Duality and Large dimensions in Closed strings

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based on work in collaboration
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Scales and strings

- Phenomenological string models have for long been built in weakly coupled heterotic $E_8 \times E_8$ theory, where

$$l_{\mathrm{P}} = g_{\mathrm{YM}} l_{\mathrm{H}} \ , \qquad g_{\mathrm{H}}^2 = g_{\mathrm{YM}}^2 V / l_{\mathrm{H}}^6$$

  The weak coupling implies $m_{\mathrm{P}} < g_{\mathrm{YM}}^{4/3} m_{GUT}$ which fails by a factor 25. In addition, issues like SUSY breaking, moduli stabilisation are not believed to be resolved perturbatively. String dualities leave the hope for controlling strong coupling regime. The string scale remains however tied to the Planck scale.

  Witten

- In type I strings for instance, due to the different coupling of RR gauge fields, the situation is quite different :

$$l_{\mathrm{P}} = \sqrt{g_{\mathrm{I}}} g_{\mathrm{YM}} l_{\mathrm{I}} \ , \qquad g_{\mathrm{I}} = g_{\mathrm{YM}}^2 V / l_{\mathrm{I}}^6$$

  so that the weak coupling bound $m_{\mathrm{P}} < g_{\mathrm{YM}}^{4/3} m_{GUT} / \sqrt{g_{\mathrm{I}}}$ can be fulfilled for arbitrary weak coupling, lowering the string scale at the same time. This also comes out in the Horava-Witten description of strongly coupled heterotic string.

- Why not lower the string scale all the way to the electroweak scale, 250 GeV to a 1 TeV ?

  Lykken

Type I TeV string

- One may want to lower the string scale all the way to the electroweak scale, 250 GeV to a 1 TeV. This implies internal dimensions much smaller than the string length :

$$V = \frac{l_{\mathsf{P}}^2 l_{\mathrm{I}}^4}{g_{\mathsf{YM}}^4} = \begin{cases} (10^{33} GeV) & l_{\mathrm{I}}^5 \\ (10^{18} GeV)^2 & l_{\mathrm{I}}^4 \\ (10^{8} GeV)^6 & \end{cases}$$

  with very light winding states propagating around them. In a T-dual description, this is a type I' theory on a D3-brane, with coupling $g_{\mathrm{I}} = g_{\mathsf{YM}}^2$ and internal volume

$$V = \frac{l_{\mathsf{P}}^{-2} l_{\mathrm{I}}^8}{g_{\mathsf{YM}}^4} = \begin{cases} (10^{-18} eV) & l_{\mathrm{I}}^5 \\ (10^{-3} eV)^2 & l_{\mathrm{I}}^4 \\ (10 MeV)^6 & \end{cases}$$

- Internal dimensions of size $<$ 250 GeV are experimentally excluded, *except if not felt by gauge interactions* : The bounds on deviations from Newton law at submillimiter distances are very weak. Indeed, only gravity propagates in the directions transverse to the D-brane, while gauge interactions are confined in 4D. *Antoniadis,Arkani,Dimopoulos,Dvali*

- Weakly coupled Type II strings offer an alternative framework where the string length can be lowered to a TeV *without introducing large dimensions*, by considering a singular internal manifold. *Antoniadis,P*

Outline

- U-duality and asymptotic regimes of M-theory on torii

  Obers,P;Banks,Fischler,Susskind

- Triality in theories with N=16 SUSY and truly strong coupling

  Witten,Horava;(Dine,Shirman)

- Large dimensions and TeV scale in type I strings

  Antoniadis,Arkani-Hamed,Dimopoulos,Dvali

- TeV scale in type II strings with(out) large dimensions

  Antoniadis,P

Ur-theory and U-duality

- Any Lorentz invariant theory compactified on a torus T^d has a discrete global symmetry, the *mapping class group* of the torus :

$$S_{IJ} \ : \ \gamma_I \leftrightarrow \gamma_J \ , \ \ R_I \leftrightarrow R_J$$

$$\Gamma_{IJ} \ : \ \gamma_I \to \gamma_I + \gamma_J \ , \ \ A_J^{(I)} \to A_J^{(I)} + 1$$

where $ds^2 = R_I^2 (dx^I + A_J^{(I)} dx^J)^2 + dx^J g_{JK} dx^K$

$$S_{IJ} \bowtie \Gamma_{IJ} = Sl(d, \mathbb{Z})$$

- Perturbative type IIA theory on a torus T^d has in addition a discrete global symmetry valid to all orders, namely (double) T-duality :

$$T_{ij} \ : \ (R_i, R_j, g_s) \to \left(\frac{l_s^2}{R_j}, \frac{l_s^2}{R_i}, \frac{g_s l_s^2}{R_i R_j} \right)$$

$$\Delta_{ij} \ : \ B_{ij} \to B_{ij} + 1$$

$$Sl(d, \mathbb{Z}) \bowtie T_{ij} \bowtie \Delta_{ij} = SO(d, d, \mathbb{Z})$$

- M-theory on a torus T^d should exhibit both Lorentz invariant on T^d and T-duality invariance on T^{d-1} :

$$Sl(d, \mathbb{Z}) \bowtie SO(d-1, d-1, \mathbb{Z}) = E_{d(d)}(\mathbb{Z})$$

This defines *a* discrete subgroup of Cremmer–Julia hidden symmetry group of 11D SUGRA on T^d, which has to remain valid at the quantum level.

