Closed Strings in the Misner Universe

aka the Lorentzian orbifold

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based on hep-th/0307280 w/ M. Berkooz

and work in progress w/ M. Berkooz, B. Durin, D. Reichmann, M. Rozali

slides available from

`http://www.lpthe.jussieu.fr/~pioline/seminars.html`

Motivational string cosmology

- Observational Cosmology is now challenging string theory with high-precision data:

$$\Omega_{baryon} = 0.047\ , \quad \Omega_{darkm} = 0.243\ , \quad \Omega_{\Lambda} = 0.71\ , \quad w = -0.98 \pm .12\ , \ldots$$

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- With LHC still far in the future, understanding StringY Cosmology may be the only way to make contact with reality...

Brandenberger Vafa

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- Perturbative string theory requires an Euclidean worldsheet, hence Euclidean target space. The analytic continuation may be ambiguous or ill-defined, Lorentzian observables may be very different from their Euclidean counterparts.
- String theory is not content on a finite time interval, and one is frequently forced into Big Bang / Big Crunch singularities, CTC in the process of maximally extending the geometry.

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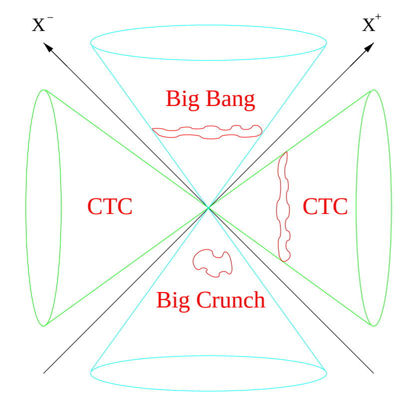

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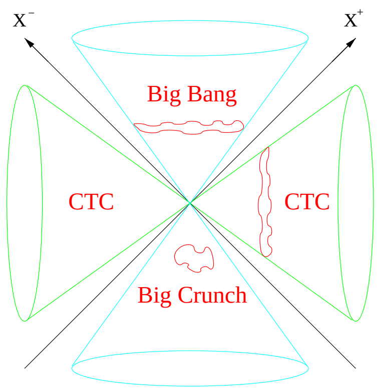


- Our main focus will be on the topological excitations which wind around the collapsing dimension: can the production of winding states resolve the singularity ?

Outline of the talk

1. Introduction

2. The Lorentzian orbifold and its avatars

Misner, Taub-NUT, Grant...

3. Closed strings in Misner space: first pass

Nekrasov

3. A detour: Open strings in electric fields

Berkooz BP

4. Closed strings in Misner space: second pass

Berkooz BP; Berkooz Durin BP Reichmann Rozali

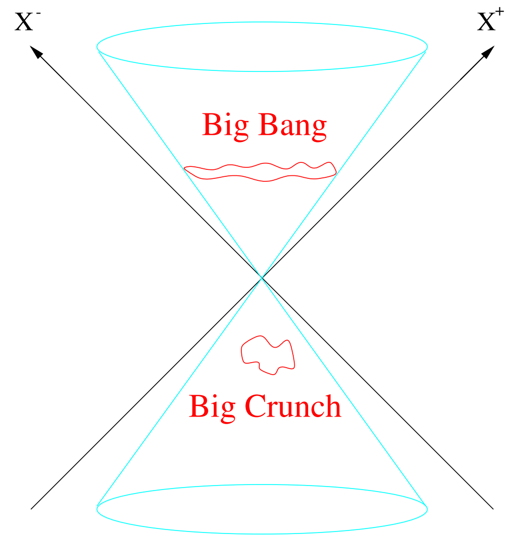
5. Comments on cosmological production of winding strings

The Lorentzian orbifold

- One of the simplest examples of space-like singularities is the quotient of flat Minkowski space by a discrete boost, also known as Misner space (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^\pm \sim e^{\pm 2\pi\beta} X^\pm$$



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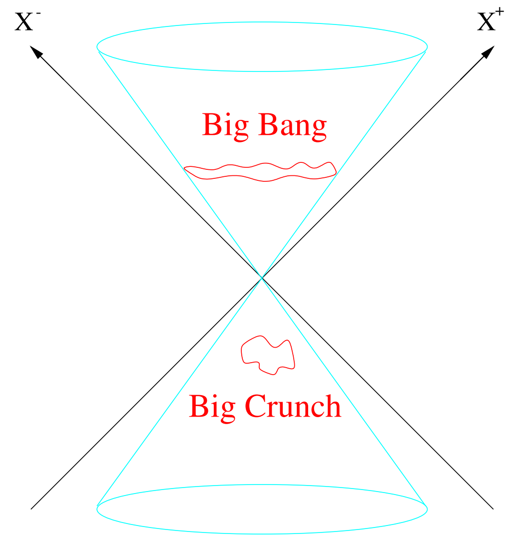

- The future (past) regions $X^+X^- > 0$ describes a cosmological universe often known as the Milne universe (1932), linearly expanding away from a Big Bang singularity (or contracting into a Big Crunch singularity):

$$ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + (dX^i)^2 \ , \quad \theta \equiv \theta + 2\pi \ , \quad X^\pm = Te^{\pm\beta\theta}/\sqrt{2}$$

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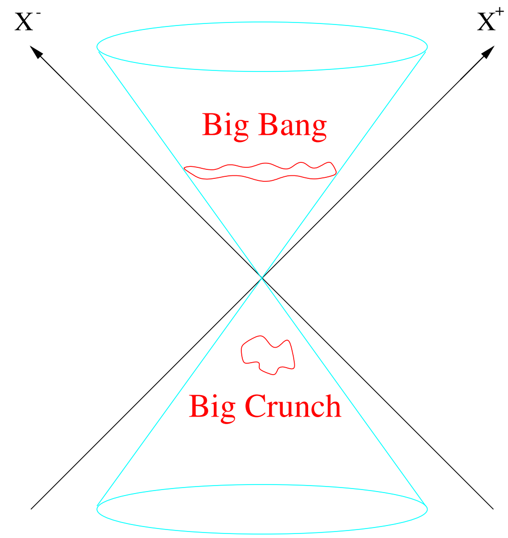

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This is a (degenerate) Kasner singularity, everywhere flat, except for a delta-function curvature at $T = 0$.

