Closed Strings in the Misner Universe

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LPTHE, Paris

séminaire du GreCo

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based on hep-th/0307280 w/ M. Berkooz

and work in progress w/ M. Berkooz, B. Durin, D. Reichmann, M. Rozali

slides available from

`http://www.lpthe.jussieu.fr/~pioline/seminars.html`

Motivational string cosmology

- Observational Cosmology is now challenging string theory with high-precision data:

$$\Omega_{baryon} = 0.047 \; , \quad \Omega_{darkm} = 0.243 \; , \quad \Omega_{\Lambda} = 0.71 \; , \quad w = -0.98 \pm .12 \; , \ldots$$

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- With the expected improved accuracy of cosmological measurements, it is conceivable that distinctive features of string theory may reveal themselves:
  1. UV softness, Regge behavior
  2. exponentially large density of states, limiting Hagedorn temperature $T_H \sim 1/l_s$
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- With LHC still far in the future, understanding StringY Cosmology may be the only way to make contact with reality...

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- Perturbative string theory requires an Euclidean worldsheet, hence Euclidean target space. The analytic continuation may be ambiguous or ill-defined, Lorentzian observables may be very different from their Euclidean counterparts.
- String theory is not content on a finite time interval, and one is frequently forced into Big Bang / Big Crunch singularities, CTC in the process of maximally extending the geometry.

String theory and cosmological singularities

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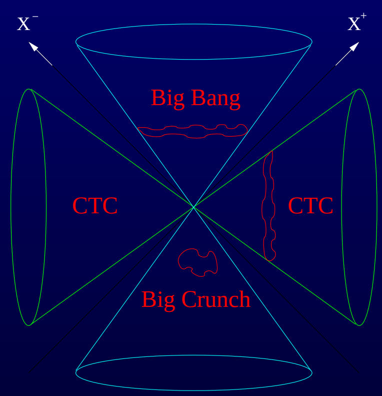
- In this talk, we shall discuss the "Lorentzian" orbifold of flat Minkowski space by a discret boost, as a toy model of a singular cosmological universe where string theory can in principle be solved explicitely.



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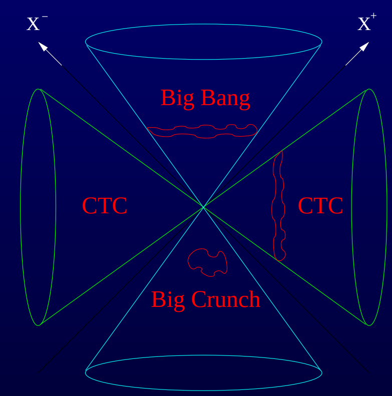


- We shall focus in particular on the topological excitations which wind around the collapsing dimension: can the production of winding states resolve the singularity ?

Outline of the talk

1. Introduction
2. The Lorentzian orbifold and its avatars

Misner, Taub-NUT, Grant...

3. Closed strings in Misner space: first pass

Nekrasov

3. A detour: Open strings in electric fields

Berkooz BP

4. Closed strings in Misner space: second pass

Berkooz BP; Berkooz Durin BP Reichmann Rozali

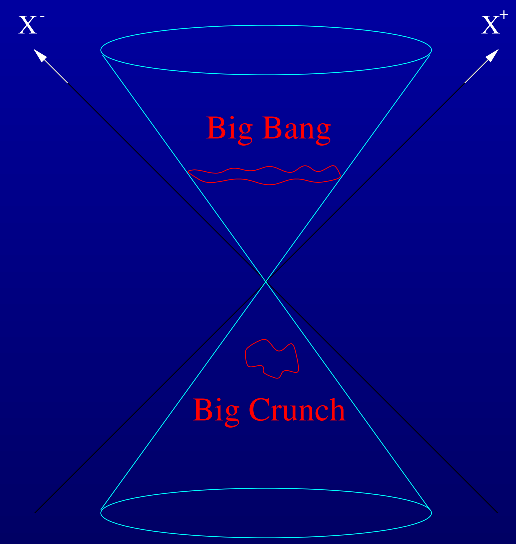
5. Conclusions, speculations

The Lorentzian orbifold

- One of the simplest examples of space-like singularities is the quotient of flat Minkowski space by a discrete boost, also known as Misner space (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^\pm \sim e^{\pm 2\pi\beta} X^\pm$$



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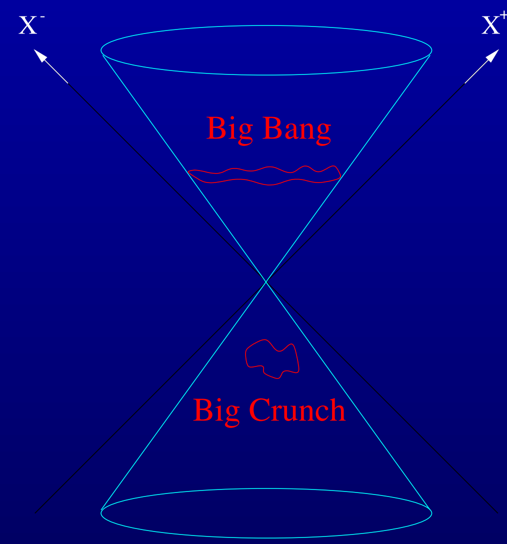

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$$ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + (dX^i)^2 \,, \quad \theta \equiv \theta + 2\pi \,, \quad X^\pm = T e^{\pm\beta\theta}/\sqrt{2}$$

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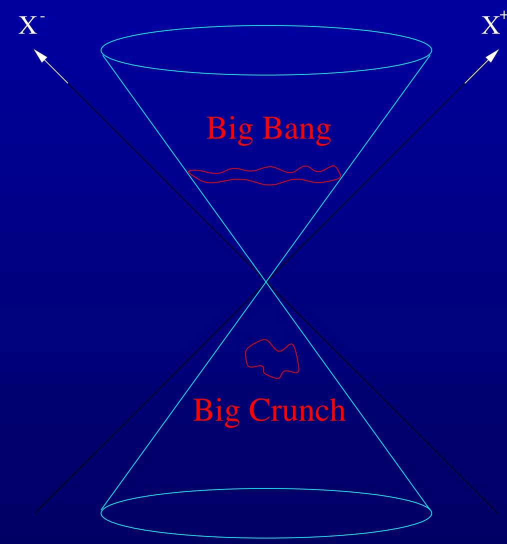

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This is a (degenerate) Kasner singularity, everywhere flat, but for a delta-function curvature at $T = 0$.

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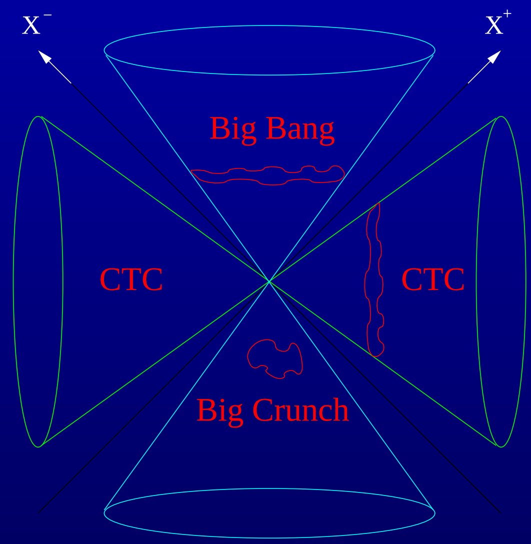

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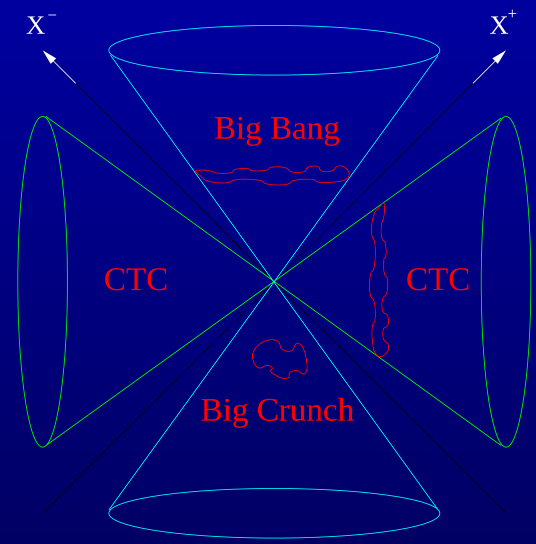

- In addition, the spacelike regions $X^+X^- < 0$ describe two Rindler wedges with compact time, often known as whiskers, leading to closed time-like curves:

$$ds^2 = dr^2 - \beta^2 r^2 d\eta^2 + (dX^i)^2 \quad , \eta \equiv \eta + 2\pi \; , \quad X^\pm = \pm r e^{\pm\beta\eta}/\sqrt{2}$$

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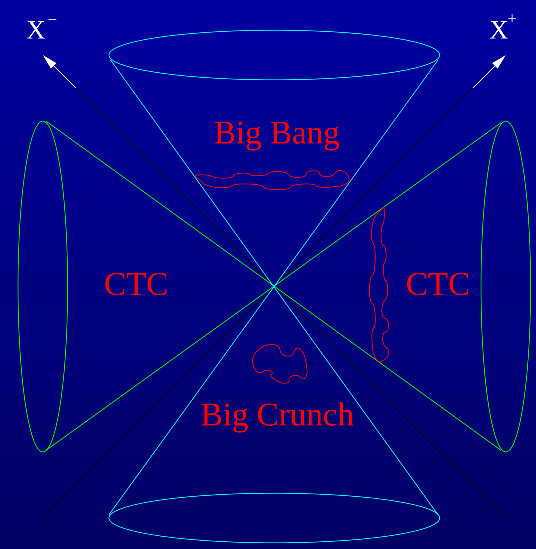

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- Finally, the lightcone $X^+X^- = 0$ gives rise to non-Hausdorff sets with a degenerate metric, attached to the singularities.

