

Closed Strings in the Misner Universe

aka the Lorentzian orbifold

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based on hep-th/0307280 w/ M. Berkooz
and hep-th/0405xxx w/ M. Berkooz, and M. Rozali

slides available from

<http://www.lpthe.jussieu.fr/~pioline/seminars.html>

Motivational string cosmology

- **Observational Cosmology** is now challenging string theory with high-precision data:

$$\Omega_{baryon} = 0.047, \quad \Omega_{darkm} = 0.243, \quad \Omega_{\Lambda} = 0.71, \quad w = -0.98 \pm .12, \dots$$

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- With LHC still far in the future, understanding **StringY Cosmology** may be the only way to make contact with reality...

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- Perturbative string theory requires an Euclidean worldsheet, hence Euclidean target space. The **analytic continuation** may be ambiguous or ill-defined, **Lorentzian observables** may be very different from their Euclidean counterparts.
- String theory is not content on a finite time interval, and one is frequently forced into **Big Bang / Big Crunch singularities, CTC** in the process of maximally extending the geometry.

String theory and cosmological singularities

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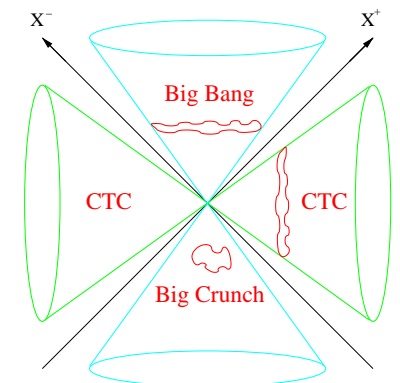
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- Usual Euclidean (static) orbifolds can often be resolved by **condensation of twisted sector fields**. Can production of winding strings resolve the singularity ?
- In this talk, we shall discuss the **“Lorentzian” orbifold** of flat Minkowski space by a discrete boost, as a toy model of a **singular cosmological universe** where string theory can in principle be solved explicitly.



Outline of the talk

1. Introduction
2. The Lorentzian orbifold and its avatars
3. Closed strings in Misner space: first pass
3. A detour: Open strings in electric fields
4. Closed strings in Misner space: second pass
5. Comments on backreaction from winding strings

Misner, Taub-NUT, Grant...

Nekrasov

Bachas Porrati; Berkooz BP

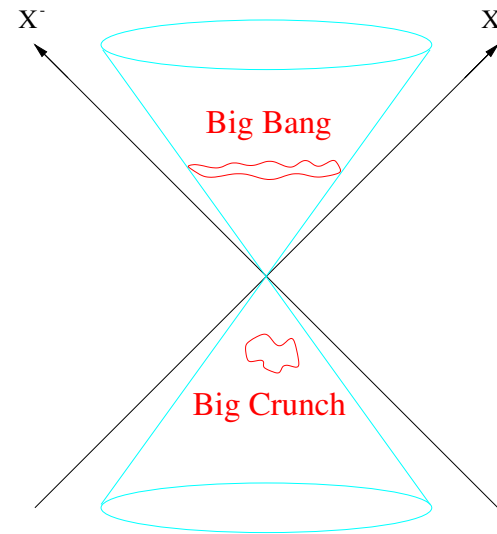
Berkooz BP; Berkooz BP Rozali

The Lorentzian orbifold

- One of the simplest examples of space-like singularities is the **quotient of flat Minkowski space by a discrete boost**, also known as **Misner space** (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^\pm \sim e^{\pm 2\pi\beta} X^\pm$$

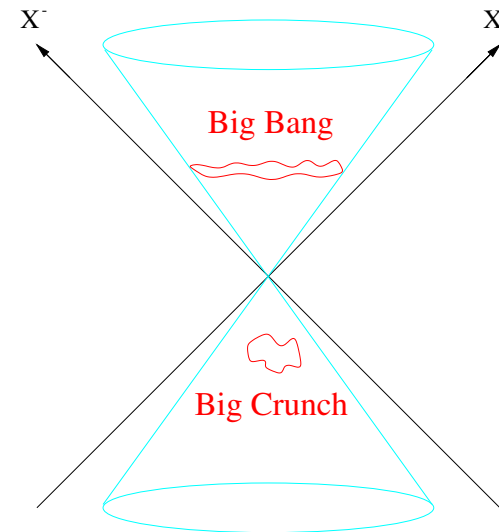


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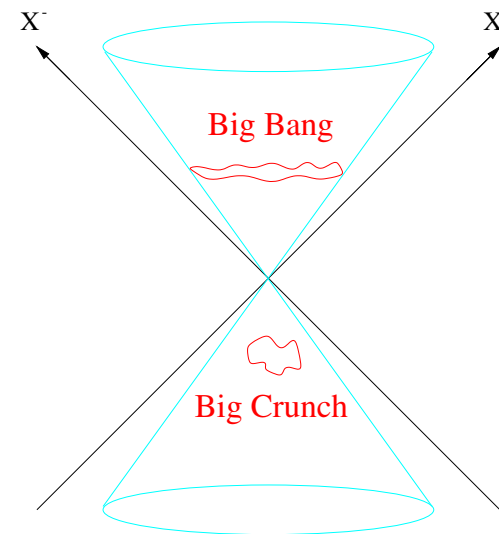
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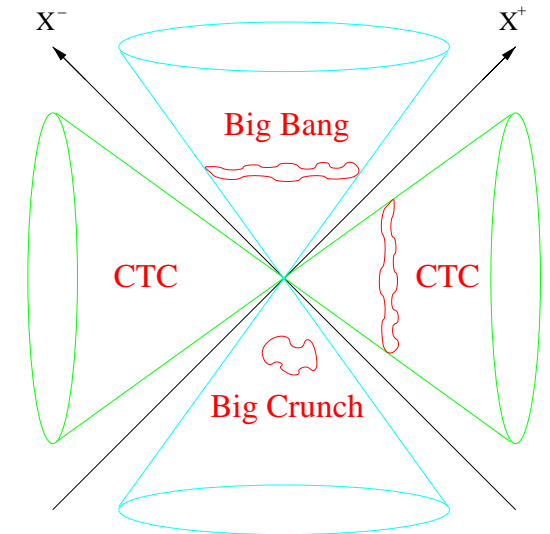
This is a (degenerate) **Kasner singularity**, everywhere **flat**, except for a **delta-function curvature** at $T = 0$.

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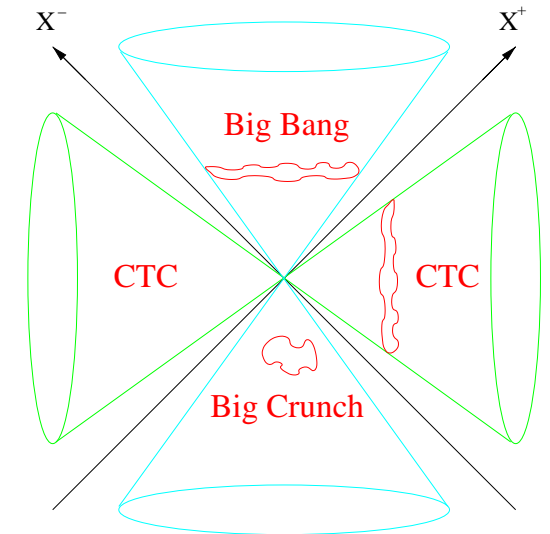
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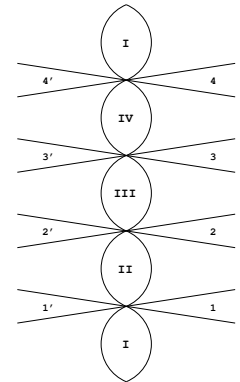
- Finally, the **lightcone** $X^+X^- = 0$ gives rise to a **null, non-Hausdorff** locus attached to the singularity.

Close relatives of the Misner Universe

- Misner space was first introduced as a local model of **Lorentzian Taub-NUT** space:

$$ds^2 = 4l^2 U(t) \sigma_3^2 + 4l \sigma_3 dt + (t^2 + l^2) (\sigma_1^2 + \sigma_2^2), \quad U(t) = -1 + \frac{2mt + l^2}{t^2 + l^2}$$

A **bouncing** universe, isomorphic to $R^{1,1}/boost \times S^2$ around each singularity.



