Closed Strings in the Misner Universe

aka the Lorentzian orbifold

Boris Pioline
LPTHE, Paris

Imperial College, London
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based on hep-th/0307280 w/ M. Berkooz
and hep-th/0405xxx w/ M. Berkooz, and M. Rozali

slides available from
`http://www.lpthe.jussieu.fr/~pioline/seminars.html`

Motivational string cosmology

- Observational Cosmology is now challenging string theory with high-precision data:

$$\Omega_{baryon} = 0.047 \; , \quad \Omega_{darkm} = 0.243 \; , \quad \Omega_{\Lambda} = 0.71 \; , \quad w = -0.98 \pm .12 \; , \ldots$$

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- With LHC still far in the future, understanding StringY Cosmology may be the only way to make contact with reality...

Brandenberger Vafa

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- Perturbative string theory requires an Euclidean worldsheet, hence Euclidean target space. The analytic continuation may be ambiguous or ill-defined, Lorentzian observables may be very different from their Euclidean counterparts.
- String theory is not content on a finite time interval, and one is frequently forced into Big Bang / Big Crunch singularities, CTC in the process of maximally extending the geometry.

String theory and cosmological singularities

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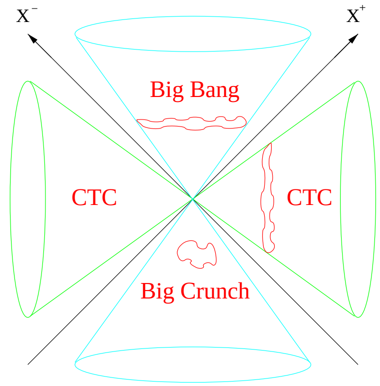
- In this talk, we shall discuss the "Lorentzian" orbifold of flat Minkowski space by a discrete boost, as a toy model of a singular cosmological universe where string theory can in principle be solved explicitely.



Outline of the talk

1. Introduction

2. The Lorentzian orbifold and its avatars

Misner, Taub-NUT, Grant...

3. Closed strings in Misner space: first pass

Nekrasov

3. A detour: Open strings in electric fields

Bachas Porrati; Berkooz BP

4. Closed strings in Misner space: second pass

Berkooz BP; Berkooz BP Rozali

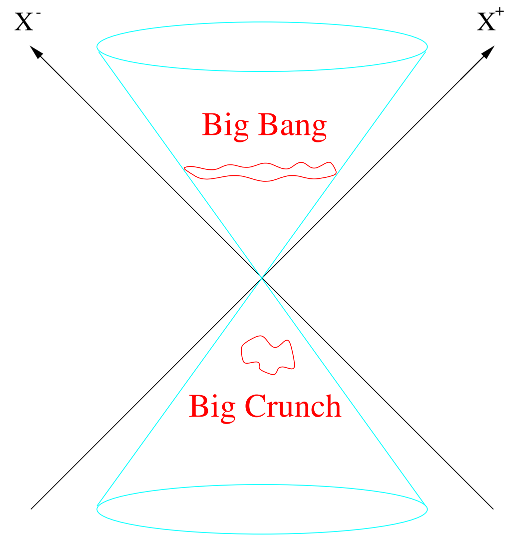
5. Comments on backreaction from winding strings

The Lorentzian orbifold

- One of the simplest examples of space-like singularities is the quotient of flat Minkowski space by a discrete boost, also known as Misner space (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^\pm \sim e^{\pm 2\pi\beta} X^\pm$$



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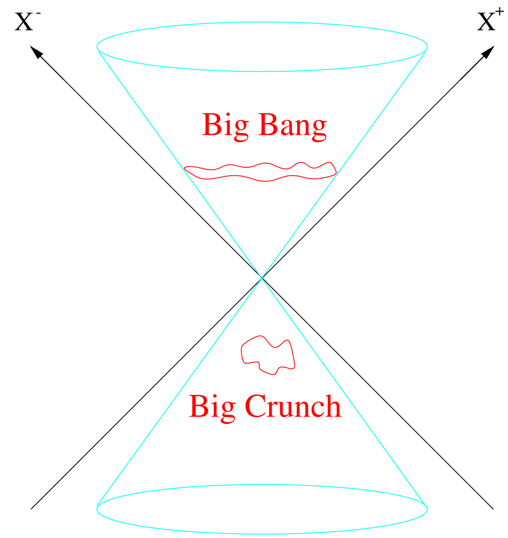

- The future (past) regions $X^+X^- > 0$ describes a cosmological universe often known as the Milne universe (1932), linearly expanding away from a Big Bang singularity (or contracting into a Big Crunch singularity):

$$ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + (dX^i)^2 , \quad \theta \equiv \theta + 2\pi , \quad X^\pm = T e^{\pm\beta\theta}/\sqrt{2}$$

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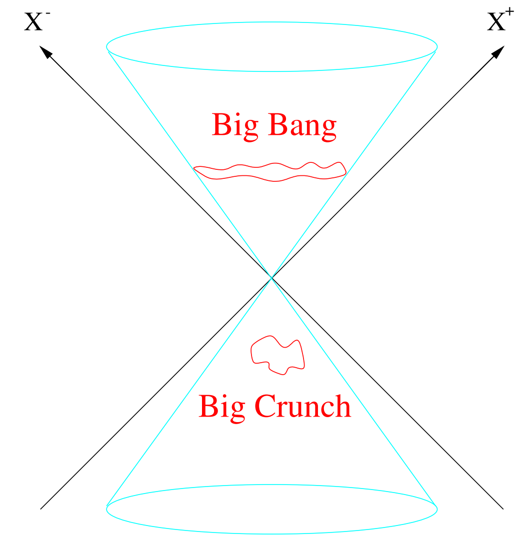

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This is a (degenerate) Kasner singularity, everywhere flat, except for a delta-function curvature at $T = 0$.

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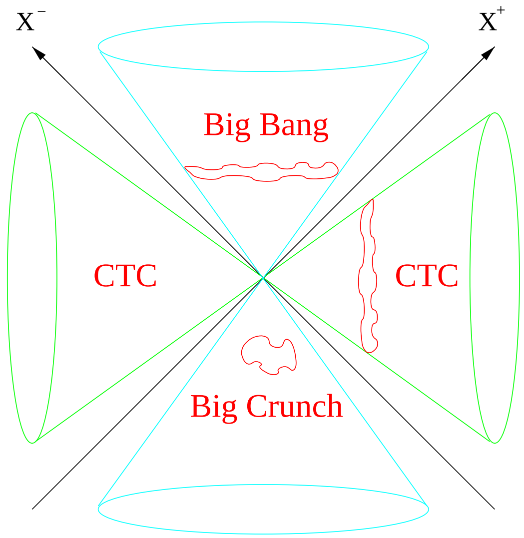

- In addition, the spacelike regions $X^+X^- < 0$ describe two Rindler wedges with compact time, often known as whiskers, leading to closed time-like curves:

$$ds^2 = dr^2 - \beta^2 r^2 d\eta^2 + (dX^i)^2 \quad , \eta \equiv \eta + 2\pi \; , \quad X^\pm = \pm r e^{\pm\beta\eta}/\sqrt{2}$$

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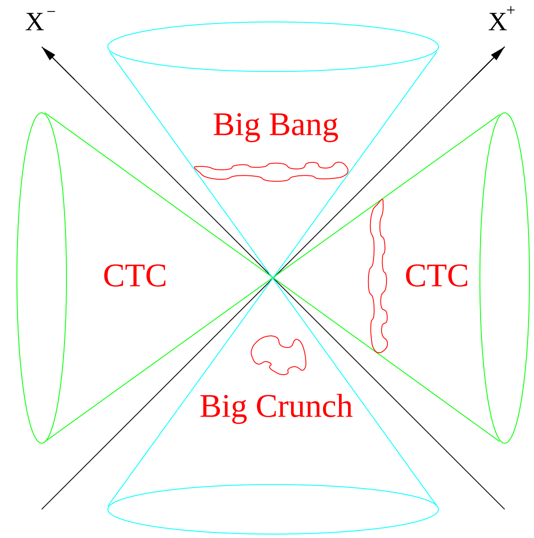

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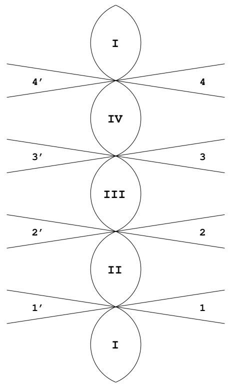
- Finally, the lightcone $X^+X^- = 0$ gives rise to a null, non-Hausdorff locus attached to the singularity.