T-duality, Weyl and Borel generators

- Restricting 11D SUGRA/T^{d+1} to the Neveu-Schwarz sector, we find a very similar structure :

$$\mathcal{H} = \mathbb{R}^+ \times \frac{SO(d,d,\mathbb{R})}{SO(d) \times SO(d)} \ni (g_s, \mathcal{V})$$

$$\mathcal{V} = \left(\begin{array}{cc|cc} 1/R_1 & & & \\ & 1/R_2 & & \\ \hline & & R_1 & \\ & & & R_2 \end{array}\right) \cdot \left(\begin{array}{cc|cc} 1 & -A^1_2 & B_{11} & B_{12} \\ & 1 & B_{21} & B_{22} \\ \hline & & 1 & A^1_2 \\ & & & 1 \end{array}\right) = \mathcal{A} \cdot \mathcal{N}$$

- The *Weyl* group of $SO(d,d)$ preserves the diagonal form of $\mathcal{A}$. It acts on $\mathcal{V}$ from the right and permutes R_i with R_j or $1/R_j$. A minimal subset is given by :

$$S_i : R_i \leftrightarrow R_{i+1} \ , \quad i = 1 \dots d-1$$

$$T : (g_s, R_1, R_2) \leftrightarrow \left(\frac{g_s}{R_1 R_2}, \frac{1}{R_2}, \frac{1}{R_1}\right)$$

- The *Borel* group of upper triangular matrices with 1's on the diagonal (and appropriate syms) also acts from the right, leaving R_i invariant but shifting the 0-form potentials. A basis is

$$\Gamma_{ij} : A^{(i)}_j \to A^{(i)}_j + 1$$

$$\Delta_{ij} : B_{ij} \to B_{ij} + 1$$

T-duality Weyl group

- The action of the Weyl group can be represented on a weight space :

  $$\mathcal{T} = g_s^{x^0} R_1^{x^1} R_2^{x^2} \dots R_d^{x^d} \to \lambda = x^0 e_0 + x^1 e_1 + \dots + x^d e_d$$

- The Weyl generators T and S_i act as *orthogonal reflections* $\rho_{\alpha_0}, \rho_{\alpha_i}, i = 1 \dots d-1$

  $$\lambda \to \rho_\alpha(\lambda) = \lambda - 2\frac{\alpha \cdot \lambda}{\alpha \cdot \alpha}\alpha$$

  w.r.t. the signature $(-++\dots)$ metric

  $$\mathrm{d}s^2 = -(\mathrm{d}x^0)^2 + (\mathrm{d}x^i)^2 + \mathrm{d}x^0(dx^1 + \dots + \mathrm{d}x^d)$$

- The Planck length $V/g_s^2 l_s^8$ is invariant, so the action restricts to the spacelike hyperplane $\delta \cdot x = x^0 = 0$ normal to

  $$\delta = e_1 + \dots + e_d - 2e_0$$

- The Coxeter group is characterized by its Cartan matrix or Dynkin diagram

  $$\begin{array}{ccccccc} T & & & & & & \\ & \diagdown & & & & & \\ & & S_2 & - & S_3 & - \cdots - & S_{d-1} \\ & \diagup & & & & & \\ S_1 & & & & & & \end{array}$$

  This is the Dynkin diagram of $SO(d,d)$.

T-duality Weyl multiplets

- The representation of the finite group $\mathcal{W}(SO(d,d))$ can be obtained from the fundamental weights dual to the simple roots :

$$
\begin{array}{ccccccc}
\frac{1}{g} & & & & & & \\
& \diagdown & & & & & \\
& & \frac{R_1 R_2}{g_s^2} & - & \frac{R_1 R_2 R_3}{g_s^2} & - \cdots - & \frac{1}{R_d} \\
& \diagup & & & & & \\
\frac{R_1}{g} & & & & & &
\end{array}
$$

- Depending on the power of the invariant string length we include, this gives the T-duality Weyl multiplets of particle masses, or string tensions.

- The Kaluza-Klein modes on the torus together with the string winding states transform as a *vector* of $SO(d,d)$, with highest weight $\mathcal{M} = 1/R_{d-1}$.

- The D0-branes transform as a *spinor* of $SO(d,d)$ with highest weight $\mathcal{M} = 1/gl_s$, just as the RR gauge fields :

$$\mathcal{R} \leftrightarrow \mathrm{d}x^i \cdot \mathcal{R} + \mathrm{d}x^i \wedge \mathcal{R} \ , \ \ \mathcal{R} = \sum_p \mathcal{R}_p$$

  which generates a Clifford algebra. The same is true for the D2-brane tension, with highest weight $\mathcal{T}_2 = 1/gl_s^3$.

- The D-strings obtained by wrapping a D2-brane on a circle transform as a spinor of opposite chirality and highest weight $\mathcal{T}_1 = R_1/gl_s^3$.

U-duality Weyl and Borel generators

- In M-theory variables, the T-duality $(R_i, R_j) \to (1/R_j, 1/R_i)$ reads

$$T_{ij} : R_i \to \frac{l_M^3}{R_j R_s} \ , \ \ R_j \to \frac{l_M^3}{R_s R_i} \ , \ \ R_s \to \frac{l_M^3}{R_i R_j} \ , \ \ l_M^3 \to \frac{l_M^6}{R_i R_j R_s}$$

- By a $Sl(d, \mathbb{Z})$ M-theory Lorentz boost, it can be conjugated to

$$T_{IJK} : R_I \to \frac{l_M^3}{R_J R_K} \ , \quad l_M^3 \to \frac{l_M^6}{R_I R_J R_K}$$

which involves a set of *three* directions. A minimal set of Weyl transformations can be chosen as $T = T_{123}$ and $S_I \ : R_I \leftrightarrow R_{I+1}$.