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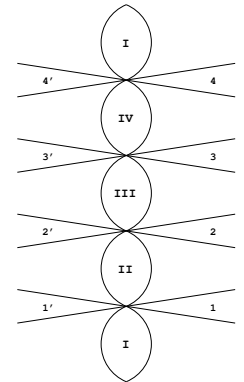
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- A close variant of Misner space is the quotient of flat space by the **combination of a discrete boost and a translation** on an extra direction, often known as the **Grant space**:

$$ds^2 = -2dX^+ dX^- + dX^2 + (dX^i)^2, \quad (X^\pm, X) \sim (e^{\pm 2\pi\beta} X^\pm, X + 2\pi R)$$

This describes the space away from two **moving cosmic strings**. The cosmological singularity is smoothed out, but regions with CTC remain.



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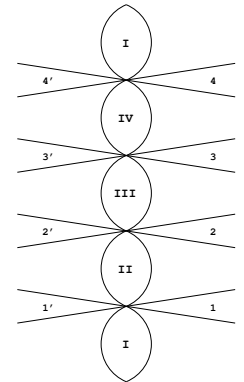
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Gott 91, Grant 93; Cornalba, Costa, Kounnas

- The Misner geometry arose again more recently as the **M-theory** lift of a simple (**ekpyrotic**) cosmological solution of Einstein-dilaton gravity with no potential.

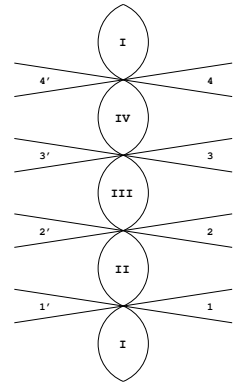
Khoury Ovrut Seiberg Steinhard Turok



Close relatives of the Misner Universe (cont)

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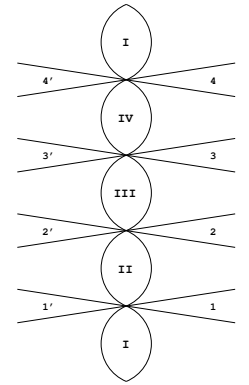
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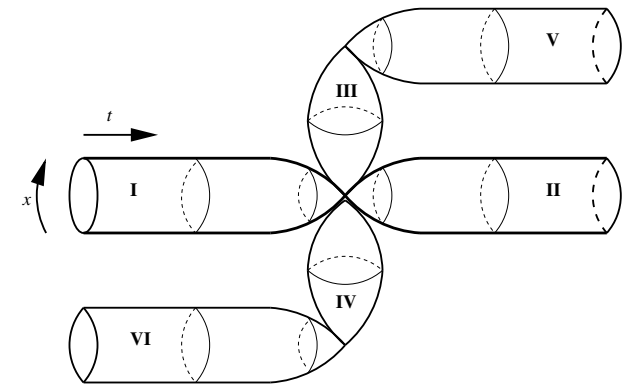
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- The gauged WZW model $Sl(2)/U(1)$ at **negative level orbifolded by a boost J** describes two parallel Universes with a curvature and a Milne singularity, and compact whiskers.

Tseytlin Vafa; Craps Kutasov Rajesh; Craps Ovrut

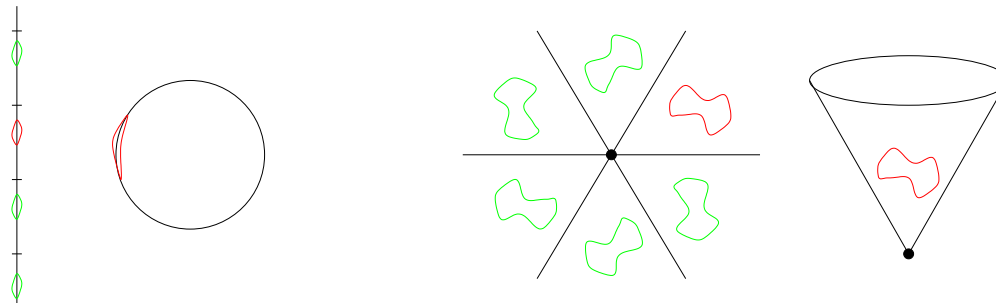


Strings on Euclidean orbifolds - untwisted states

- Well-known examples of orbifolds are the **circle**, R/Z , and the **rotation orbifold** R^2/Z_k .

Dixon Harvey Vafa Witten

- The spectrum of the quotient theory contains closed string states of the parent theory which are invariant under G : **untwisted states**.

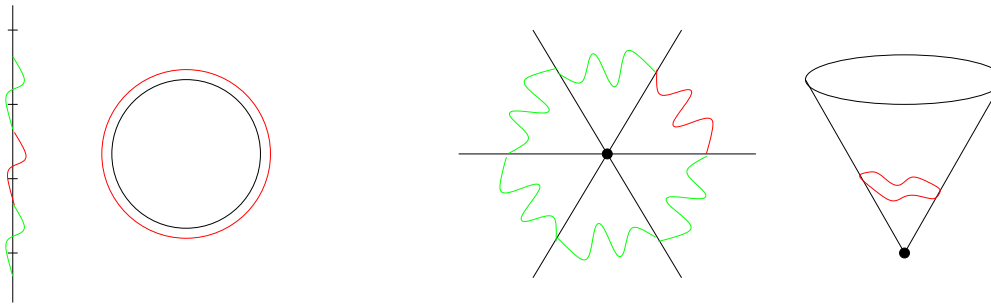


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- Modular invariance** requires that the spectrum should also include closed strings in the quotient theory which **close up to the action of G** in the parent theory: **twisted states**.



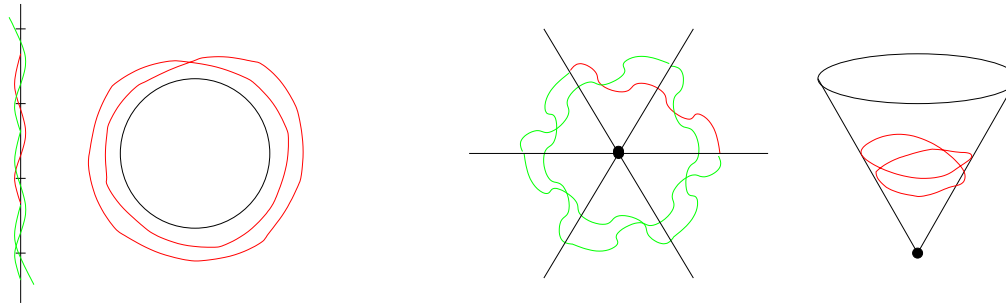
- When G acts non-freely, the twisted sector states are **localized at the fixed points**. They yield new localized degrees of freedom, which ensure the consistency of the background: anomaly free, divergence free...

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- Twisted sectors are labelled by **conjugacy classes** of G . Higher twisted sectors correspond to multiply wound states.

