle) **exactly solvable** in this regime.
- **Bulk string modes** can be identified with a class of nearly BPS **gauge invariant observables**:

$$a_n^{j\dagger} a_{j,-n}^\dagger |0\rangle = \sum_{l=1}^J \text{Tr} [Z^l \cdot \phi^i \cdot Z^{J-l} \partial_j Z] e^{2\pi i n l / J}$$

- It corresponds to a new **double scaling** limit of the gauge theory, different from the 't Hooft limit,

$$\mu\alpha' p_+ = J / \sqrt{g_{YM}^2 N} = \text{fixed} , \quad g_{\text{eff}} = J^2 / N = \text{fixed}$$

- The 't Hooft coupling $g_{YM}^2 N$ goes to ∞ but interactions are further suppressed by $1/J$ for **near-BPS** states. The genus counting parameter N^{2-2h} goes to zero but **combinatorics** provide an extra J^{4h-4} which keeps interaction finite.

*Kristjansen Plefka Semenoff Staudacher
Gross Mikhailov Roiban
Constable Freedman Headrick Minwalla Motl Postnikov Skiba*

This allows for detailed checks of the AdS/CFT correspondence that have so far been impractical in the $AdS_5 \times S^5$ case.

Intro 2: holography in flat space ?

Another reason for interest in this set-up is that it provides a **new class** of geometries with in principle a well-defined holographic dual description.

- One may hope to learn more about background independent features of holography, and possibly about **holography for flat space**.
- Even though curvature invariants are all zero, the **effective curvature** $\rho = \mu\alpha'p_+$ is **momentum dependent**. Flat space is recovered as $p_+ \rightarrow 0$.
- The spectrum of the light-cone Hamiltonian matches on smoothly to that of flat Minkowski space:

$$p_- = \sum_{n=-\infty}^{+\infty} \sqrt{\mu^2 + \frac{n^2}{\alpha'p_+}} = \mu \sum_{n=-\infty}^{+\infty} \sqrt{1 + \frac{g_{YM}N}{J^2} n^2}$$

- The structure of **conformal infinity** shares similarities with Minkowski, in particular the induced metric on the boundary is **degenerate**.

PP-waves may be a more convenient starting point for getting at holography in flat space than $AdS \times S$.

Polchinski; Balasubramanian Giddings

Intro 3: Shortcomings of large J SYM

Despite these attractive features, this new ppwave/CFT holographic duality in its present stage has several shortcomings:

- At finite J , there are many more gauge-invariant operators that remember that the theory lived on $AdS \times S$: could we keep only the dof relevant for the pp-wave geometry ?
- Correlators in SYM do not have a well-defined limit as $J \rightarrow \infty$. States are really **gauge invariant words** rather than local operators. Only **s-wave** modes on $R_t \times S^3$ should be kept. Could the gauge theory reduce to a large- N **M(atrix) model** for ϕ^I and $\psi_i \sim \partial_i Z$?
- **Symmetries** of the bulk are **not manifest** on the dual side, e.g. the discrete symmetry exchanging AdS and S . The realization of the bulk isometries, namely the **Wigner contraction** of both factors in $SO(2,4) \times SO(6)$, is obscure, except for the trivial $SO(4) \times SO(4)$.
- It does not come yet with a prescription for computing **boundary correlators** from bulk S-matrix elements. Rather, one should compute correlators in usual AdS and then take the limit.

Summary of results

Here, we attempt to construct an holographic correspondence for pp-wave backgrounds, without prejudices about the possible AdS origin of that background (yet taking inspiration from AdS experience).

- We identify the **Poincaré patch** of the pp-wave, i.e. the patch conformal to flat Minkowski. The analogue of the radial direction is found to be the null direction x^+ , i.e. the light cone time. The holographic dual lives on a **null** slice at fixed x^+ .
- We find a realization of the bulk isometries as symmetry acting on the boundary fields: the conformal group is replaced by an **extended Heisenberg algebra**. Ward identities restrict the form of 2 and 3-point functions.
- We evaluate the 2 and 3 point correlation functions for a minimally coupled massive scalar field using a boundary to bulk correspondence.
- Boundary data at fixed $x^+ \rightarrow -\infty$ are insufficient to determine the full evolution: one also needs a **complementary screen** at $x^- = -\infty$. This provides the missing “normalizable” modes controlling the vacuum.