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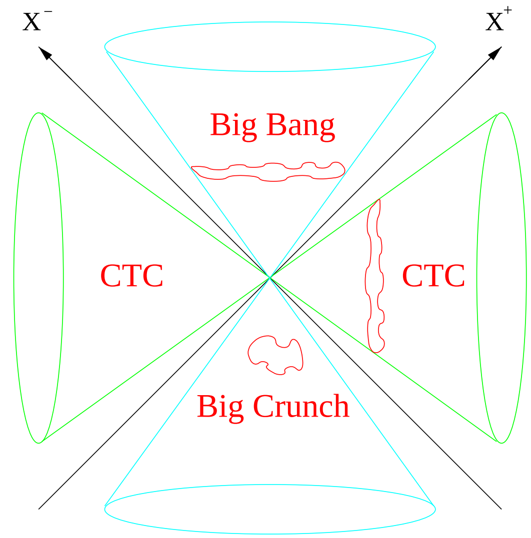

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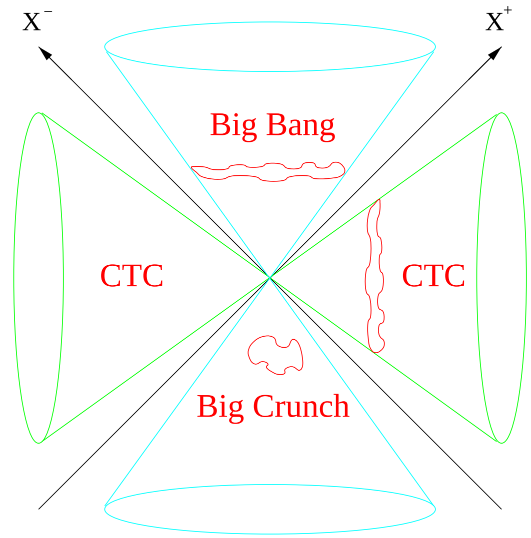

- In addition, the spacelike regions $X^+X^- < 0$ describe two Rindler wedges with compact time, often known as whiskers, leading to closed time-like curves:

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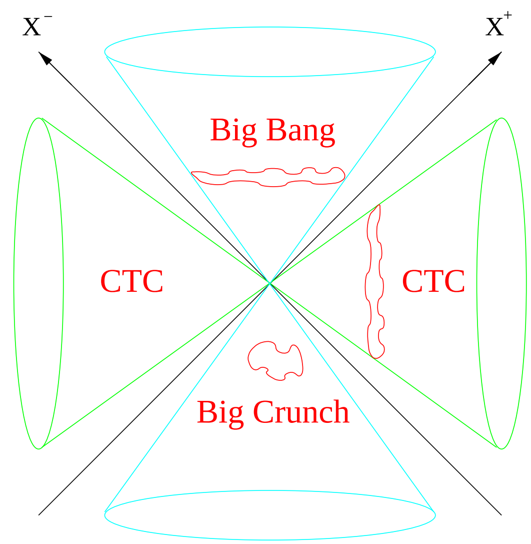

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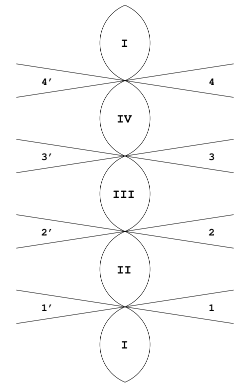
- Finally, the lightcone $X^+X^- = 0$ gives rise to a null, non-Hausdorff locus attached to the singularity.

Close relatives of the Misner Universe

- Misner space was first introduced as a local model of Lorentzian Taub-NUT space:



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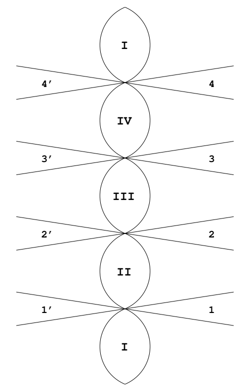

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This describes the space away from two moving cosmic strings. The cosmological singularity is smoothed out, but regions with CTC remain.

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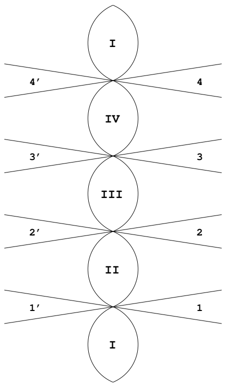

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- The Misner geometry arose again more recently as the M-theory lift of a simple (ekpyrotic) cosmological solution of Einstein-dilaton gravity with no potential.

Khoury Ovrut Seiberg Steinhard Turok

Close relatives of the Misner Universe (cont)

- The gauged WZW model $Sl(2) \times Sl(2)/U(1) \times U(1)$ describes a bouncing 4-dimensional Universe, with singularities analogous to the Lorentzian orbifold.

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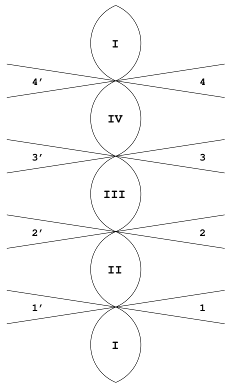

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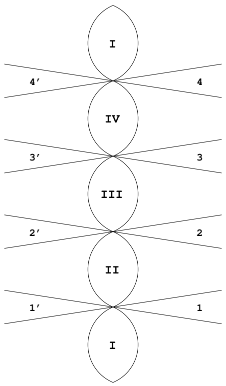

- The gauged WZW model $Sl(2)/U(1)$ at negative level orbifolded by a boost J describes two parallel Universes with a curvature and a Milne singularity, and compact whiskers.