- The Borel generators include as usual the Dehn twists $\gamma_I \to \gamma_I + \gamma_J$ in $Sl(d, \mathbb{Z})$, as well as the $B_{ij} = \mathcal{C}_{sij}$ shifts in $SO(d-1, d-1, \mathbb{Z})$. These can be conjugated into

$$C_{IJK} : \mathcal{C}_{IJK} \to \mathcal{C}_{IJK} + 1$$

and we also need, for $d \geq 6$ ($d \geq 8$),

$$E_{IJKLMN} \ : \mathcal{E}_{IJKLMN} \to \mathcal{E}_{IJKLMN} + 1$$

$$K_{I;JKLMNPQR} \ : \mathcal{K}_{I;JKLMNPQR} \to \mathcal{K}_{I;JKLMNPQR} + 1$$

U-duality Weyl group

- The action of the Weyl group can again be represented on a weight space :

  $\mathcal{T} = l_M^{3x^0} R_1^{x^1} R_2^{x^2} \dots R_d^{x^d} \to \lambda = x^0 e_0 + x^1 e_1 + \dots + x^d e_d$

- The Weyl generators T and S_i act as *orthogonal reflections*

  $$\lambda \to \rho_\alpha(\lambda) = \lambda - 2\frac{\alpha \cdot \lambda}{\alpha \cdot \alpha}\alpha$$

  for the signature $(- + + \dots)$ metric

  $$\mathrm{d}s^2 = -(\mathrm{d}x^0)^2 + (\mathrm{d}x^i)^2$$

- The Planck length V/l_M^3 being invariant, the action restricts to the spacelike hyperplane $\delta \cdot x = 0$ normal to

  $$\delta = e_1 + \dots + e_d - 3e_0 \ ,$$

  except when $d = 9$ *where δ is null*.

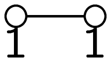
- The Coxeter group is characterized by its Cartan matrix or Dynkin diagram

  $$\begin{array}{ccccccccc} & & & & T & & & & \\ & & & & | & & & & \\ S_1 & - & S_2 & - & S_3 & - & S_4 & - \cdots - & S_{d-1} \end{array}$$

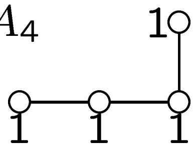
  This is the Dynkin diagram of E_d, or the *extended* Dynkin diagram of $\hat{E}_8 = E_9$ for $d = 9$.

$E_2 = A_1$

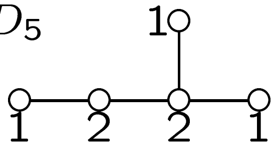
$E_3 = A_2 \oplus A_1$ 



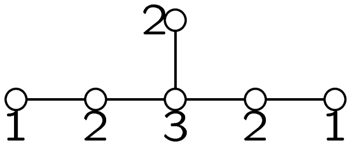
$E_4 = A_4$



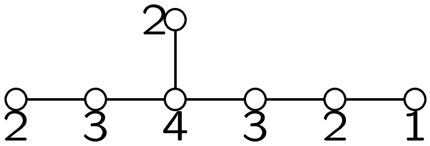
$E_5 = D_5$



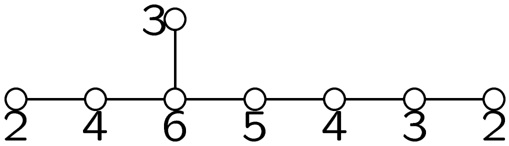
E_6



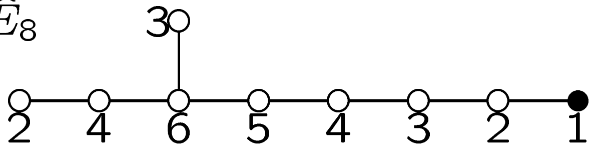
E_7



E_8



$E_9 = \widehat{E}_8$



U-duality Weyl multiplets

- The representation of the finite group $\mathcal{W}(E_{d(d)})$ ($d < 9$) can be obtained from the fundamental weights dual to the simple roots :

$$
\begin{array}{ccccccccc}
 & & & & \frac{1}{l_M^3} & & & & \\
 & & & & | & & & & \\
\frac{R_1}{l_M^3} & - & \frac{R_1 R_2}{l_M^6} & - & \frac{R_1 R_2 R_3}{l_M^9} & - & \frac{R_1 R_2 R_3 R_4}{l_M^9} & - \cdots - & \frac{1}{R_d}
\end{array}
$$

- The *particle multiplet* with highest weight $\mathcal{M} = 1/R_{d-1}$ transforms as the representation on the rightmost node.

- The *string multiplet* with highest weight $\mathcal{T}_1 = R_1/l_M^3$ transforms as the representation on the leftmost node.

- The *membrane multiplet* with highest weight $\mathcal{T}_2 = 1/l_M^3$ appears on the T-duality node.

- The other nodes do not correspond to other simple multiplets, but to Weyl submultiplets.

- For $d = 9$, the Weyl orbit becomes infinite. The level and degree are given by

$$k = \delta \cdot \lambda = x^1 + \cdots + x^9 + 3x^0$$

$$d = \kappa \cdot \lambda = x^9 - k/2$$

and correspond to the scaling dimension and the number of 9-th directions.

The particle multiplet

In $d = 8$: a $\mathbf{248}$ of $E_{8(8)}$.

mass $\mathcal{M}$	$Sl(8)$ irrep	charge
$\frac{1}{R_I}$	$\mathbf{8}$	m_1
$\frac{R_I R_J}{l_M^3}$	$\mathbf{28}$	m^2
$\frac{R_I R_J R_K R_L R_M}{l_M^6}$	$\mathbf{56}$	m^5
$\frac{R_I^2 R_J R_K R_L R_M R_N R_P}{l_M^9}$, $8\ \frac{V_R}{l_M^9}$	$\mathbf{1{+}63}$	$m^{1;7}$
$\frac{R_I^2 R_J^2 R_K^2 R_L R_M R_N R_P R_Q}{l_M^{12}}$	$\mathbf{56}$	$m^{3;8}$
$\frac{R_I^2 R_J^2 R_K^2 R_L^2 R_M^2 R_N^2 R_P R_Q}{l_M^{15}}$	$\mathbf{28}$	$m^{6;8}$
$\frac{R_I^3 R_J^2 R_K^2 R_L^2 R_M^2 R_N^2 R_P^2 R_Q^2}{l_M^{18}}$	$\mathbf{8}$	$m^{1;8;8}$

In lower d, drop the charges with two many antisymmetric indices :