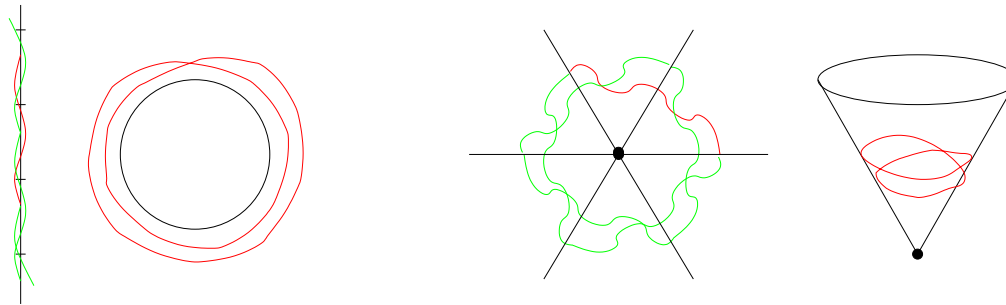


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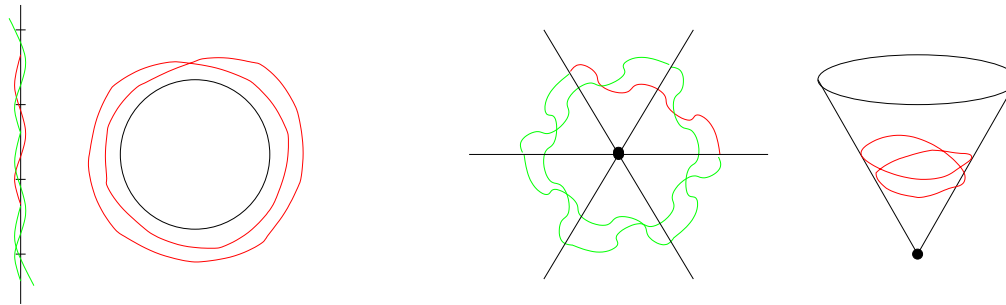
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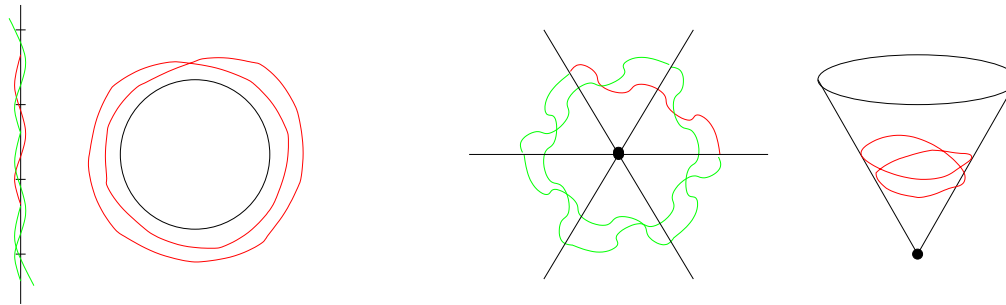
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- The **condensation** of these twisted states changes the vacuum, and effectively **resolves the singularity**: $R^2/Z_k \rightarrow R^2/Z_{k-1} \rightarrow \dots$ (tachyon), $R^4/Z_k \rightarrow$ multi-centered Eguchi-Hanson (massless mode).

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- The Lorentzian orbifold shares features with both examples: an **infinite number of winding sectors**, and a, non compact, **fixed locus**.

Closed strings in Misner space - untwisted states

- As usual in standard orbifolds, part of the spectrum involves closed strings on Minkowski covering space, which are **invariant under the orbifold projection**. In conformal gauge,

$$X^\pm(\sigma + 2\pi, \tau) = X^\pm(\sigma, \tau), \quad (\partial_\tau^2 - \partial_\sigma^2)X^\pm = 0$$

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- Vertex operators (or states) can be obtained by (infinite) **sum over images**, e.g.

$$\sum_{l=-\infty}^{\infty} \partial X^+ \bar{\partial} X^- \exp\left(ik^+ X^- e^{-2\pi\beta l} + ik^- X^+ e^{2\pi\beta l} + ik_i X^i\right)$$

with the physical state condition $2k^+ k^- = M^2$.

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- Vertex operators (or states) can be obtained by (infinite) **sum over images**, e.g.

$$\sum_{l=-\infty}^{\infty} \partial X^+ \bar{\partial} X^- \exp \left(ik^+ X^- e^{-2\pi\beta l} + ik^- X^+ e^{2\pi\beta l} + ik_i X^i \right)$$

with the physical state condition $2k^+ k^- = M^2$.

- Equivalently, after **Poisson resummation over l** , this is a superposition of states with **integer boost momentum $j = i(x^+ \partial_+ - x^- \partial_-)$** ,

$$\left(\sum_{j=-\infty}^{\infty} \right) \partial X^+ \bar{\partial} X^- \int_{-\infty}^{\infty} dv \exp \left(ik^+ X^- e^{-2\pi\beta v} + ik^- X^+ e^{2\pi\beta v} + ik_i X^i + ivj \right)$$

Closed strings in Misner space - untwisted states

- As usual in standard orbifolds, part of the spectrum involves closed strings on Minkowski covering space, which are **invariant under the orbifold projection**. In conformal gauge,

$$X^\pm(\sigma + 2\pi, \tau) = X^\pm(\sigma, \tau), \quad (\partial_\tau^2 - \partial_\sigma^2)X^\pm = 0$$

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- The resulting eigenfunctions describe (topologically trivial) **closed strings traveling around the Milne circle** with integer momentum j .

Quantum fluctuations in field theory

- In the **Minkowski vacuum** (inherited from the covering space), the renormalized propagator can be obtained as a sum over images,

$$G(x; x') = \sum_{l=-\infty, l \neq 0}^{\infty} \int_0^{\infty} d\tau \int dp^{\mu} \exp \left(-ip^{-} (x^{+} - e^{2\pi\beta l} x^{+'}) - ip^{+} (x^{-} - e^{2\pi\beta l} x^{-'}) - ip^i (x^i - x^{i'}) \right)$$

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- The one-loop stress-energy tensor follows from $G(x, x)$, e.g for a conformally coupled scalar,

$$\langle T_{ab} \rangle = \lim_{x \rightarrow x'} \left[(1 - 2\xi) \nabla_a \nabla'_b - 2\xi \nabla_a \nabla_b + (2\xi - \frac{1}{2}) g_{ab} \nabla_c \nabla'^c \right] G(x, x')$$

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This leads to a **divergent quantum backreaction** (worse if the spin $|s| > 1$):

$$\langle T_{\mu}^{\nu} \rangle = \frac{K}{12\pi^2} T^{-4} \text{diag}(1, -3, 1, 1), \quad K = \sum_{l=1}^{\infty} \cosh(2\pi\beta l s) \frac{2 + \cosh 2\pi l \beta}{[\cosh 2\pi l \beta - 1]^2}$$

One-loop vacuum amplitude in field and string theory

- On the other hand, in string theory $\langle T_{\mu}^{\nu} \rangle(x)$ is an **off-shell** quantity, and only its integral over space-time is well defined:

$$\int dx^+ dx^- G(x, x) = \sum_{l=-\infty}^{+\infty} \int_0^{\infty} \frac{d\rho}{\rho^{D/2}} \frac{e^{-m^2 \rho}}{\sinh^2(\pi \beta l)}$$

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- This reproduces the zero-mode contribution to the string one-loop vacuum amplitude in the untwisted sector:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l, w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2 \rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho) \theta_1(i\beta(l + w\rho); \rho)|^2}$$

$$\theta_1(v; \rho) = 2q^{1/8} \sin \pi v \prod_{n=1}^{\infty} (1 - e^{2\pi i v} q^n)(1 - q^n)(1 - e^{-2\pi i v} q^n), \quad q = e^{2\pi i \rho}$$

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- The local divergence in $\langle T_\mu^\nu \rangle(x)$ is integrable and yields a finite free energy.
- The existence of **Regge trajectories** with arbitrary high spin implies new (log) **divergences in the bulk of the moduli space** which resemble long string poles in AdS_3 .

Scattering of untwisted states

- Tree-level scattering amplitudes of untwisted sector states can be computed from those in flat space by the inheritance principle,

$$\langle V(j_1, k_1) \dots V(j_n, k_n) \rangle_{Misner} = \int dv_1 \dots dv_n e^{i(j_1 v_1 + \dots + j_n v_n)}$$

$$\langle V(e^{\beta v_1} k_1^+, e^{-\beta v_1} k_1^-, k_1^i) \dots V(e^{\beta v_n} k_n^+, e^{-\beta v_n} k_n^-, k_n^i) \rangle_{Minkowski}$$

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- The integral diverges due to Regge behavior in the large momentum, fixed angle regime. E.g, the four-tachyon scattering amplitude in bosonic string leads to

$$\int dv v^{-\frac{1}{2}(k_1^i - k_3^i)^2 + i(j_2 - j_4)}$$

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Berkooz Craps Rajesh Kutasov

- Could higher order corrections, e.g. resummation of ladder diagrams, lead to a finite amplitude ?

Deser McCarthy Steif; Cornalba Costa

Closed string in Misner space - twisted sectors

- In addition, there is **an infinite set of twisted sectors**, corresponding to strings on the covering space that close **up to the action of the orbifold group**:

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau) , \quad \nu = 2\pi\omega\beta$$

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- They have a normal mode expansion:

$$X_R^\pm(\tau - \sigma) = \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha_n^\pm e^{-i(n \pm i\nu)(\tau - \sigma)}$$

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- We will focus on the **quasi zero-mode** sector, which consists of two commuting pairs of real (i.e. hermitian) canonically conjugate operators,

$$[\alpha_0^+, \alpha_0^-] = -i\nu, \quad [\tilde{\alpha}_0^+, \tilde{\alpha}_0^-] = i\nu$$

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated by half of them, e.g.

$$\alpha_{n>0}^{\pm}, \quad \tilde{\alpha}_{n>0}^{\pm}, \quad \alpha_0^-, \quad \tilde{\alpha}_0^+$$

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- The worldsheet Hamiltonian, **normal-ordered wrt to this vacuum**, reads

$$L_0^{l.c.} = - \sum_{n=0}^{\infty} (\alpha_n^+)^* \alpha_n^- - \sum_{n=1}^{\infty} (\alpha_n^-)^* \alpha_n^+ + \frac{1}{2} i\nu(1 - i\nu) - 1 + L_{int}$$

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- This is the familiar result for the vacuum energy $\frac{1}{2}\theta(1 - \theta)$ in the **Euclidean rotation orbifold**, after analytically continuing $\theta \rightarrow i\nu$.
- Due to the $i\nu/2$ term in the ground state energy, all states obtained by acting on $|0\rangle$ by creation operators $\alpha_{n<0}^{\pm}$ and by α_0^+ will have **imaginary energy**, hence **the physical state conditions** $L_0 = \tilde{L}_0 = 0$ seem to have no solutions.