Close relatives of the Misner Universe

- Misner space was first introduced as a local model of Lorentzian Taub-NUT space:



$$ds^2 = 4l^2U(t)\sigma_3^2 + 4l\sigma_3 dt + (t^2 + l^2)(\sigma_1^2 + \sigma_2^2)\ , \quad U(t) = -1 + \frac{2mt + l^2}{t^2 + l^2}$$

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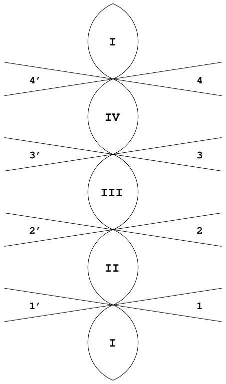

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- A close variant of Misner space is the quotient of flat space by the combination of a discrete boost and a translation on an extra direction, often known as the Grant space:

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This describes the space away from two moving cosmic strings. The cosmological singularity is smoothed out, but regions with CTC remain.

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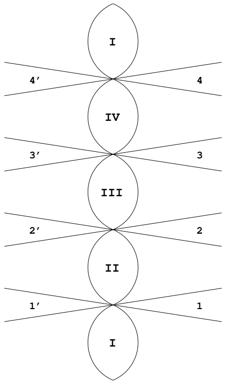

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- The Misner geometry arose again more recently as the M-theory lift of a simple (ekpyrotic) cosmological solution of Einstein-dilaton gravity with no potential.

Khoury Ovrut Seiberg Steinhard Turok

Close relatives of the Misner Universe (cont)

- The gauged WZW model $Sl(2) \times Sl(2)/U(1) \times U(1)$ describes a bouncing 4-dimensional Universe, with singularities analogous to the Lorentzian orbifold.

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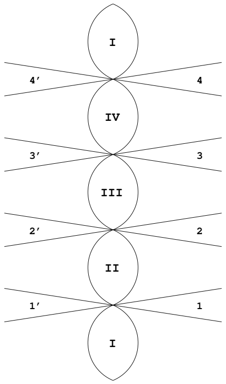

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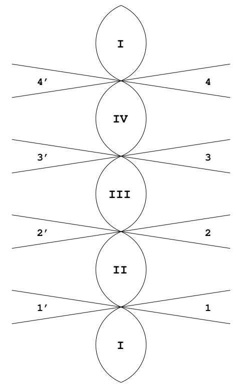

- The gauged WZW model $Sl(2)/U(1)$ at negative level orbifolded by a boost J describes two parallel Universes with a curvature and a Milne singularity, and compact whiskers.

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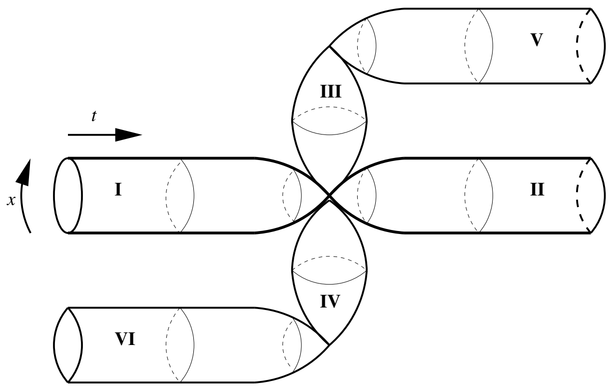

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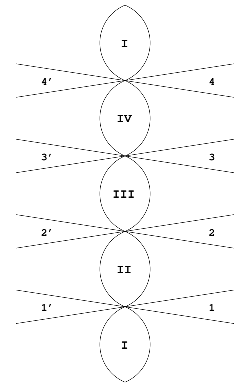

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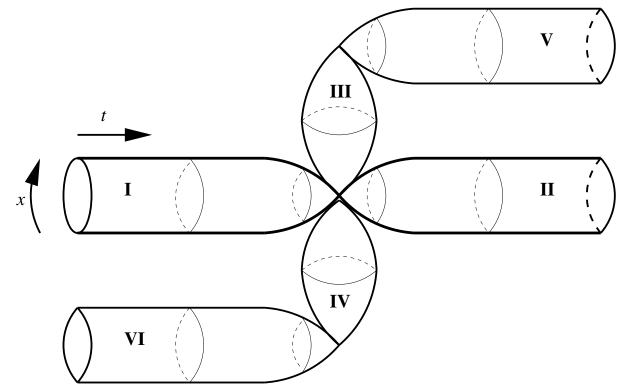


- The Lorentzian orientifold $IIB/[(-)^F boost]/[\Omega(-)^{F_L}]$ was also recently argued to describe orientifolds of non-supersymmetric strings with non-vanishing Neveu-Schwarz tadpoles.

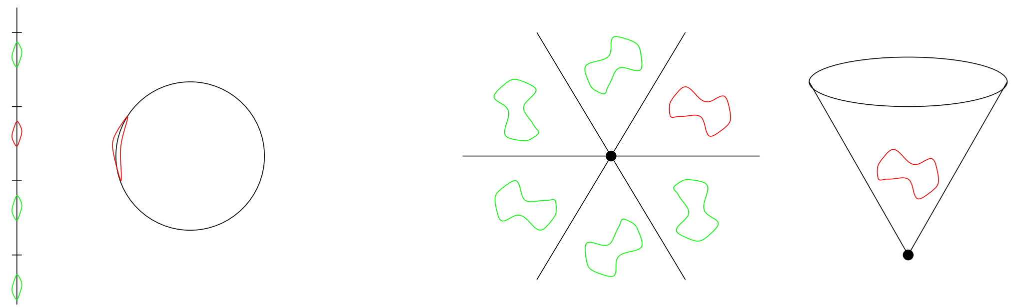
  Dudas Mourad Timirgaziu

Strings on Euclidean orbifolds - untwisted states

- Well-known examples of orbifolds are the circle, R/Z, and the rotation orbifold R^2/Z_k.

Dixon Harvey Vafa Witten

- The spectrum of the quotient theory contains closed string states of the parent theory which are invariant under G: untwisted states.



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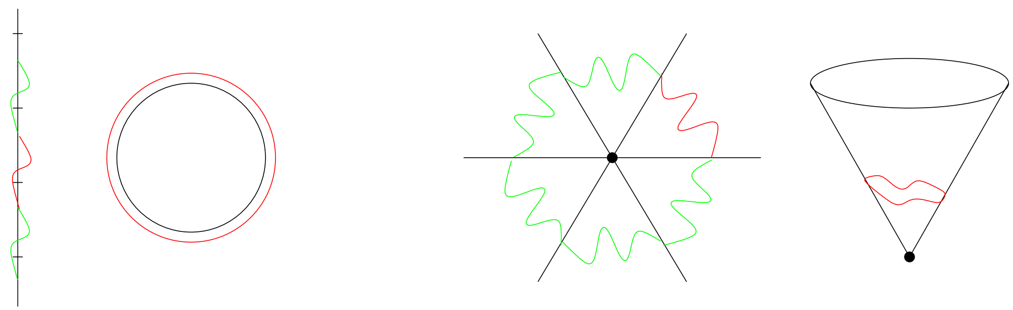

- When G acts non-freely, the twisted sector states are localized at the fixed points. They yield new localized degrees of freedom, which ensure the consistency of the background: anomaly free, divergence free...

Strings on Euclidean orbifolds - twisted sectors (cont.)

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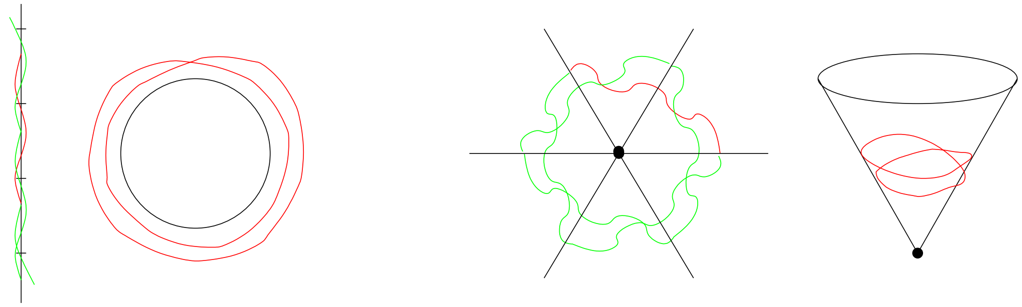
- Twisted sectors are labelled by conjugacy classes of G. Higher twisted sectors correspond to multiply wound states.