D	d	U-duality group	irrep	$Sl(d)$ content
10	1	1	$\mathbf{1}$	$\mathbf{1}$
9	2	$Sl(2,\mathbb{Z})$	$\mathbf{3}$	$\mathbf{2} + \mathbf{1}$
8	3	$Sl(3,\mathbb{Z}) \times Sl(2,\mathbb{Z})$	$(\mathbf{3},\mathbf{2})$	$\mathbf{3} + \mathbf{3}$
7	4	$Sl(5,\mathbb{Z})$	$\mathbf{10}$	$\mathbf{4} + \mathbf{6}$
6	5	$SO(5,5,\mathbb{Z})$	$\mathbf{16}$	$\mathbf{5} + \mathbf{10} + \mathbf{1}$
5	6	$E_{6(6)}(\mathbb{Z})$	$\mathbf{27}$	$\mathbf{6} + \mathbf{15} + \mathbf{6}$
4	7	$E_{7(7)}(\mathbb{Z})$	$\mathbf{56}$	$\mathbf{7} + \mathbf{21} + \mathbf{21} + \mathbf{7}$
3	8	$E_{8(8)}(\mathbb{Z})$	$\mathbf{248}$	$2(\mathbf{8} + \mathbf{28} + \mathbf{56}) + \mathbf{63} + \mathbf{1}$

Large dimensions in M-theory on torii

- The boundaries of the moduli space correspond to a scaling limit of the dilatonic scalars R_I, l_M parametrising the Cartan "torus". Modulo the action of the Weyl group, we can choose them to lie in a *Weyl chamber* such that $\alpha_+ \cdot \lambda > 0$:

$$R_1 < R_2 < \cdots < R_d \ , \qquad R_1 R_2 R_3 > l_M^3$$

- The 11D SUGRA description is valid provided all radii are larger than the Planck length :

$$11D\ SUGRA \ : \qquad l_M < R_1 < R_2 < \ldots$$

- If one of the radii is smaller, then we may have a type IIA description with weak coupling $g_s^2 = (R_1/l_M)^3$, provided all radii are larger than the string length $l_s^2 = l_M^3/R_1$:

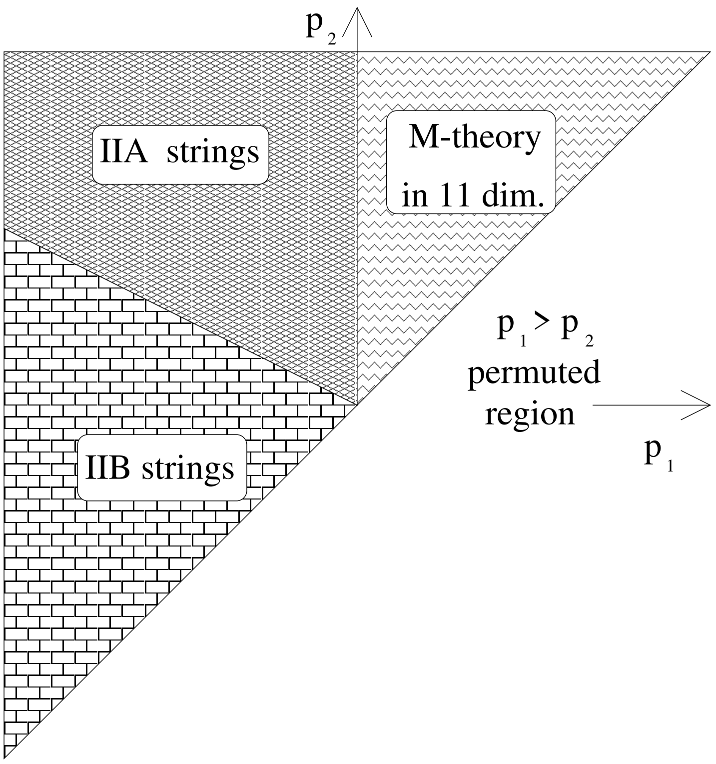
$$R_1 < l_M \ , \qquad R_1 R_2^2 > l_M^3$$

- If one of the radii is smaller than the above string length, we may instead try a type IIB description with weak coupling $g_s = R_1/R_2$, same string length $l_s^2 = l_M^3/R_1$ and 10-th radius $R_B = l_M^3/(R_1 R_2)$. The IIB radii are smaller than the string length provided

$$R_1 R_i^2 > l_M^3 \ , \qquad R_1 R_2^2 < l_M^3$$

The first relation is automatically satisfied if $R_1 R_2 R_3 > l_M^3$, and the second implies $R_1 < l_M$.

- The 11D SUGRA, type IIA and type IIB therefore cover the complete asymptotic region of the moduli space.