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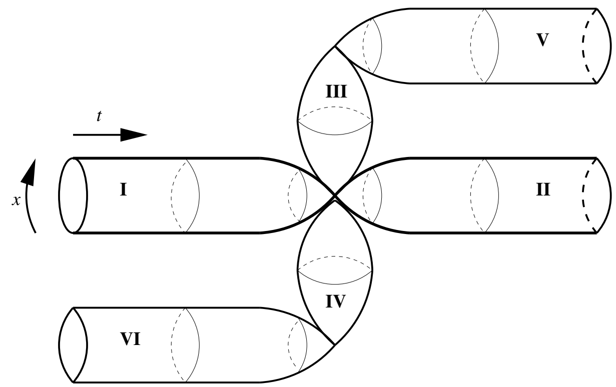

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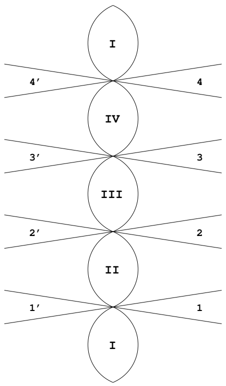

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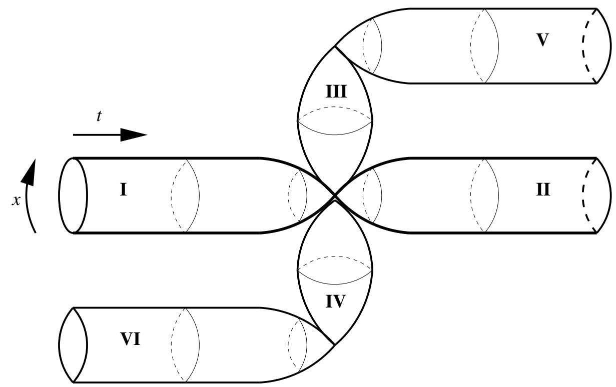


- The Lorentzian orientifold $IIB/[(-)^F boost]/[\Omega(-)^{F_L}]$ was also recently argued to describe orientifolds of non-supersymmetric strings with non-vanishing Neveu-Schwarz tadpoles.

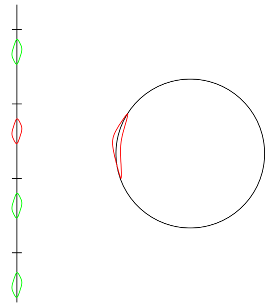
Dudas Mourad Timirgaziu

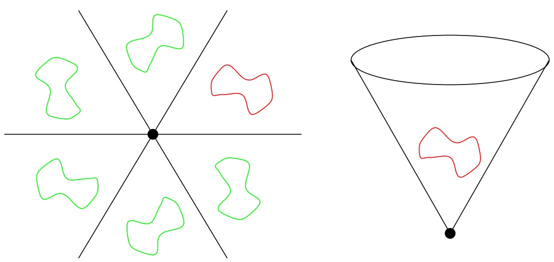
Strings on Euclidean orbifolds - untwisted states

- Well-known examples of orbifolds are the circle, R/Z, and the rotation orbifold R^2/Z_k.

Dixon Harvey Vafa Witten

- The spectrum of the quotient theory contains closed string states of the parent theory which are invariant under G: untwisted states.





Strings on Euclidean orbifolds - twisted states

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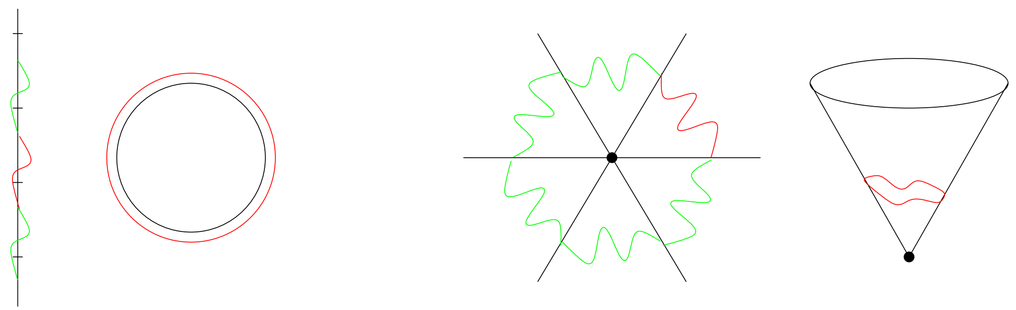

- When G acts non-freely, the twisted sector states are localized at the fixed points. They yield new localized degrees of freedom, which ensure the consistency of the background: anomaly free, divergence free...

Strings on Euclidean orbifolds - twisted sectors (cont.)

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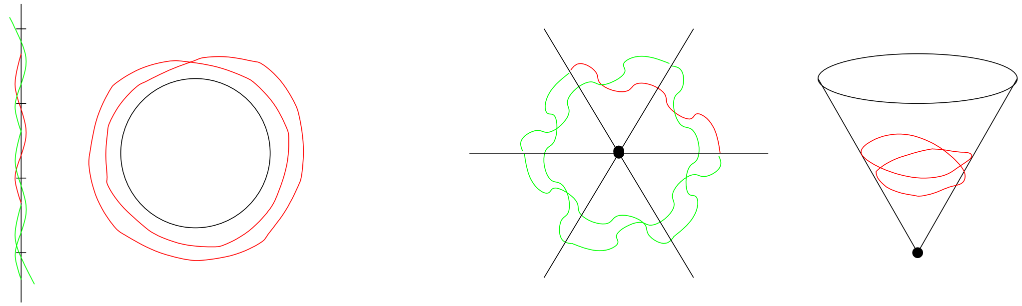
- Twisted sectors are labelled by conjugacy classes of G. Higher twisted sectors correspond to multiply wound states.



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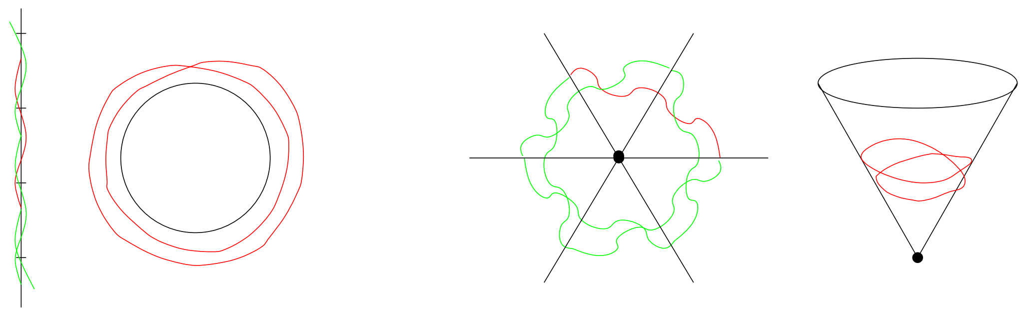

- Additionally, each twisted sector admits excited levels. The ground state can be thought as a Gaussian wave function centered at the origin.

