Closed Strings in the Misner Universe

aka the Lorentzian orbifold

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LPTHE and LPTENS, Paris

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Talk based on

hep-th/0307280 w/ M. Berkooz
hep-th/0405126 w/ M. Berkooz, and M. Rozali
hep-th/0406xxx w/ M. Berkooz, B. Durin and D. Reichmann

slides available from
`http://www.lpthe.jussieu.fr/~pioline/seminars.html`

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- With the expected improved accuracy of cosmological measurements, it is possible that distinctive features of string theory may reveal themselves: exponentially large density of states, limiting Hagedorn temperature, winding states and other extended states, fundamental cosmic strings...
- Most importantly, inflation does not get rid of the initial singularity. Can string theory evade the usual divergences of perturbative gravity and “no-bounce theorems´´ ?

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- Even before quantum (g_s) corrections, string theory backgrounds undergo classical (α') corrections. Very few examples of classical cosmological solutions are known.
- In this talk, I will discuss an example of a classically exact cosmological background with a

space-like singularity: Misner space, aka the “Lorentzian” orbifold. We will compute tree-level particle/string production rates, and ask what they imply for the singularity.

Outline of the talk

1. Euclidean and Lorentzian orbifolds, and their avatars

Misner, Taub-NUT, Grant...

2. Untwisted strings in Misner space

Hiscock, Konkowski; Berkooz Craps Kutasov Rajesh, . . .

3. Twisted strings in Misner space: first pass

Nekrasov

3. A detour: Open strings in electric fields

Bachas Porrati; Berkooz BP

4. Twisted strings in Misner space: second pass

Berkooz BP Rozali

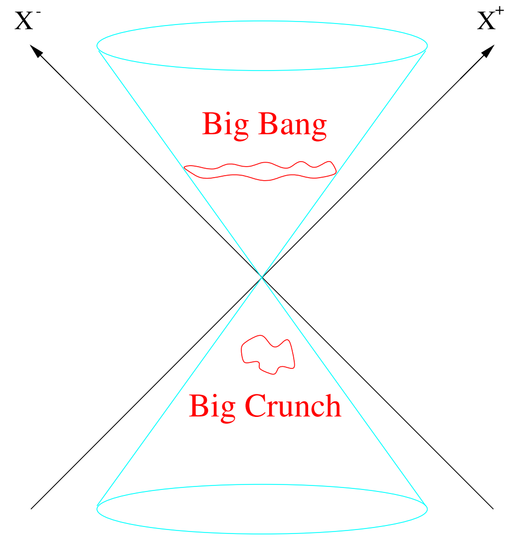
5. Comments on backreaction from winding strings

The Lorentzian orbifold

- One of the simplest examples of space-like singularities is the quotient of flat Minkowski space by a discrete boost, also known as Misner space (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^\pm \sim e^{\pm 2\pi\beta} X^\pm$$



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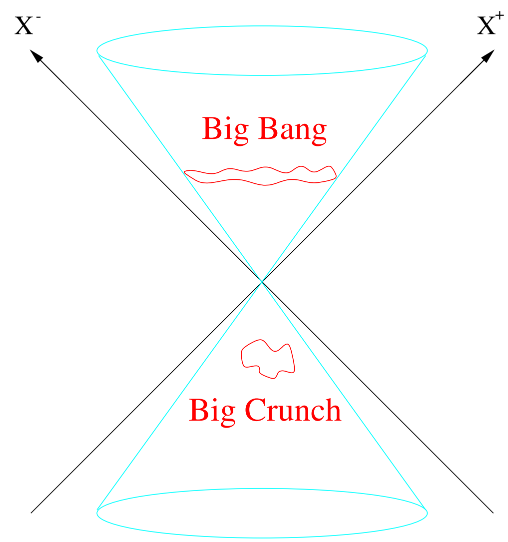

- The future (past) regions $X^+X^- > 0$ describes a cosmological universe often known as the Milne universe (1932), linearly expanding away from a Big Bang singularity (or contracting into a Big Crunch singularity):

$$ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + (dX^i)^2\,, \quad \theta \equiv \theta + 2\pi\,, \quad X^\pm = Te^{\pm\beta\theta}/\sqrt{2}$$

This is a (degenerate) Kasner singularity, everywhere flat, except for a delta-function curvature at $T = 0$.

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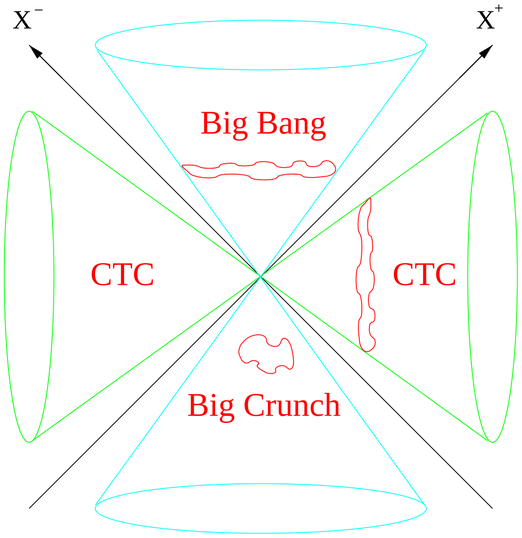

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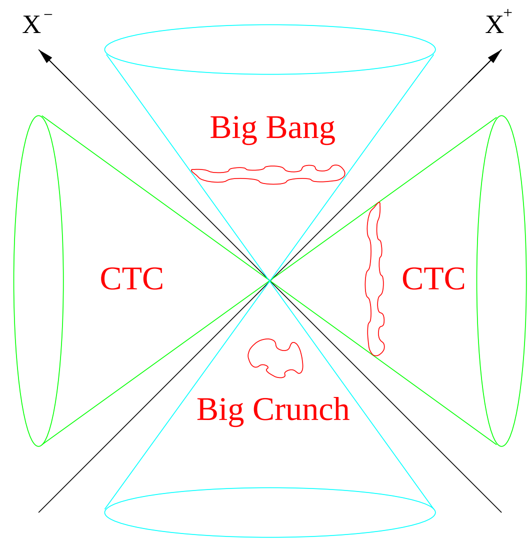

- In addition, the spacelike regions $X^+X^- < 0$ describe two Rindler wedges with compact time, often known as whiskers, leading to closed time-like curves:

$$ds^2 = dr^2 - \beta^2 r^2 d\eta^2 + (dX^i)^2 \quad , \eta \equiv \eta + 2\pi \ , \quad X^\pm = \pm r e^{\pm\beta\eta}/\sqrt{2}$$

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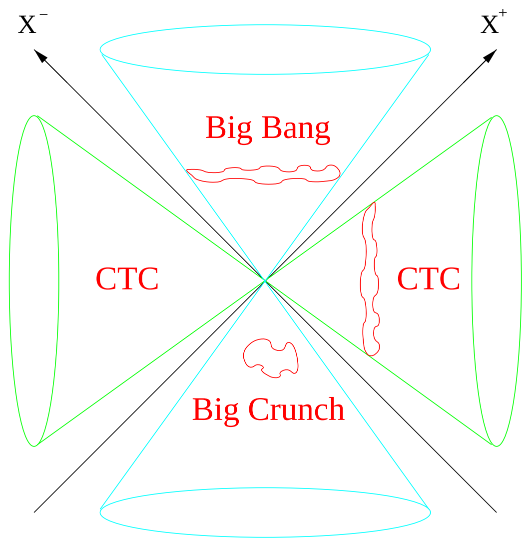

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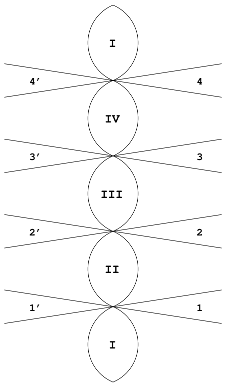
- Finally, the lightcone $X^+X^- = 0$ gives rise to a null, non-Hausdorff locus attached to the singularity.