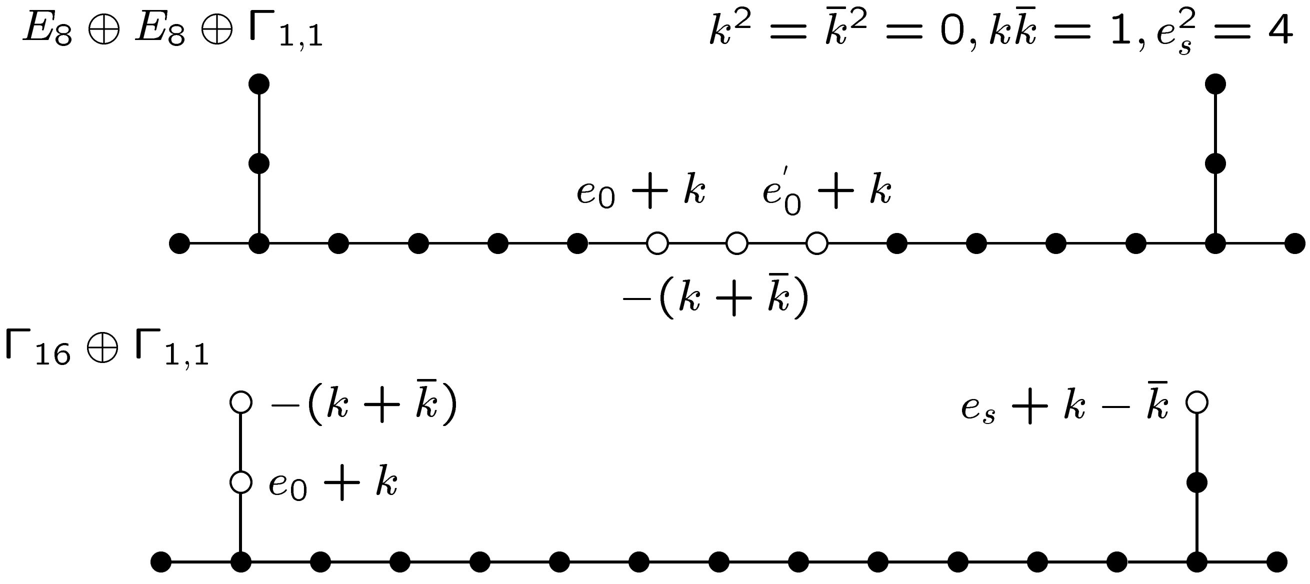
- For $d > 9, 10$, the metric is now Lorentzian, and the above Weyl chamber generates the *future light cone* only. There is a truly strongly coupled space-like region, excluded by the holographic principle.

Banks,Fischler,Motl

Heterotic $E_8 \times E_8$ - $Spin(32)/Z_2$ duality

- The two ten-dimensional heterotic strings HE and HO are built out of the only two euclidean even self-dual lattices of dimension 16.

- Upon compactification, the gauge lattice combines with a $\Gamma_{1,1}$ lattice to make a Lorentzian $\Gamma_{1,17}$ lattice. The radius R together with the 16 Wilson lines make a $SO(1,17)/SO(17)$ moduli space, that allows to boost one lattice into the other :



- At zero Wilson line, each theory is *self-dual* under $R \to 1/R$. At the $SO(16) \times SO(16)$ enhanced symmetry point, T-duality maps one theory to the other.

Heterotic $Spin(32)/Z_2$ - Type I duality

- The two ten-dimensional theories have the same gauge group and effective action, upon identifying

$$g_I = \frac{1}{g_{HO}} \; , \qquad l_I = \sqrt{g_{HO}} l_H$$

This duality maps KK states to KK states, the fundamental heterotic string to the type I D-string, the heterotic 5-brane to the type I D5-brane. Non-BPS heterotic states in the spinor conjugacy class map to Non-BPS type I D0-particles. The non-BPS type I string has no known representation on the heterotic side.

- Using the Horava-Witten picture of the heterotic string $E_8 \times E_8$, one can "derive" this duality by considering M-theory on $I(R_{I'}) \times S_1(R_1)$ in two different ways :

$$HE/S_1(R_1) \left\{ \begin{array}{l} R_I = g_{\mathsf{HE}} l_{\mathsf{H}} \\ l_M^3 = g_{\mathsf{HE}} l_{\mathsf{H}}^3 \end{array} \right. = I'/I(R'_I) \left\{ \begin{array}{l} l_M^3 = g_{I'} l_I^3 \\ R_1 = g_{I'} l_I \end{array} \right.$$

Applying a T-duality on each side, we find heterotic $SO(32)$ and type I with inverse coupling compactified on a common circle :

$$g_{HO} = \frac{g_{\mathsf{HE}} l_{\mathsf{H}}}{R_1} = \frac{R_{I'}}{R_H} = \frac{R_{I'}}{g_{I'} l_I} = \frac{1}{g_I}$$

$$R_{HO} = \frac{l_{\mathsf{H}}^2}{R_1} = \frac{l_M^3}{R_{I'} R_H} = \frac{l_I^2}{R_{I'}} = R_{I'}$$

The 10D duality is obtained by decompactification.

Heterotic $E_8 \times E_8$ - Type II/K_3

- The heterotic-type II duality is usually presented in 6 dimensions, where the two theories have the same effective action, upon identifying

$$g_{I6} = \frac{1}{g_{\text{HE6}}} \,, \qquad l_I = g_{\text{HE6}} l_H$$

This duality maps het KK and winding states to type II D-branes wrapped on K_3, the het NS5-brane wrapped on T^4 to the fundamental type IIA string, the type II NS5 wrapped on K_3 to the fundamental het string.

- In particular, by identifying the volume of K_3 as the radius (square) of heterotic circle,

$$\frac{R_1^2}{l_H^2} = \frac{V_{K3}}{l_{\rm II}^4}$$

we see that type II D0-branes are mapped to Het KK states along R_1, so the duality decompactifies to a Het/T^3-M/K_3 7D duality.

- This duality can in fact be derived in the Horava-Witten picture by considering M-theory on $I(R_I) \times T^3(R_1, R_2, R_3)$ and performing a double T-duality on T^3 :

$$\frac{R_1R_2R_3R_4}{g_{\text{HE}}^2 l_{\text{H}}^4} = \frac{R_1R_2R_3R_4}{R_I l_M^3} \to \frac{R_4 l_M^3}{R_I R_1 R_2 R_3} = \frac{g_{\rm II}^2 l_{\rm II}^4}{R_1 R_2 R_3 R_I}$$

- This identifies the $E_8 \times E_8$ heterotic string with M-theory compactified on K_3 at an $E_8 \times E_8$ singularity :

$$K_3 = I(R_I) \times T^3(\tilde{R}_2, \tilde{R}_3, \tilde{R}_4)$$

and gives the complete mapping of the dilatonic $SO(4,4)$ moduli:

$$l_{\rm II} = \frac{g_{\sf HE} l_{\sf H}^3}{\sqrt{R_1 R_2 R_3 R_4}} \ , \qquad g_{\rm II} = \frac{\sqrt{R_1^3 R_2 R_3 R_4}}{g_{\sf HE} l_{\sf H}^3} \ ,$$

$$R_I = g_{\sf HE} l_{\sf H} \ , \qquad \tilde{R}_i = \frac{g_{\sf HE} l_{\sf H}^3}{R_j R_k} \ , \ i, j, k = 2, 3, 4$$