The nature of the **HFT (Heisenberg Field Theory)** living on the boundary will be left unanswered.

Outline

1. Geometry of homogeneous pp-waves
2. The “Poincaré” patch of pp-wave backgrounds
3. Holography at work
4. Summary / outlook

1. Geometry of pp-waves

The Nappi-Witten background

Nappi Witten

The simplest example of pp-wave background can be obtained as the **group manifold** (WZW model) of the solvable (non-semi-simple) group $E_2^c := H_4$:

$$[P_1, P_2] = K, \quad [J, P_i] = \epsilon_{ij} P_j, \quad [K, \cdot] = 0$$

Any group element can be parameterized as

$$g = \exp(uJ + vK) \cdot \exp(a_1 P_1 + a_2 P_2)$$

- H_4 possesses an invariant non-degenerate bilinear form, giving rise to a metric invariant under $H_{4L} \times H_{4R}$:

$$\begin{aligned} ds^2 &= 2dudv + da_1^2 + da_2^2 + (a_2 da_1 - a_1 da_2) du \\ H &= du \wedge da_{12} \end{aligned}$$

- If u was a spacelike compact direction, that would be an (exact) **constant magnetic background** for the KK gauge field $g_{\mu u}$. The electric charge is the momentum p_+ along v . Classical trajectories are Larmor orbits in the transverse plane (a_1, a_2) . In this frame the ground state has a **large degeneracy**.
- Going to the **rotating frame** $a_1 + ia_2 = e^{iu/2}(x_1 + ix_2)$ yields the more familiar metric

$$\begin{aligned} ds^2 &= 2dudv + dx_1^2 + dx_2^2 - \frac{1}{4}\mu^2(x_1^2 + x_2^2) du^2 \\ H &= du \wedge dx_{12} \end{aligned}$$

- This is an **exact** conformal background with $c = 4$ and 16 SUSY. It can be completed by flat \mathbb{R}^6 to $c = 10$.

Nappi-Witten as the Penrose limit of NS5

Let us consider the Penrose limit of the near horizon geometry of the NS5-brane, for a null geodesic **spinning along the sphere** S^3 :

$$\begin{aligned} ds^2 &= -dt^2 + dr^2 + N(d\psi^2 \cos^2 \theta + d\theta^2 + \sin^2 \theta d\phi^2) + (dx^i)^2 \\ g_s^2 &= e^{-2r/(l_s \sqrt{N})} \end{aligned}$$

Define

$$\begin{aligned} t + \psi\sqrt{N} &= u\sqrt{2}, \\ t - \psi\sqrt{N} &= v\sqrt{2}/\alpha^2, \quad \theta = \alpha\rho, \quad x^i = \alpha y_i, \quad r = \alpha y_6 \end{aligned}$$

In the limit $\alpha \rightarrow \infty$:

$$\begin{aligned} ds^2 &= -dt^2 + N d\psi^2 (1 - \frac{1}{6} \alpha^2 \rho^2) + \alpha^2 (dr^2 + r^2 d\phi^2) + \alpha^2 (dy^i)^2 \\ &= \alpha^2 \left[2dudv - \frac{1}{12} r^2 du^2 + N(dr^2 + r^2 d\phi^2) + (dy^i)^2 \right] \end{aligned}$$

The dilaton gradient disappears in this limit. This is just the Nappi-Witten geometry ! Only states with $E \sim \sqrt{N}$, $J \sim N$ are kept in this limit. This probes the **high energy regime** of little string theory.

also Gomis Ooguri

[Had we considered a geodesic along the radial direction, we would have found a flat geometry with a **null dilaton profile**, $\phi = \phi(u)$. This is the simplest wave background in string theory !]

Other avatars of Nappi-Witten

The WZW construction of NW can be generalized to any even dimension by considering the Heisenberg algebra $[P_i, Q^j] = \delta_j^i K$ extended by a generator J rotating positions and momenta :

$$\begin{aligned} ds^2 &= 2dudv + \sum dx_i^2 - \frac{1}{4}\mu^2 x_i^2 du^2 \\ H &= du \wedge (dx_{12} + dx_{34} + \dots) \end{aligned}$$

Kehagias Maessen

This gives a CFT realization of all Lorentzian maximally symmetric **Cahen-Wallach spaces**, supported by NS flux.