Close relatives of the Misner Universe

- Misner space was first introduced as a local model of Lorentzian Taub-NUT space:



$$ds^2 = 4l^2 U(t)\sigma_3^2 + 4l\sigma_3 dt + (t^2+l^2)(\sigma_1^2+\sigma_2^2)\,, \quad U(t) = -1 + \frac{2mt+l^2}{t^2+l^2}$$

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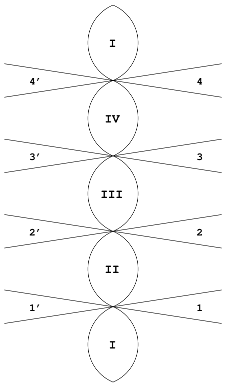

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- A close variant of Misner space is the quotient of flat space by the combination of a discrete boost and a translation on an extra direction, often known as the Grant space:

$$ds^2 = -2dX^+dX^- + dX^2 + (dX^i)^2\,, \quad (X^\pm, X) \sim (e^{\pm 2\pi\beta}X^\pm, X + 2\pi R)$$

  This describes the space away from two moving cosmic strings. The cosmological singularity is smoothed out, but regions with CTC remain.

Gott 91, Grant 93; Cornalba, Costa, Kounnas

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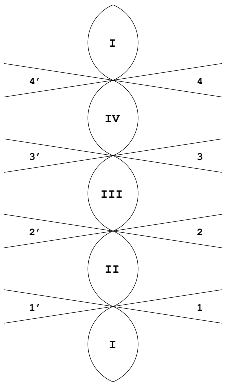

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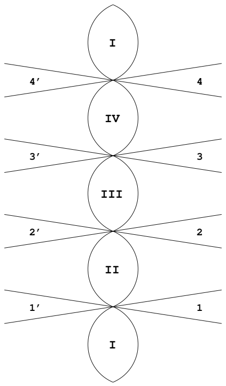
- The Misner geometry arose again more recently as the M-theory lift of a simple (ekpyrotic) cosmological solution of Einstein-dilaton gravity with no potential.

Khoury Ovrut Seiberg Steinhard Turok

Close relatives of the Misner Universe (cont)

- The gauged WZW model $Sl(2) \times Sl(2)/U(1) \times U(1)$ describes a bouncing 4-dimensional Universe, with singularities analogous to the Lorentzian orbifold.

Nappi Witten; Elitzur Giveon Kutasov Rabinovici



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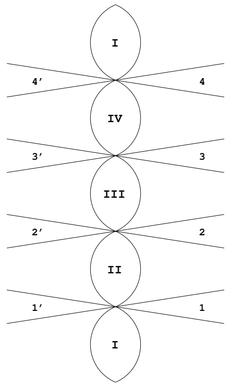

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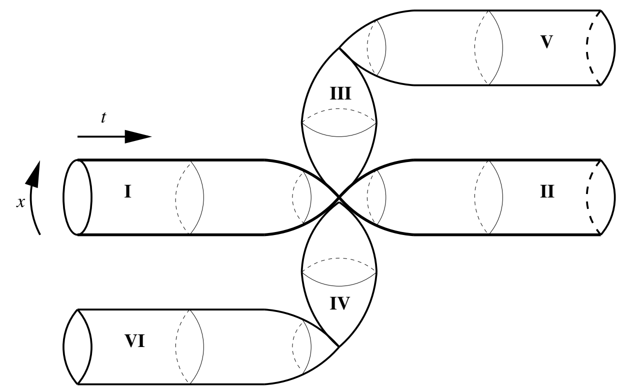

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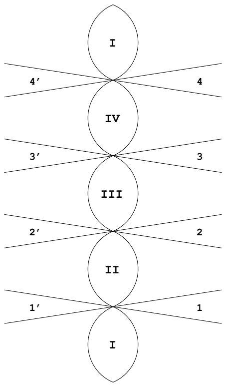

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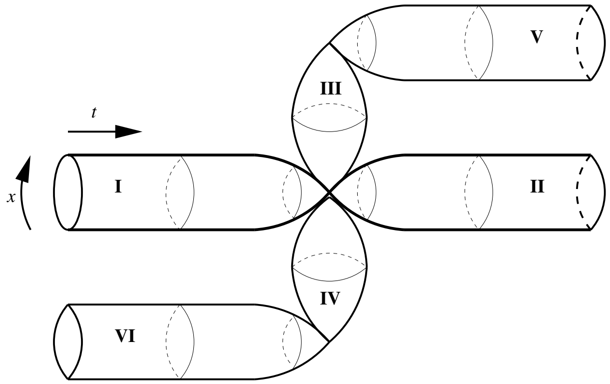

- The Lorentzian orientifold $IIB/[(-)^F boost]/[\Omega(-)^{F_L}]$ was also recently argued to describe orientifolds of non-supersymmetric strings with non-vanishing Neveu-Schwarz tadpoles.

Dudas Mourad Timirgaziu

Classical particles in the Misner Universe

- Classical Particles propagate as straight lines on the covering space:

$$\begin{aligned} X^{\pm} &= x_0^{\pm} + p^{\pm}\tau \\ 2p^{+}p^{-} &= M^2 \\ j &= p^{+}x_0^{-} - p^{-}x_0^{+} \end{aligned}$$

- As the particle approaches the singularity from the past, it starts spinning faster and faster, $\theta \sim \log |T|$, implying large gravitational backreaction.
- In the Rindler wedges, the particle winds infinitely many times around the time direction: at any fixed Rindler time, there is an infinity of copies of the particle, each with energy j: the total Rindler energy is infinite.

Quantum particles in the Misner Universe

- Quantum mechanically, the radial motion, for fixed boost momentum j, is governed by a Liouville wall potential:

$$
\begin{aligned}
\frac{1}{r}\partial_r r \partial_r + \frac{j^2}{r^2} &= M^2 , \quad r = e^y , \quad V(y) = M^2 e^{2y} - j^2 \equiv 0 \\
-\frac{1}{T}\partial_T \partial_T - \frac{j^2}{T^2} &= M^2 , \quad T = e^x , \quad V(x) = -j^2 - M^2 e^{2x} \equiv 0
\end{aligned}
$$

  The singularity is at infinite distance in the canonical x or y coordinate.

- Wave functions are Bessel functions, and can be expressed as superpositions of plane waves on the covering space ($s =$spin)

$$
f_{j,M^2,s}(x^+, x^-) = \int_{-\infty}^{\infty} dv \exp\left(ik^+ X^- e^{-2\pi\beta v} + ik^- X^+ e^{2\pi\beta v} + ik_i X^i + ivj + vs \right)
$$

- Wave functions can be defined globally by continuing across the horizons. The *in* and *out* states defined at $T = -\infty$ and $T = +\infty$ are identical, hence no overall particle production. However, there is particle production around $T = 0$:

$$f(x^+ > 0, x^- > 0) = e^{-ij\theta} H^{(1)}_{-ij}(2MT) \sim \alpha (x^+)^{-ij} + \beta (x^-)^{ij}$$

Tree-level scattering of untwisted states

- For strings on an orbifold, part of the spectrum consists of closed strings in the parent theory, invariant under the orbifold projection. These topologically trivial states behave at low energy just like ordinary point particles.
- Tree-level scattering amplitudes of untwisted sector states can be computed from those in flat space by the inheritance principle,

$$\langle V(j_1,k_1)\dots V(j_n,k_n)\rangle_{Misner} = \int dv_1\dots dv_n\, e^{i(j_1 v_1+\dots+j_n v_n)}$$
$$\langle V(e^{\beta v_1}k_1^+, e^{-\beta v_1}k_1^-, k_1^i)\dots V(e^{\beta v_n}k_n^+, e^{-\beta v_n}k_n^-, k_n^i)\rangle_{Minkowski}$$

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- String amplitudes are exponentially suppressed in the high energy regime $s\to\infty$ at fixed s/t, s/u. However, in the Regge regime $s\to\infty$ with fixed t, $A\sim s^t$ as if the size of the strings were growing like $\sqrt{\ln s}$. This leads to

$$\int dv\; v^{-\frac{1}{2}(k_1^i-k_3^i)^2+i(j_2-j_4)}$$

This diverges if $(k_1^i-k_3^i)^2\le 2$. This can be understood as large graviton exchange near the cosmological singularity.