- This is nothing but a SO(4,4) triality transforming the vector of heterotic KK and windings into the spinor of type II D-branes :

$$\frac{R_1}{l_{\sf H}} \to \sqrt{\frac{R_1 R_2 R_3 R_4}{l_{\rm II}^4}} \ , \qquad \frac{R_2}{l_{\sf H}} \to \sqrt{\frac{R_1 R_2}{R_3 R_4}}$$

Aspinwall;Polchinski;Dine,Shirman

which can in fact be extended to the full $SO(4,4) \subset SO(4,20)$ moduli :

$$M(vect, g_{ij}, B_{ij}) = M(spin, V g^{ij}, \epsilon^{ijkl} B_{kl})$$

Heterotic-type I-type II triality

The four theories with $N = 16$ supersymmetries are thus unified in the M-theory setting as

$$
\begin{tikzcd}
\begin{array}{c}\text{Type I } SO(32) \text{ on} \\ T^4(\tilde{R}_I, R_2, R_3, R_4)\end{array} & & \begin{array}{c}\text{Het } SO(32) \text{ on} \\ T^4(\tilde{R}_1, R_2, R_3, R_4)\end{array} \\
& \begin{array}{c}\text{M-theory on} \\ S_1(R_1) \times I(R_I) \times T^3(R_2, R_3, R_4)\end{array} \arrow[ul, "T_{1I}"] \arrow[ur, "T_{I1}"'] \arrow[dl, "T_{234}"'] \arrow[dr] & \\
\begin{array}{c}\text{Type IIA on } K_3 = \\ I(R_I) \times T^3(\tilde{R}_2, \tilde{R}_3, \tilde{R}_4)\end{array} & & \begin{array}{c}\text{Het } E_8 \times E_8 \text{ on} \\ T^4(R_1, R_2, R_3, R_4)\end{array}
\end{tikzcd}
$$

Truly strongly coupled Het theories

As an application, we can study the truly strongly coupled regions of the moduli space. Consider the heterotic $SO(32)$ scaling limit $g_{\mathsf{H}} \to \infty$, $R \sim g_{\mathsf{H}}^{\alpha}$

- The type I dual has $R/l_{\mathrm{I}} = g_{\mathsf{H}}^{\alpha-1/2}$. For $\alpha > 1/2$, this is truly weakly coupled. For $\alpha < 1/2$, T-dualize the 6 radii :
$$g_{\mathrm{I}} \sim g_{\mathsf{H}}^{2-6\alpha} \ , R/l_{\mathrm{I}} = g_{\mathsf{H}}^{1/2-\alpha}$$
which is strongly coupled if $\alpha < 1/3$.

- The HE dual has 5 radii of size R and 1 of size $1/R$. The type II dual has coupling $g^{3\alpha-1}$ but 3 small radii of size $g, g^{\alpha}, g^{-\alpha}$. It is strongly coupled for all α.

In the $E_8 \times E_8$ case, the same scaling gives :

- a type IIA dual with coupling $g \sim g_{\mathsf{H}}^{3\alpha-1}$, small at $\alpha < 1/3$, but in this region the $R_{5,6}$ are much smaller than $l_{\mathrm{II}} \sim g_{\mathsf{H}}^{1-2\alpha}$. The T-dual type II theory is non-perturbative, except at the marginal case $\alpha = 1/3$.

- a type I dual with coupling $g_{\mathrm{I}} \sim g_{\mathsf{H}}^{\alpha-1}$ but small radii $R/l_{\mathrm{I}} = g_{\mathsf{H}}^{(3\alpha-1)/2}$, $R/l_{\mathrm{I}} = g_{\mathsf{H}}^{-(\alpha+1)/2}$. The T-dual type I string is always strongly coupled.

Type IIA theories with low string scale

Consider for simplicity $IIA/K^3 \times T^2(R_5, R_6)$:

$$S_{\rm IIA} = \int d^4x\sqrt{-g}\left(\frac{1}{g^2_{\rm 6IIA}}\frac{R_5R_6}{l^4_{\rm II}}\mathcal{R} + \frac{R_5R_6}{l^2_{\rm II}}F^2\right)$$

gives the gauge coupling and 4D Planck length

$$\frac{1}{g^2_{\rm YM}} = \frac{R_5R_6}{l^2_{\rm II}} \qquad , \qquad g_{\rm 6IIA} = \frac{1}{g_{\rm YM}}\frac{l_{\rm P}}{l_{\rm II}}\,.$$

- R_5R_6 is of order $l^2_{\rm II}$ to keep the gauge coupling of order unity, while $g_{\rm 6IIA}$ is a free parameter that allows to move the string length away from the Planck scale :
$$l^{-1}_{\rm II} \sim TeV \ , \qquad g_{\rm II} \sim 10^{-14}$$
- A non-Abelian symmetry only arises at singularities of K_3 from D2-branes wrapping vanishing two-cycles, even though the coupling is very weak. Gauge interactions are localized on a NS5-brane.
- String interactions are suppressed by the Planck mass. The only signal is $N = 4$ KK excitations of gauge bosons. Matter fields are further localized on the base and have no KK excitations.
- The decoupling of string modes may not be so obvious and logarithmic coupling dependent threshold may allow a dynamical determination of the hierarchy.