Close relatives of the Misner Universe

- Misner space was first introduced as a local model of Lorentzian Taub-NUT space:

$$ds^2 = 4l^2 U(t)\sigma_3^2 + 4l\sigma_3 dt + (t^2 + l^2)(\sigma_1^2 + \sigma_2^2) \ , \quad U(t) = -1 + \frac{2mt + l^2}{t^2 + l^2}$$

A bouncing universe, isomorphic to $R^{1,1}/boost \times S^2$ around each singularity.

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This describes the space away from two moving cosmic strings. Due to the absence of fixed point, the cosmological singularity is smoothed out.

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- The Misner geometry arose again more recently as the M-theory lift of a simple (ekpyrotic) cosmological solution of Einstein-dilaton gravity with no potential.

  Khoury Ovrut Seiberg Steinhard Turok

Close relatives of the Misner Universe (cont)

- The gauged WZW model $Sl(2) \times Sl(2)/U(1) \times U(1)$ describes a bouncing 4-dimensional Universe, with singularities analogous to the Lorentzian orbifold.

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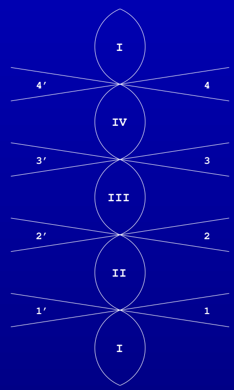

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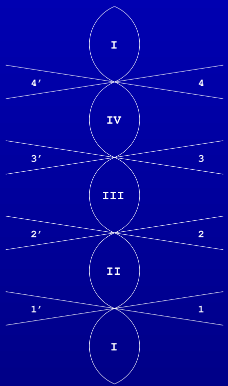

- The gauged WZW model $Sl(2)/U(1)$ at negative level orbifolded by a boost J describes two parallel Universes with a curvature and a Milne singularity, and compact whiskers.

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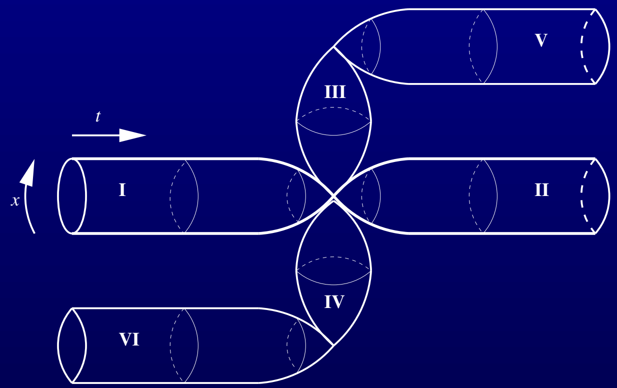

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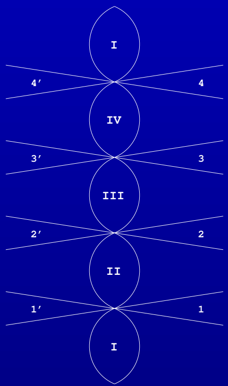

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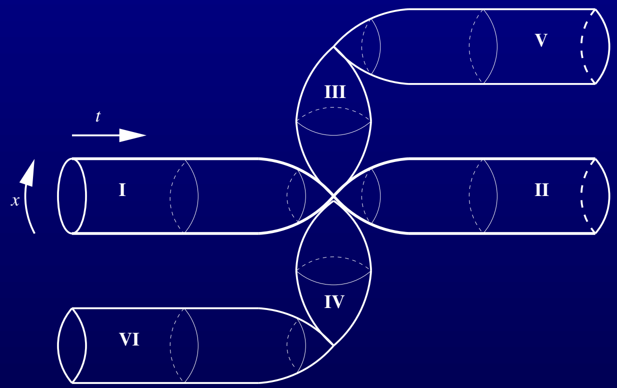
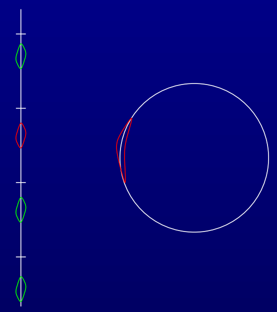


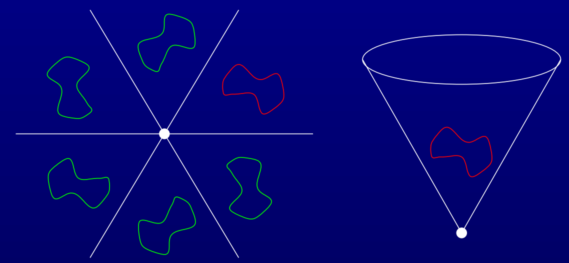
- The Lorentzian orientifold $IIB/[(-)^F boost]/[\Omega(-)^{F_L}]$ was also recently argued to describe orientifolds of non-supersymmetric strings with non-vanishing Neveu-Schwarz tadpoles.

  Dudas Mourad Timirgaziu

Strings on Euclidean orbifolds - untwisted states

- One way to obtain non-trivial yet solvable backgrounds in string theory is the orbifold construction: to a CFT with a discrete global symmetry G, associate a CFT' with only G-invariant states. Simple examples are the circle, R/Z, and the rotation orbifold R^2/Z_k.
- The spectrum of the quotient theory contains closed string states of the parent theory which are invariant under G: untwisted states.





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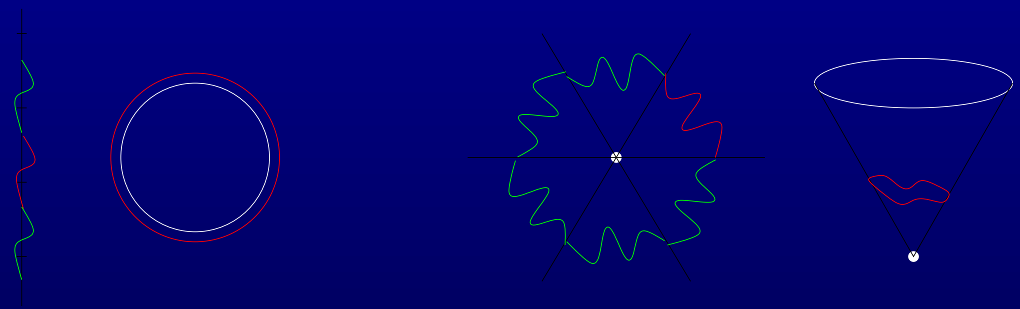
- Modular invariance requires that the spectrum should also include closed strings in the quotient theory which close up to the action of G in the parent theory: twisted states.



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- Twisted sectors are labelled by conjugacy classes of G. Higher twisted sectors correspond to multiply wound states

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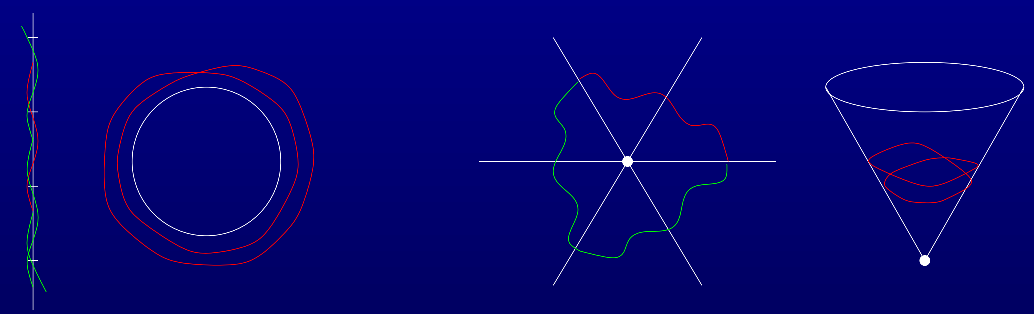