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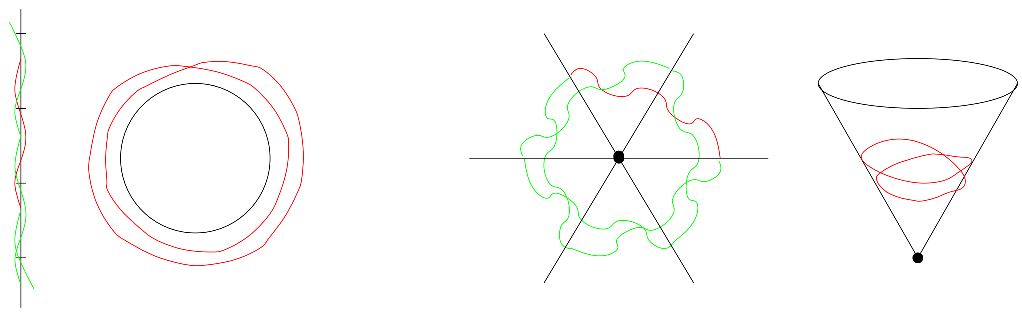

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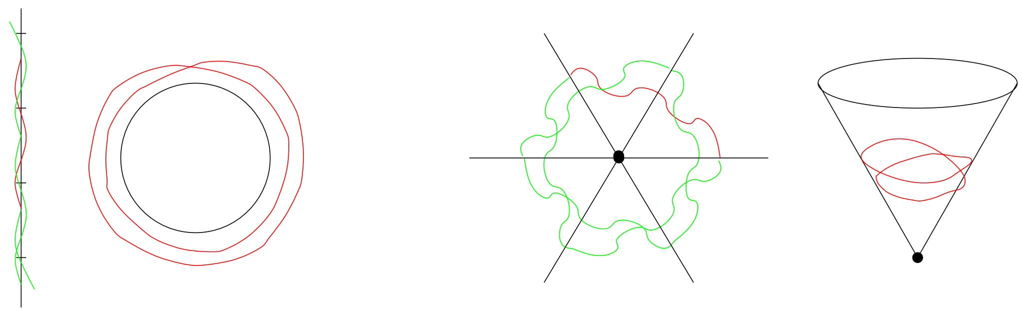

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- The condensation of these twisted states changes the vacuum, and effectively resolves the singularity: $R^2/Z_k \to R^2/Z_{k-1} \to \ldots$ (tachyon), $R^4/Z_k \to$ multi-centered Eguchi-Hanson (massless mode).

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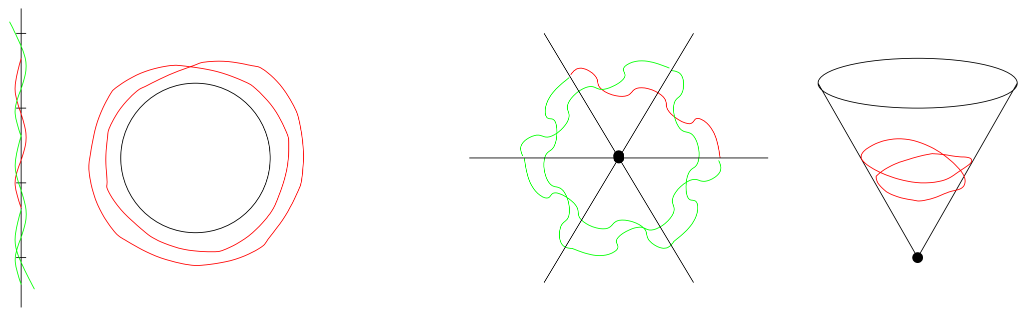

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- The Lorentzian orbifold shares features with both examples: an infinite number of winding sectors, and a, non compact, fixed locus.

Closed strings in Misner space - untwisted states

- As usual in standard orbifolds, part of the spectrum involves closed strings on Minkowski covering space, which are invariant under the orbifold projection. In conformal gauge,

$$X^{\pm}(\sigma + 2\pi, \tau) = X^{\pm}(\sigma, \tau) \ , \quad (\partial_{\tau}^{2} - \partial_{\sigma}^{2})X^{\pm} = 0$$

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- Vertex operators (or states) can be obtained by (infinite) sum over images, e.g.

$$\sum_{l=-\infty}^{\infty} \partial X^+ \bar{\partial} X^- \exp\left(i k^+ X^- e^{-2\pi\beta l} + i k^- X^+ e^{2\pi\beta l} + i k_i X^i \right)$$

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- Equivalently, after Poisson resummation over l, this is a superposition of states with integer boost momentum $j = i(x^+\partial_+ - x^-\partial_-)$,

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- The resulting eigenfunctions describe (topologically trivial) closed strings traveling around the Milne circle with integer momentum j.

Quantum fluctuations in field theory

- In the Minkowski vacuum (inherited from the covering space), the renormalized propagator can be obtained as a sum over images,

$$\begin{aligned} G(x;x') &= \sum_{l=-\infty, l\neq 0}^{\infty} \int_0^{\infty} d\tau \int dp^{\mu} \\ & \exp\Big(-ip^-(x^+ - e^{2\pi\beta l}x^{+'}) - ip^+(x^- - e^{2\pi\beta l}x^{-'}) - ip^i(x^i - x^{i'})\Big) \end{aligned}$$

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- The one-loop stress-energy tensor follows from $G(x,x)$, e.g for a conformally coupled scalar,

$$\langle T_{ab}\rangle = \lim_{x\to x'}\left[(1-2\xi)\nabla_a\nabla'_b - 2\xi\nabla_a\nabla_b + (2\xi - \frac{1}{2})g_{ab}\nabla_c\nabla'^c\right]G(x,x')$$

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This leads to a divergent quantum backreaction (worse if the spin $|s|>1$):

$$\langle T_\mu^\nu\rangle = \frac{K}{12\pi^2}T^{-4}\text{diag}(1,-3,1,1)\ , \quad K = \sum_{l=1}^{\infty}\cosh(2\pi\beta l s)\frac{2+\cosh 2\pi l\beta}{[\cosh 2\pi l\beta - 1]^2}$$

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One-loop vacuum amplitude in field and string theory

- On the other hand, in string theory $\langle T_\mu^\nu \rangle(x)$ is an off-shell quantity, and only its integral over space-time is well defined:

$$\int dx^+ dx^- G(x,x) = \sum_{l=-\infty}^{+\infty} \int_0^\infty \frac{d\rho}{\rho^{D/2}} \frac{e^{-m^2\rho}}{\sinh^2(\pi\beta l)}$$

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- The local divergence in $\langle T_\mu^\nu \rangle(x)$ is integrable and yields a finite free energy.
- The existence of Regge trajectories with arbitrary high spin implies new (log) divergences in the bulk of the moduli space which resemble long string poles in AdS_3.

Scattering of untwisted states

- Tree-level scattering amplitudes of untwisted sector states can be computed from those in flat space by the inheritance principle,

$$\langle V(j_1,k_1)\dots V(j_n,k_n)\rangle_{Misner} = \int dv_1\dots dv_n\; e^{i(j_1 v_1+\dots+j_n v_n)}$$
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- The integral diverges due to Regge behavior in the large momentum, fixed angle regime. E.g, the four-tachyon scattering amplitude in bosonic string leads to

$$\int dv \; v^{-\frac{1}{2}(k_1^i - k_3^i)^2 + i(j_2 - j_4)}$$

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Berkooz Craps Rajesh Kutasov

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- Could higher order corrections,e.g. resummation of ladder diagrams, lead to a finite amplitude ?