Quantum fluctuations in field theory

- In the Minkowski vacuum (inherited from the covering space), the renormalized propagator can be obtained as a sum over images,

$$
\begin{aligned}
G(x;x') \quad = \quad & \sum_{l=-\infty, l\neq 0}^{\infty} \int_0^{\infty} d\tau \int dp^{\mu} \\
& \exp\left(-ip^-(x^+ - e^{2\pi\beta l}x^{+\prime}) - ip^+(x^- - e^{2\pi\beta l}x^{-\prime}) - ip^i(x^i - x^{i\prime})\right)
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- The one-loop stress-energy tensor follows from $G(x,x)$, e.g for a conformally coupled scalar,

$$\langle T_{ab}\rangle = \lim_{x\to x'}\left[(1-2\xi)\nabla_a\nabla'_b - 2\xi\nabla_a\nabla_b + (2\xi - \frac{1}{2})g_{ab}\nabla_c\nabla'^c\right]G(x,x')$$

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This leads to a divergent quantum backreaction (worse if the spin $|s| > 1$):

$$\langle T^{\nu}_{\mu}\rangle = \frac{K}{12\pi^2}T^{-4}\text{diag}(1,-3,1,1)\ , \quad K = \sum_{l=1}^{\infty}\cosh(2\pi\beta l s)\frac{2+\cosh 2\pi l\beta}{[\cosh 2\pi l\beta - 1]^2}$$

Hiscock Konkowski 82

One-loop vacuum amplitude in field and string theory

- On the other hand, in string theory $\langle T_\mu^\nu \rangle(x)$ is an off-shell quantity, and only its integral over space-time is well defined:

$$\int dx^+ dx^- G(x,x) = \sum_{l=-\infty}^{+\infty} \int_0^\infty \frac{d\rho}{\rho^{D/2}} \frac{e^{-m^2\rho}}{\sinh^2(\pi\beta l)}$$

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- This reproduces the zero-mode contribution to the string one-loop vacuum amplitude in the untwisted sector:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho)\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

$$\theta_1(v;\rho) = 2q^{1/8} \sin \pi v \prod_{n=1}^{\infty} (1-e^{2\pi i v} q^n)(1-q^n)(1-e^{-2\pi i v} q^n) \,, \quad q = e^{2\pi i \rho}$$

Nekrasov, Cornalba Costa

- The local divergence in $\langle T_\mu^\nu \rangle(x)$ is integrable and yields a finite free energy.

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- The existence of Regge trajectories with arbitrary high spin implies new (log) divergences in the bulk of the moduli space which resemble long string poles in AdS_3.

Closed string in Misner space - twisted sectors

- In addition, there is an infinite set of twisted sectors, corresponding to strings on the covering space that close up to the action of the orbifold group:

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau) \ , \quad \nu = 2\pi w \beta$$

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- They have a normal mode expansion:

$$\begin{aligned} X_R^{\pm}(\tau - \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha_n^{\pm} \, e^{-i(n \pm i\nu)(\tau - \sigma)} \\ X_L^{\pm}(\tau + \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \mp i\nu)^{-1} \tilde{\alpha}_n^{\pm} \, e^{-i(n \mp i\nu)(\tau + \sigma)} \end{aligned}$$

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$$\begin{aligned} X_R^{\pm}(\tau - \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha_n^{\pm} \, e^{-i(n \pm i\nu)(\tau - \sigma)} \\ X_L^{\pm}(\tau + \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \mp i\nu)^{-1} \tilde{\alpha}_n^{\pm} \, e^{-i(n \mp i\nu)(\tau + \sigma)} \end{aligned}$$

with canonical commutation relations

$$[\alpha_m^+, \alpha_n^-] = -(m + i\nu)\delta_{m+n} \quad , \quad [\tilde{\alpha}_m^+, \tilde{\alpha}_n^-] = -(m - i\nu)\delta_{m+n}$$

$$(\alpha_m^{\pm})^* = \alpha_{-m}^{\pm} \quad , \quad (\tilde{\alpha}_m^{\pm})^* = \tilde{\alpha}_{-m}^{\pm}$$

Closed string in Misner space - twisted sectors

- In addition, there is an infinite set of twisted sectors, corresponding to strings on the covering space that close up to the action of the orbifold group:

$$X^{\pm}(\sigma+2\pi,\tau)=e^{\pm\nu}X^{\pm}(\sigma,\tau)\ ,\quad \nu=2\pi w\beta$$

- They have a normal mode expansion:

$$\begin{aligned} X^{\pm}_R(\tau-\sigma) &= \frac{i}{2}\sum_{n=-\infty}^{\infty}(n\pm i\nu)^{-1}\alpha^{\pm}_n\, e^{-i(n\pm i\nu)(\tau-\sigma)} \\ X^{\pm}_L(\tau+\sigma) &= \frac{i}{2}\sum_{n=-\infty}^{\infty}(n\mp i\nu)^{-1}\tilde{\alpha}^{\pm}_n\, e^{-i(n\mp i\nu)(\tau+\sigma)} \end{aligned}$$

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$$[\alpha^+_m,\alpha^-_n]=-(m+i\nu)\delta_{m+n}\quad ,\quad [\tilde{\alpha}^+_m,\tilde{\alpha}^-_n]=-(m-i\nu)\delta_{m+n}$$
$$(\alpha^{\pm}_m)^*=\alpha^{\pm}_{-m}\quad ,\quad (\tilde{\alpha}^{\pm}_m)^*=\tilde{\alpha}^{\pm}_{-m}$$

- We will focus on the quasi zero-mode sector, which consists of two commuting pairs of real (i.e. hermitian) canonically conjugate operators,

$$[\alpha_0^+, \alpha_0^-] = -i\nu \ , \quad [\tilde{\alpha}_0^+, \tilde{\alpha}_0^-] = i\nu$$

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated by half of them, e.g.

$$\alpha^{\pm}_{n>0}\,, \quad \tilde{\alpha}^{\pm}_{n>0}\,, \qquad \alpha^{-}_{0}\,, \quad \tilde{\alpha}^{+}_{0}$$

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- The worldsheet Hamiltonian, normal-ordered wrt to this vacuum, reads

$$L_0^{l.c.} = -\sum_{n=0}^{\infty}(\alpha_n^+)^*\alpha_n^- \ - \sum_{n=1}^{\infty}(\alpha_n^-)^*\alpha_n^+ + \frac{1}{2}i\nu(1-i\nu) - 1 + L_{int}$$

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- This is the familiar result for the vacuum energy $\frac{1}{2}\theta(1-\theta)$ in the Euclidean rotation orbifold, after analytically continuing $\theta \to i\nu$.
- Due to the $i\nu/2$ term in the ground state energy, all states obtained by acting on $|0\rangle$ by creation operators $\alpha^{\pm}_{n<0}$ and by α_0^+ will have imaginary energy, hence the physical state conditions $L_0 = \tilde L_0 = 0$ seem to have no solutions.