One-loop amplitude, twisted sector

- Independently of this fact, one may compute the one-loop path integral on an **Euclidean worldsheet and Minkowskian target-space**:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho) x \theta_1(i\beta(l+w\rho); \rho)|^2}$$

where θ_1 is the Jacobi theta function,

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- In the **twisted** sector, the left-moving zero-modes contribute

$$\frac{1}{2 \sinh(\beta w \rho)} = \sum_{n=1}^{\infty} q^{i(n+\frac{1}{2})\beta w}$$

in accordance with the quantization scheme based on a **Fock vacuum** annihilated by α_0^- .

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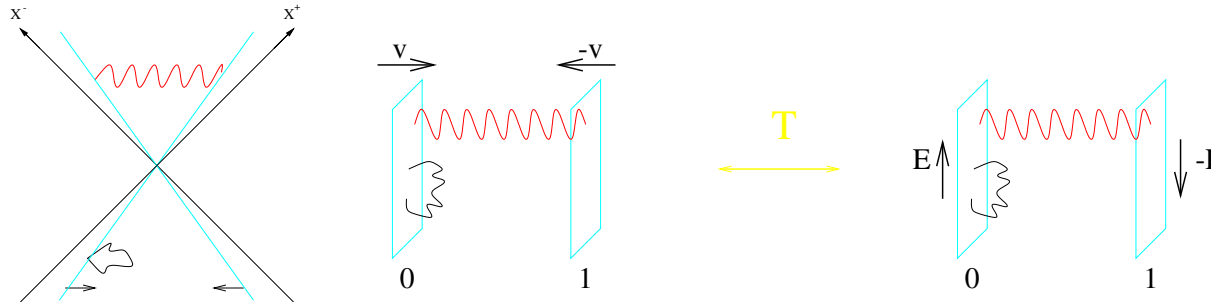
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- The absence of physical twisted states crushes our hopes for resolving the singularity... yet does not sound very sensible. An important point: α_0^+ and α_0^- are not hermitian conjugate to each other, but rather **self-hermitian**...

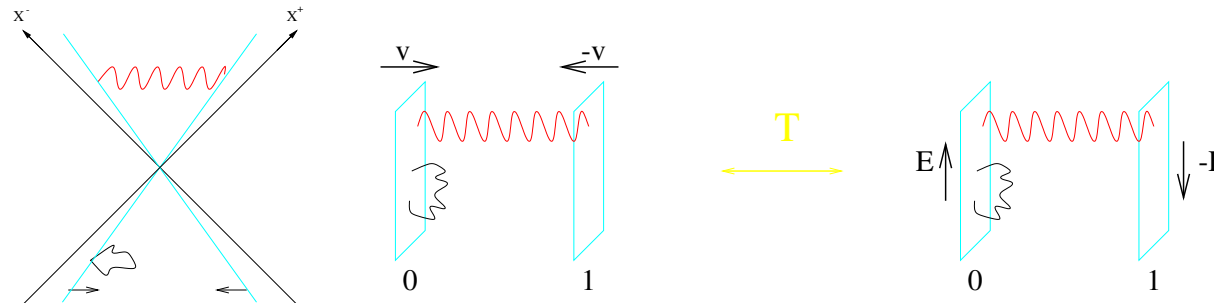
A detour via Open strings in electric field

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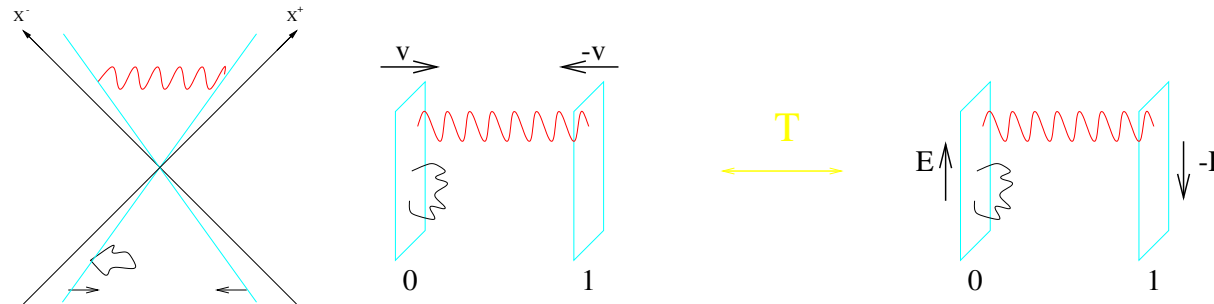
- Recall that for open strings stretched between two D-branes with electromagnetic fields F_0 and F_1 , proper frequencies satisfy

$$e^{-2\pi i \omega \eta} = \frac{1 + F_0}{1 - F_0} \cdot \frac{1 - F_1}{1 + F_1}$$

For $F_0 \neq F_1$, the open string carries a net electric charge, and the motion of its center of motion is **that of a charged particle**.

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- Recall that for open strings stretched between two D-branes with electromagnetic fields F_0 and F_1 , proper frequencies satisfy

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- In the case of an **electric field** $F_1 = E dx^+ \wedge dx^-$, $F_0 = 0$, the resulting spectrum is

$$\omega_n = n + i\nu, \quad \nu := \text{Arctanh} E = w\beta$$

just as in the **Lorentzian orbifold** case. The large winding number limit $w \rightarrow \infty$ amounts to a **near critical electric field** $E \rightarrow 1$.

Open string mode expansion

- The light-cone embedding coordinates have the normal mode expansion

$$X^\pm = x_0^\pm + i \sum_{n=-\infty}^{+\infty} (-)^n (n \pm i\nu)^{-1} a_n^\pm e^{-i(n \pm i\nu)\tau} \cos[(n \pm i\nu)\sigma]$$

with reality conditions $(a_n^\pm)^* = a_{-n}^\pm$, $(x_0^\pm)^* = x_0^\pm$

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- In particular, **open and closed strings have isomorphic zero-mode structures**, upon identifying $\alpha_0^\pm \equiv a_0^\pm$ and $\tilde{\alpha}_0^\pm \equiv \pm\sqrt{\nu E}x_0^\pm$.

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with reality conditions $(a_n^\pm)^* = a_{-n}^\pm$, $(x_0^\pm)^* = x_0^\pm$

- The canonical commutation relations read

$$[a_m^+, a_n^-] = -(m + i\nu)\delta_{m+n}, \quad [x_0^+, x_0^-] = -\frac{i}{E}$$

- In particular, **open and closed strings have isomorphic zero-mode structures**, upon identifying $\alpha_0^\pm \equiv a_0^\pm$ and $\tilde{\alpha}_0^\pm \equiv \pm\sqrt{\nu E}x_0^\pm$.
- The world-sheet Hamiltonian, **normal ordered with respect to the vacuum** annihilated by $a_{n>0}^+$, $a_{n>0}^-$ and a_0^+ , takes the form

$$L_0^{l.c.} = - \sum_{m=0}^{\infty} a_{-m}^+ a_m^- - \sum_{m=1}^{\infty} a_{-m}^- a_m^+ + \frac{i\nu}{2}(1 - i\nu) - \frac{1}{12}$$

Open string mode expansion

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- By the same token, charged open strings should have no physical states... yet electrons and positrons certainly do exist.