Type IIB theories with low string scale

On the type IIB/K_3 side,

$$S_{\rm IIB} = \int d^4x\sqrt{-g}\left(\frac{1}{g^2_{\rm 6IIB}}\frac{R_5R_6}{l^4_{II}}\mathcal{R} + \frac{R_5}{R_6}F^2\right) ,$$

leads to the identifications:

$$\frac{1}{g^2_{\rm YM}} = \frac{R_5}{R_6} \qquad , \qquad g_{\rm 6IIB} = \frac{l_{\rm P}}{l_{\rm II}}\sqrt{\frac{R_5R_6}{l^2_{\rm II}}} = g_{\rm YM}\frac{Rl_{\rm P}}{l^2_{\rm II}} .$$

- Keeping the Yang-Mills coupling of order unity now implies that R_5 and R_6 are of the same order, $R_5 = R_6/g^2_{\rm YM} \equiv R$, while $g_{\rm 6IIB}$ is a free parameter.

- Gauge interactions propagate in the two large dimensions $R_{5,6}$, and are described by 6D tensionless strings.

- Weak 10D coupling $g_{\rm II} = g_{\rm 6IIB}(V_{K_3}/l^4_{\rm II})^{1/2} < 1$ implies $l_{\rm II} \geq \sqrt{g_{\rm YM}Rl_{\rm P}}$,when all K_3 radii have string scale size, so

  $$R^{-1} \sim TeV \ , \quad l_{\rm I} \sim 10^{11}GeV \ , \quad g_{\rm II} \sim 1$$

  Gravity becomes strong at the intermediate string scale.

Large dimensions in heterotic string

Consider the scaling

$$g_{\mathsf{H}} = g_{\mathsf{YM}} \left(\frac{R}{l_{\mathsf{H}}}\right)^{n/2} \gg 1 \ , \qquad l_{\mathsf{P}} = g_{\mathsf{YM}} l_{\mathsf{H}}$$

The type I dual has

$$l_{\mathrm{I}} = g_{\mathsf{YM}}^{1/2} R^{\frac{n}{4}} l_{\mathsf{H}}^{1-\frac{n}{4}} \ , \qquad g_{\mathrm{I}} = \frac{1}{g_{\mathsf{YM}}} \left(\frac{l_{\mathsf{H}}}{R}\right)^{\frac{n}{2}} \ ,$$

$$\text{Type I:} \quad \left\{ \begin{array}{lccccc} n = 1,2,3,4: & l_{\mathsf{H}} & < & l_{\mathrm{I}} & < & R \\ n = 5,6: & l_{\mathsf{H}} & < & R & < & l_{\mathrm{I}} \end{array} \right.$$

There are dimensions ($6-n$ or 6 respectively) with size smaller than the type I string length, which should be T-dualised to trade light winding modes for KK states. The type I' dual has

$$\widehat{l_{\mathsf{H}}} = g_{\mathsf{YM}} R^{\frac{n}{2}} l_{\mathsf{H}}^{\frac{1-n}{2}} \ , \quad \widehat{R} = g_{\mathsf{YM}} R^{\frac{n-2}{2}} l_{\mathsf{H}}^{\frac{2-n}{2}}$$

$$\left\{ \begin{array}{lllccccc} n = 1,2 & : & g_{\mathrm{I}'} = g_{\mathsf{YM}}^{\frac{4-n}{2}} \left(\frac{R}{l_{\mathsf{H}}}\right)^{\frac{n(4-n)}{4}} \ , & l_{\mathrm{I}} & < & \widehat{l_{\mathsf{H}}} & < & R \\ n = 3,4 & : & g_{\mathrm{I}'} = g_{\mathsf{YM}}^{\frac{4-n}{2}} \left(\frac{R}{l_{\mathsf{H}}}\right)^{\frac{n(4-n)}{4}} \ , & l_{\mathrm{I}} & < & R & < & \widehat{l_{\mathsf{H}}} \\ n = 5,6 & : & g_{\mathrm{I}'} = g_{\mathsf{YM}}^{2} \ , & l_{\mathrm{I}} & < & \widehat{R} & < & \widehat{l_{\mathsf{H}}} \end{array} \right.$$

Weakly coupled type I' dual

$$
\begin{array}{c|cccc}
\text{HO } n=6 & 1 & \text{R} & g_{\text{YM}}^{1/2}R^{3/2} & g_{\text{YM}}R^2 \\
\updownarrow & \longrightarrow & & & \\
\mathrm{I}'_3, g_{\mathrm{I}'} = g_{\text{YM}}^2 & l_{\text{H}} & & l_{\mathrm{I}} & \widehat{R}_{1,2,3,4,5,6}
\end{array}
$$

is the type I' with 6 fermi-size transverse dimensions.

$$
\begin{array}{c|ccccc}
\text{Het } n=5 & 1 & & g_{\text{YM}}^{1/2}R^{5/4} & g_{\text{YM}}R^{3/2} & g_{\text{YM}}R^{5/2} \\
\updownarrow & \longrightarrow & & & & \\
\mathrm{I}'_3, g_{\mathrm{I}'} = g_{\text{YM}}^2 & l_{\text{H}}, R_6 & \text{R} & l_{\mathrm{I}} & \widehat{R}_{1,2,3,4,5} & \widehat{R}_6
\end{array}
$$

is a type I' with 1+5 large transverse dimensions.

$$
\begin{array}{c|cccc}
\text{Het } n=4 & 1 & g_{\text{YM}}^{1/2}R & R & g_{\text{YM}}R^2 \\
\updownarrow & \longrightarrow & & & \\
\mathrm{I}'_7, g_{\mathrm{I}'} = 1 & l_{\text{H}} & l_{\mathrm{I}} & R_{1,2,3,4} & \widehat{R}_{5,6}
\end{array}
$$

is the type I' theory with 2 mm-size transverse dimensions.

$$
\begin{array}{c|ccccc}
\text{Het } n=4 & 1 & R & g_{\text{YM}}^{1/2}R & g_{\text{YM}}R & g_{\text{YM}}R^2 \\
\updownarrow & \longrightarrow & & & & \\
\mathrm{I}'_3, g_{\mathrm{I}'} = g_{\text{YM}}^2 > 1 & l_{\text{H}} & & l_{\mathrm{I}} & \widehat{R}_{1,2,3,4} & \widehat{R}_{5,6}
\end{array}
$$

is the same theory with 2 mm-size transverse directions, after T-dualising 4 string-size dimensions.