Deser McCarthy Steif; Cornalba Costa

Closed string in Misner space - twisted sectors

- In addition, there is an infinite set of twisted sectors, corresponding to strings on the covering space that close up to the action of the orbifold group:

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- They have a normal mode expansion:

$$\begin{aligned} X_R^{\pm}(\tau - \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha_n^{\pm} \, e^{-i(n \pm i\nu)(\tau - \sigma)} \\ X_L^{\pm}(\tau + \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \mp i\nu)^{-1} \tilde{\alpha}_n^{\pm} \, e^{-i(n \mp i\nu)(\tau + \sigma)} \end{aligned}$$

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with canonical commutation relations

$$[\alpha_m^+, \alpha_n^-] = -(m + i\nu)\delta_{m+n} \quad , \quad [\tilde{\alpha}_m^+, \tilde{\alpha}_n^-] = -(m - i\nu)\delta_{m+n}$$

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- We will focus on the quasi zero-mode sector, which consists of two commuting pairs of real (i.e. hermitian) canonically conjugate operators,

$$[\alpha_0^{+}, \alpha_0^{-}] = -i\nu \ , \quad [\tilde{\alpha}_0^{+}, \tilde{\alpha}_0^{-}] = i\nu$$

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated by half of them, e.g.

$$\alpha^{\pm}_{n>0}\,, \quad \tilde{\alpha}^{\pm}_{n>0}\,, \qquad \alpha^{-}_{0}\,, \quad \tilde{\alpha}^{+}_{0}$$

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$$L_0^{l.c.} = -\sum_{n=0}^{\infty}(\alpha_n^{+})^{*}\alpha_n^{-} \; - \sum_{n=1}^{\infty}(\alpha_n^{-})^{*}\alpha_n^{+} + \frac{1}{2}i\nu(1-i\nu) - 1 + L_{int}$$

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- This is the familiar result for the vacuum energy $\frac{1}{2}\theta(1-\theta)$ in the Euclidean rotation orbifold, after analytically continuing $\theta \to i\nu$.
- Due to the $i\nu/2$ term in the ground state energy, all states obtained by acting on $|0\rangle$ by creation operators $\alpha^{\pm}_{n<0}$ and by α^{+}_{0} will have imaginary energy, hence the physical state conditions $L_0 = \tilde{L}_0 = 0$ seem to have no solutions.

One-loop amplitude, twisted sector

- Independently of this fact, one may compute the one-loop path integral on an Euclidean worldsheet and Minkowskian target-space:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho)x\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

where θ_1 is the Jacobi theta function,

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- In the twisted sector, the left-moving zero-modes contribute

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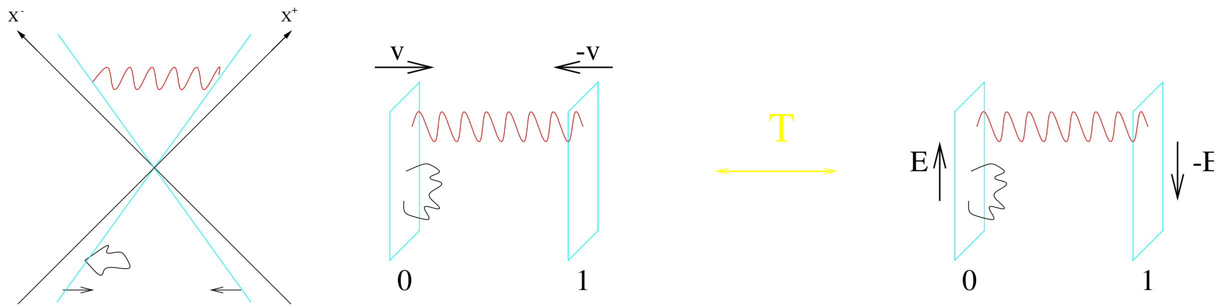
- The absence of physical twisted states crushes our hopes for resolving the singularity... yet does not sound very sensible. An important point: α_0^+ and α_0^- are not hermitian conjugate to each other, but rather self-hermitian...

A detour via Open strings in electric field

- A very similar puzzle is faced in the case of colliding D-branes, or in the T-dual process of charged open strings in a constant electric field:



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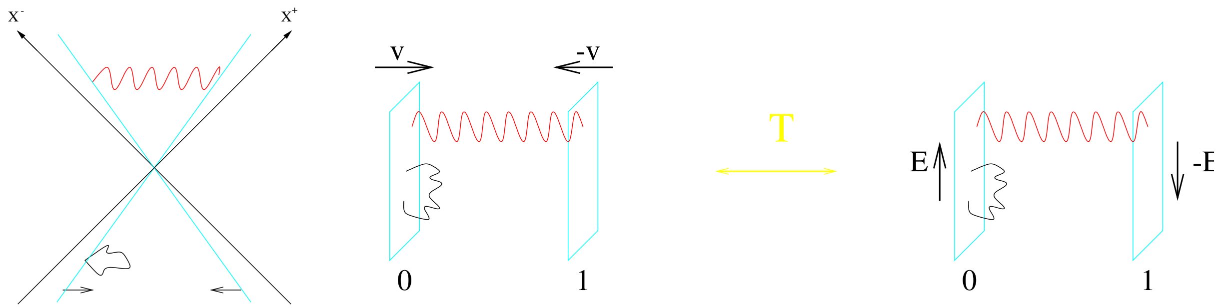

- Recall that for open strings stretched between two D-branes with electromagnetic fields F_0 and F_1, proper frequencies satisfy

$$e^{-2\pi i \omega_n} = \frac{1+F_0}{1-F_0} \cdot \frac{1-F_1}{1+F_1}$$

For $F_0 \neq F_1$, the open string carries a net electric charge, and the motion of its center of motion is that of a charged particle.

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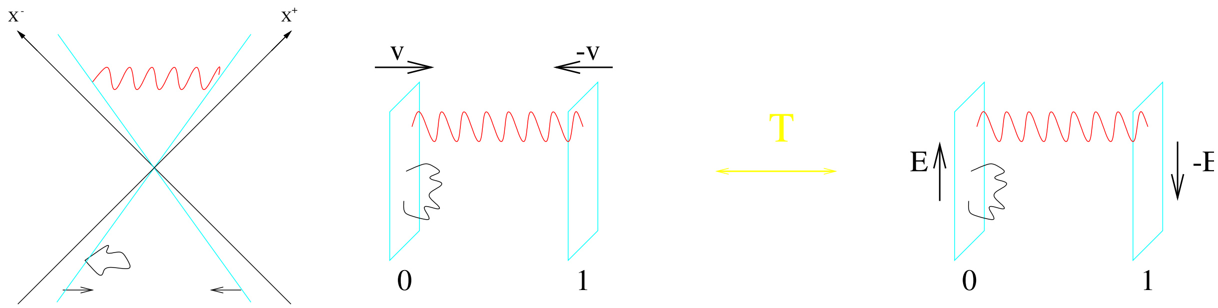

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- In the case of an electric field $F_1 = E dx^+ \wedge dx^-$, $F_1 = 0$, the resulting spectrum is

$$\omega_n = n + i\nu \ , \qquad \nu := \text{Arctanh} E = w\beta$$

just as in the Lorentzian orbifold case. The large winding number limit $w \to \infty$ amounts to a near critical electric field $E \to 1$.

Open string mode expansion

- The light-cone embedding coordinates have the normal mode expansion

$$X^{\pm} = x_0^{\pm} + i \sum_{n=-\infty}^{+\infty} (-)^n (n \pm i\nu)^{-1} a_n^{\pm} e^{-i(n \pm i\nu)\tau} \cos[(n \pm i\nu)\sigma]$$

with reality conditions $(a_n^{\pm})^* = a_{-n}^{\pm}\ , \quad (x_0^{\pm})^* = x_0^{\pm}$

Open string mode expansion

- The light-cone embedding coordinates have the normal mode expansion

$$X^{\pm} = x_0^{\pm} + i \sum_{n=-\infty}^{+\infty} (-)^n (n \pm i\nu)^{-1} a_n^{\pm} e^{-i(n \pm i\nu)\tau} \cos[(n \pm i\nu)\sigma]$$

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- By the same token, charged open strings should have no physical states... yet electrons and positrons certainly do exist.