Charged particle and open string zero-modes

- Let us recall the quantization of a **charged particle in an electric field**:

$$L = \frac{1}{2}m \left(-2\partial_\tau X^+ \partial_\tau X^- + (\partial_\tau X^i)^2 \right) + \frac{\nu}{2} \left(X^+ \partial_\tau X^- - X^- \partial_\tau X^+ \right)$$

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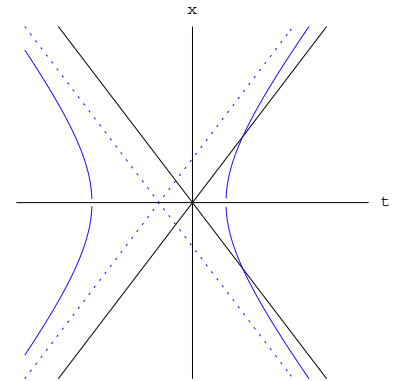
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$$X^\pm = x_0^\pm \pm \frac{1}{\nu} a_0^\pm e^{\pm\nu\tau}$$

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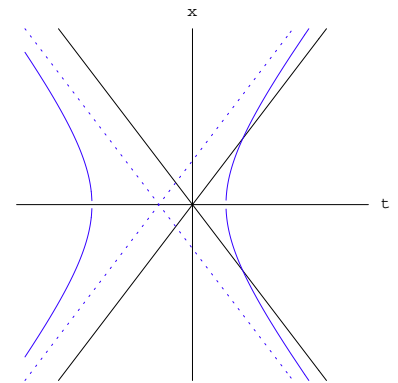
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- Starting from the canonical equal-time commutation rules

$$[\pi^+, x^-] = [\pi^-, x^+] = i, \quad [\pi^i, x^j] = i\delta_{ij}$$

one recovers the open string zero-mode commutation relations

$$[a_0^+, a_0^-] = -i\nu, \quad [x_0^+, x_0^-] = -\frac{i}{\nu}$$



Charged particle and ppen string zero-modes

- Quantum mechanically, one may represent $\pi^\pm = i\partial/\partial x^\mp$ hence obtain a_0^\pm, x_0^\pm as **covariant derivatives**

$$a_0^\pm = i\partial_\mp \pm \frac{\nu}{2}x^\pm, \quad x_0^\pm = \mp \frac{1}{\nu} \left(i\partial_\mp \mp \frac{\nu}{2}x^\pm \right)$$

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acting on wave functions $f(x^+, x^-)$.

- The zero-mode piece of L_0 , **including the bothersome** $\frac{i\nu}{2}$,

$$L_0^{(0)} = -a_0^+ a_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\nabla_0^+ \nabla_0^- + \nabla_0^- \nabla_0^+)$$

is just the **Klein-Gordon operator** of a particle of charge ν .

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^\pm = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator can be rewritten as an **inverted harmonic oscillator**:

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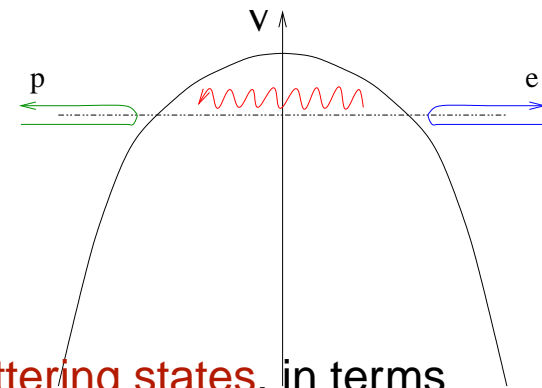
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- More explicitly, in terms of $u = (\tilde{p} + \nu x)\sqrt{2/\nu}$,

$$\left(-\partial_u^2 - \frac{1}{4}u^2 + \frac{M^2}{2\nu} \right) \psi_{\tilde{p}}(u) = 0$$

- The latter admits a respectable **delta-normalizable spectrum of scattering states**, in terms of **parabolic cylinder functions**, e.g:

$$\phi_{in}^+(x, t) = D_{-\frac{1}{2} + i\frac{M^2}{2\nu}} \left(e^{-\frac{3i\pi}{4}} u \right) e^{-i\tilde{p}t} e^{i\nu x t/2}$$



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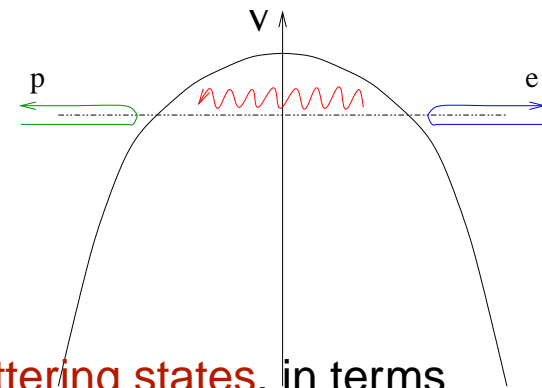
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- These correspond to **non-compact** trajectories of charged particles in the electric field. **Tunnelling** is just (stimulated) **Schwinger pair creation**,

$$e^- \rightarrow (1 + \eta) e^- + \eta e^+, \quad \eta \sim e^{-\pi M^2/\nu}$$



Lorentzian vs Euclidean states

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- The zero-mode contribution to the one-loop amplitude can be interpreted either way,

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The density of states is obtained from the **reflection phase shift**,

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- The physical spectrum can be explicitly worked out at low levels, and is **free of ghosts**: a tachyon at level 0, a **transverse gauge boson** at level 1, ...

Charged particle in Rindler space

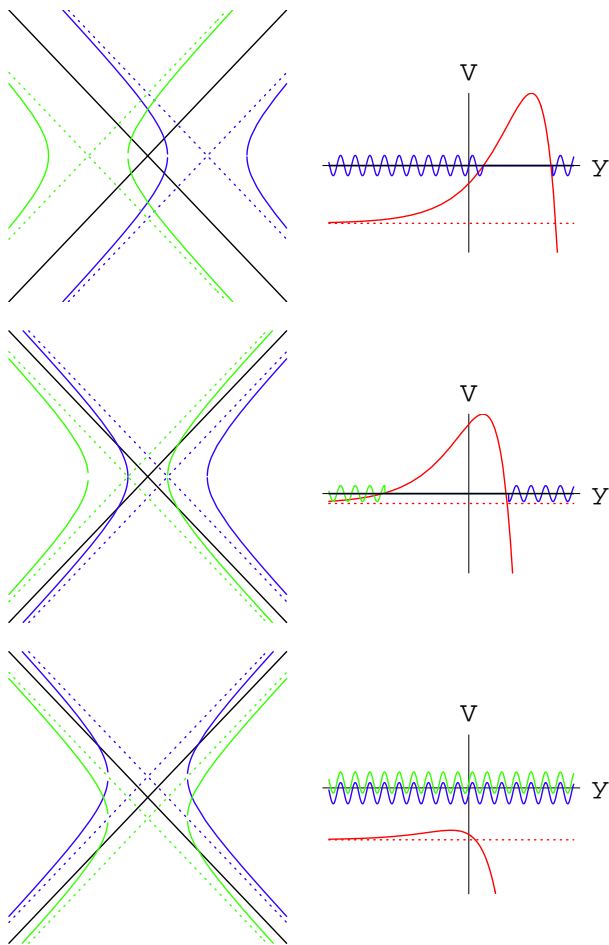
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Gabriel Spindel; Mottola Cooper