Weakly coupled type II dual

$$l_{\rm II} = g_{\sf YM}\sqrt{R_5 R_6}\ , \qquad g_{\rm II} = \frac{1}{g_{\sf YM}}\frac{R_1}{\sqrt{R_5 R_6}}$$

$$R_I = g_{\sf H} l_{\sf H}\ , \qquad \tilde{R}_i = \frac{g_{\sf H} l_{\sf H}^3}{R_j R_k}\ ,\ i,j,k = 2,3,4$$

Choose R_5, R_6 as the possibly large radii:

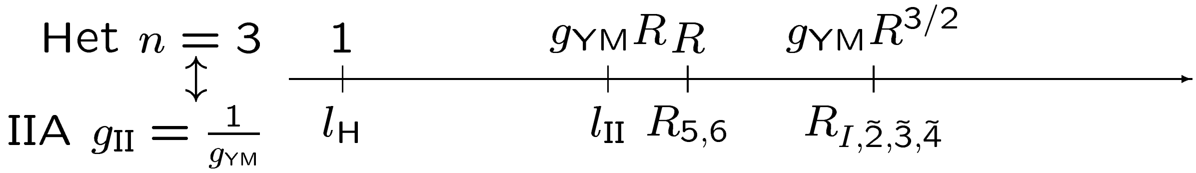
$$\begin{array}{lccccc} \text{Het } n=1 & 1 & g_{\sf YM}R^{1/2} & g^2_{\sf YM}R & R & \\ \updownarrow & | & | & | & | & \longrightarrow \\ \text{IIB } g_{\rm II}=1 & l_{\sf H} & l_{\rm II}, R_{I,\tilde{2},\tilde{3},\tilde{4}} & \hat{R}_6 & R_5 & \end{array}$$

is the type IIB model with two TeV radii and string scale at intermediate energy 10^{11} GeV.

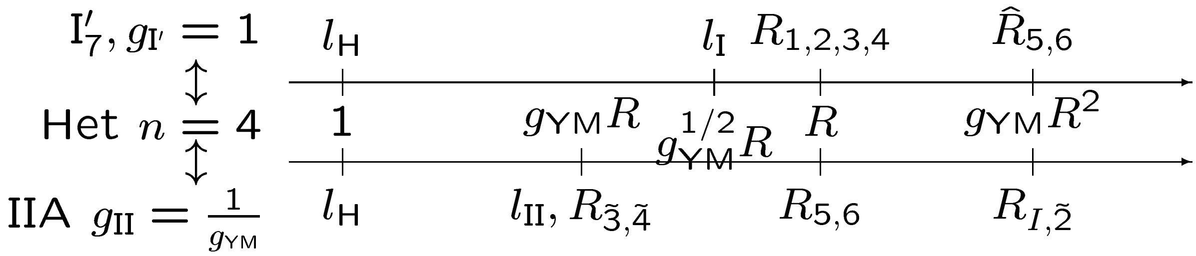
$$\begin{array}{lcccc} \text{Het } n=2 & 1 & g_{\sf YM}R & R & \\ \updownarrow & | & | & | & \longrightarrow \\ \text{IIA } g_{\rm II}=\frac{1}{g_{\sf YM}R} & l_{\sf H} & l_{\rm II}, R_{I,\tilde{2},\tilde{3},\tilde{4}} & R_{5,6} & \end{array}$$

is the type IIA model with string scale and all internal dimensions at the TeV, and $g_{\rm II} \sim 10^{-14}$

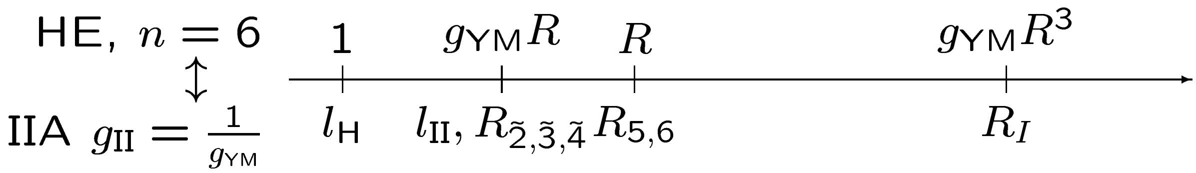
For $n > 2$, choose $R_1, R_{2,3,4}$ as the large radii :



has the string scale and two longitudinal directions at TeV, and an isotropic K_3 at a fermi.



gives two perturbative descriptions of the same theory with 2 mm-size transverse directions.



is notably different from the HO $n = 6$ case: the bound $R_I < 1$ mm implies a string scale $l_{\mathrm{II}} > 10^8$ GeV.

Conclusions

- Duality relates descriptions with different couplings and geometrical hierarchy. The moduli space of $N = 32$ theories in $D \geq 4$ is entirely covered by weak coupling descriptions.

- Type I' strings offer a tractable framework to bring the string scale within reach of next round accelerators. It can also be tested by table-top experiments.
Cavendish 1798

- Type IIA strings at a singularity offer an alternative that does not require large radii. The string coupling is very weak, even though the gauge symmetry is non-perturbative in origin. Type IIB strings can accomodate two longitudinal TeV size radii, with intermediate string length.

- Bringing the string scale to a TeV nullifies the issue of supersymmetry breaking. Proton decay needs to be adressed. The gauge hierarchy problem is turned into a geometric hierarchy issue, and moduli stabilisation is as problematic as ever. The incomplete decoupling of transverse dimensions may provide a solution.
Antoniadis, Bachas

- Gauge unification may still occur above the string scale. Large internal radii do not spoil unification even though the running is no more log, and SUSY may still protect the running above the string scale.
Dienes,Dudas,Ghergetta;Bachas