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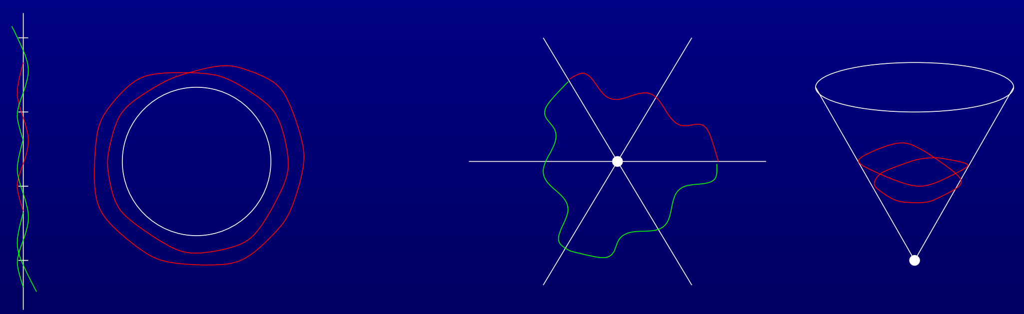

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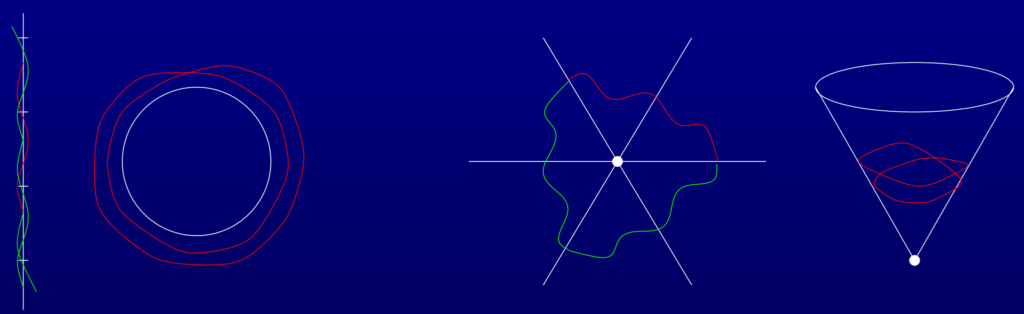

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- The condensation of these twisted states changes the vacuum, and effectively resolves the singularity: $R^2/Z_k \to R^2/Z_{k-1} \to \dots$ (tachyon), $R^4/Z_k \to$ multi-centered Eguchi-Hanson (massless mode).

Closed strings in Misner space - untwisted states

- As usual in standard orbifolds, part of the spectrum involves closed strings on Minkowski covering space, which are invariant under the orbifold projection. In conformal gauge,

$$X^{\pm}(\sigma+2\pi,\tau)=X^{\pm}(\sigma,\tau)\ ,\quad (\partial_\tau^2-\partial_\sigma^2)X^{\pm}=0$$

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- Vertex operators (or states) can be obtained by (infinite) sum over images, e.g.

$$\sum_{n=-\infty}^{\infty} \partial X^+ \bar{\partial} X^- \exp\left(i k^+ X^- e^{-2\pi\beta n} + i k^- X^+ e^{2\pi\beta n} + i k_i x^i \right)$$

with the physical state condition $2k^+k^- = M^2$.

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  with the physical state condition $2k^+k^-=M^2$.
- Equivalently, after Poisson resummation over n, this is a superposition of states with integer boost momentum $j=x^+\partial_+-x^-\partial_-$,

$$(\sum_{j=-\infty}^{\infty})\partial X^+\bar{\partial}X^-\int_{-\infty}^{\infty}dv\exp\left(+ik^+X^-e^{-2\pi\beta v}+ik^-X^+e^{2\pi\beta v}+ik_iX^i+2\pi ivj\right)$$

Closed strings in Misner space - untwisted states

- As usual in standard orbifolds, part of the spectrum involves closed strings on Minkowski covering space, which are invariant under the orbifold projection. In conformal gauge,

$$X^{\pm}(\sigma + 2\pi, \tau) = X^{\pm}(\sigma, \tau) \ , \quad (\partial_\tau^2 - \partial_\sigma^2) X^{\pm} = 0$$

satisfying the Virasoro (physical state) condition $(\dot{X}^\mu \pm X'^\mu)^2 = 0$.

- Vertex operators (or states) can be obtained by (infinite) sum over images, e.g.

$$\sum_{n=-\infty}^{\infty} \partial X^+ \bar{\partial} X^- \exp\left(ik^+ X^- e^{-2\pi\beta n} + ik^- X^+ e^{2\pi\beta n} + ik_i x^i \right)$$

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- The resulting eigenfunctions describe closed strings traveling around the Milne circle with integer momentum j.

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$$H^{(1)}_{-ij}(mT) e^{-ij\theta} \sim e^{-ij\theta - imT}/\sqrt{T}$$

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Quantum fluctuations and backreaction

- In the Minkowski vacuum (inherited from the covering space), the renormalized propagator can be obtained as a sum over images, e.g in D=4

$$G(x;x') = \sum_{n=-\infty, n\neq 0}^{\infty} [-2(X^+ - e^{2\pi\beta n}X^{+'})(X^- - e^{2\pi\beta n}X^{-'}) + (X^i - X^{i'})^2]^{-1}$$

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- The one-loop stress-energy tensor follows from $G(x,x)$, e.g for a conformally coupled scalar,

$$\langle T_{ab} \rangle = \lim_{x\to x'} \left[(1-2\xi)\nabla_a \nabla'_b - 2\xi \nabla_a \nabla_b + (2\xi - \frac{1}{2}) g_{ab} \nabla_c {\nabla'}^c \right] G(x,x')$$

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leading to a divergent quantum backreaction:

$$\langle T_\mu^\nu \rangle = \frac{K}{12\pi^2} T^{-4} \text{diag}(1,-3,1,1) \; , \quad K = \sum_{n=1}^{\infty} \frac{2 + \cosh 2\pi n\beta}{[\cosh 2\pi n\beta - 1]^2}$$

Hiscock Konkowski 82

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- In the case of the Grant space, the one-loop energy momentum tensor diverges as $1/(R^2T^2)$ on the chronological horizon, and $1/(T-T_n)^3$ on the polarized hypersurfaces. This is at the basis of Hawking's chronology protection conjecture.

Scattering of untwisted states

- Scattering amplitudes of untwisted sector states can be computed from those in flat space by the inheritance principle,

$$\langle V(j_1,k_1)\dots V(j_n,k_n)\rangle_{Misner} = \int dv_1\dots dv_n\, e^{i(j_1 v_1+\dots+j_n v_n)}$$
$$\langle V(e^{\beta v_1}k_1^+, e^{-\beta v_1}k_1^-, k_1^i)\dots V(e^{\beta v_n}k_n^+, e^{-\beta v_n}k_n^-, k_n^i)\rangle_{Minkowski}$$

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- The integral diverges due to Regge behavior in the large momentum, fixed angle regime. E.g, the four-tachyon scattering amplitude in bosonic string leads to

$$\int dv\; v^{-\frac{1}{2}(k_1^i-k_3^i)^2+i(j_2-j_4)}$$

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- The divergence disappears for the Grant space, except for a localized contribution at $k_1^i = k_3^i$. The amplitude is also finite for transverse gravitons in type II superstring on Misner space, but reappears for longitudinal gravitons.

Berkooz Durin Pioline Reichmann, unpublished

Closed string in Misner space - twisted sectors

- In addition, there is an infinite set of twisted sectors, corresponding to strings on the covering space that close up to the action of the orbifold group:

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau) , \quad \nu = 2\pi w \beta$$

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- They have a normal mode expansion:

$$\begin{aligned} X_R^{\pm}(\tau - \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha_n^{\pm} e^{-i(n \pm i\nu)(\tau - \sigma)} \\ X_L^{\pm}(\tau + \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \mp i\nu)^{-1} \tilde{\alpha}_n^{\pm} e^{-i(n \mp i\nu)(\tau + \sigma)} \end{aligned}$$

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$$(\alpha^{\pm}_m)^*=\alpha^{\pm}_{-m}\quad ,\quad (\tilde{\alpha}^{\pm}_m)^*=\tilde{\alpha}^{\pm}_{-m}$$

- There are no translational zero-modes, instead two pairs of quasi zero-modes which are canonically conjugate real operators:

$$[\alpha^{+}_0,\alpha^{-}_0]=-i\nu\ ,\quad [\tilde{\alpha}^{+}_0,\tilde{\alpha}^{-}_0]=i\nu$$

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated, e.g., by

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- This is the familiar result for the vacuum energy $\frac{1}{2}\theta(1-\theta)$ a rotation orbifold, after analytic continuation $\theta \to i\nu$...
- Due to the $i\nu/2$ term in the ground state energy, all states obtained by acting on $|0\rangle$ by creation operators $\alpha^{\pm}_{n<0}$ and by α_0^+ will have imaginary energy, hence the physical state condition $L_0 = 0$ has no solutions.