Charged particle and open string zero-modes

- Let us recall the quantization of a charged particle in an electric field:

$$L = \frac{1}{2}m\left(-2\partial_\tau X^+\partial_\tau X^- + (\partial_\tau X^i)^2\right) + \frac{\nu}{2}\left(X^+\partial_\tau X^- - X^-\partial_\tau X^+\right)$$

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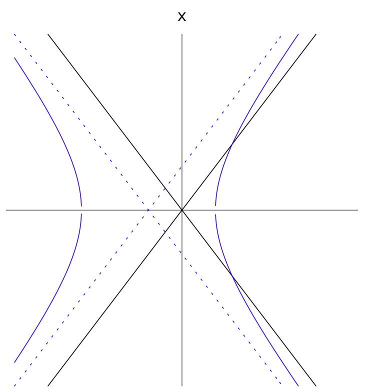

- The classical trajectories are identical to the open string zero-mode:

$$X^\pm = x_0^\pm \pm \frac{1}{\nu}a_0^\pm e^{\pm\nu\tau}$$

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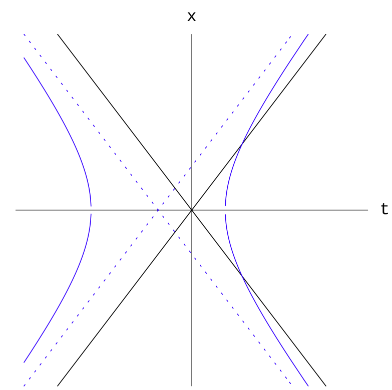

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- Starting from the canonical equal-time commutation rules

$$[\pi^+, x^-] = [\pi^-, x^+] = i \ , \quad [\pi^i, x^j] = i\delta_{ij}$$

one recovers the open string zero-mode commutation relations

$$[a_0^+, a_0^-] = -i\nu \ , \quad [x_0^+, x_0^-] = -\frac{i}{\nu}$$

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- Quantum mechanically, one may represent $\pi^{\pm} = i\partial/\partial x^{\mp}$ hence obtain $a_0^{\pm}, x_0^{\pm}$ as covariant derivatives

$$a_0^{\pm} = i\partial_{\mp} \pm \frac{\nu}{2} x^{\pm} \,, \quad x_0^{\pm} = \mp \frac{1}{\nu} \left(i\partial_{\mp} \mp \frac{\nu}{2} x^{\pm} \right)$$

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- The zero-mode piece of L_0, including the bothersome $\frac{i\nu}{2}$,

$$L_0^{(0)} = -a_0^+ a_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\nabla_0^+ \nabla_0^- + \nabla_0^- \nabla_0^+)$$

is just the Klein-Gordon operator of a particle of charge ν.

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^{\pm} = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator can be rewritten as an inverted harmonic oscillator:

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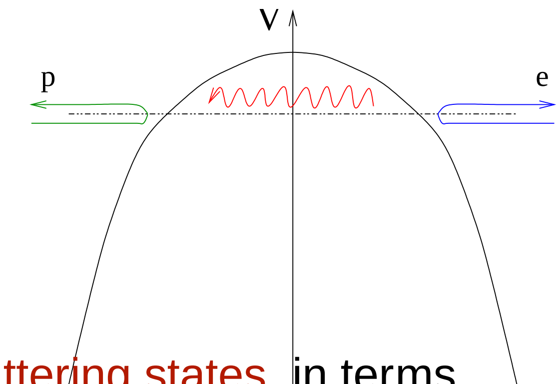
$$\left(-\partial_u^2 - \frac{1}{4}u^2 + \frac{M^2}{2\nu} \right) \psi_{\tilde{p}}(u) = 0$$



- The latter admits a respectable delta-normalizable spectrum of scattering states, in terms of parabolic cylinder functions, e.g:

$$\phi_{in}^+(x,t) = D_{-\frac{1}{2}+i\frac{M^2}{2\nu}}(e^{-\frac{3i\pi}{4}}u)e^{-i\tilde{p}t}e^{i\nu xt/2}$$

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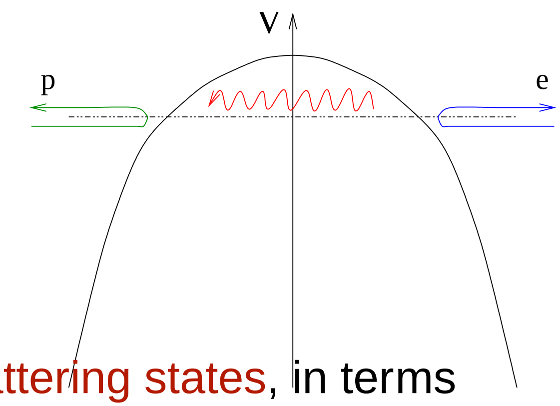

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- These correspond to non-compact trajectories of charged particles in the electric field. Tunnelling is just (stimulated) Schwinger pair creation,

$$e^- \to (1+\eta)\, e^- + \eta\, e^+\,, \quad \eta \sim e^{-\pi M^2/\nu}$$

Brezin Itzykson; Brout Massar Parentani Spindel

Lorentzian vs Euclidean states

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$$\frac{1}{2i\sin(\nu t/2)} = \sum_{n=1}^{\infty} e^{-i(n+\frac{1}{2})\nu t} = \int dM^2 \rho(M^2) e^{-M^2 t/2}$$

The density of states is obtained from the reflection phase shift,

$$\rho(M^2) = \frac{1}{\nu}\log\Lambda - \frac{1}{2\pi i}\frac{d}{dM^2}\log\frac{\Gamma\left(\frac{1}{2}+i\frac{M^2}{2\nu}\right)}{\Gamma\left(\frac{1}{2}-i\frac{M^2}{2\nu}\right)}$$

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- The physical spectrum can be explicitely worked out at low levels, and is free of ghosts: a tachyon at level 0, a transverse gauge boson at level 1, . . .

Charged particle in Rindler space

- For applications to the Milne universe, one should diagonalize the boost momentum J, ie consider an accelerated observer.

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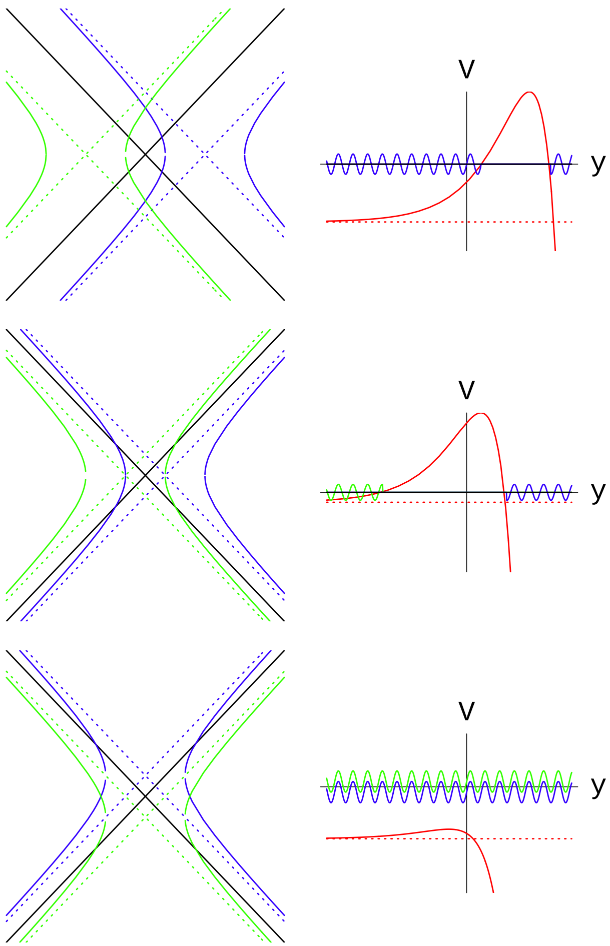

- In the Rindler patch R, letting $f(r, \eta) = e^{-ij\eta} f_j(r)$ and $r = e^y$, one gets a Schrödinger equation for a particle in a potential