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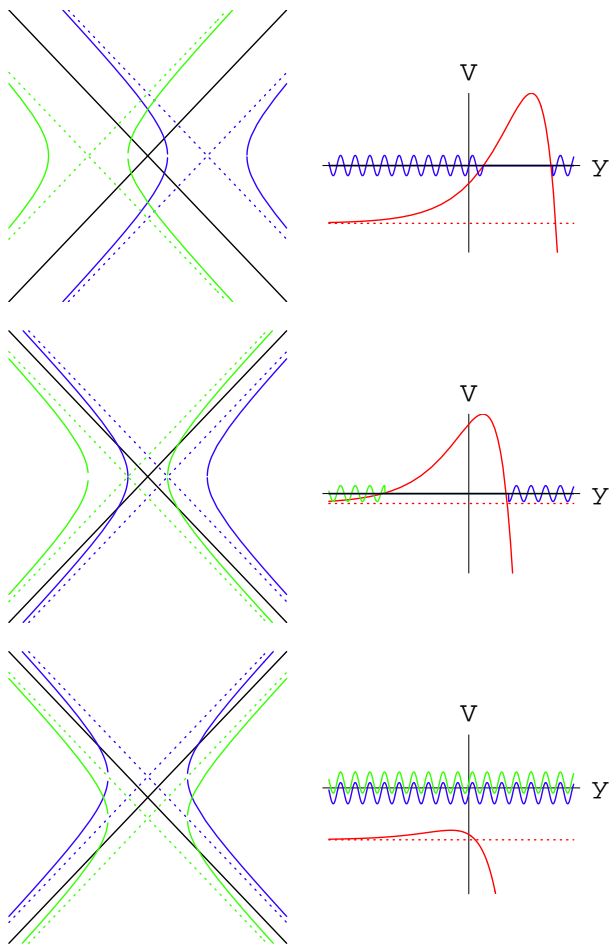
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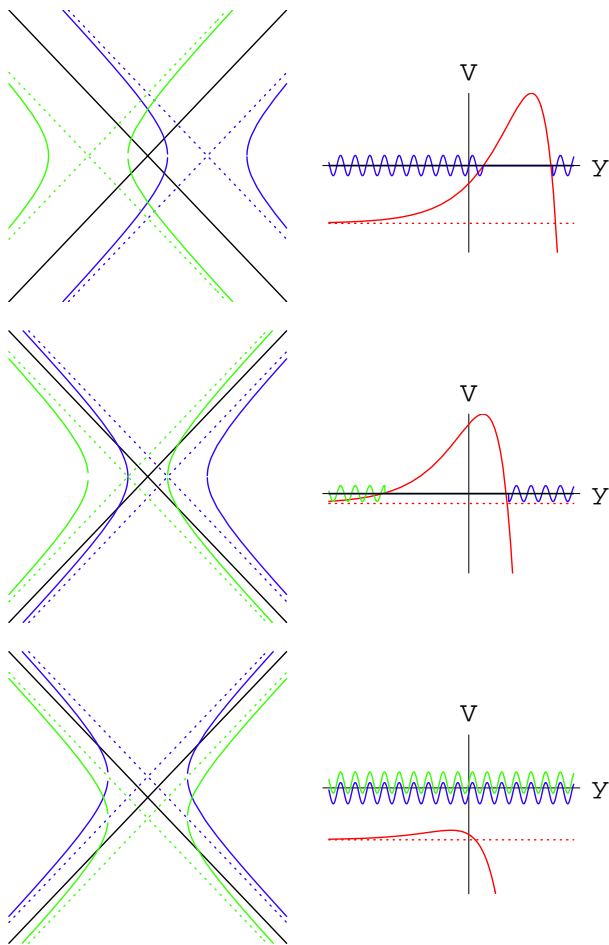
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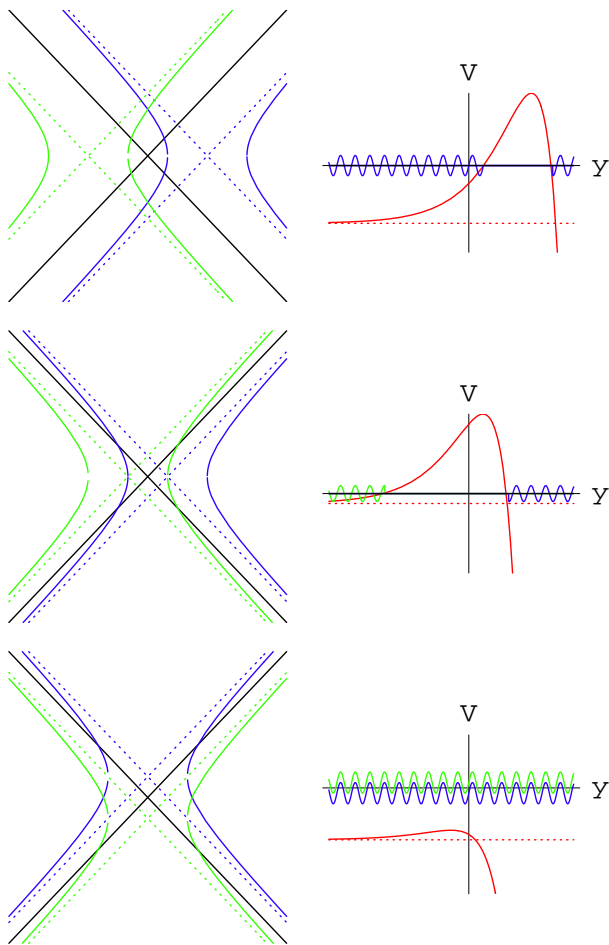
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- If $j > M^2/(2\nu)$, the electron branches extend in the Milne regions. There is **no tunneling**, but partial reflection amounts to a combination of **Schwinger** and **Hawking** emission.

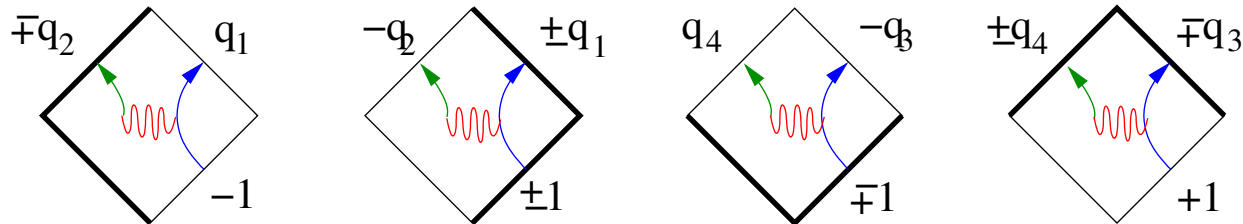
Rindler modes

- Incoming modes from Rindler infinity I_R^- read, in terms of parabolic cylinder functions:

$$\mathcal{V}_{in,R}^j = e^{-ij\eta} r^{-1} M_{-i(\frac{j}{2} - \frac{m^2}{2\nu}), -\frac{ij}{2}}(i\nu r^2/2)$$

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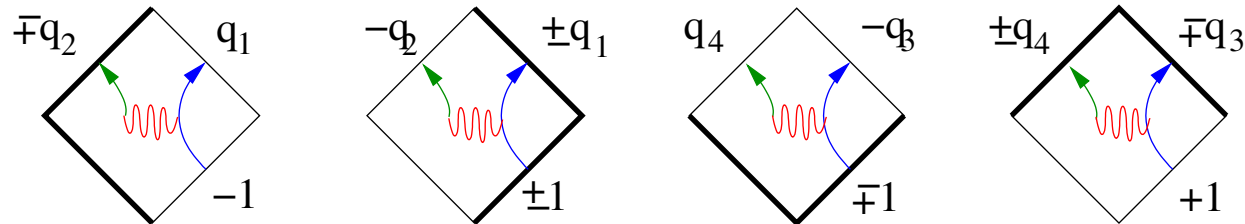
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- The reflection coefficients can be computed:

$$q_1 = e^{-\pi j} \frac{\cosh\left[\pi \frac{M^2}{2\nu}\right]}{\cosh\left[\pi \left(j - \frac{M^2}{2\nu}\right)\right]}, \quad q_3 = e^{\pi \left(j - \frac{M^2}{2\nu}\right)} \frac{\cosh\left[\pi \frac{M^2}{2\nu}\right]}{|\sinh \pi j|}$$

and $q_2 = 1 - q_1$, $q_4 = q_3 - 1$, by charge conservation.

Global Charged Unruh Modes

- Global modes may be defined by patching together Rindler modes, ie by **analytic continuation across the horizons**. **Unruh modes** are those which are superposition of **positive energy** Minkowski modes,

$$\Omega_{in,+}^j = \mathcal{V}_{in,P}^j = (-i\nu X^+ X^-) [X^+ / X^-]^{-ij/2} W_{-i(\frac{j}{2} - \frac{m^2}{2\nu}), \frac{ij}{2}}$$