Nekrasov

One-loop amplitude

- Independently of this fact, one may compute the one-loop (Euclidean ws, Minkowskian target) free energy using path integral methods:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{\left|\eta^{21}(\rho)x\ \theta_1(i\beta(l+w\rho);\rho)\right|^2}$$

where θ_1 is the Jacobi theta function,

$$\theta_1(v;\rho) = 2q^{1/8} \sin\pi v \prod_{n=1}^{\infty} (1-e^{2\pi i v}q^n)(1-q^n)(1-e^{-2\pi i v}q^n)\ , \quad q = e^{2\pi i \rho}$$

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- In the untwisted sector, this reproduces the integrated vacuum free energy found by the method of images:

$$\int dx^+ dx^- G(x,x) = \sum_{l=-\infty}^{+\infty} \int_0^{\infty} \frac{d\rho}{\rho^{D/2}} \frac{e^{-m^2\rho}}{\sinh^2(\pi\beta l)}$$

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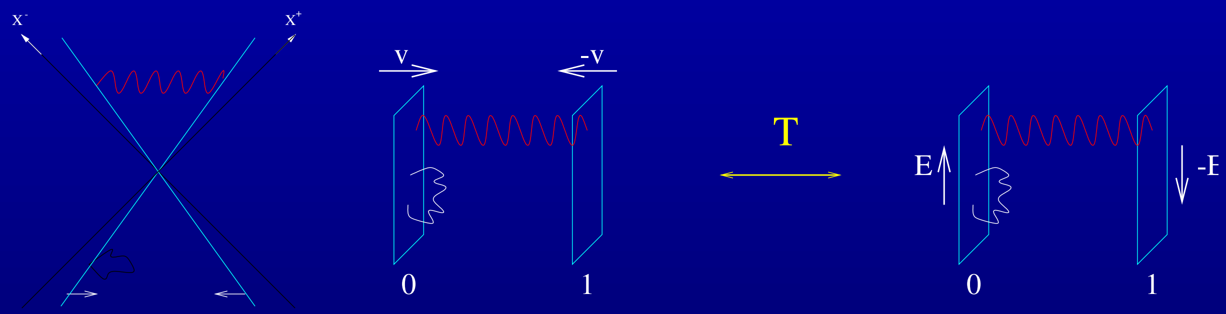
- The absence of physical twisted states crushes our hopes for resolving the singularity... yet it is hard to swallow. In particular, α_0^+ and α_0^- are not hermitian conjugate to each other, but rather self-hermitian...

Open strings in electric field vs Lorentzian orbifold

- A very similar puzzle is faced in the case of colliding D-branes, or in the T-dual process of charged open strings in a constant electric field:



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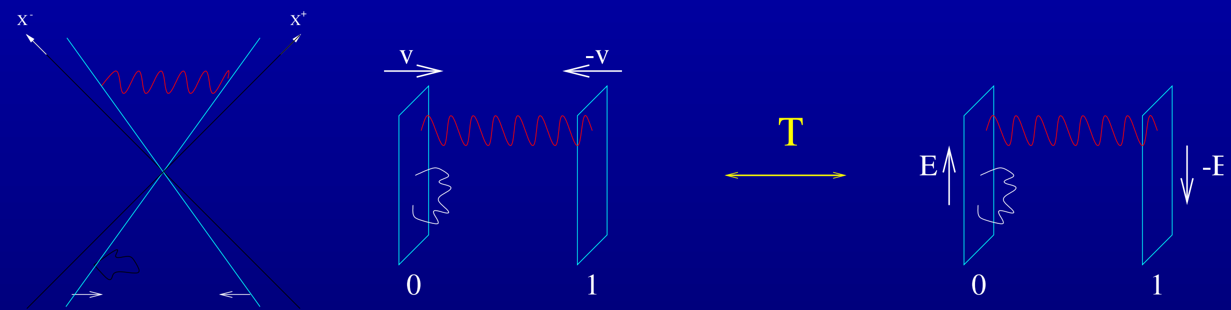

- Recall that for open strings stretched between two D-branes with electromagnetic fields F_0 and F_1, proper frequencies satisfy

$$e^{-2\pi i\omega_n} = \frac{1+F_0}{1-F_0} \cdot \frac{1-F_1}{1+F_1}$$

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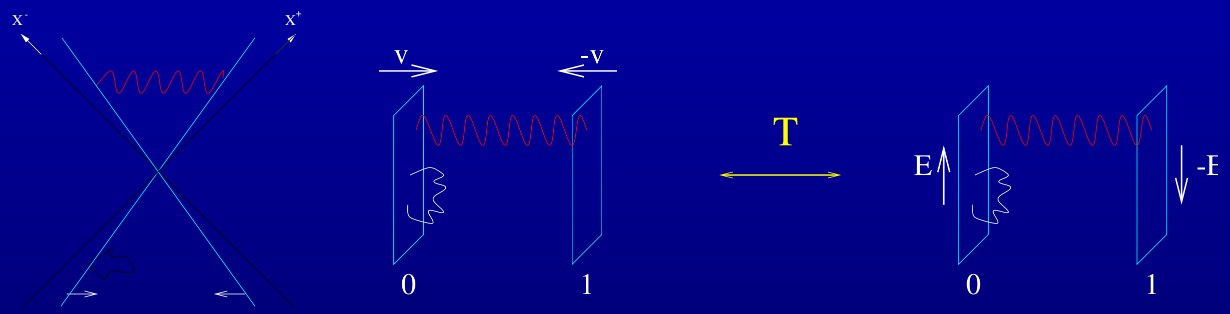

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- In the case of an electric field $F_1 = E dx^+ \wedge dx^-$, $F_1 = 0$, the resulting spectrum is

$$\omega_n = n + i\nu \ , \qquad \nu := \text{Arctanh} E = w\beta$$

just as in the Lorentzian orbifold case. More precisely, the charged open string has half as many excited modes than the twisted closed strings, and isomorphic quasi-zero modes.

Open string mode expansion

- The light-cone embedding coordinates have the normal mode expansion

$$X^{\pm} = x_0^{\pm} + i \sum_{n=-\infty}^{+\infty} (-)^n (n \pm i\nu)^{-1} a_n^{\pm} e^{-i(n\pm i\nu)\tau} \cos[(n \pm i\nu)\sigma]$$

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- The world-sheet Hamiltonian, normal ordered with respect to the vacuum annihilated by $a_{n>0}^+, a_{n>0}^-$ and a_0^+, takes the form

$$L_0^{l.c.} = -\sum_{m=0}^{\infty} a_{-m}^+ a_m^- - \sum_{m=1}^{\infty} a_{-m}^- a_m^+ + \frac{i\nu}{2}(1 - i\nu) - \frac{1}{12}$$

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- By the same token, charged open strings should have no physical states...

One-loop amplitude and Schwinger pair production

- Using this quantization scheme, the one-loop (Euclidean worldsheet, Minkowskian target) vacuum free energy reads

$$A_{bos} = \frac{i\pi V_{26}(e_0+e_1)}{2} \int_0^\infty \frac{dt}{(4\pi^2 t)^{13}} \frac{e^{-\pi\nu^2 t/2}}{\eta^{21}(it/2)\ \theta_1(t\nu/2; it/2)}$$

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- Each of the poles at $t = 2k/\nu$ contributes to the imaginary part, yielding the production rate of charged open strings,

$$\mathcal{W} = \frac{1}{2(2\pi)^{25}} \frac{(e_0+e_1)}{\nu} \sum_{k=1}^{\infty} (-)^{k+1} \left(\frac{|\nu|}{k}\right)^{13} \sum_{N=-1}^{\infty} c_b(N) \exp\left(-2\pi k \frac{N}{|\nu|} - 2\pi k|\nu|\right)$$

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where $\eta^{-24}(q) = \sum_{N=-1}^{\infty} c_b(N) q^N$. This can be viewed as the sum of the Schwinger production rates for each state in the spectrum, of mass $m^2 = 2N + \nu^2$.

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- This seems to support the quantization scheme based on a vacuum, hence the absence of physical states. But physical states do exist classically, how could quantization make them disappear altogether ?