Blau Figueroa-O'Farrill Papadopoulos

- For $N = 6$, we get the Penrose limit of the **NS5-F1** near horizon geometry. The D1-D5 system is obtained by trading the H_{NS} flux for an H_{RR} .
- For $N = 10$, we get the same geometry as that of the Penrose limit of $AdS_5 \times S_5$, but with H_{NS} flux instead of RR 5-form $F = du(dx_{1234} + dx_{5678})$. This has 8 SUSY instead of 32.

While all these backgrounds are in principle solvable, those with NS flux allow for a **covariant** quantization of the NSR string, whereas those with RR flux can only be considered in the Green-Schwarz **light-cone** string. General features of holography should be essentially the same.

Geometry of pp-wave backgrounds

- Like any group manifold, pp-wave spaces are invariant under the **left and right actions** of the group H . This is most easily computed using the (non-unitary) matrix representation

$$g = \begin{pmatrix} 1 & a_1 \sin u - a_2 \cos u & a_1 \cos u + a_2 \sin u & 2v \\ 0 & \cos u & -\sin u & a_1 \\ 0 & \sin u & \cos u & a_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Killing vectors are best expressed in a complex basis, $z = x_1 + ix_2$, $P^\pm = P_1 \pm iP_2$:

$$\begin{aligned} K_L &= K_R = \partial_v, \\ J_L &= \partial_u + (x_1 \partial_2 - x_2 \partial_1), \quad J_R = \partial_u + (x_1 \partial_2 - x_2 \partial_1), \\ P_L^+ &= e^{iu/2} (4\partial_{\bar{z}} - iz\partial_v), \quad P_L^- = e^{-iu/2} (4\partial_z + i\bar{z}\partial_v) \\ P_R^+ &= e^{-iu/2} (4\partial_{\bar{z}} + iz\partial_v), \quad P_R^- = e^{iu/2} (4\partial_z - i\bar{z}\partial_v) \\ J_L \text{ and } J_R &\text{ are null, but } J_L + J_R = 2\partial_u \text{ is timelike.} \end{aligned}$$

- For pp-waves supported by RR higher-form flux there may be more **accidental isometries**, such as $SO(4) \times SO(4)$ in the Penrose limit of $AdS_5 \times S_5$.

Geodesics in pp-wave backgrounds

- Geodesics are generated by **one-parameter subgroups** of H , $g(\tau) = U \exp(\tau h) V$. For those passing through the origin at $\tau = 0$ ($UV = 1$):

$$\begin{aligned} u(\tau) &= 2p_+\tau \\ v(\tau) &= \left(2p_- + \frac{p_1^2 + p_2^2}{p_+}\right)\tau - \frac{\sin 2p_+\tau}{2p_+^2}(p_1^2 + p_2^2) \\ z(\tau) &= 2\sin(p_+\tau)(p_1 + ip_2)/p_+ \end{aligned}$$

All geodesics **come back** to (minus) their original position in transverse space after $u \rightarrow u + 2\pi$. The net motion along v depends on the sign of $p^2 = 2p_+p_- + p_i^2$. At shorter time scales it exhibits **Zitterbewegung**. Lightlike geodesics are trapped along v .

- The geodesic (Lorentzian) distance between 2 points is

$$\begin{aligned} r^2 &= 2(u - u')(v - v') + \\ &+ \frac{u - u'}{2\sin\left(\frac{u-u'}{2}\right)} \left[(x_i^2 + x_i'^2) \cos\left(\frac{u - u'}{2}\right) - 2x_i x_i' \right] \end{aligned}$$

Points with $u' = u + 2n\pi$ are at **infinite geodesic distance** away unless $x_i = \pm x_i'$.

Classical dispersion relation

- Free scalar fields propagating on pp-wave backgrounds satisfy $(\Delta - m^2)\phi = 0$ where

$$\Delta = 2\partial_u\partial_v + \frac{1}{4}x_i^2\partial_v^2 + \partial_i^2 + \Delta_\perp$$

- Acting on a plane wave $e^{i(p_-u + p_+v)}$, this is the harmonic oscillator with frequency $|p_+|/4$. The ground state is **localized** in a ball of radius $1/\sqrt{\mu p_+}$:

$$\phi(u, v, z) = \exp \left[i(p_+v + p_-u + p_\perp x_\perp) - \frac{1}{4}|p_+|z\bar{z} \right]$$

- Other states in the same multiplet are obtained by acting n_L times (resp. n_R) with the creation operator P_L^- (resp. P_R^+). The **dispersion relation** is

$$2p_+p_- + |p_+|(n_L + n_R + 1) + p_\perp^2 + m^2 = 0 ,$$

while the helicity in transverse plane is $h = \epsilon(p_+)(n_L - n_R)$.