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Dixon Harvey Vafa Witten

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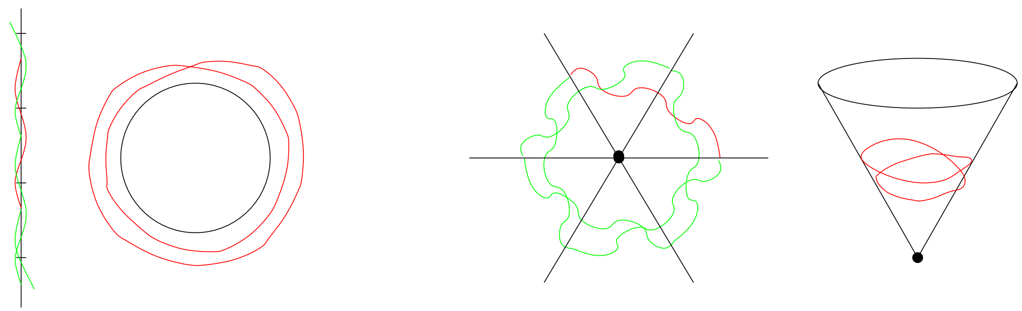

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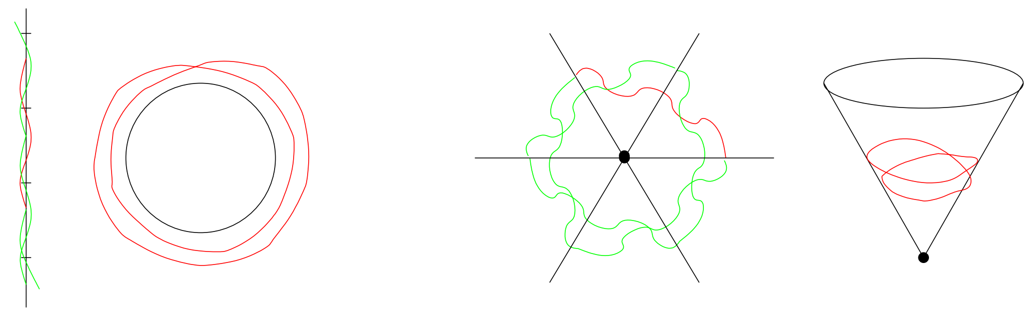

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- The Lorentzian orbifold share features with both examples: an infinite number of winding sectors, and a, non compact, fixed locus.

Closed strings in Misner space - untwisted states

- As usual in standard orbifolds, part of the spectrum involves closed strings on Minkowski covering space, which are invariant under the orbifold projection. In conformal gauge,

$$X^{\pm}(\sigma + 2\pi, \tau) = X^{\pm}(\sigma, \tau) \ , \quad (\partial_\tau^2 - \partial_\sigma^2) X^{\pm} = 0$$

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$$\sum_{n=-\infty}^{\infty} \partial X^+ \bar{\partial} X^- \exp\left(ik^+ X^- e^{-2\pi\beta n} + ik^- X^+ e^{2\pi\beta n} + ik_i x^i \right)$$

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- Equivalently, after Poisson resummation over n, this is a superposition of states with integer boost momentum $j = x^+\partial_+ - x^-\partial_-$,

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- The resulting eigenfunctions describe closed strings traveling around the Milne circle with integer momentum j.

Quantum fluctuations in field theory

- In the Minkowski vacuum (inherited from the covering space), the renormalized propagator can be obtained as a sum over images,

$$
\begin{aligned}
G(x;x') \quad = \quad & \sum_{n=-\infty, n\neq 0}^{\infty} \int_0^{\infty} d\tau \int dp^{\mu} \\
& \exp\left(-ip^-(x^+ - e^{2\pi\beta n}x^{+\prime}) - ip^+(x^- - e^{2\pi\beta n}x^{-\prime}) - ip^i(x^i - x^{i\prime})\right)
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- The one-loop stress-energy tensor follows from $G(x,x)$, e.g for a conformally coupled scalar,

$$\langle T_{ab}\rangle = \lim_{x\to x'}\left[(1-2\xi)\nabla_a\nabla'_b - 2\xi\nabla_a\nabla_b + (2\xi - \frac{1}{2})g_{ab}\nabla_c\nabla'^c\right] G(x,x')$$

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leading to a divergent quantum backreaction:

$$\langle T^\nu_\mu\rangle = \frac{K}{12\pi^2}T^{-4}\text{diag}(1,-3,1,1)\ , \quad K = \sum_{n=1}^\infty \frac{2+\cosh 2\pi n\beta}{[\cosh 2\pi n\beta - 1]^2}$$

Hiscock Konkowski 82

One-loop vacuum amplitude in field and string theory

- On the other hand, in string theory $\langle T^{\nu}_{\mu}\rangle(x)$ is an off-shell quantity, and only its integral over space-time is well defined:

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- The local divergence in $\langle T_\mu^\nu \rangle(x)$ is integrable and yields a finite free energy.
- The existence of Regge trajectories with arbitrary high spin implies new (log) divergences in the bulk of the moduli space, not unlike long string poles in AdS_3.

Scattering of untwisted states

- Tree-level scattering amplitudes of untwisted sector states can be computed from those in flat space by the inheritance principle,

$$\langle V(j_1,k_1)\dots V(j_n,k_n)\rangle_{Misner} = \int dv_1\dots dv_n\; e^{i(j_1 v_1+\dots+j_n v_n)}$$
$$\langle V(e^{\beta v_1}k_1^+, e^{-\beta v_1}k_1^-, k_1^i)\dots V(e^{\beta v_n}k_n^+, e^{-\beta v_n}k_n^-, k_n^i)\rangle_{Minkowski}$$

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- The integral diverges due to Regge behavior in the large momentum, fixed angle regime. E.g, the four-tachyon scattering amplitude in bosonic string leads to

$$\int dv\; v^{-\frac{1}{2}(k_1^i-k_3^i)^2+i(j_2-j_4)}$$

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Scattering of untwisted states

- It could be that eikonal resummation of ladder diagrams may lead to a finite result, e.g.

$$\mathcal{A} \sim -G\frac{s^2}{t} \quad \rightarrow \quad -G\frac{s^2}{t+(2\pi G s)^2} \qquad \text{(3D gravity)}$$

Yet this remains to be demonstrated.