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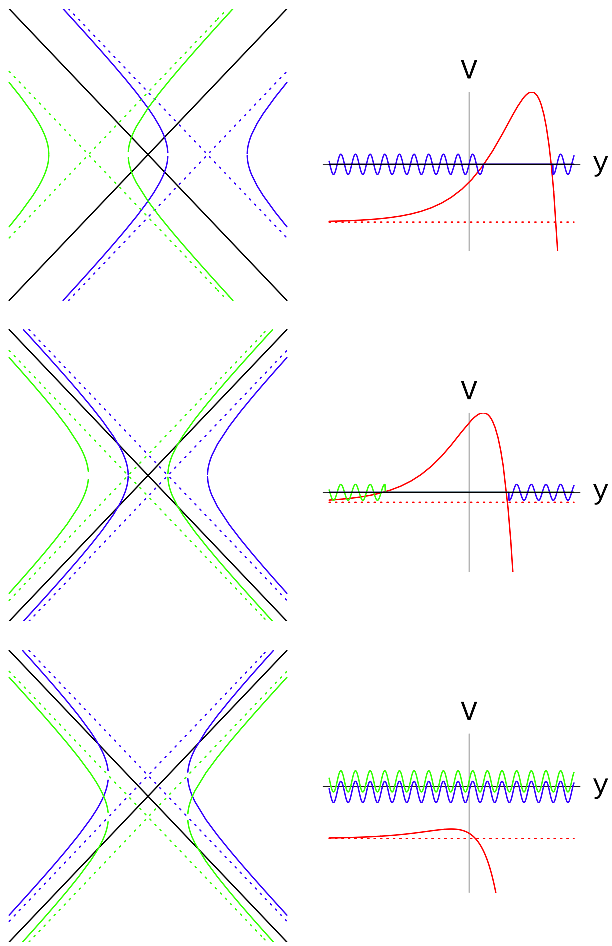

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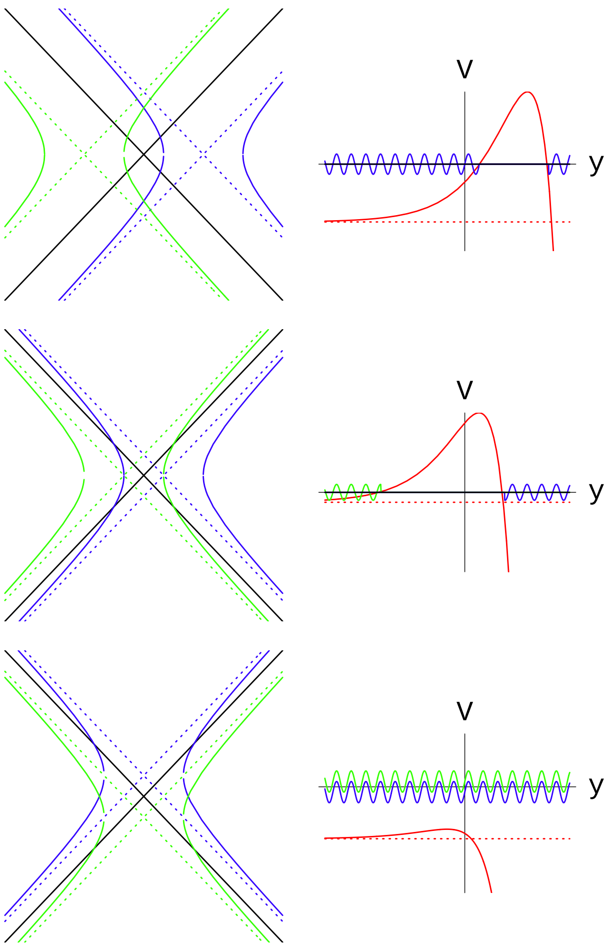

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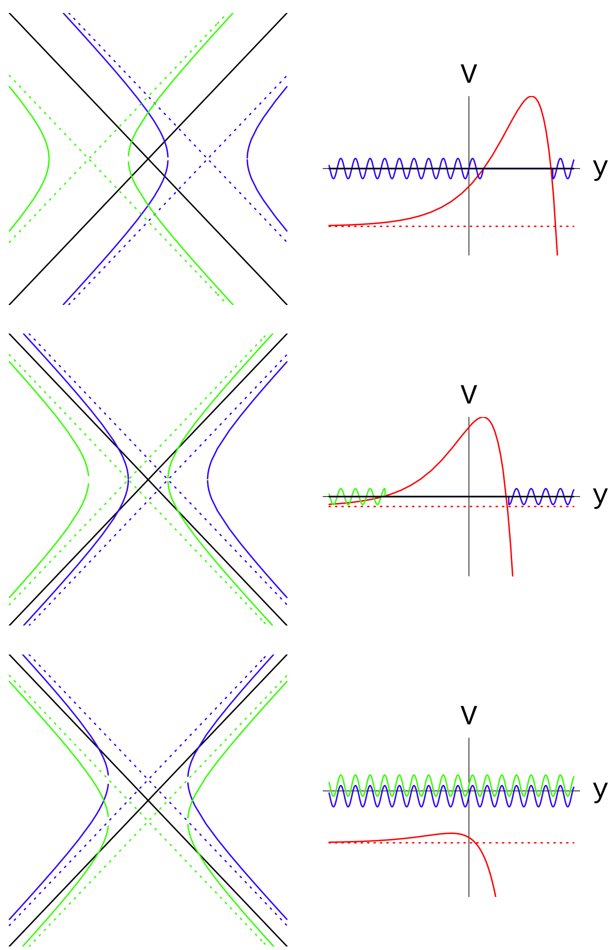

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- If $j > M^2/(2\nu)$, the electron branches extend in the Milne regions. There is no tunneling, but partial reflection amounts to a combination of Schwinger and Hawking emission.

Rindler modes

- Incoming modes from Rindler infinity I_R^- read, in terms of parabolic cylinder functions:

$$\mathcal{V}^j_{in,R} = e^{-ij\eta} r^{-1} M_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),-\frac{ij}{2}}(i\nu r^2/2)$$

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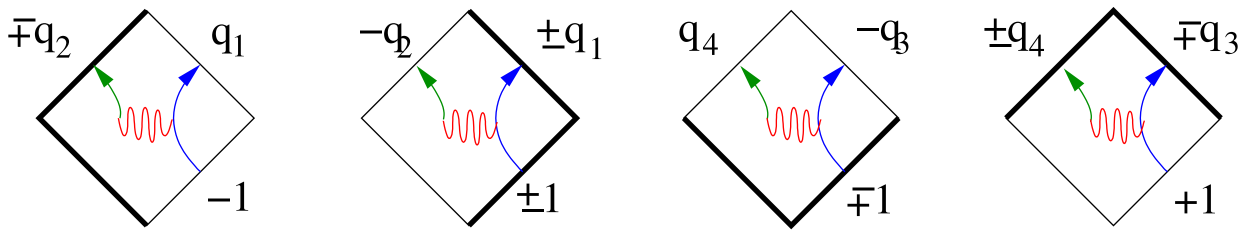

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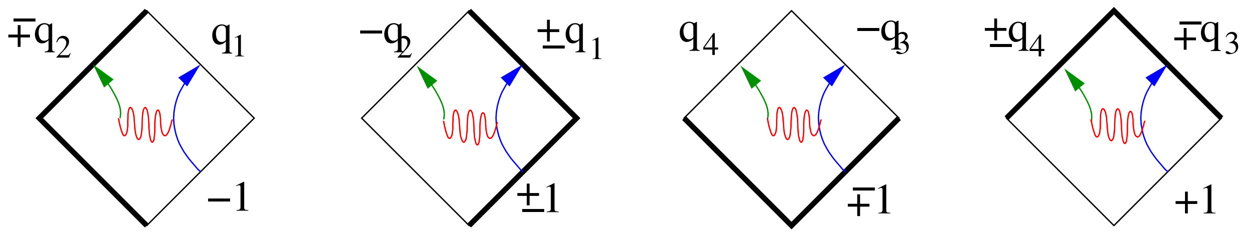

- The reflection coefficients can be computed:

$$q_1 = e^{-\pi j} \frac{\cosh\left[\pi \frac{M^2}{2\nu}\right]}{\cosh\left[\pi\left(j - \frac{M^2}{2\nu}\right)\right]}, \quad q_3 = e^{\pi\left(j-\frac{M^2}{2\nu}\right)} \frac{\cosh\left[\pi \frac{M^2}{2\nu}\right]}{|\sinh \pi j|}$$

and $q_2 = 1 - q_1$, $q_4 = q_3 - 1$, by charge conservation.