- For **$p_+ = 0$** and $m^2 = 0$, the solution is an arbitrary profile in u , it is a mode **traveling along the wave**. There can also be tachyonic modes with $p_+ = 0$ and p_i non vanishing.

Covariant quantization of strings on NW

Kiritsis Kounnas

- String theory on group manifolds can be solved by standard WZW techniques. Even better, chiral currents can be rewritten in terms of **free bosons**:

$$K = \partial u , \quad J = \partial v , \quad P_L^+ = e^{-iu} \partial w , \quad P_L^- = e^{iu} \partial \bar{w} ,$$

- For $0 < p_+ < 1$, chiral primaries can be written as

$$R_{p_+, p_-} = \exp [i(p_- u + p_+ v)] H_{p_+}$$

where H_{p_+} is a **twist field** modifying the periodicity of the transverse coordinate w by an (irrational) angle p_+ ,

$$w(e^{2\pi i} z) = e^{-2\pi i p_+} w(z) , \quad \bar{w}(e^{2\pi i} z) = e^{2\pi i p_+} \bar{w}(z).$$

Spectral flow and long strings

- For $p_+ \in \mathbb{Z}$, the effect of the twist disappears, and one expects a new zero mode: this is where **long strings** arise. They can be obtained by **spectral flow** from a spacelike geodesic at $p_+ = 0$:

$$z(\tau, \sigma) = e^{iw\sigma}(x_1 + ix_2) \left[1 \pm 2\tau \sqrt{\frac{h - 2p_- w}{x_1^2 + x_2^2}} \right]$$

- At the critical value $p_- = h/(2w)$, this becomes a **static** string, whose radius can stretch in the transverse directions at **no cost in energy**, thanks to a cancellation between **tension** and NS **flux**.

Ooguri Maldacena

- More generally, the spectrum shares many similarities with that of strings on AdS_3 . Indeed, one can construct $Sl(2)$ in the enveloping algebra of H_4 ,

$$E_+ = \frac{i}{2K} P_1^2, \quad E_- = \frac{i}{2K} P_2^2, \quad H = -\frac{1}{2K} (P_1 P_2 + P_2 P_1)$$

2. Holography for pp-wave backgrounds ?

- While the **holographic principle** is expected to be of general validity in quantum gravity, its most useful implementation to date is the AdS/CFT correspondence, which relates **off shell** correlators of boundary fields with **on shell** amplitudes in the bulk.
- It relies on having a well defined Cauchy problem. Boundary values of **non-normalizable** modes correspond to deformations of the boundary theory, while **normalizable modes** correspond to vevs.
- A crucial ingredient is the identification of the **radial** evolution in the bulk with with RG flow on the boundary. This is particularly obvious in the **Poincaré patch** of AdS:

$$ds^2 = (-dt^2 + dx_i^2 + dz^2)/z^2$$

Motion along z generates scaling transformations of the holographic dual. $z = 0$ corresponds to the **UV** of the gauge theory, while $z \rightarrow \infty$ is the **horizon** of the Poincaré patch, or the **IR** regime of the holographic dual.

Q: can one construct a similar correspondence for pp-waves ?

The “Poincaré” patch

- It is known that the homogeneous pp-wave background is **conformally flat**. The conformal factor $1/\sin^2[(u - u_0)/2]$ is non-singular on any **interval** in u of length 2π .
- The change of coordinates mapping the slice $u \in [0, 2\pi]$ of NW conformally to Minkowski is

$$\begin{aligned} x^+ &= -\cot(u/2) , & v &= x^- - \frac{1}{2} \sin u \sum y_i^2 \\ x_i &= 2y_i \sin(u/2) \end{aligned}$$

In these coordinates, the metric and flux read

$$\begin{aligned} ds^2 &= \frac{4}{1 + x^{+2}} (dx^+ dx^- + \sum dy_i^2) \\ H &= \frac{1}{(1 + x^{+2})^2} dx^+ (dy_1 dy_2 + \dots) \end{aligned}$$

The motion along x^+ rescales the metric of a fixed x^+ slice.