One-loop amplitude, twisted sector

- Independently of this fact, one may compute the one-loop path integral on an Euclidean worldsheet and Minkowskian target-space:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho)x\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

where θ_1 is the Jacobi theta function,

$$\theta_1(v;\rho) = 2q^{1/8}\sin\pi v \prod_{n=1}^{\infty}(1-e^{2\pi i v}q^n)(1-q^n)(1-e^{-2\pi i v}q^n)\ , \quad q = e^{2\pi i \rho}$$

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- In the twisted sector, the left-moving zero-modes contribute

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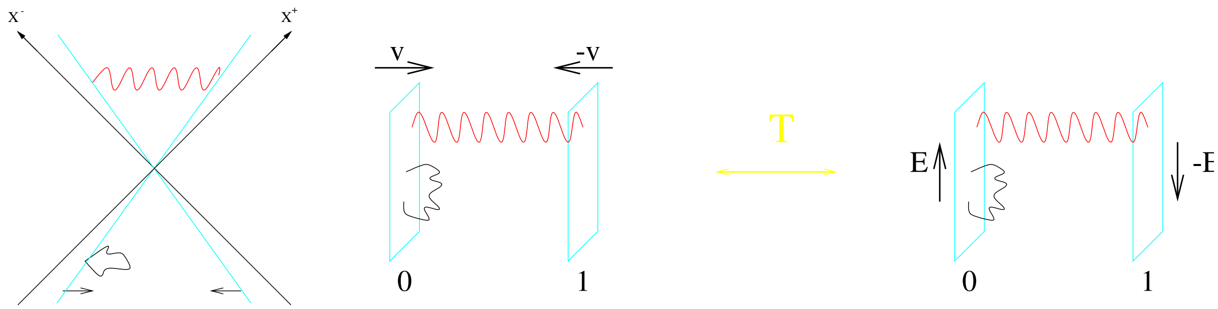
A detour via Open strings in electric field

- A very similar puzzle is faced in the case of colliding D-branes, or in the T-dual process of charged open strings in a constant electric field:



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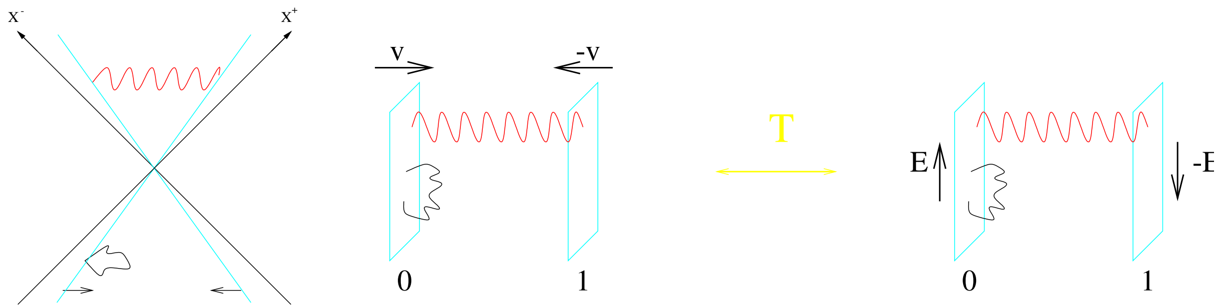

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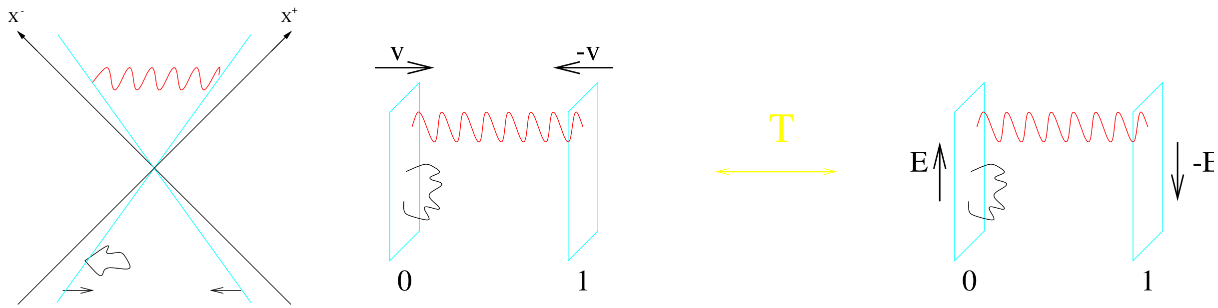

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- This reproduces (one half of) the spectrum of Closed strings in Misner space upon identifying $\nu = w\beta$. The large winding number limit $w \to \infty$ amounts to a near critical electric field $E \to 1$.
- In particular, the open string zero-modes describe the motion of a charged particle in an electric field, and have a structure isomorphic to the closed string case.

Charged particle and open string zero-modes

- Recall the first quantized charged particle in an electric field:

$$L = \frac{1}{2}m\left(-2\partial_\tau X^+\partial_\tau X^- + (\partial_\tau X^i)^2\right) + \frac{\nu}{2}\left(X^+\partial_\tau X^- - X^-\partial_\tau X^+\right)$$

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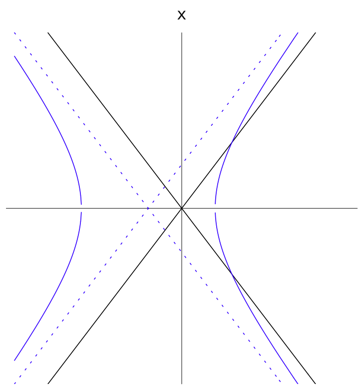

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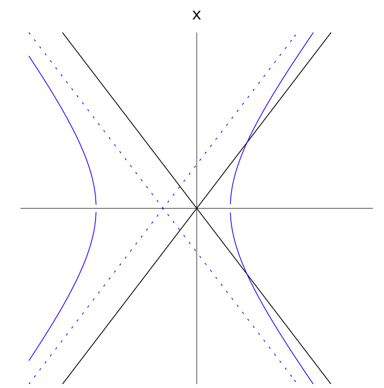

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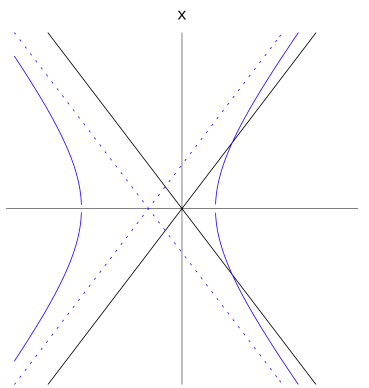

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- Upon quantizing $a_0^\pm$ as creation/annihilation operators in a Fock space, electrons and positrons would have no physical state...

Charged particle and Klein-Gordon equation

- Quantum mechanically, one represents the canonical momenta as derivatives, $\pi^{\pm} = i\partial/\partial x^{\mp}$, hence $a_0^{\pm}$, $x_0^{\pm}$ as covariant derivatives

$$a_0^{\pm} = i\partial_{\mp} \pm \frac{\nu}{2} x^{\pm} \, , \quad x_0^{\pm} = \mp \frac{1}{\nu} \left(i\partial_{\mp} \mp \frac{\nu}{2} x^{\pm} \right)$$

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- The zero-mode piece of L_0, including the bothersome $\frac{i\nu}{2}$,

$$L_0^{(0)} = -a_0^+ a_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\nabla^+\nabla^- + \nabla^-\nabla^+)$$

is just the Klein-Gordon operator of a particle of charge ν, and has well-behaved eigenmodes $L_0 = -m^2$ for any $m^2 > 0$.

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^{\pm} = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator can be rewritten as an inverted harmonic oscillator:

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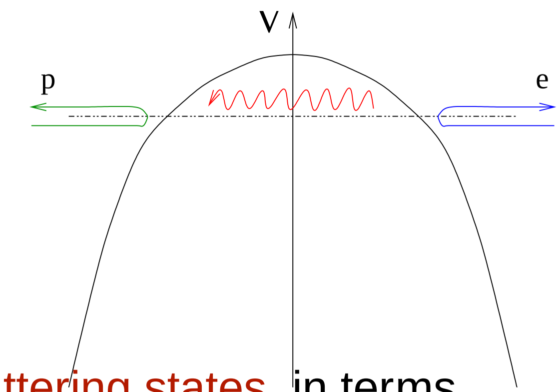
$$\left(-\partial_u^2 - \frac{1}{4}u^2 + \frac{M^2}{2\nu} \right) \psi_{\tilde{p}}(u) = 0$$



- The latter admits a respectable delta-normalizable spectrum of scattering states, in terms of parabolic cylinder functions, e.g:

$$\phi_{in}^{+}(x,t) = D_{-\frac{1}{2}+i\frac{M^2}{2\nu}}(e^{-\frac{3i\pi}{4}}u)e^{-i\tilde{p}t}e^{i\nu xt/2}$$

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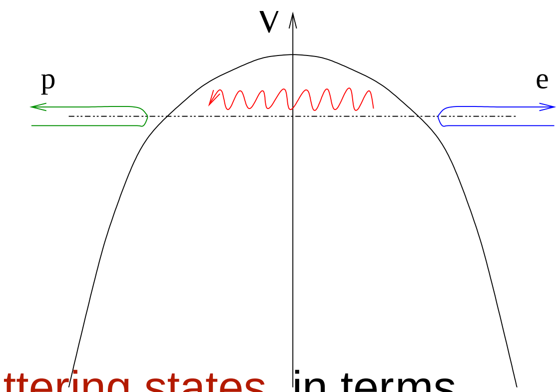

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- These correspond to non-compact trajectories of charged particles in the electric field. Tunnelling is just (stimulated) Schwinger pair creation,

$$e^- \longrightarrow (1+\eta)\, e^- + \eta\, e^+ \,, \quad \eta \sim e^{-\pi M^2/\nu}$$

Brezin Itzykson; Brout Massar Parentani Spindel

Lorentzian vs Euclidean states

- The Wick rotation $X^0 \to -iX^0$, $\nu \to i\nu$ turns the electric field (Schwinger) problem in $R^{1,1}$ into the magnetic field (Landau) problem in R^2.