Charged particle and open string zero-modes

- Let us recall the quantization of a charged particle in an electric field:

$$L = \frac{1}{2} m \left(-2\partial_\tau X^+ \partial_\tau X^- + (\partial_\tau X^i)^2 \right) + \frac{e}{2} \left(X^+ \partial_\tau X^- - X^- \partial_\tau X^+ \right)$$

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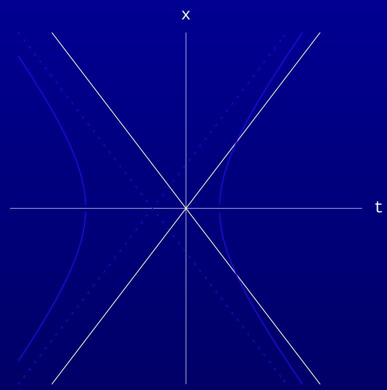

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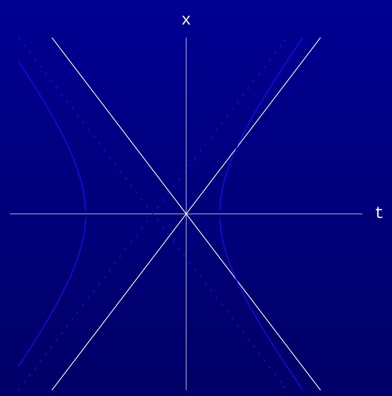

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- Starting from the canonical equal-time commutation rules

$$[\pi^+, x^-] = [\pi^-, x^+] = i \ , \quad [\pi^i, x^j] = i\delta_{ij}$$

one obtains the open string zero-mode commutation relations ($\nu = e$),

$$[a_0^+, a_0^-] = -i\nu \ , \quad [x_0^+, x_0^-] = -\frac{i}{\nu}$$

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- Quantum mechanically, one may represent $\pi^{\pm} = i\partial/\partial x^{\mp}$ hence obtain $a_0^{\pm}, x_0^{\pm}$ as covariant derivatives

$$a_0^{\pm} = i\partial_{\mp} \pm \frac{\nu}{2} x^{\pm} \,, \quad x_0^{\pm} = \mp\frac{1}{\nu}\left(i\partial_{\mp} \mp \frac{\nu}{2} x^{\pm} \right)$$

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- The zero-mode piece of L_0, including the evil $\frac{i\nu}{2}$,

$$L_0^{(0)} = -a_0^+ a_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\nabla_0^+ \nabla_0^- + \nabla_0^- \nabla_0^+)$$

is just the Klein-Gordon operator of a particle of charge ν.

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^{\pm} = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator can be rewritten as an inverted harmonic oscillator:

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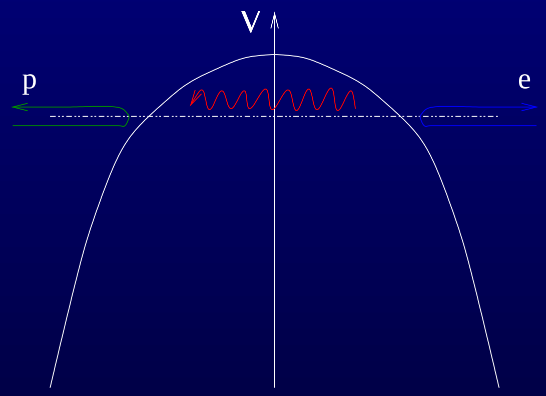
- The latter admits a respectable delta-normalizable spectrum of scattering states, in terms of parabolic cylinder functions, e.g:

$$\phi_{in}^+ = D_{-\frac{1}{2}+i\frac{M^2}{2\nu}}(e^{-\frac{3i\pi}{4}}u)e^{-i\tilde{p}t}e^{i\nu xt/2}$$



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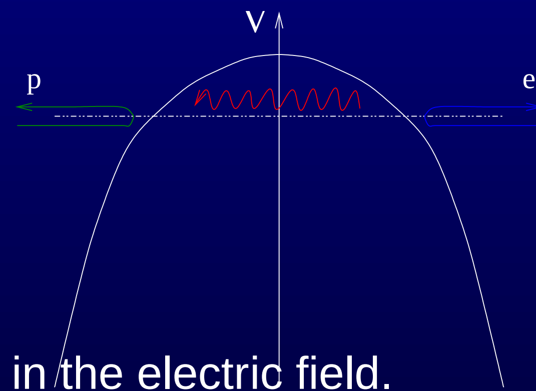

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- These correspond to non-compact trajectories of charged particles in the electric field. Tunnelling is just (stimulated) Schwinger pair creation,

$$e^- \longrightarrow (1+\eta)\, e^- + \eta\, e^+ \,, \quad \eta \sim e^{-\pi M^2/\nu}$$

Brezin Itzykson; Brout Massar Parentani Spindel

Lorentzian vs Euclidean states

- Analytic continuation $X^0 \to -iX^0$, $\nu \to i\nu$ turns an electric field in $R^{1,1}$ into a magnetic field in R^2. At the same time, one should Wick rotate the worldsheet time.

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$$\frac{1}{2i\sin(\nu t/2)} = \sum_{n=1}^{\infty} e^{-i(n+\frac{1}{2})\nu t} = \int dM^2 \rho(M^2) e^{-M^2 t/2}$$

The density of states is obtained from the reflection phase shift,

$$\rho(M^2) = \frac{1}{\nu}\log\Lambda - \frac{1}{2\pi i}\frac{d}{dM^2}\log\frac{\Gamma\left(\frac{1}{2}+i\frac{M^2}{2\nu}\right)}{\Gamma\left(\frac{1}{2}-i\frac{M^2}{2\nu}\right)}$$

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- The physical spectrum can be explicitely worked out at low levels, and is free of ghosts: a tachyon at level 0, a transverse gauge boson at level 1, . . .

Physical spectrum at low level

- The ground state tachyon

$$|T\rangle = \phi(x^+, x^-)|0_{ex}, k\rangle$$

should satisfy the Virasoro constraint

$$L_0|T\rangle = \left[-\frac{1}{2}\left(a_0^+ a_0^- + a_0^- a_0^+\right) + \frac{1}{2}\nu^2 - 1 + \frac{1}{2}k_i^2\right]|T\rangle$$

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$$|A\rangle = \left(-f^+\, a_{-1}^- - f^-\, a_{-1}^+ + f^i\, a_{-1}^i\right)|0_{ex}, k\rangle$$
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$$[M^2 - k_i^2 - \nu^2]f^i = 0\ , \quad [M^2 - k_i^2 - \nu^2 \mp 2i\nu]f^\pm = 0$$
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- The L_1 Virasoro constraint eliminates one polarization. Despite the non-vanishing two-dimensional mass $k_i^2 - \nu^2$, the spurious state $L_{-1}\phi|0\rangle$ is still physical, eliminating an extra polarization.
- One thus has $D-2$ transverse degrees of freedom, ie a massless gauge boson in D dimensions.

Charged particle in Rindler space

- For applications to the Milne universe, one should diagonalize the boost momentum J, ie consider an accelerated observer.

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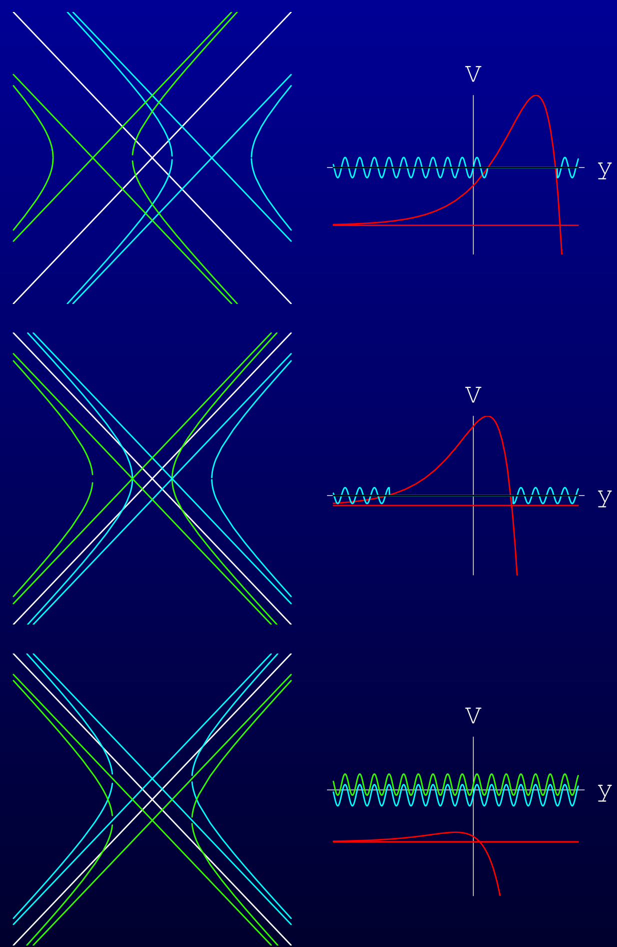