Global Charged Unruh Modes

- Global modes may be defined by patching together Rindler modes, ie by analytic continuation across the horizons. Unruh modes are those which are superposition of positive energy Minkowski modes,

$$\begin{aligned}
\Omega^j_{in,+} &= \mathcal{V}^j_{in,P} = (-i\nu X^+X^-)[X^+/X^-]^{-ij/2}W_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}} \\
\omega^j_{in,-} &= \mathcal{U}^j_{in,P} = (i\nu X^+X^-)[X^+/X^-]^{-ij/2}M_{i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}
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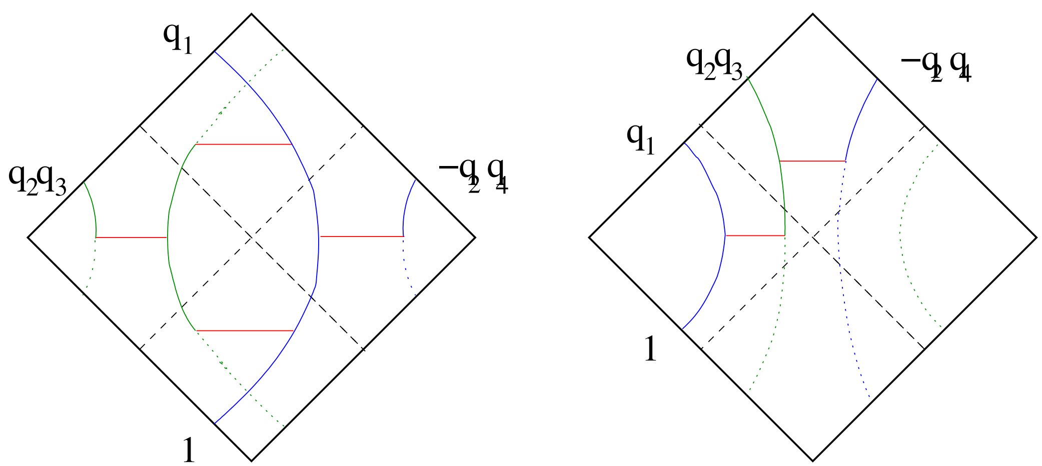
- There are two types of Unruh modes, involving 2 or 4 tunelling events:



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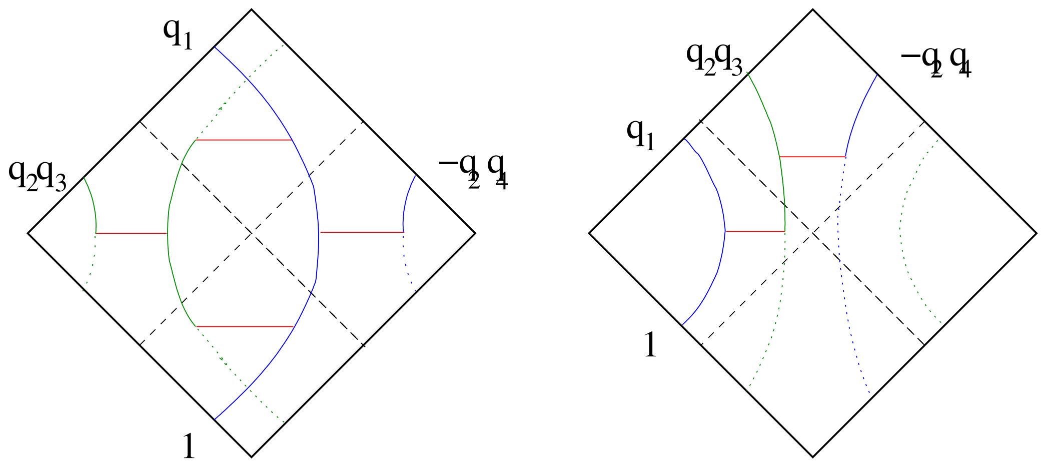

- Any state in Minkowski space can be represented as a state in the tensor product of the Hilbert spaces of the left and right Rindler patches. In contrast to neutral fields in Rindler space, Boulware-Fulling modes that vanish in L or R have positive Minkowski energy.

Closed string zero-modes

- Let us reanalyze the classical solutions for the closed string zero modes

$$X^{\pm}(\tau,\sigma) = e^{\mp\nu\sigma}\left[\pm\frac{1}{2\nu}\alpha_0^{\pm}e^{\pm\nu\tau} \mp \frac{1}{2\nu}\tilde{\alpha}_0^{\pm}e^{\mp\nu\tau}\right] \;, \quad \alpha_0^{\pm}, \tilde{\alpha}_0^{\pm} \in R$$

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- The behavior at early/late proper time now depends on $\epsilon\tilde{\epsilon}$: For $\epsilon\tilde{\epsilon} = 1$, the string begins/ends in the Milne regions. For $\epsilon\tilde{\epsilon} = -1$, the string begins/ends in the Rindler regions.

Short and long strings ($j = 0$)

Choosing $j = 0$ for simplicity, we have two very different types of solutions:

- $\epsilon = 1$, $\tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma, \tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma} \, , \quad T = \frac{M}{\nu} \sinh(\nu\tau) \, , \quad \theta = \nu\sigma$$

is a short string winding around the Milne circle from $T = -\infty$ to $T = +\infty$.

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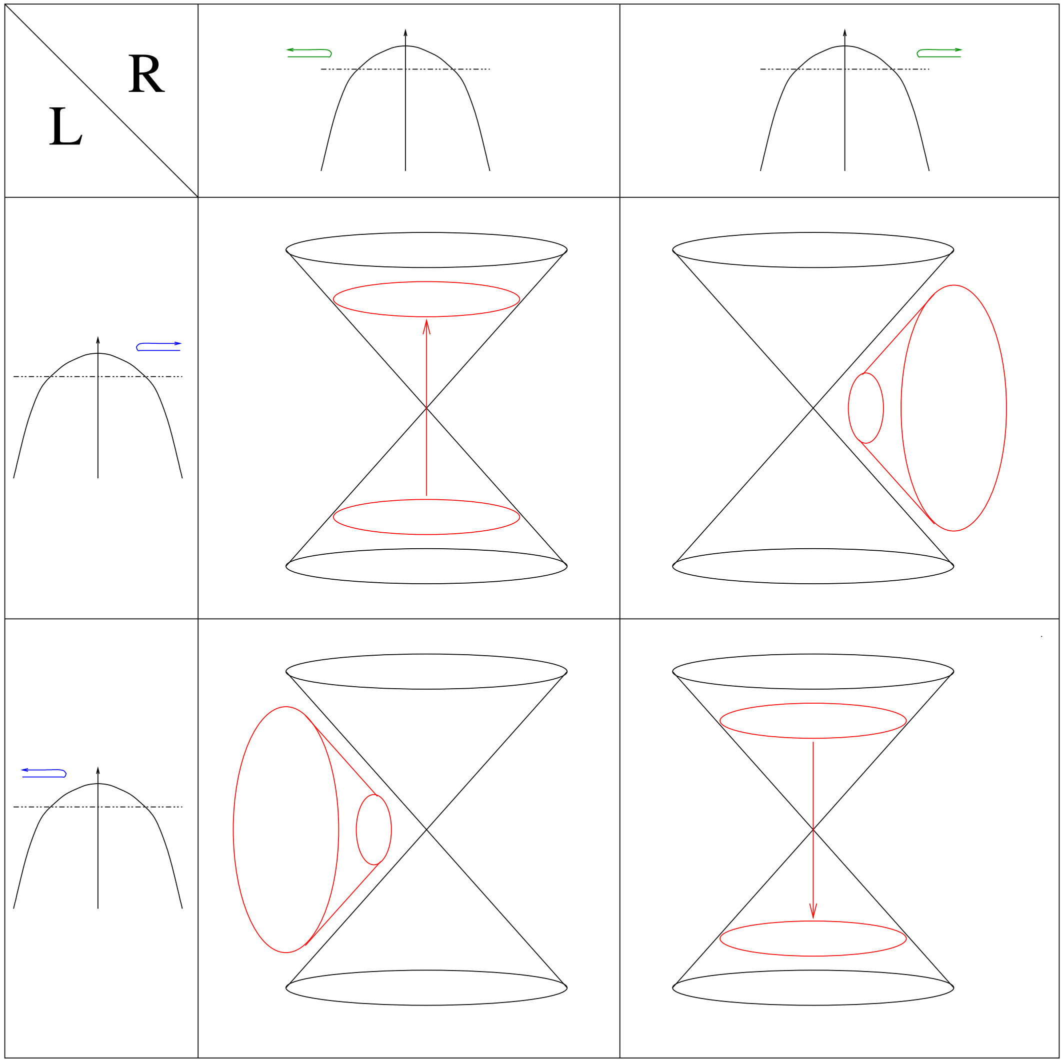
$\epsilon = -1$, $\tilde{\epsilon} = 1$ is the analogue in the left Rindler patch.