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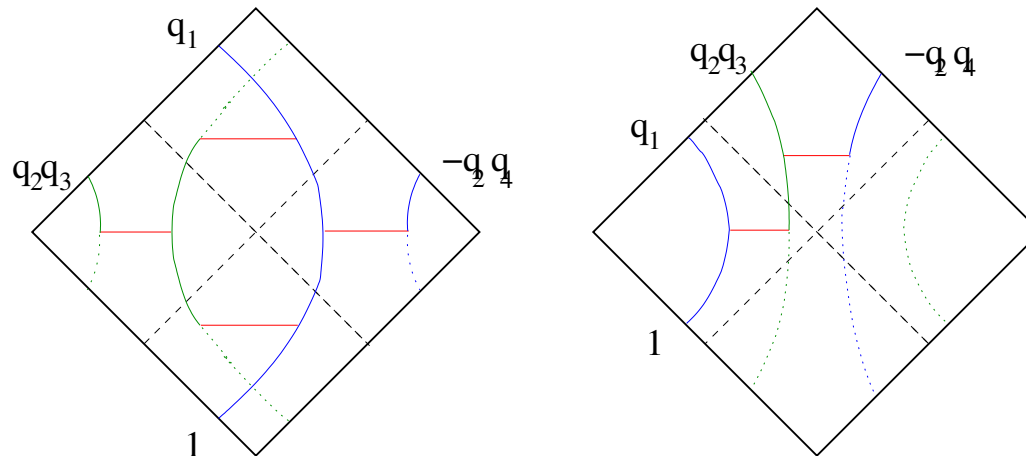
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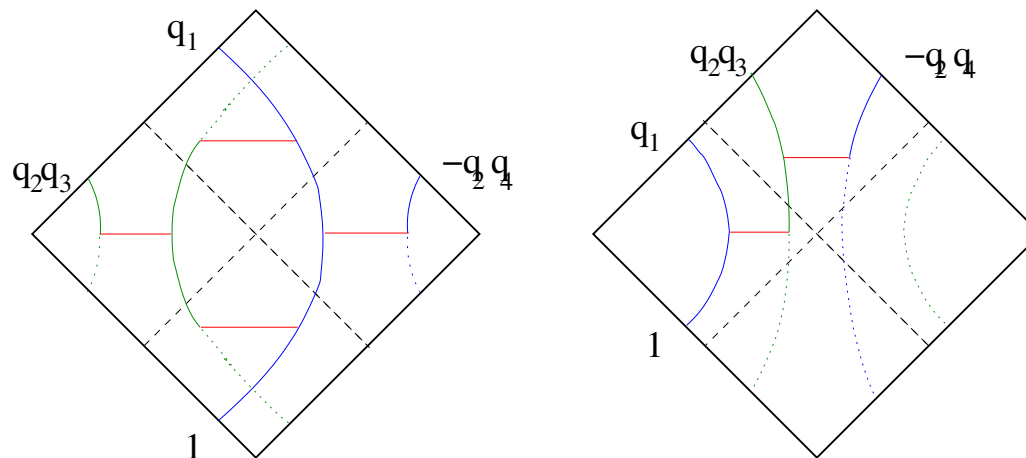
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- Any state in Minkowski space can be represented as a state in the **tensor product of the Hilbert spaces of the left and right Rindler patches**. In contrast to neutral fields in Rindler space, Boulware-Fulling modes that vanish in L or R have positive Minkowski energy.

Closed string zero-modes

- Let us reanalyze the classical solutions for the closed string zero modes

$$X^\pm(\tau, \sigma) = e^{\mp\nu\sigma} \left[\pm \frac{1}{2\nu} \alpha_0^\pm e^{\pm\nu\tau} \mp \frac{1}{2\nu} \tilde{\alpha}_0^\pm e^{\mp\nu\tau} \right], \quad \alpha_0^\pm, \tilde{\alpha}_0^\pm \in R$$

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- The behavior at early/late proper time now depends on $\epsilon\tilde{\epsilon}$: For $\epsilon\tilde{\epsilon} = 1$, the string begins/ends in the **Milne** regions. For $\epsilon\tilde{\epsilon} = -1$, the string begins/ends in the **Rindler** regions.

Short and long strings ($j = 0$)

Choosing $j = 0$ for simplicity, we have two very different types of solutions:

- $\epsilon = 1, \tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma, \tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma}, \quad T = \frac{M}{\nu} \sinh(\nu\tau), \quad \theta = \nu\sigma$$

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is a **long string stretched in the right Rindler patch**, from $r = \infty$ to $r = M/\nu$ and back to $r = \infty$; σ is now the proper time direction in the induced metric.

Short and long strings ($j = 0$)

Choosing $j = 0$ for simplicity, we have two very different types of solutions:

- $\epsilon = 1, \tilde{\epsilon} = 1$:

$$X^\pm(\sigma, \tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma}, \quad T = \frac{M}{\nu} \sinh(\nu\tau), \quad \theta = \nu\sigma$$

is a **short string winding around the Milne circle** from $T = -\infty$ to $T = +\infty$.

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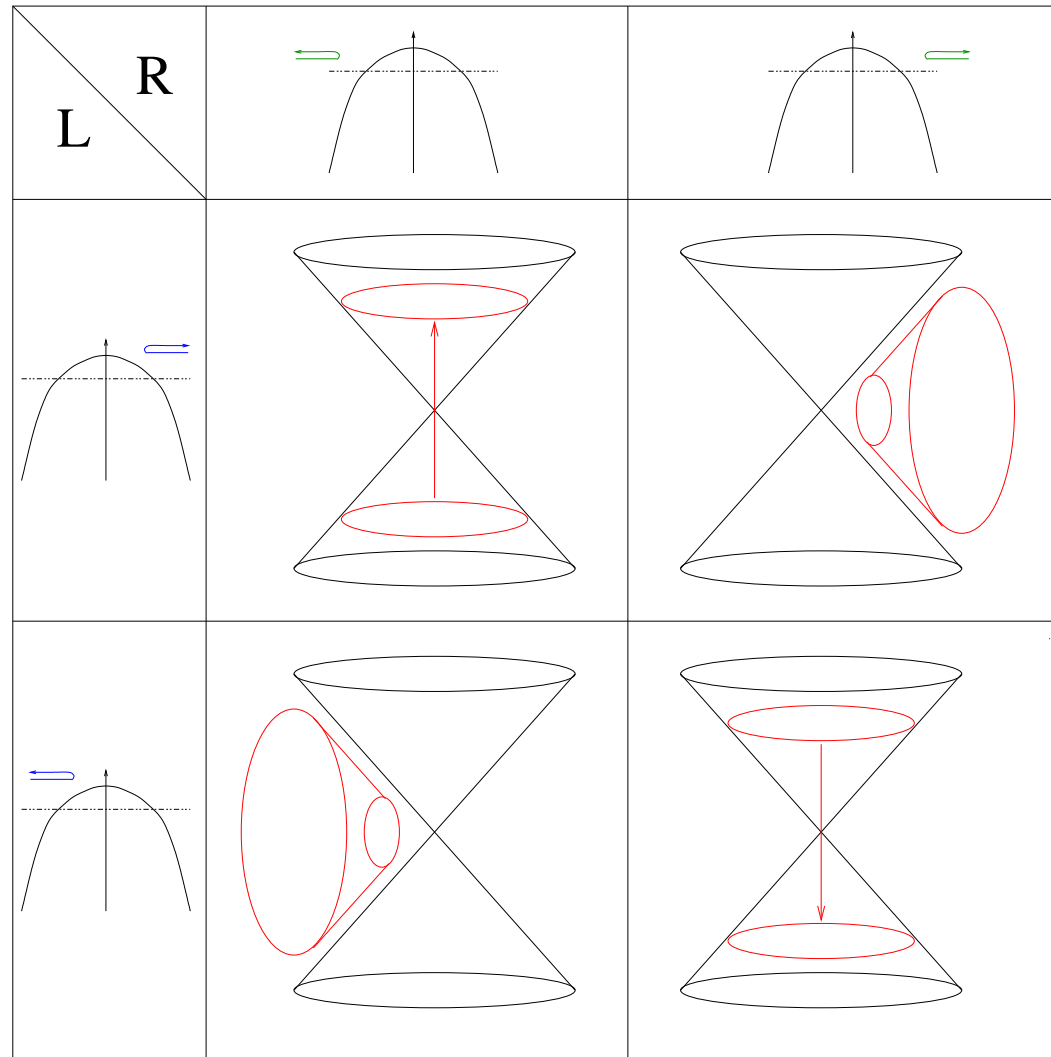
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$\epsilon = -1, \tilde{\epsilon} = 1$ is the analogue in the left Rindler patch.