Deser McCarthy Steif; Cornalba Costa

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$$X^{\pm}_{R}(\tau - \sigma) = \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha^{\pm}_{n} \, e^{-i(n \pm i\nu)(\tau - \sigma)}$$

$$X^{\pm}_{L}(\tau + \sigma) = \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \mp i\nu)^{-1} \tilde{\alpha}^{\pm}_{n} \, e^{-i(n \mp i\nu)(\tau + \sigma)}$$

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with canonical commutation relations

$$[\alpha_m^+, \alpha_n^-] = -(m + i\nu)\delta_{m+n} \quad , \quad [\tilde{\alpha}_m^+, \tilde{\alpha}_n^-] = -(m - i\nu)\delta_{m+n}$$

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$$(\alpha^{\pm}_m)^* = \alpha^{\pm}_{-m} \quad , \quad (\tilde{\alpha}^{\pm}_m)^* = \tilde{\alpha}^{\pm}_{-m}$$

- There are no translational zero-modes, instead two pairs of quasi zero-modes which are canonically conjugate real operators:

$$[\alpha^{+}_0, \alpha^{-}_0] = -i\nu \ , \quad [\tilde{\alpha}^{+}_0, \tilde{\alpha}^{-}_0] = i\nu$$

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated by half of them, e.g.

$$\alpha^{\pm}_{n>0}\,, \quad \tilde{\alpha}^{\pm}_{n>0}\,, \qquad \alpha^{-}_{0}\,, \quad \tilde{\alpha}^{+}_{0}$$

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- The worldsheet Hamiltonian, normal-ordered wrt to this vacuum, reads

$$L_0^{l.c.} = -\sum_{n=0}^{\infty}(\alpha_n^+)^*\alpha_n^- \;-\sum_{n=1}^{\infty}(\alpha_n^-)^*\alpha_n^+ + \frac{1}{2}i\nu(1-i\nu) - 1 + L_{int}$$

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- This is the familiar result for the vacuum energy $\frac{1}{2}\theta(1-\theta)$ in the Euclidean rotation orbifold, after analytic continuation $\theta \to i\nu$.
- Due to the $i\nu/2$ term in the ground state energy, all states obtained by acting on $|0\rangle$ by creation operators $\alpha^{\pm}_{n<0}$ and by α^{+}_0 will have imaginary energy, hence the physical state condition $L_0 = 0$ seem to have no solutions.

One-loop amplitude, twisted sector

- Independently of this fact, one may compute the one-loop path integral on an Euclidean worldsheet and Minkowskian target-space:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho)x\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

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- In the twisted sector, the left-moving zero-modes contribute

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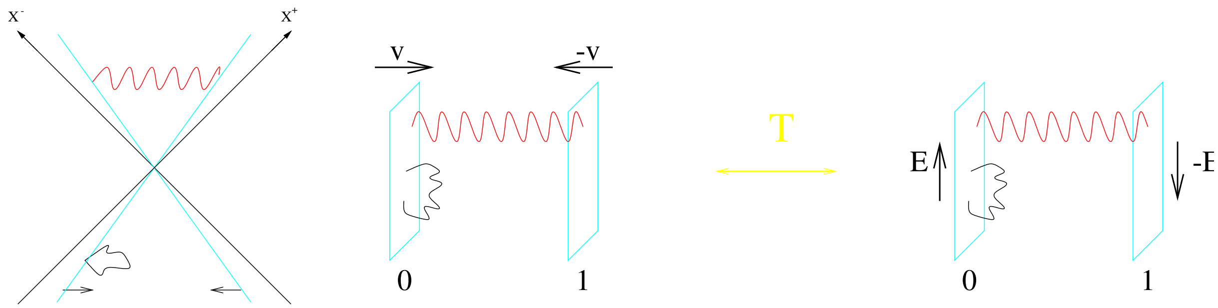
- The absence of physical twisted states crushes our hopes for resolving the singularity... yet does not sound very sensible. An important point: α_0^+ and α_0^- are not hermitian conjugate to each other, but rather self-hermitian...

A detour via Open strings in electric field

- A very similar puzzle is faced in the case of colliding D-branes, or in the T-dual process of charged open strings in a constant electric field:



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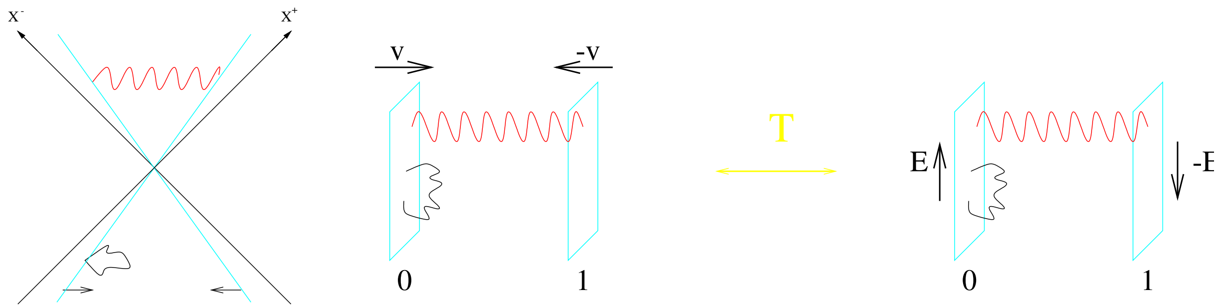

- Recall that for open strings stretched between two D-branes with electromagnetic fields F_0 and F_1, proper frequencies satisfy

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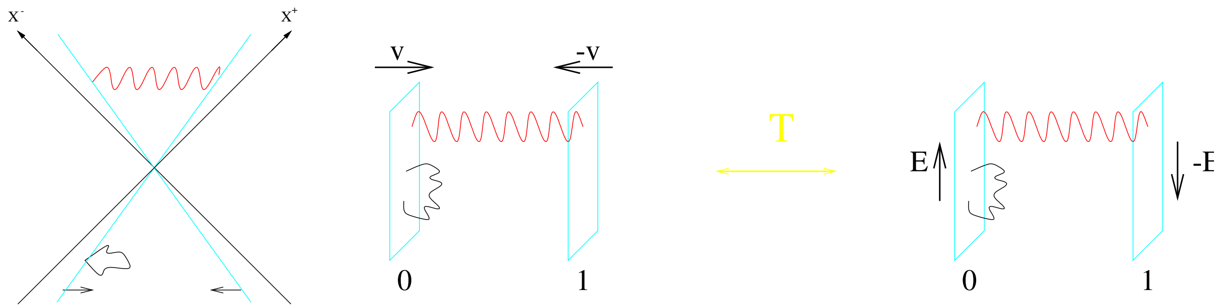
For $F_0 \neq F_1$, the open string carries a net electric charge, and the motion of its center of motion is that of a charged particle.