Short and long strings

Closed string trajectories are thus generated by the motion of two decoupled particles in inverted harmonic oscillators:



Relation to open string modes

- Instead of following the motion of a point at fixed σ, one may consider instead a point at fixed $\sigma + \tau$: this is precisely the trajectory of the open string zero-mode.
- Using the covariant derivative representation

$$\alpha_0^{\pm} = i\partial_{\mp} \pm \frac{\nu}{2} x^{\pm} \,, \quad \tilde{\alpha}_0^{\pm} = i\partial_{\mp} \mp \frac{\nu}{2} x^{\pm}$$

we observe that $x^{\pm}$ is the Heisenberg operator corresponding to the location of the closed string (at $\sigma = 0$):

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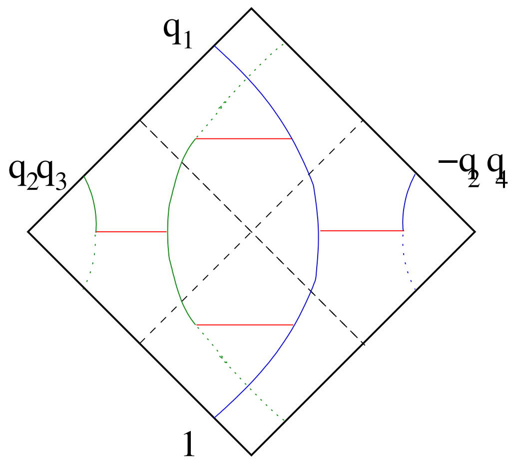

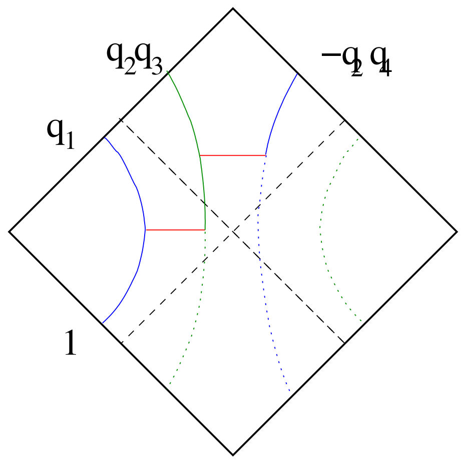

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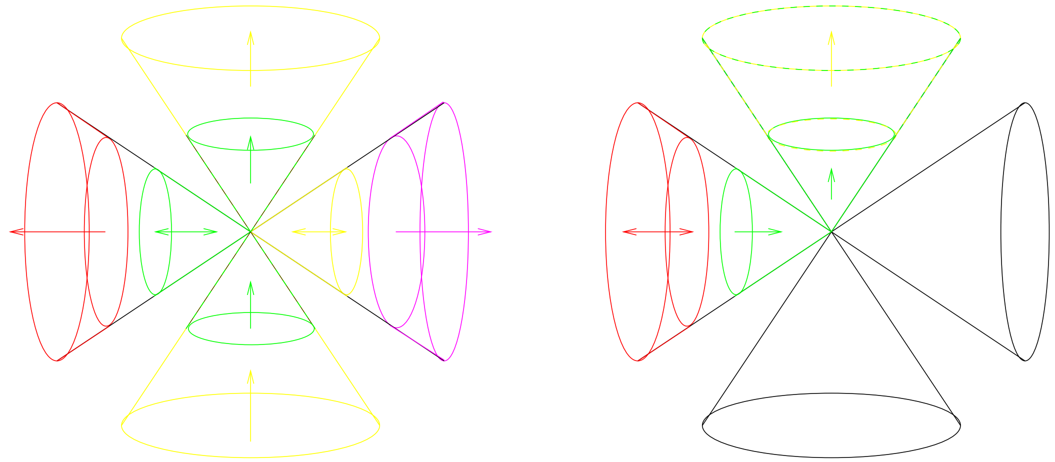

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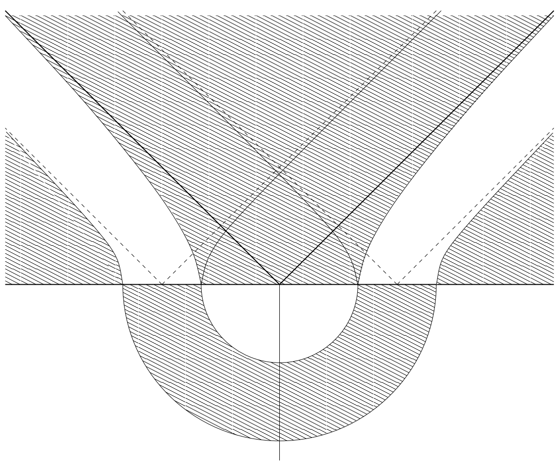
- Spontaneous pair production of winding strings can be described by cutting open a periodic trajectory in the Euclidean rotation orbifold: Hartle-Hawking vacuum



A few words on second quantization

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- Instead, one may (in two different ways) identify the Hilbert space with that of a single charged particle, including its center of motion. The wave function is thus a state in the tensor product of the left and right Rindler patches. One can define in and out vacua, and find global pair production.

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- Finally, motivated by holography, one may try to quantize with respect to the radial evolution in Rindler space. Short and long strings would be analogous to normalizable / non-normalizable modes.

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$$W = -\int_{-\infty}^{\infty} d\tau \left(X^+ \partial_\sigma X^- - X^- \partial_\sigma X^+ \right) = \int_{-\infty}^{\infty} d\tau \; r^2 \partial_\sigma \eta$$

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- The total Rindler energy of a long string is infinite, due to its extension towards $r \to \infty$. The energy density by unit of radial distance

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- The spectrum is thus unbounded from below (and above): Can CTC prevent the vacuum to decay ?

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- A bounce in dimension i requires $H_i' > 0$ at the point where $H_i = 0$, i.e.

$$(D-2) p_i + \rho \geq \sum_{j \neq i} p_j$$

The most efficient solution is a gas of scalar momentum states, with $p = \rho$: provides enough pressure for the bounce.

Effective gravity analysis (cont.)

- However, consider fundamental strings wrapped around dimension i,

$$\rho = \frac{T}{V}\,, \quad p_i = -\rho\,, \quad p_{j\neq i} = 0\,, \quad V = \prod_{j\neq i} a_j \quad \Rightarrow D \leq 3$$

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- We assumed a constant number of wound strings: one should incorporate the dependence of the production rate on the Hubble parameters.

Effective gravity analysis (cont.)

- Einstein's equations imply that the quantity

$$\mu = \left(\frac{H_k}{H_i} - 1 \right) \Big/ \left(\frac{H_j}{H_i} - \frac{3}{4 - D} \right)$$

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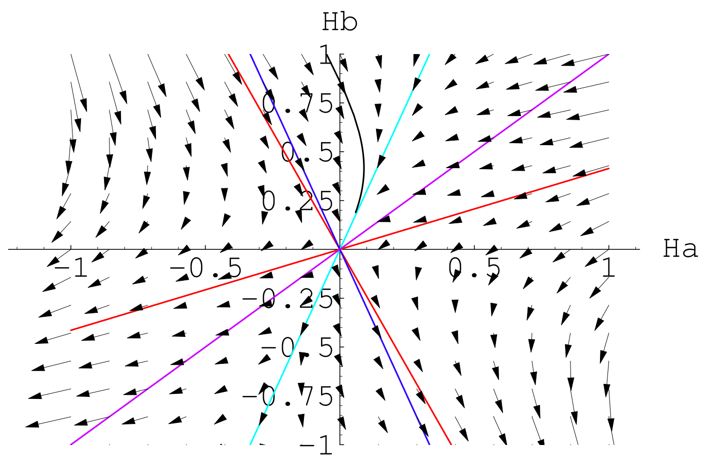
A bounce for direction i in units of the eleven-dimensional frame therefore takes place for any initial condition such that $2\mu + D - 3 > 0$ and $2 < D < 4$.



Conclusions

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- The production rate for winding strings in a singular geometry diverges at large winding number. Can the resolved geometry be determined self-consistently so that the divergences at one-loop cancel those at tree-level ?