Short and long strings

Closed string trajectories are thus generated by the motion of **two decoupled particles in inverted harmonic oscillators**:



Relation to open string modes

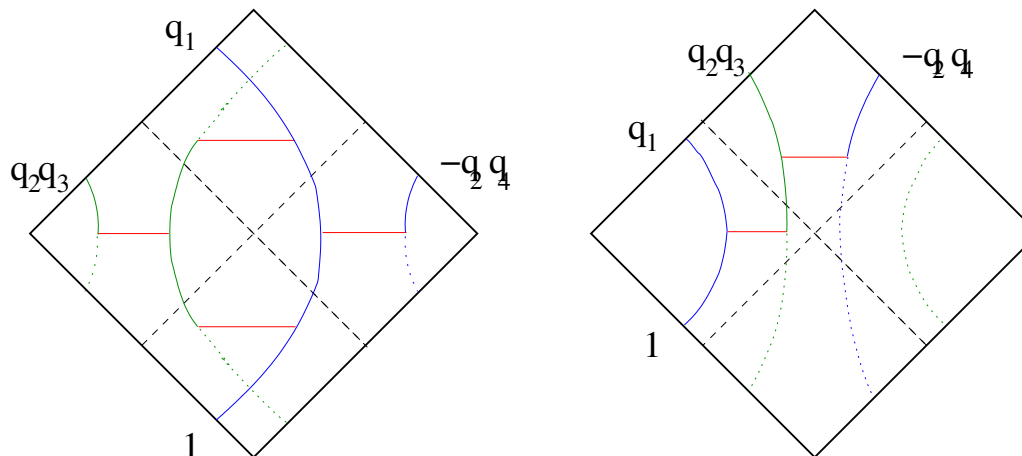
- Instead of following the motion of a point at fixed σ , one may consider instead a point at fixed $\sigma + \tau$: this is precisely the **trajectory of the open string zero-mode**.
- Using the covariant derivative representation

$$\alpha_0^\pm = i\partial_\mp \pm \frac{\nu}{2}x^\pm, \quad \tilde{\alpha}_0^\pm = i\partial_\mp \mp \frac{\nu}{2}x^\pm$$

we observe that x^\pm is the **Heisenberg operator** corresponding to the location of the closed string (at $\sigma = 0$):

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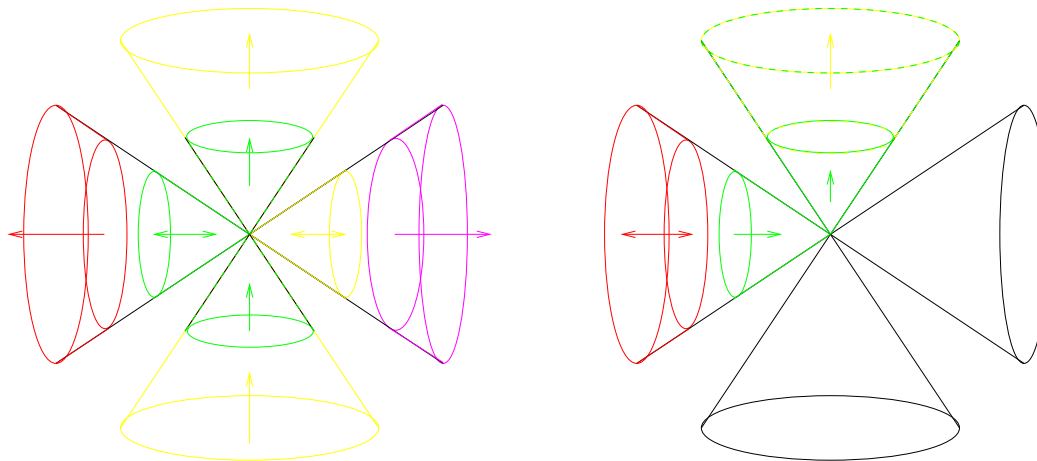
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- The open string global wave functions are also the closed string wave functions. . .



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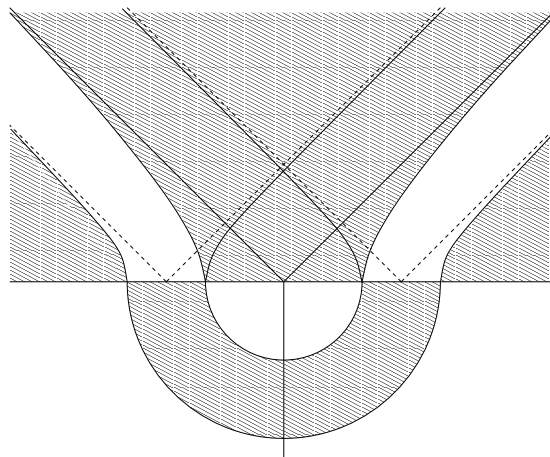
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- Similarly, in the closed string problem, **tunneling under the barrier** corresponds to **induced pair production** of winding strings.
- **Spontaneous pair production** of winding strings can be described by cutting open a periodic trajectory in the **Euclidean rotation orbifold**: **Hartle-Hawking vacuum**



A few words on second quantization

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- Instead, one may (in two different ways) identify the Hilbert space with that of a **single charged particle**, including its center of motion. The wave function is thus a state in the **tensor product of the left and right Rindler patches**. One can define in and out vacua, and find global pair production.

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- Finally, motivated by holography, one may try to quantize with respect to the radial evolution in Rindler space. Short and long strings would be analogous to normalizable / non-normalizable modes.

Quantization in the Rindler patch

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- If so we should quantize the string with respect to the “time” coordinate σ rather than τ . The Rindler energy is given by the canonical generator associated to boosts,

$$W = - \int_{-\infty}^{\infty} d\tau \left(X^+ \partial_{\sigma} X^- - X^- \partial_{\sigma} X^+ \right) = \int_{-\infty}^{\infty} d\tau r^2 \partial_{\sigma} \eta$$

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$$w(r) = \frac{4\nu^2 r^3 \operatorname{sgn}(\nu)}{\sqrt{(M^2 + \tilde{M}^2 - 4\nu^2 r^2)^2 - 4M^2 \tilde{M}^2}}$$

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- For short strings, the energy stored **in the Rindler patch** is finite, however **not conserved**. The integral over the entire worldsheet is **infinite** and negative.
- The spectrum is thus **unbounded from below** (and above): Can CTC prevent the vacuum to decay ?

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- A bounce in dimension i requires $H_i' > 0$ at the point where $H_i = 0$, i.e.

$$(D-2)p_i + \rho \geq \sum_{j \neq i} p_j$$

The most efficient solution is a gas of scalar momentum states, with $p = \rho$: provides enough pressure for the bounce.

Effective gravity analysis (cont.)

- However, consider fundamental strings wrapped around dimension i ,

$$\rho = \frac{T}{V}, \quad p_i = -\rho, \quad p_{j \neq i} = 0, \quad V = \prod_{j \neq i} a_j \quad \Rightarrow \quad D \leq 3$$

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- We assumed a constant number of wound strings: one should incorporate the dependence of the production rate on the Hubble parameters.

Effective gravity analysis (cont.)

- Einstein's equations imply that the quantity

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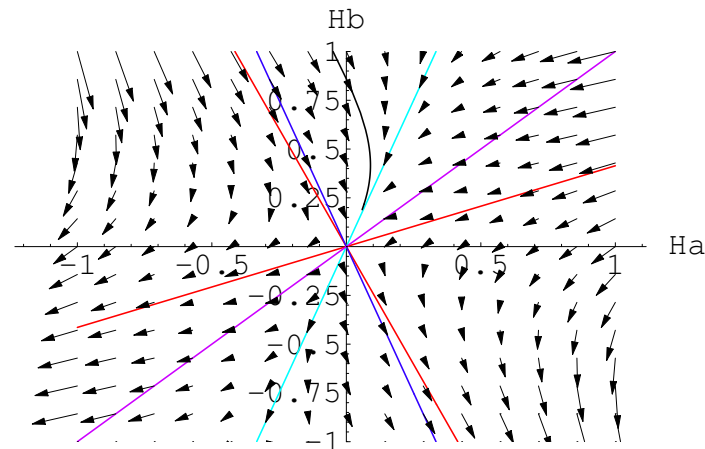
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A bounce for direction i in units of the eleven-dimensional frame therefore takes place for any initial condition such that $2\mu + D - 3 > 0$ and $2 < D < 4$.



Conclusions

We discussed closed strings in a toy model of a cosmological singularities. However, some of the features we uncovered should carry over to more general geometries:

- Winding string production can be understood semi-classically as **tunneling under the barrier** in regions with compact time, or **scattering over the barrier** in cosmological regions. In general, it can be computed as a **tree-level two-point function** in an appropriate basis depending on the choice of vacuum.

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- The production rate for winding strings in a singular geometry diverges at large winding number. **Can the resolved geometry be determined self-consistently so that the divergences at one-loop cancel those at tree-level ?**