A detour via Open strings in electric field

- A very similar puzzle is faced in the case of colliding D-branes, or in the T-dual process of charged open strings in a constant electric field:



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- In the case of an electric field $F_1 = E dx^+ \wedge dx^-$, $F_1 = 0$, the resulting spectrum is

$$\omega_n = n + i\nu \ , \qquad \nu := \text{Arctanh} E = w\beta$$

just as in the Lorentzian orbifold case. The large winding limit amounts to a near critical electric field.

Open string mode expansion

- The light-cone embedding coordinates have the normal mode expansion

$$X^{\pm} = x_0^{\pm} + i \sum_{n=-\infty}^{+\infty} (-)^n (n \pm i\nu)^{-1} a_n^{\pm} e^{-i(n \pm i\nu)\tau} \cos[(n \pm i\nu)\sigma]$$

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$$L_0^{l.c.} = -\sum_{m=0}^{\infty} a_{-m}^+ a_m^- - \sum_{m=1}^{\infty} a_{-m}^- a_m^+ + \frac{i\nu}{2}(1 - i\nu) - \frac{1}{12}$$

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- By the same token, charged open strings should have no physical states... yet electrons and positrons certainly do exist.

Charged particle and open string zero-modes

- Let us recall the quantization of a charged particle in an electric field:

$$L = \frac{1}{2}m\left(-2\partial_\tau X^+\partial_\tau X^- + (\partial_\tau X^i)^2\right) + \frac{e}{2}\left(X^+\partial_\tau X^- - X^-\partial_\tau X^+\right)$$

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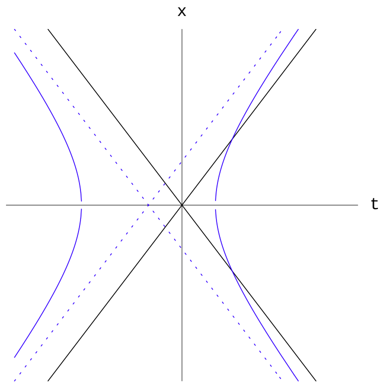

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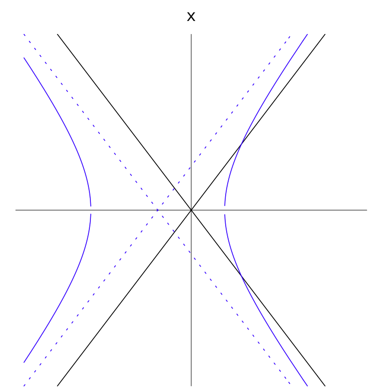

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- Starting from the canonical equal-time commutation rules

$$[\pi^+, x^-] = [\pi^-, x^+] = i\ , \quad [\pi^i, x^j] = i\delta_{ij}$$

one recovers the open string zero-mode commutation relations ($\nu = e$),

$$[a_0^+, a_0^-] = -i\nu\ , \quad [x_0^+, x_0^-] = -\frac{i}{\nu}$$

Charged particle and ppen string zero-modes

- Quantum mechanically, one may represent $\pi^{\pm} = i\partial/\partial x^{\mp}$ hence obtain $a_0^{\pm}, x_0^{\pm}$ as covariant derivatives

$$a_0^{\pm} = i\partial_{\mp} \pm \frac{\nu}{2} x^{\pm} \, , \quad x_0^{\pm} = \mp \frac{1}{\nu} \left(i\partial_{\mp} \mp \frac{\nu}{2} x^{\pm} \right)$$

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- The zero-mode piece of L_0, including the bothersome $\frac{i\nu}{2}$,

$$L_0^{(0)} = -a_0^+ a_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\nabla_0^+ \nabla_0^- + \nabla_0^- \nabla_0^+)$$

is just the Klein-Gordon operator of a particle of charge ν.

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^{\pm} = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator can be rewritten as an inverted harmonic oscillator:

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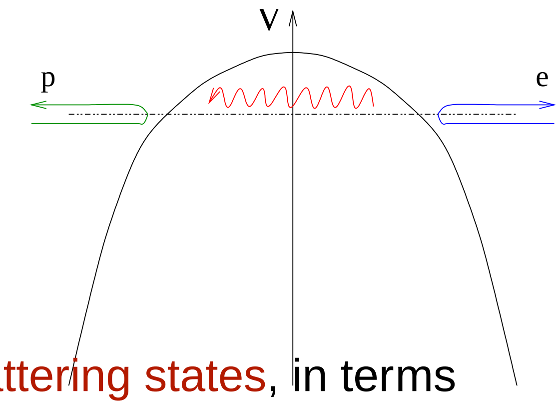
$$\left(-\partial_u^2 - \frac{1}{4}u^2 + \frac{M^2}{2\nu} \right) \psi_{\tilde{p}}(u) = 0$$



- The latter admits a respectable delta-normalizable spectrum of scattering states, in terms of parabolic cylinder functions, e.g:

$$\phi_{in}^+ = D_{-\frac{1}{2}+i\frac{M^2}{2\nu}}(e^{-\frac{3i\pi}{4}}u)e^{-i\tilde{p}t}e^{i\nu x t/2}$$