- In the Rindler patch R, letting $f(r,\eta) = e^{-iJ\eta} f_J(r)$ and $r = e^y$, one gets a Schrödinger equation for a particle in a potential

$$V(y) = M^2 e^{2y} - (J + \frac{1}{2}\nu\, e^{2y})^2$$

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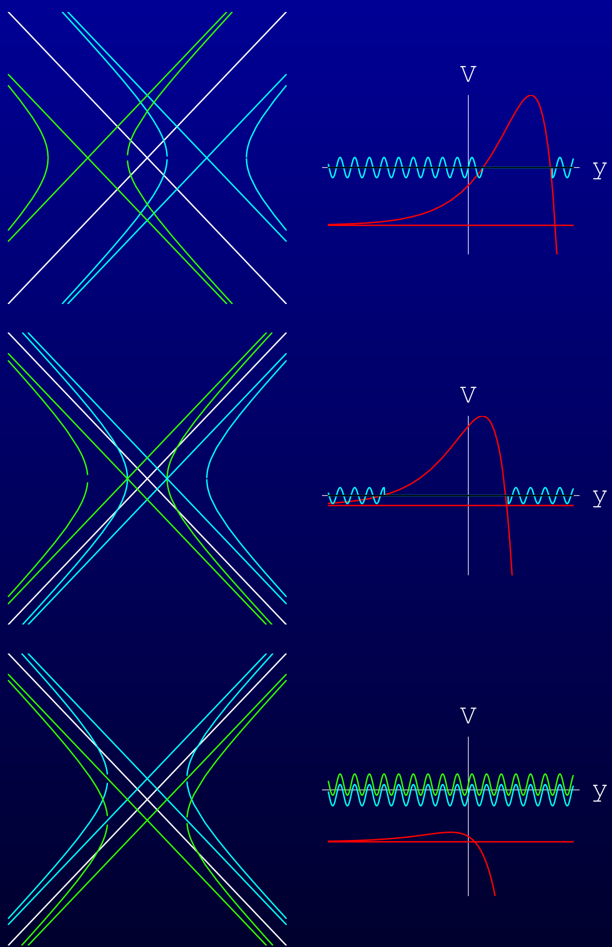

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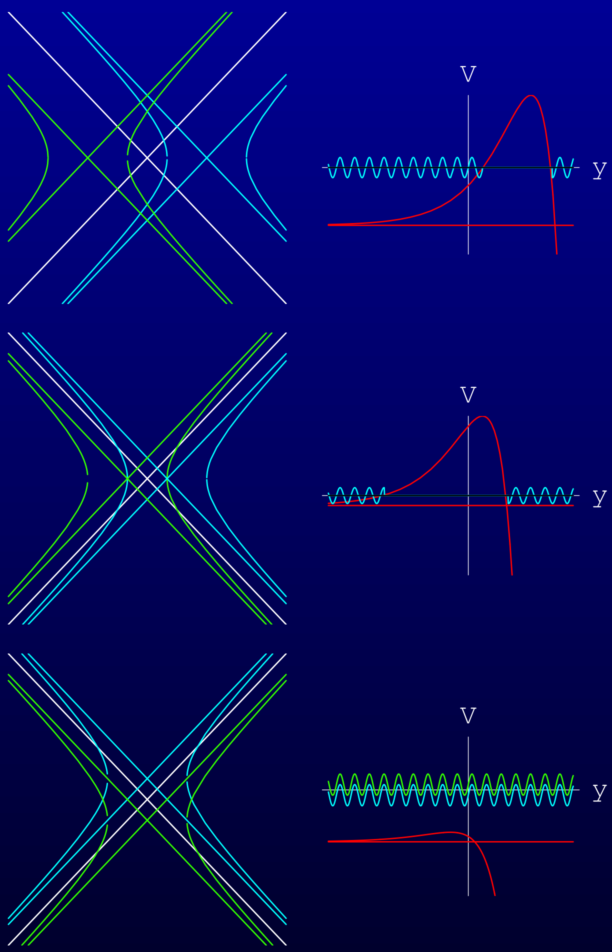
Charged particle in Rindler space

- For applications to the Milne universe, one should diagonalize the boost momentum J, ie consider an accelerated observer.

Gabriel Spindel; Mottola Cooper



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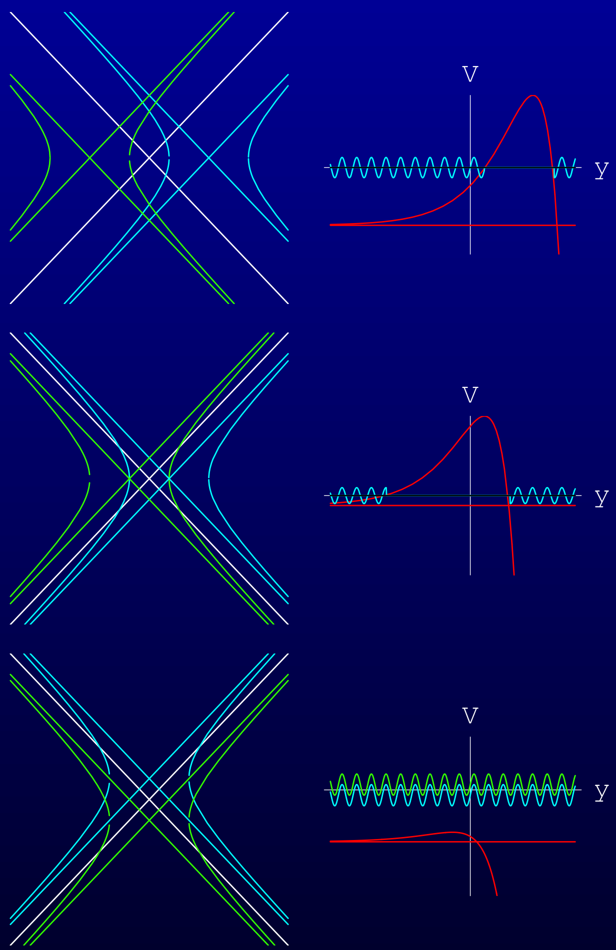

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- If $j > M^2/(2\nu)$, the electron branches extend in the Milne regions. There is no tunneling, but partial reflection amounts to a combination of Schwinger and Hawking emission.

Rindler modes

- Incoming modes from Rindler infinity I_R^- read, in terms of parabolic cylinder functions:

$$\mathcal{V}^j_{in,R} = e^{-ij\eta} r^{-1} M_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),-\frac{ij}{2}}(i\nu r^2/2)$$

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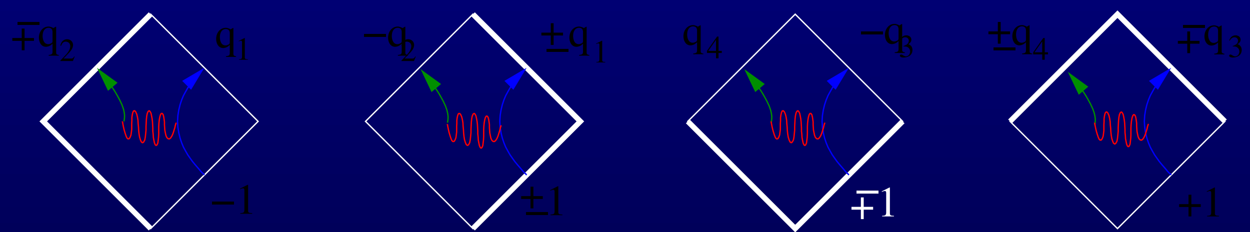

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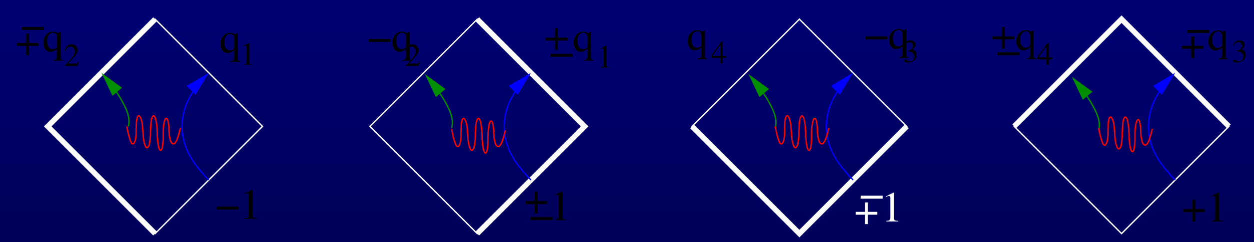

- The reflection coefficients can be computed:

$$q_1 = e^{-\pi j} \frac{\cosh\left[\pi \frac{M^2}{2\nu}\right]}{\cosh\left[\pi\left(j - \frac{M^2}{2\nu}\right)\right]}, \quad q_3 = e^{\pi\left(j-\frac{M^2}{2\nu}\right)} \frac{\cosh\left[\pi \frac{M^2}{2\nu}\right]}{|\sinh \pi j|}$$

and $q_2 = 1 - q_1$, $q_4 = q_3 - 1$, by charge conservation.