- We propose that there should exist **a holographic dual to string theory in a pp-wave background of type H_D , with degrees of freedom living on the null space spanned by the (x^-, y_i) coordinates at $x^+ \rightarrow -\infty$ together with the other spectator coordinates.**
- The holographic correspondence relates **S-matrix** elements between $x^+ = \pm\infty$ to **correlators** of a putative dual gauge theory at fixed x^+ .

compare to Das Gomez Rey, Leigh Rozali Okada

Comments on the proposal

- The pp-wave geometry is intermediate between AdS (where the conformal factor depends on a **spacelike** coordinate) and dS (where the conformal factor depends on a **timelike** coordinate).
- The conformal factor goes to 0 at $x^+ = \pm\infty$, which corresponds to the **horizon** of the patch. The conformal factor is always < 1 , so that **UV is effectively cut-off**.
- Choosing a null surface as an holographic screen is not unprecedented: this arises naturally in **Minkowski** space, and has been attempted in the context of **horizon holography**.
Solodukhin, Sachs
- Evolution along a null direction involves a **first order** differential equation. At fixed m^2 , only one “non-normalizable” mode propagates, in contrast to AdS/dS.
- The Cauchy problem is not well defined unless one specifies boundary conditions on the **complementary half of the wedge**, $x^- \rightarrow -\infty$. These are the missing modes, they encode the vacuum structure of the theory.

Comments on the proposal (cont.)

- In contrast to the global coordinates, the Poincaré patch has **translational invariance** in the transverse coordinates y_i . The transverse motion is therefore **unbounded**, corresponding to motion along an helix in original (u, v, x_i) coordinates.
- The Killing vectors can be expressed in terms of the conformal coordinates:

$$\begin{aligned} K &= \partial_- , & J_L - J_R &= y_1 \partial_2 - y_2 \partial_1 \\ P_{iL} - P_{iR} &= \epsilon_{ij} \partial_j , & P_{iL} + P_{iR} &= 2y_i \partial_- - x^+ \partial_i \\ J_L + J_R &= (1 + x^{+2}) \partial_+ - (y_1^2 + y_2^2) \partial_- \\ &\quad + x^+ (y_1 \partial_1 + y_2 \partial_2) \end{aligned}$$

All generators leave the boundary invariant, except for $J_L + J_R$ which moves forward along x^+ . Its action can nevertheless be brought back to the boundary.

- The dual theory is no more a CFT, but a new type of (non-local) field theory with **Heisenberg symmetry** $H_L \times H_R$, which we may dub **HFT**.

3. Holography at work

- A class of deformations away from homogeneous pp-wave is the **conformal wave background**

$$ds^2 = \frac{1}{f^2(x^+)}(dx^+ dx^- + \sum dy_i^2), \quad H = \sqrt{f''/f^5} dx^+ \wedge \omega$$

This solves the eom for any **convex** profile, $f'' > 0$.

- The wave equation is **first order** in ∂_+ and can be integrated for any profile:

$$\phi = f^{\frac{D-2}{2}}(x^+) \exp \left(i \left(p_+ x^- + p_i y_i - \frac{p_T^2}{4p_+} x^+ \right) - i \frac{m^2}{4p_+} \int_{-\infty}^{x^+} \frac{du}{f^2(u)} \right)$$

- For initial conditions at x_0^+ specified by $\chi(p_+, p_i)$ in momentum space, and **assuming zero conditions at $x^- \rightarrow 0$** , the solution at arbitrary time is

$$\begin{aligned} \phi(x^+, x^-, x) &= \int dp_+ dp_i \chi(p_+, p_i) \left(\frac{f(x^+)}{f(x_0^+)} \right)^{\frac{D-2}{2}} \\ &\times \exp \left[i(p_+ x^- + p_i y_i) - i \frac{p_i^2}{4p_+} (x^+ - x_0^+) - i \frac{m^2}{4p_+} \int_{x_0^+}^{x^+} \frac{du}{f^2(u)} \right] \end{aligned}$$