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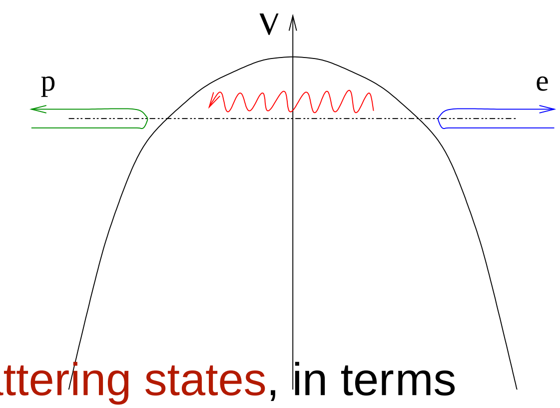

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- These correspond to non-compact trajectories of charged particles in the electric field. Tunnelling is just (stimulated) Schwinger pair creation,

$$e^- \rightarrow (1+\eta)\, e^- + \eta\, e^+ \,, \quad \eta \sim e^{-\pi M^2/\nu}$$

Brezin Itzykson; Brout Massar Parentani Spindel

Lorentzian vs Euclidean states

- Analytic continuation $X^0 \to -iX^0$, $\nu \to i\nu$ turns an electric field in $R^{1,1}$ into a magnetic field in R^2. At the same time, one should Wick rotate the worldsheet time.

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- The contribution of zero-modes to the one-loop amplitude can be interpreted either way,

$$\frac{1}{2i\sin(\nu t/2)} = \sum_{n=1}^{\infty} e^{-i(n+\frac{1}{2})\nu t} = \int dM^2 \rho(M^2) e^{-M^2 t/2}$$

The density of states is obtained from the reflection phase shift,

$$\rho(M^2) = \frac{1}{\nu}\log\Lambda - \frac{1}{2\pi i}\frac{d}{dM^2}\log\frac{\Gamma\left(\frac{1}{2}+i\frac{M^2}{2\nu}\right)}{\Gamma\left(\frac{1}{2}-i\frac{M^2}{2\nu}\right)}$$

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- The physical spectrum can be explicitely worked out at low levels, and is free of ghosts: a tachyon at level 0, a transverse gauge boson at level 1, . . .

Charged particle in Rindler space

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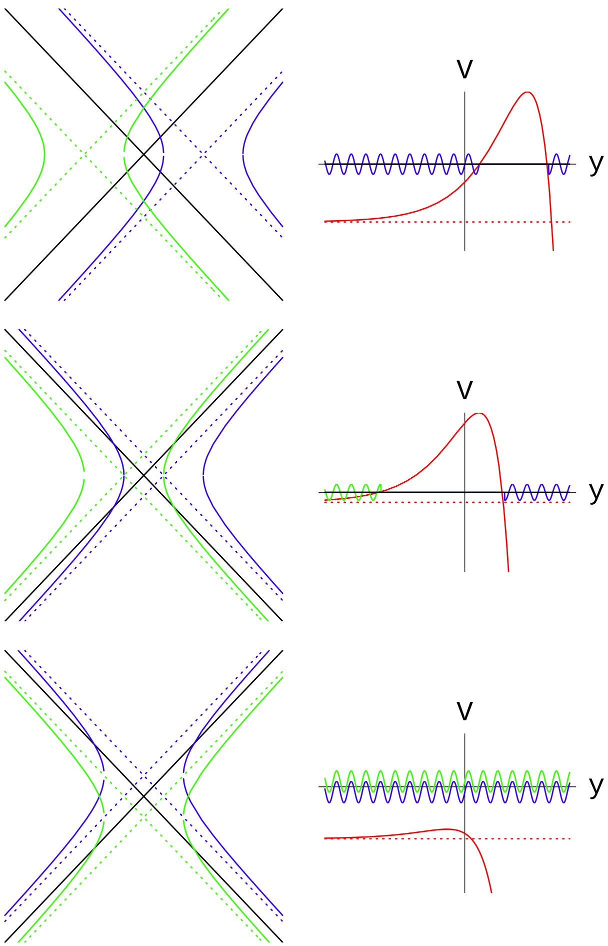

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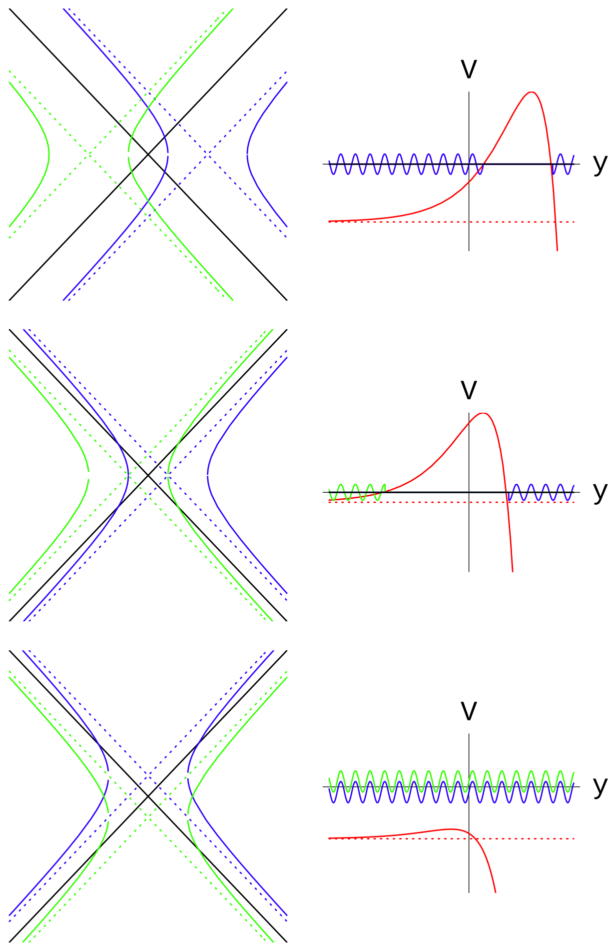