Global Charged Unruh Modes

- Global modes may be defined by patching together Rindler modes, ie by analytic continuation across the horizons. Unruh modes are those which are superposition of positive energy Minkowski modes,

$$\begin{aligned} \Omega^j_{in,+} &= \mathcal{V}^j_{in,P} = (-i\nu X^+ X^-)[X^+/X^-]^{-ij/2} W_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}} \\ \omega^j_{in,-} &= \mathcal{U}^j_{in,P} = (i\nu X^+ X^-)[X^+/X^-]^{-ij/2} M_{i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}} \end{aligned}$$

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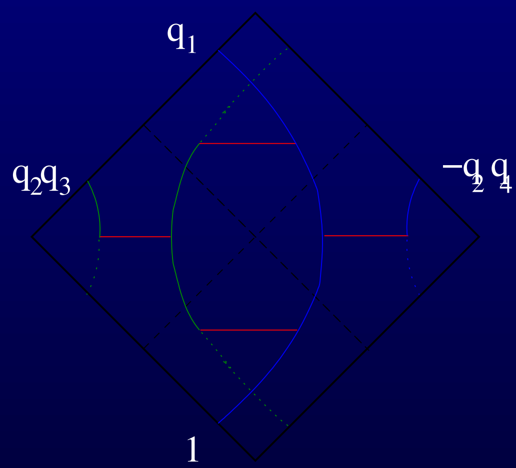
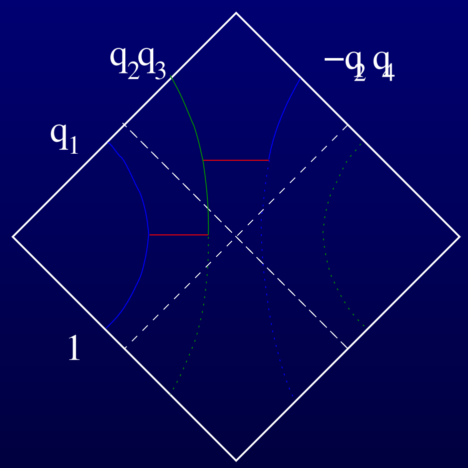
- There are two types of Unruh modes, involving 2 or 4 tunelling events:





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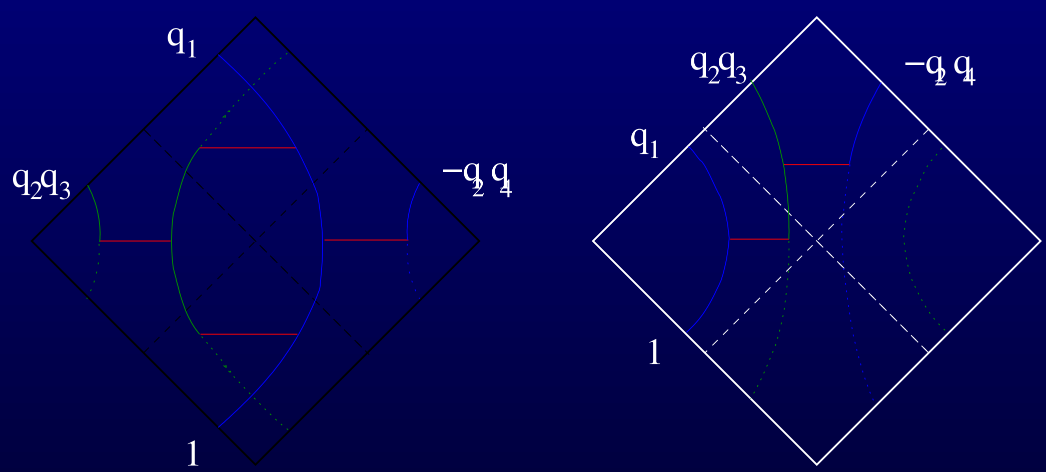

- Any state in Minkowski space can be represented as a state in the tensor product of the Hilbert spaces of the left and right Rindler patches. In contrast to neutral fields in Rindler space, Boulware-Fulling modes that vanish in L or R have positive Minkowski energy.

Closed string zero-modes

- Let us analyze the classical solutions for the closed string zero modes

$$X^{\pm}(\tau,\sigma) = e^{\mp\nu\sigma}\left[\pm\frac{1}{2\nu}\alpha_0^{\pm}e^{\pm\nu\tau} \mp \frac{1}{2\nu}\tilde{\alpha}_0^{\pm}e^{\mp\nu\tau}\right] \ , \quad \alpha_0^{\pm}, \tilde{\alpha}_0^{\pm} \in R$$

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- Up to a shift of τ and σ, the physical state conditions require

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- The behavior at early/late proper time now depends on $\epsilon\tilde{\epsilon}$: For $\epsilon\tilde{\epsilon} = 1$, the string begin/ends in the Milne regions. For $\epsilon\tilde{\epsilon} = -1$, the string begin/ends in the Rindler regions.

Short and long strings ($j = 0$)

Choosing $j = 0$ for simplicity, we have 4 solutions:

- $\epsilon = 1$, $\tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma,\tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma} \, , \quad T = \frac{M}{\nu} \sinh(\nu\tau) \, , \quad \theta = \nu\sigma$$

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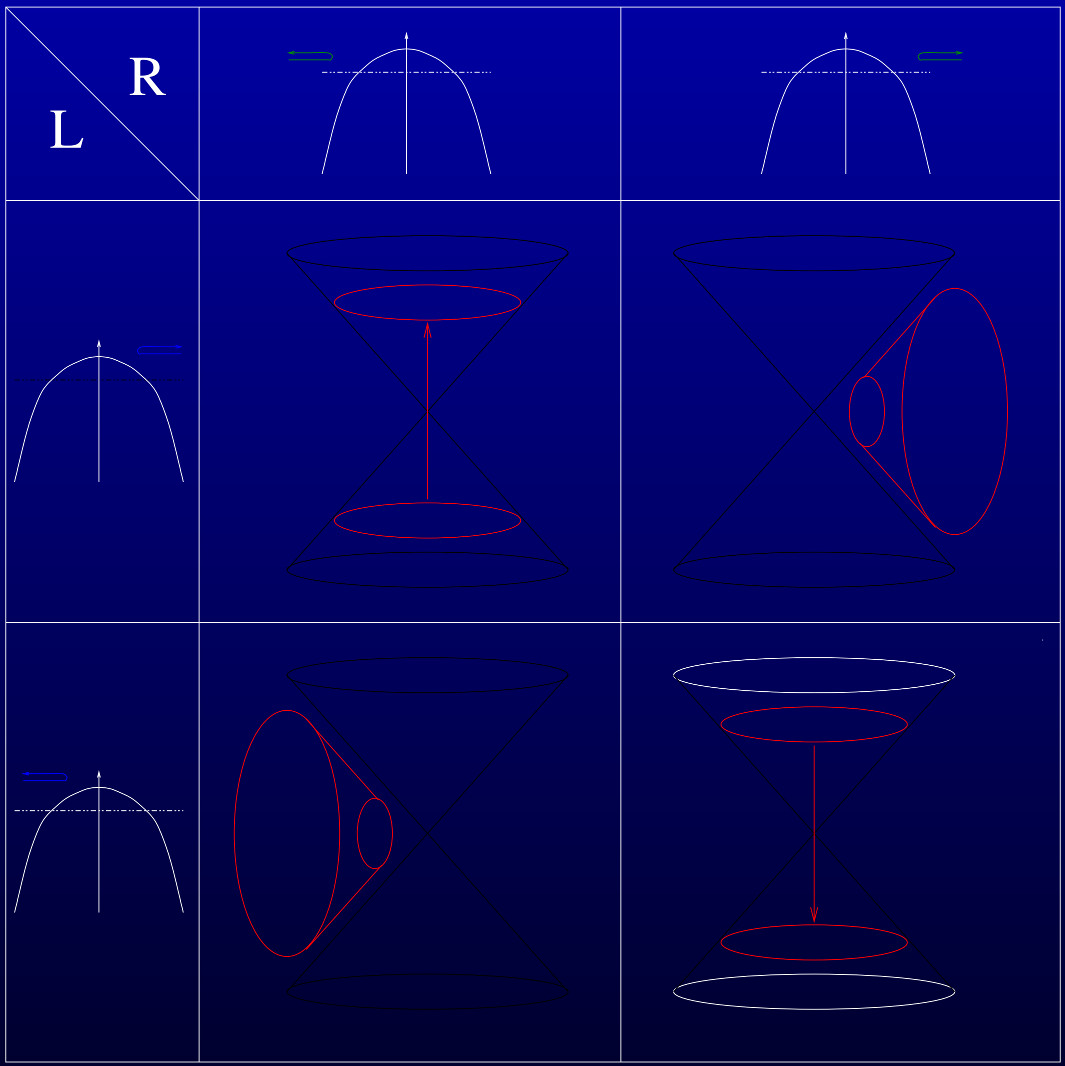
Short and long strings

Closed string trajectories are thus generated by the motion of two decoupled particles in inverted harmonic oscillators:



Relation to open string modes

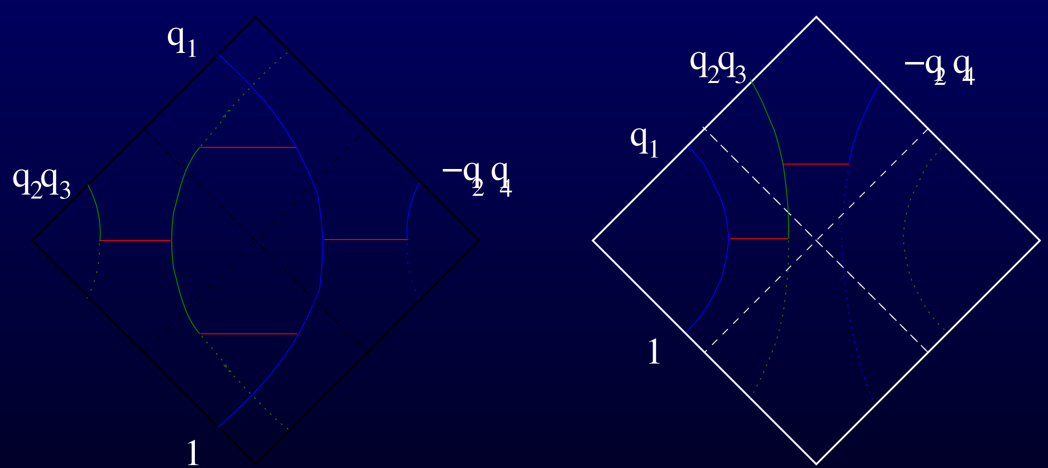
- Instead of following the motion of a point at fixed σ, one may consider instead a point a fixed $\sigma+\tau$: this is precisely the trajectory of the open string zero-mode.
- Using the covariant derivative representation

$$\alpha_0^\pm = i\partial_\mp \pm \frac{\nu}{2}x^\pm\,, \quad \tilde\alpha_0^\pm = i\partial_\mp \mp \frac{\nu}{2}x^\pm$$

we observe that $x^\pm$ is the Heisenberg operator corresponding to the location of the closed string (at $\sigma=0$):

$$X_0^\pm(\sigma,\tau) = e^{\mp\nu\sigma}\left[\cosh(\nu\tau)\,x^\pm + i\sinh(\nu\tau)\,\partial_\mp\right]$$

- The open string global wave functions...



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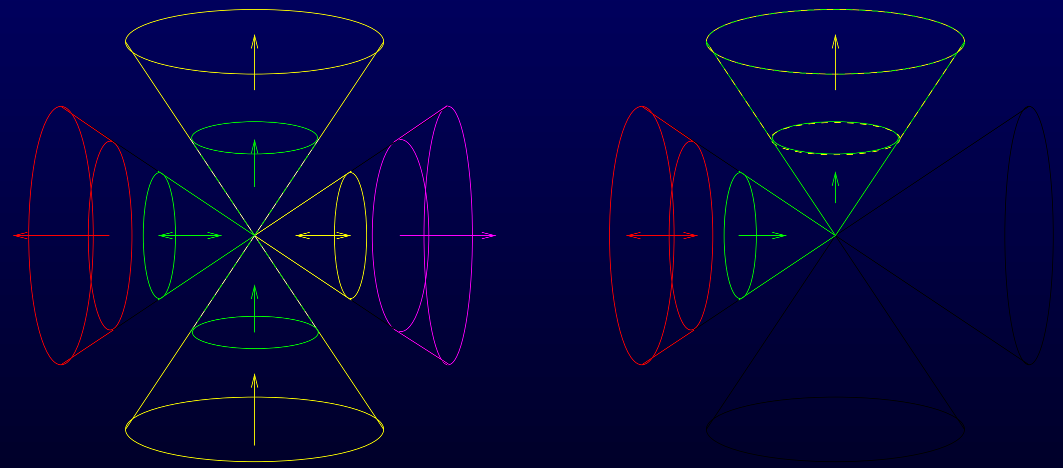
- The open string global wave functions are also the closed string wave functions. . .



Quantization in the Rindler patch

- For long strings in conformal gauge, the worldsheet time τ is in fact a spacelike coordinate wrt to the induced metric. This is also true for short strings: as they wander in the Rindler patch, the induced metric undergoes a signature flip.
- If so we should quantize the string with respect to the "time" coordinate σ rather than τ. The canonical generator of time translations

$$E = -\int_{-\infty}^{\infty} d\tau \left(X^+ \partial_\sigma X^- - X^- \partial_\sigma X^+ \right) = \int_{-\infty}^{\infty} d\tau r^2 \partial_\sigma \eta$$

  is infinite: long strings carry an infinite Rindler energy.
- Introducing a cut-off $-T \leq \tau < T$, the Rindler energy

$$E_T \sim -\frac{e^{2\nu T}}{4\nu^2} \left(\tilde{\alpha}_0^+ \alpha_0^- + \tilde{\alpha}_0^- \alpha_0^+ \right)$$

  can be understood as the tensive energy of the static stretched string.
- The Rindler energy spectrum is unbounded: long strings ($\epsilon = -1$) have $E_T > e^{2\nu T} M \tilde{M} / (4\nu^2)$ unbounded from below, while the short strings ($\epsilon = 1$) have $E_T < -e^{2\nu T} M \tilde{M} / (4\nu^2)$ unbounded from above.

The one-loop amplitude again

- Recall the (Euclidean ws, Minkowskian target) one-loop amplitude:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=0}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{\left|\eta^{21}(\rho)\ \theta_1(i\beta(l+w\rho);\rho)\right|^2}$$

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- As in the open string case, the zero mode contribution $1/\sinh^2(\pi\beta(l+w\rho))$ may be interpreted either as a sum over (Euclidean) discrete states, or a continuous integral over the continuous (Lorentzian) modes: there are physical states at each level.

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- In addition, there are poles in the bulk of the moduli space, for

$$i\beta(l+w\rho) = m + n\rho\ , \quad (l,w,m,n) \in Z$$

leading to logarithmic divergences $\int d\rho d\bar{\rho}/|\rho-\rho_0|^2 \sim log\epsilon$, analogous to the long strings in AdS_3.

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- In contrast to the open string case, these poles do not yield an imaginary part: the overall cosmological particle production seems to vanish. *This is not to say that there is no particle production at intermediate stages !*

Wick rotation to a rotation orbifold

- Note first that the (future) Milne region $ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + dx_i^2$ cannot be directly Wick-rotated to Euclidean.

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- Rotating $\beta = i\mu$, the Rindler region becomes get indeed an Euclidean metric,

$$ds^2 = dr^2 + \mu^2 r^2 d\eta^2 + (dX^i)^2 = 2\, dZ d\bar{Z} + (dX^i)^2$$

$$Z = X^+ = re^{i\mu\eta}\,, \quad \bar{Z} = -X^- = re^{-i\mu\eta} \qquad (r > 0)$$

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Wick rotation to a rotation orbifold

- Note first that the (future) Milne region $ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + dx_i^2$ cannot be directly Wick-rotated to Euclidean. Instead, the analytical continuation $T = ir$, $\theta = \eta + i\pi/(2\beta)$ leads to the (right) Rindler wedge.
- Rotating $\beta = i\mu$, the Rindler region becomes get indeed an Euclidean metric,

$$ds^2 = dr^2 + \mu^2 r^2 d\eta^2 + (dX^i)^2 = 2\, dZ d\bar{Z} + (dX^i)^2$$

$$Z = X^+ = re^{i\mu\eta}\,, \quad \bar{Z} = -X^- = re^{-i\mu\eta} \qquad (r > 0)$$

- The Milne identification $\eta \equiv \eta + 2\pi$ amounts to a rotation identification $Z \to e^{2\pi i\mu} Z$.
- This cannot be the usual rotation orbifold however, because this would imply that the physics depends on β being rational or not. In other words, Z and $e^{2\pi i} Z$ should not identified: $(Z, \bar{Z})$ really take value in $\widetilde{R^2 \backslash \{0\}}$.
- By the same token, the left Rindler wedge rotates to another copy of the Euclidean plane with the origin removed: the complete analytic continuation of Misner space is therefore

$$\widetilde{R^2 \backslash \{0\}}_L / e^{i\mu} \backslash \widetilde{R^2 \backslash \{0\}}_R$$

and states of interest are non-normalizable !

D'Appolonio Kiritsis

Conclusions - speculations

- Winding states in the Milne Universe behave in close analogy with open strings in an electric field. Using intuition from open strings, we have found that physical states do exist in the twisted sector of the Lorentzian orbifold, and can be pair produced.

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- To demonstrate this, one should take into account the production of (an infinite number) of twisted sector states are produced in correlated pairs, i.e. squeezed states: non-local deformations of the worldsheet ? string field theory ?

Conclusions - speculations (cont.)

- As a less ambitious goal, can one compute scattering amplitudes of twisted states, and check if they are better behaved than untwisted states. For this, the relation with negative level $Sl(2)/U(1)$ and double analytic continuation of the Nappi-Witten plane wave may be useful.

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- More generally, we still lack a framework to compute the production of closed strings in cosmological backgrounds. Those however are likely to lead to large departures from FRW cosmology, and possibly spectacular effects: cosmological bounce, Hagedorn phase transition...

Lawrence Martinec, Gubser