Boundary symmetries

- Killing vectors commute with the Laplacian, hence can be commuted through the propagator to act on the boundary data only. The dependence on x_0^+ can be eliminated by redefining

$$\phi(p_+, p_1, p_2) = \tilde{\phi}(p_+, p_1, p_2) \exp \left(-i \frac{p_1^2 + p_2^2}{4p_+} x_0^+ \right)$$

- The boundary generators are:

$$\begin{aligned} J_L - J_R &= y_1 \partial_2 - y_2 \partial_1 \\ K &= \partial_- , \\ P_{iL} + P_{iR} &= 2y_i \partial_- \\ P_{iL} - P_{iR} &= \epsilon_{ij} \partial_j , \\ J_L + J_R &= -(\sum y_i^2) \partial_- + (4m^2 - \Delta) \partial_-^{-1} \end{aligned}$$

where $\partial_-^{-1} = \int dx^-$, $\partial_-^{-1} \partial_- = \partial_- \partial_-^{-1} = 1$. The representation is **non-local and singular at $p_+ = 0$** , as usual in light-cone quantization.

- The **holographic generator** is the **Hamiltonian of the harmonic oscillator** with frequency $|\partial_-| = |p_+|$. At $x_0^+ = 0$ it does not involve any rescaling.

A new class of field theories ?

Boundary correlators from the bulk

- Consider a massive scalar field in the bulk,

$$S = -\frac{1}{2} \int d^D x \sqrt{-g} \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2 + \frac{2\lambda}{3} \phi^3 \right)$$

- Expanding perturbatively $\phi = \phi_0 + \lambda \phi_1 + \dots$,

$$(\Delta - m^2)\phi_0 = 0, \quad (\Delta - m^2)\phi_1 = \phi_0^2$$

ϕ_0 is determined from the boundary data by the **boundary-to-bulk** prop. ϕ_1 follows from ϕ_0 by the bulk-to-bulk propagators. We evaluate the **classical action** on the **solution** between x_0^+ and x_1^+ :

$$S_0 = \int [\partial_+(f^{2-D}\phi_0\partial_-\phi_0) + \partial_-(f^{2-D}\phi_0\partial_+\phi_0)]$$

$$S_1 = \lambda \int [\partial_+(f^{2-D}\phi_1\partial_-\phi_0) + \partial_-(f^{2-D}\phi_1\partial_+\phi_0) + \partial_+\partial_-()]$$

- Keeping the boundary terms at $x^+ = x_0^+, x_1^+$, we find $S_0 = 0$: **two-point functions vanish !**
- In general, Ward identities of the $H_L \times H_R$ symmetry allow for

$$F_2(p, p_+; q, q_+) = f(p_+)\delta(p_+ + q_+)\delta^{(D-2)}(p + q)$$

Three point functions

At next to leading order in λ , we can extract the three point function. A **renormalization** of the boundary fields is necessary,

$$\phi_0 \rightarrow \phi_0 f(x_0^+)^{\frac{2-D}{2}}$$

The result is a non-trivial function,

$$\begin{aligned} F_3(p, q, r) = & \lambda \frac{(2\pi)^{D-1}}{12} \int_{x_0^+}^{\infty} du f(u)^{\frac{D-6}{2}} \\ & \times \exp \left[-\frac{im^2}{4} \left(\frac{1}{p_+} + \frac{1}{q_+} + \frac{1}{r_+} \right) \gamma(u, x_0^+) \right] \\ & \times \exp \left[-\frac{i}{4} \left(\frac{p^2}{p_+} + \frac{q^2}{q_+} + \frac{r^2}{r_+} \right) (u - x_0^+) \right] \end{aligned}$$

with

$$\gamma(x^+, x_0^+) \equiv \int_{x_0^+}^{x^+} \frac{du}{f^2(u)}$$

It can be checked that it satisfies the Ward identity of $H_L \times H_R$.

Null Cauchy problem

We have so far assumed particular (vanishing) boundary conditions at $x^- \rightarrow -\infty$. Those however provide the missing “normalizable” modes (in AdS terminology).