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- The zero-mode contribution to the one-loop amplitude can be interpreted either way,

$$\frac{1}{2i\sin(\nu t/2)} = \sum_{n=1}^{\infty} e^{-i(n+\frac{1}{2})\nu t} = \int dM^2 \rho(M^2) e^{-M^2 t/2}$$

The density of states is obtained from the reflection phase shift,

$$\rho(M^2) = \frac{1}{\nu}\log\Lambda - \frac{1}{2\pi i}\frac{d}{dM^2}\log\frac{\Gamma\left(\frac{1}{2}+i\frac{M^2}{2\nu}\right)}{\Gamma\left(\frac{1}{2}-i\frac{M^2}{2\nu}\right)}$$

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- The physical spectrum of the charged open string can be explicitely worked out, and is free of ghosts: a tachyon at level 0, a transverse gauge boson at level 1, . . .

Charged particle in Rindler space

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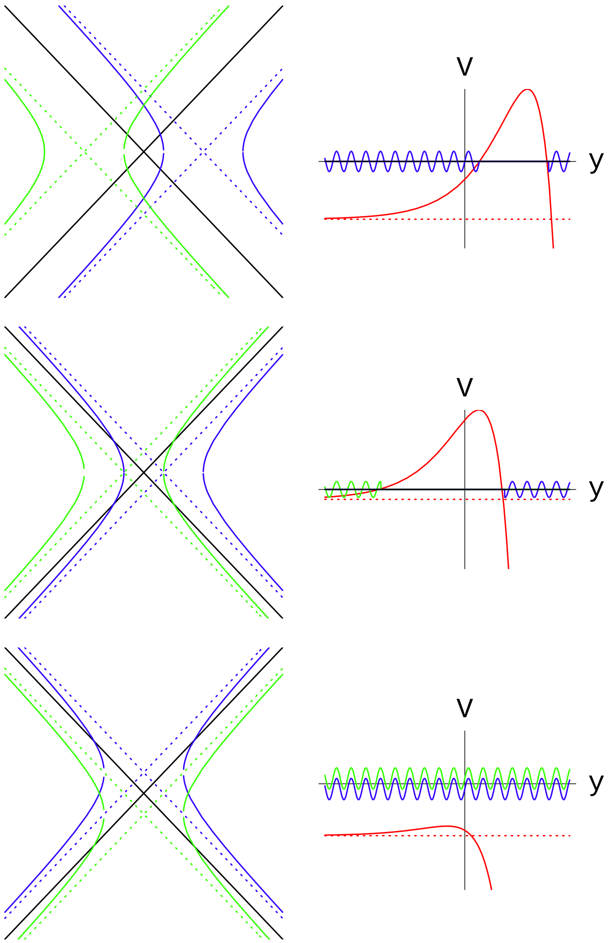

- In the Rindler patch R, letting $f(r,\eta) = e^{-ij\eta} f_j(r)$ and $r = e^y$, one gets a Schrödinger equation for a particle in a potential

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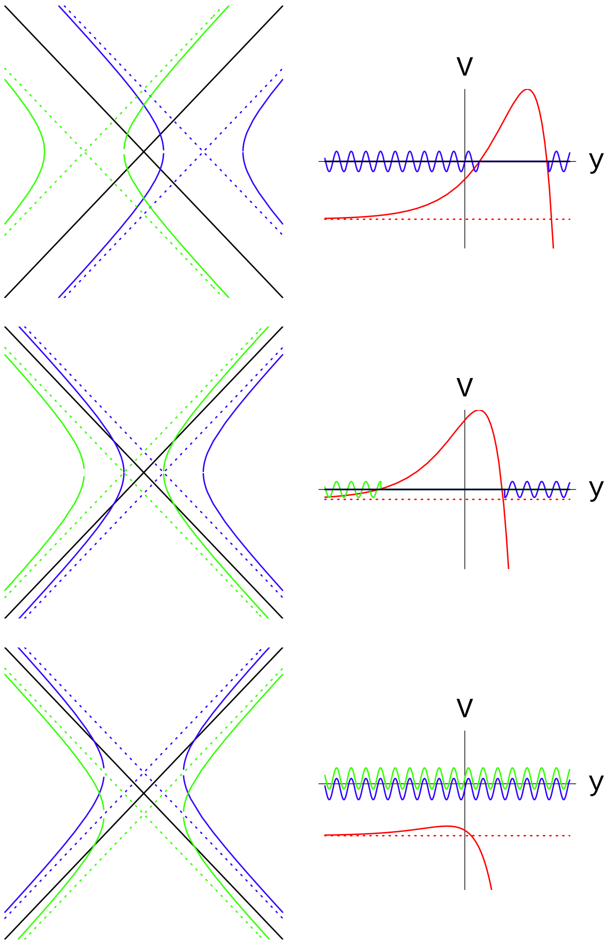

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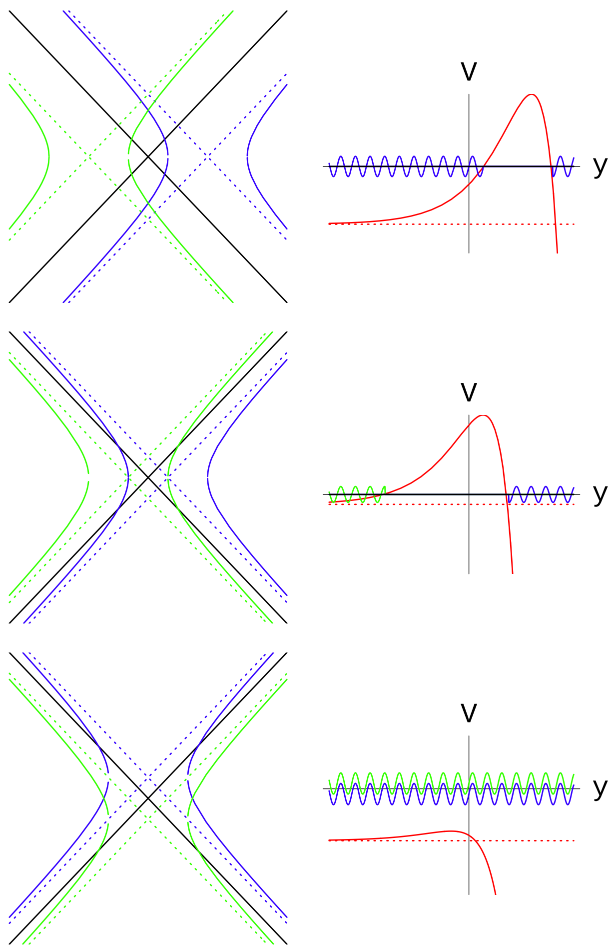