The wave equation can however be **integrated recursively** for any configuration at $x^- \rightarrow \infty$, using a procedure similar as for charged particles in a null electric field:

Tomaras, Tsamis, Woodard

Defining $\phi_- = \phi f^{(2-D)/2}$ and $\phi_+ = \partial_- \phi_-$,

$$\begin{aligned} \phi_+(x^+, x^-, y) = & \int_{-\infty}^{+\infty} \frac{dp_+}{2\pi} \left\{ \int_{x_0^-}^{+\infty} dv e^{ip_+(v-x^-)} \right. \\ & \exp \left(\frac{i}{4p_+} \int_{x_0^+}^{x^+} \left(\frac{m^2}{f^2(u)} - \Delta_T \right) du \right) \phi_+(x_0^+, v, y) \\ & + \frac{ie^{-ip_+(x^- - x_0^-)}}{4p_+} \int_{x_0^+}^{x^+} du \left(\frac{m^2}{f^2(u)} - \Delta_T \right) \\ & \left. \left[\exp \left(\frac{i}{4p_+} \int_u^{x^+} \left(\frac{m^2}{f^2(u')} - \Delta_T \right) du' \right) \right] \phi_-(u, x_0^-, y) \right\} \end{aligned}$$

where $p_+ \rightarrow p_+ + i\epsilon$ is understood.

It would be interesting to study the effect of boundary conditions at $x^- \rightarrow -\infty$ on the correlators. They would **break** the $H_N \times H_N$ invariance of the vacuum. **Particle production** can also be shown not to occur in these conformal wave backgrounds.

Summary - discussion 1

We have proposed an holographic correspondence between strings in the pp-wave bulk and a putative “gauge” theory living in a $D - 1$ dimensional degenerate space (x^-, y, x_\perp) at fixed x^+ . Important differences with usual set-up are

- Evolution along the holographic direction x^+ is **first order**. Only one set of modes, or rather there are other modes at $x^- \rightarrow -\infty$. Can one take those into account in holographic computations ?
- Conformal invariance in the dual theory is replaced by invariance under a (non-local) **Heisenberg-type** group, $H_D \times H_D$. **Ward identities** do not restrict 2 and 3-point functions as much as in CFT case, and lead to singular contributions.
- **Correlations functions** can be computed by evaluating the classical action on a solution with specified boundary conditions. 2-point functions are zero in the supergravity limit. 3-point function are non-trivial.
- Can one formulate the dual “gauge” theory microscopically ? Infinite momentum is T-dual to critical electric field: is NCOS relevant ?

Discussion 2

- Our choice of holographic screen has been guided by the relation between holographic flow and RG flow. It led us to an exotic non-local field theory where the **holographic** flow is generated by the **harmonic oscillator Hamiltonian**.
- An alternative prejudice would be to require the holographic dual to live at **conformal infinity**. Just like Minkowski, the homogeneous pp-wave can be conformally mapped to the **static Einstein universe** $R \times S^{D-1}$, except for a **null trajectory** spinning along the sphere where the conformal factor blows up:

$$\Omega^2 = v^2 + \left(1 + \frac{1}{4}x_i^2\right)^2 = |1 + \cos \alpha e^{i(\beta+t)}|^{-2}$$

Berenstein (Maldacena) Nastase

Can one define a **M(atrix)-like model** living on this null geodesic, and construct a well-defined boundary to bulk correspondence ?

Discussion 3

- By either considering **membranes** in this background or going to **infinite momentum frame**, a **M(atrix) string theory** has been proposed. Can one formulate it in an holographic way ?

Ghoshal Gopakumar Jatkar

- A M(atrix) model has also been proposed for the **11D homogeneous pp-wave** arising in the Penrose limit of $AdS_{7|4} \times S^4|7$, where the flat directions are **lifted** by mass terms. Can this model be more useful than BFSS ?

Dasgupta Sheikh-Jabbari Van Raamsdonk

PP-waves and cosmology

- In mixed coordinates (u, x^-, y_i) , the compactified pp-wave has a cosmological interpretation, as a **bouncing universe**:

$$\begin{aligned} ds^2 &= 2dudx^- + 4\sin^2(u/2)dy_i^2 \\ H &= 4\sin^2(u/2)du(dy^1dy^2 + \dots) \end{aligned}$$

The local singularity at $u = 2\pi n$ is the same as the null orbifold.

Liu Moore Seiberg

- The **H flux** provides energy density to curve back space after the big bang. It could also be used to **resolve the singularity**, using the conformal wave backgrounds introduced above.

Kiritsis P., in progress

- Need to understand the compactification of the y_i directions, ie **orbifold** the pp-wave by

$$\exp[2\pi R_i(P_{iL} - P_{iR})]$$

From null big crunch to null big bang, and beyond...