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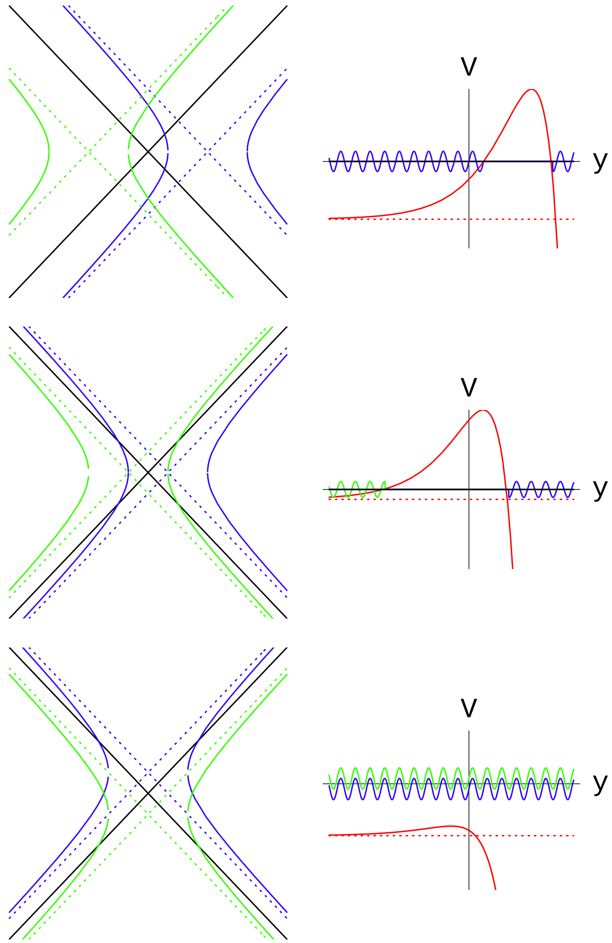

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- If $j > M^2/(2\nu)$, the electron branches cross the horizons. regions. There is no tunneling, but partial reflection amounts to a combination of Schwinger and Hawking emission.

Rindler modes

- Incoming modes from Rindler infinity I_R^- read, in terms of parabolic cylinder functions:

$$\mathcal{V}^j_{in,R} = e^{-ij\eta} r^{-1} M_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),-\frac{ij}{2}}(i\nu r^2/2)$$

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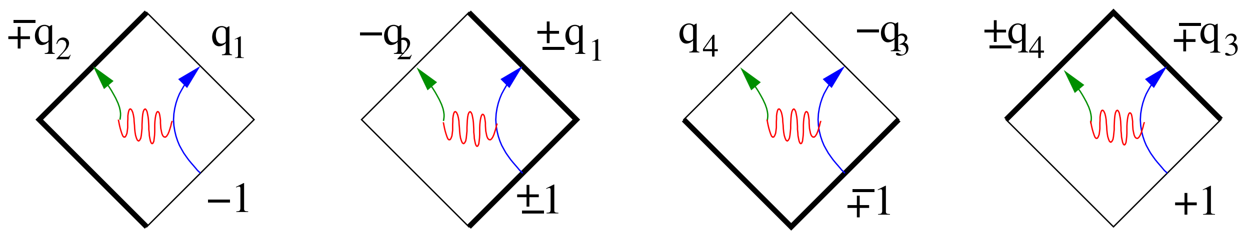

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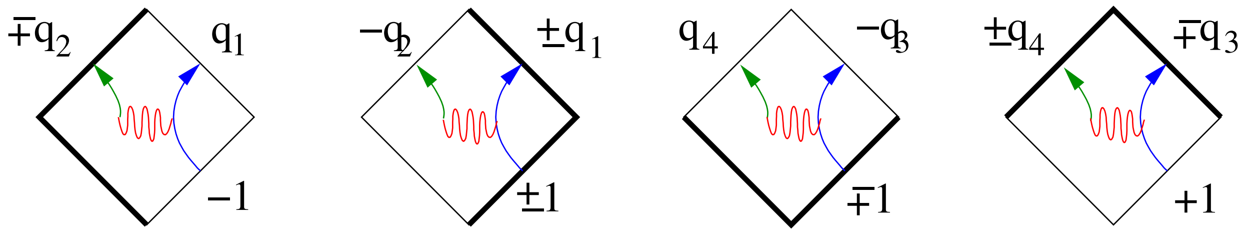

- The reflection coefficients can be computed:

$$q_2 = e^{-\frac{\pi M^2}{2\nu}} \frac{|\sinh \pi j|}{\cosh\left[\pi\left(j-\frac{M^2}{2\nu}\right)\right]}\,, \quad q_4 = e^{-\frac{\pi M^2}{2\nu}} \frac{\cosh\left[\pi\left(j-\frac{M^2}{2\nu}\right)\right]}{|\sinh \pi j|}$$

and $q_1 = 1 - q_2$, $q_3 = q_4 + 1$, by charge conservation.

Global Charged Unruh Modes

- Global modes may be defined by patching together Rindler modes, ie by analytic continuation across the horizons. Unruh modes are those which are superposition of positive energy Minkowski modes,

$$\Omega^j_{in,+} = \mathcal{V}^j_{in,P} = (-i\nu X^+X^-)[X^+/X^-]^{-ij/2}W_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}$$

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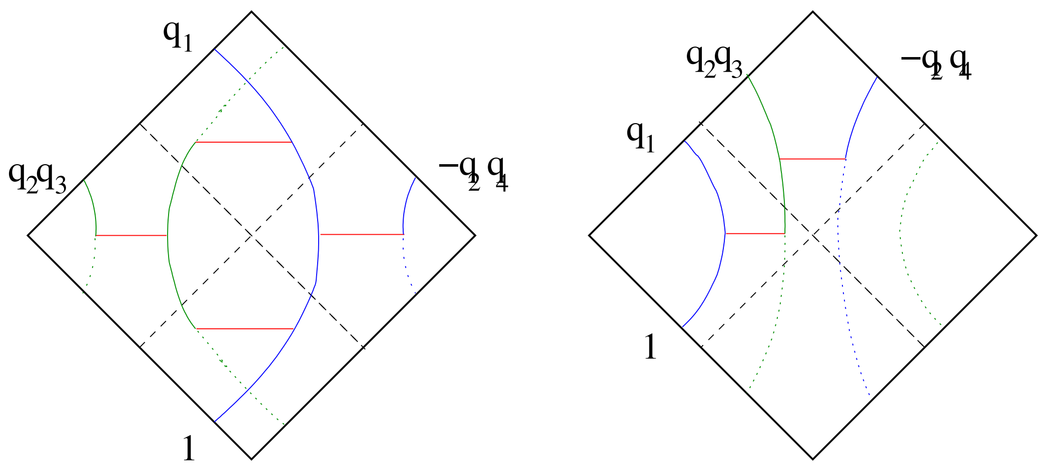
- There are two types of Unruh modes, involving 2 or 4 tunelling events:



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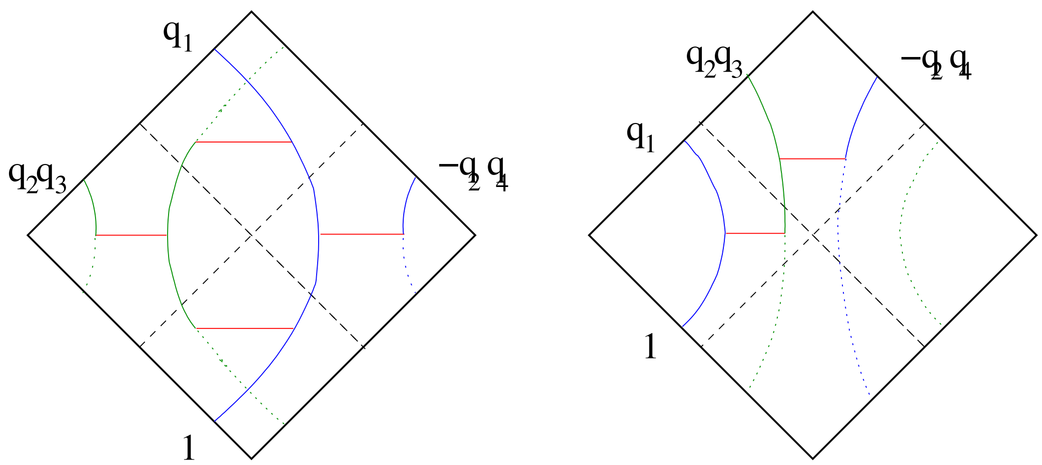

- Any state in Minkowski space can be represented as a state in the tensor product of the Hilbert spaces of the left and right Rindler patches. This gives a static description of the cosmological dynamics.

Closed string zero-modes

- Let us reanalyze the classical solutions for the closed string zero modes

$$X^{\pm}(\tau,\sigma) = e^{\mp\nu\sigma}\left[\pm\frac{1}{2\nu}\alpha_0^{\pm}e^{\pm\nu\tau} \mp \frac{1}{2\nu}\tilde{\alpha}_0^{\pm}e^{\mp\nu\tau}\right] \ , \quad \alpha_0^{\pm}, \tilde{\alpha}_0^{\pm} \in R$$

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- The Milne time, or Rindler radius, is independent of σ:

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$$\alpha_0^+ = \alpha_0^- = \epsilon\frac{M}{\sqrt{2}} , \quad \tilde{\alpha}_0^+ = \tilde{\alpha}_0^- = \tilde{\epsilon}\frac{\tilde{M}}{\sqrt{2}} , \quad M^2 - \tilde{M}^2 = 2\nu j \in Z$$

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- The behavior at early/late proper time now depends on $\epsilon\tilde{\epsilon}$: For $\epsilon\tilde{\epsilon} = 1$, the string begins/ends in the Milne regions. For $\epsilon\tilde{\epsilon} = -1$, the string begins/ends in the Rindler regions.

Short and long strings

Choosing $j = 0$ for simplicity, we have two very different types of solutions:

- $\epsilon = 1$, $\tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma, \tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma} \; , \quad T = \frac{M}{\nu} \sinh(\nu\tau) \; , \quad \theta = \nu\sigma$$

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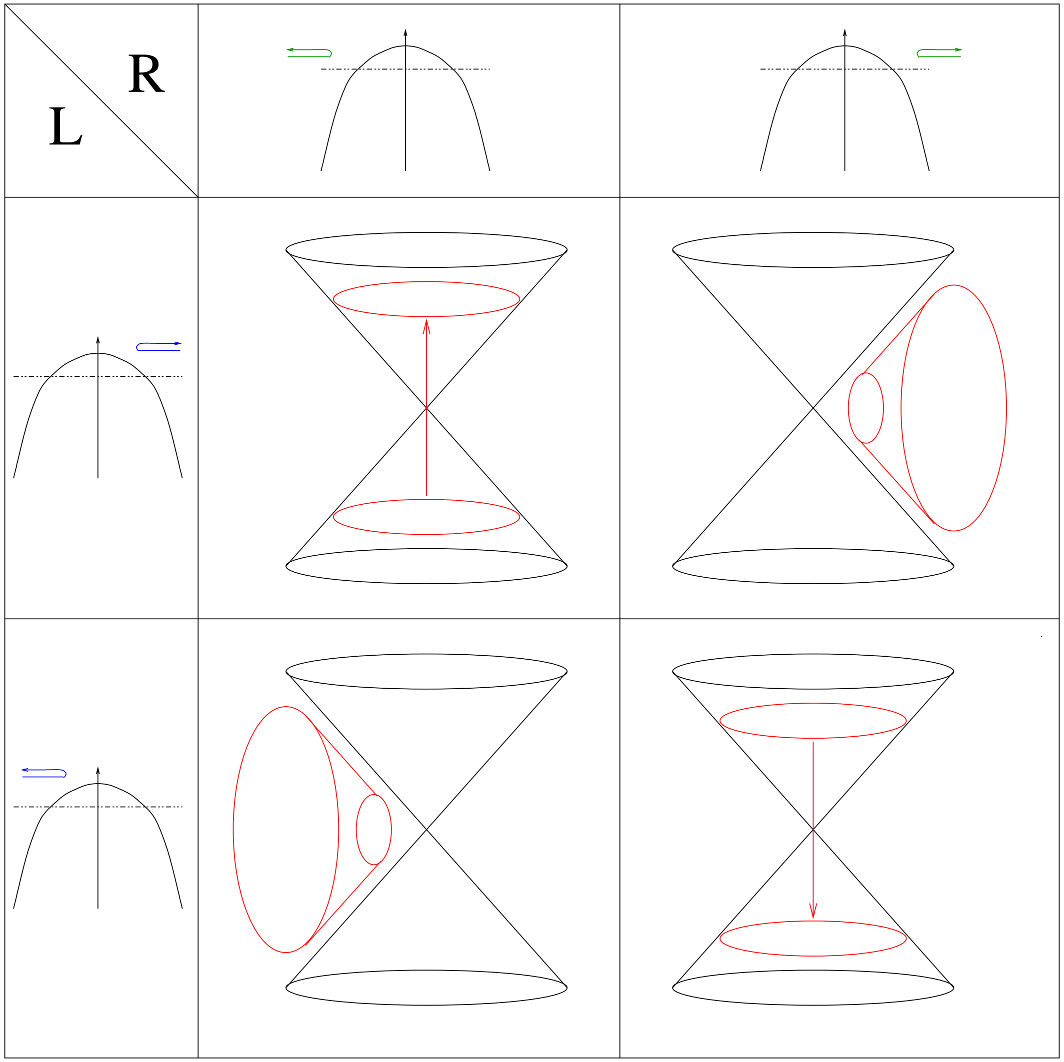
  $\epsilon = -1$, $\tilde{\epsilon} = 1$ is the analogue in the left Rindler patch.

Short and long strings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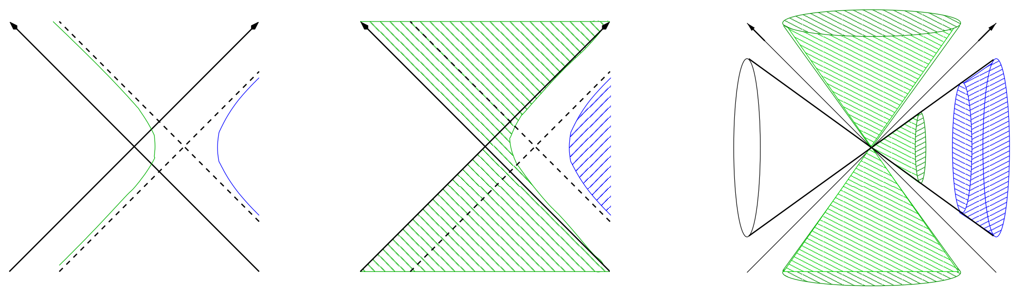

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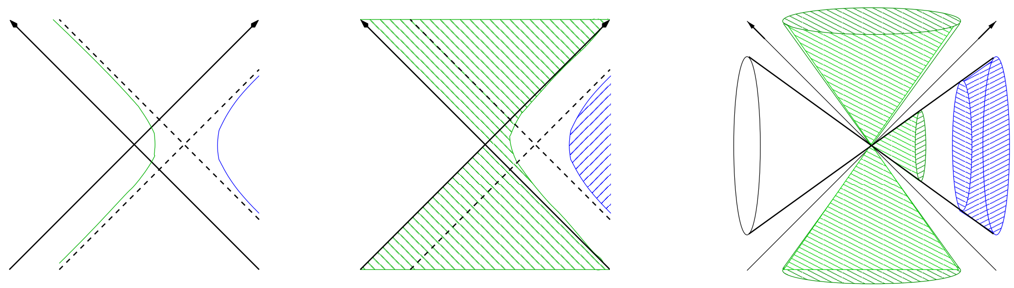

- Using the covariant derivative representation, we observe that $x^{\pm}$ is the Heisenberg operator corresponding to the location of the closed string (at $\sigma = 0$):

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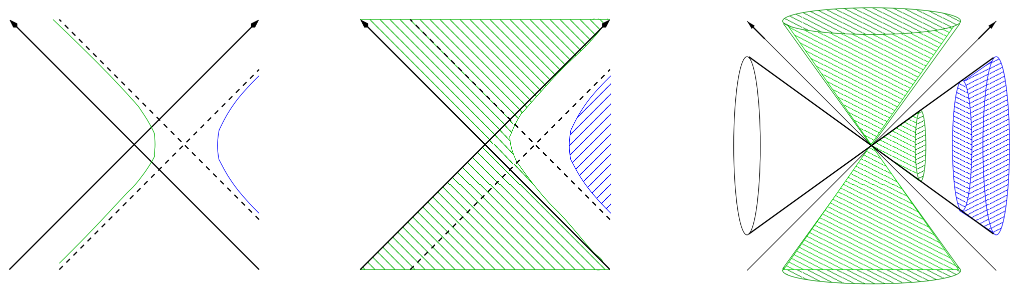

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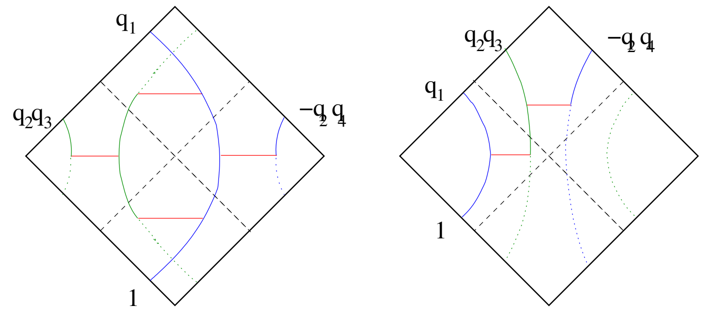
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q2q3
−q2 q4
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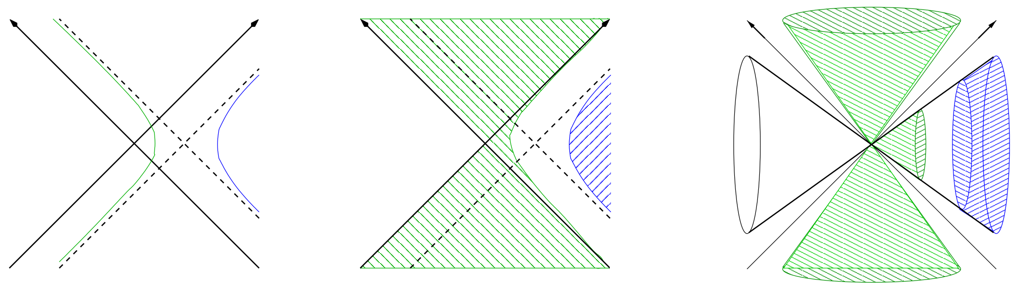

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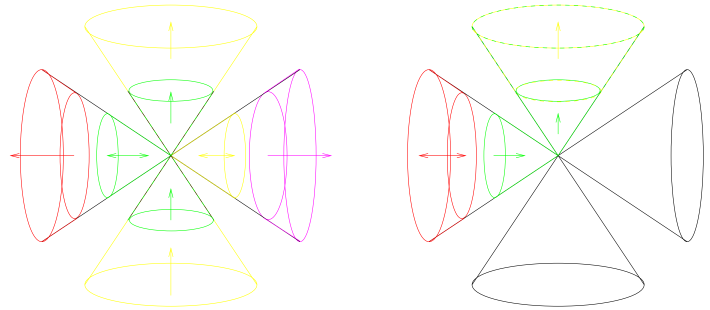

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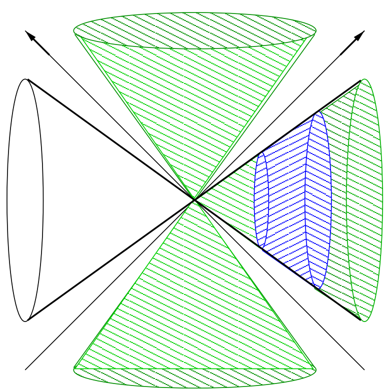

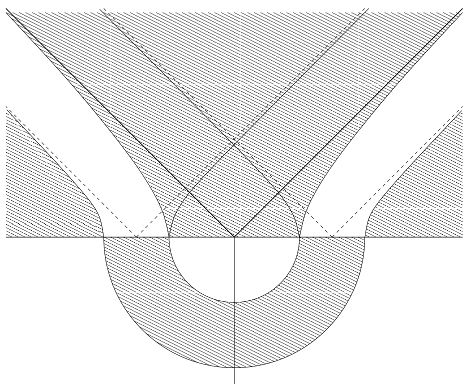

A few words on second quantization

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- Instead, one may (in two different ways) identify the Hilbert space with that of a single charged particle, including its center of motion. The wave function is thus a state in the tensor product of the left and right Rindler patches. One can define in and out vacua, and find global pair production.

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- Finally, motivated by holography, one may try to quantize with respect to the radial evolution in Rindler space. Short and long strings would be analogous to normalizable / non-normalizable modes.

Quantization in the Rindler patch

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$$W = -\int_{-\infty}^{\infty} d\tau \left(X^+ \partial_\sigma X^- - X^- \partial_\sigma X^+ \right) = \int_{-\infty}^{\infty} d\tau \; r^2 \partial_\sigma \eta$$

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- The total Rindler energy of a long string is infinite, due to its extension towards $r \to \infty$. The energy density by unit of radial distance

$$w(r) = \frac{4\nu^2 r^3 \text{sgn}\,(\nu)}{\sqrt{(M^2 + \tilde{M}^2 - 4\nu^2 r^2)^2 - 4M^2 \tilde{M}^2}}$$

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- The spectrum is thus unbounded from below (and above): Can CTC prevent the vacuum to decay ?

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- A bounce in dimension i requires $H_i' > 0$ at the point where $H_i = 0$, i.e.

$$(D-2) p_i + \rho \geq \sum_{j \neq i} p_j$$

The most efficient solution is a gas of scalar momentum states, with $p = \rho$: provides enough pressure for the bounce.

Effective gravity analysis (cont.)

- However, consider fundamental strings wrapped around dimension i,

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- Modelling the dilaton as the radius of the $\sharp$ th direction, the strings become membranes wrapped around $(i, \sharp)$:

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Effective gravity analysis (cont.)

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- We assumed a constant number of wound strings: one should incorporate the dependence of the production rate on the Hubble parameters.

Effective gravity analysis (cont.)

- Einstein's equations imply that the quantity

$$\mu = \left(\frac{H_k}{H_i} - 1 \right) / \left(\frac{H_j}{H_i} - \frac{3}{4 - D} \right)$$

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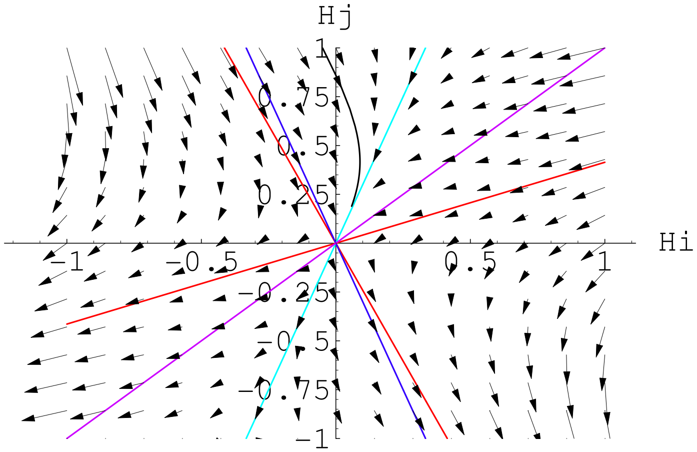
A bounce for direction i in units of the eleven-dimensional frame therefore takes place for any initial condition such that $2\mu + D - 3 > 0$ and $2 < D < 4$.


Hj
1
0.75
0.5
0.25
-1
-0.5
0.5
1
Hi
-0.25
-0.5
-0.75
-1

Conclusions

We discussed closed strings in a toy model of a cosmological singularities. However, some of the features we uncovered should carry over to more general geometries:

- Winding string production can be understood semi-classically as tunneling under the barrier in regions with compact time, or scattering over the barrier in cosmological regions. In general, it can be computed as a tree-level two-point function in an appropriate basis depending on the choice of vacuum.

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- Less ambitiously, what is the effective geometry corresponding to deforming the action with a marginal winding state ?