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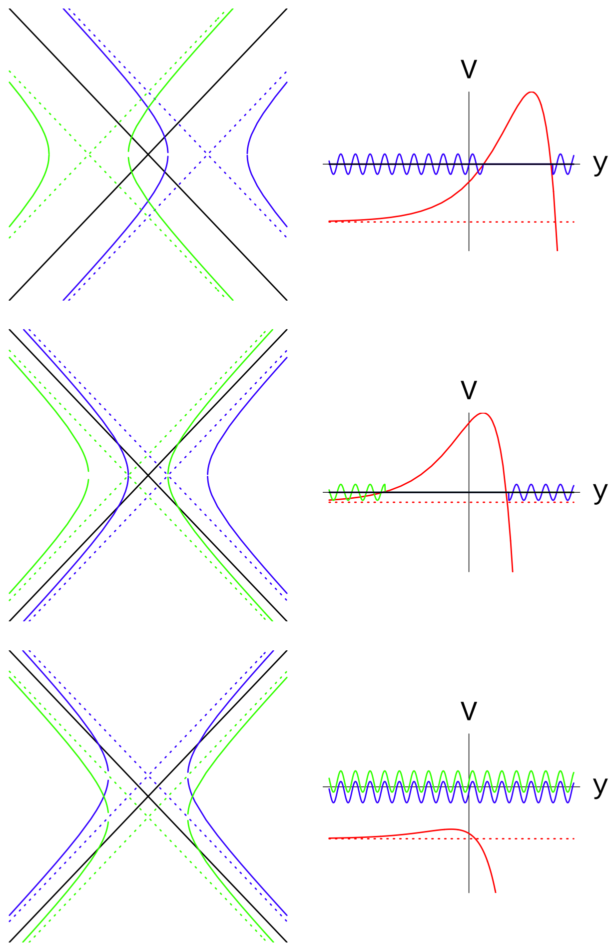

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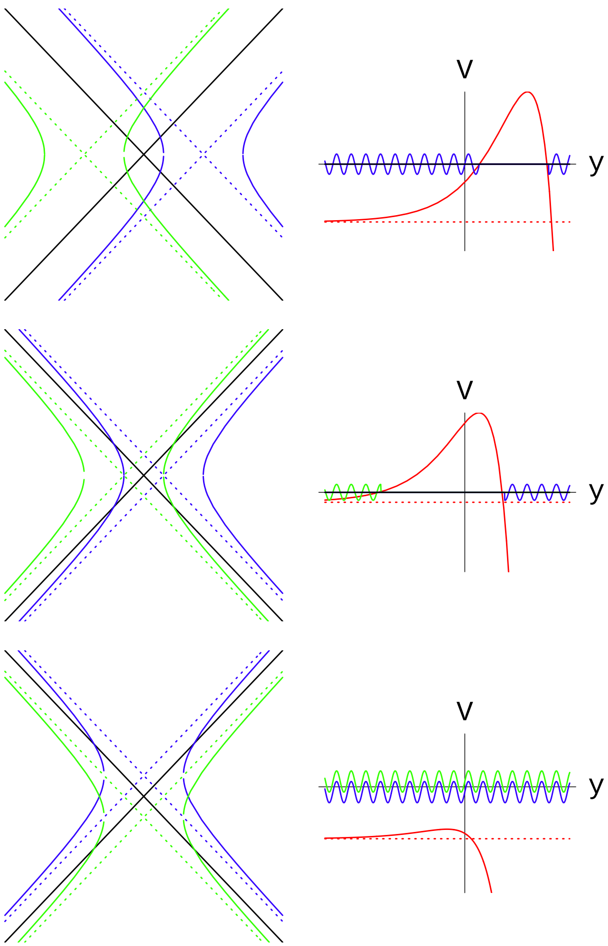

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- If $j > M^2/(2\nu)$, the electron branches extend in the Milne regions. There is no tunneling, but partial reflection amounts to a combination of Schwinger and Hawking emission.

Rindler modes

- Incoming modes from Rindler infinity I_R^- read, in terms of parabolic cylinder functions:

$$\mathcal{V}^j_{in,R} = e^{-ij\eta} r^{-1} M_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),-\frac{ij}{2}}(i\nu r^2/2)$$

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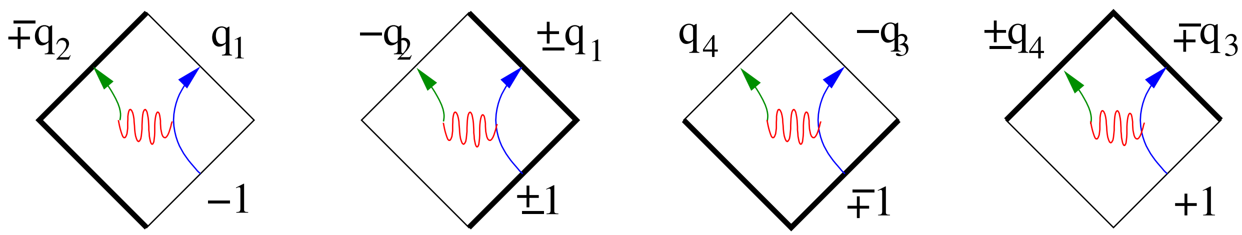

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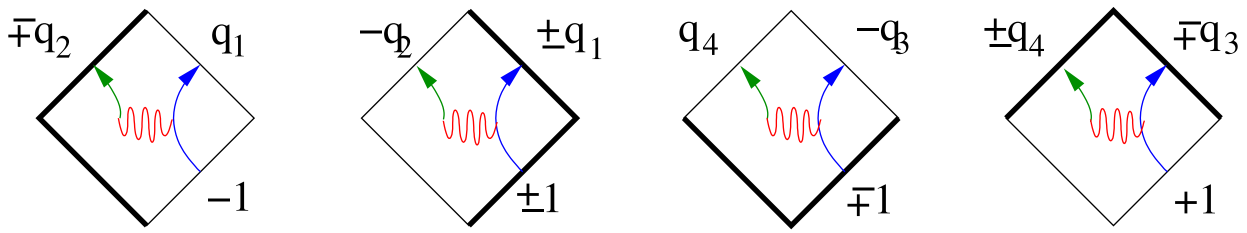

- The reflection coefficients can be computed:

$$q_1 = e^{-\pi j} \frac{\cosh\left[\pi \frac{M^2}{2\nu}\right]}{\cosh\left[\pi\left(j - \frac{M^2}{2\nu}\right)\right]}, \quad q_3 = e^{\pi\left(j-\frac{M^2}{2\nu}\right)} \frac{\cosh\left[\pi\frac{M^2}{2\nu}\right]}{|\sinh \pi j|}$$

and $q_2 = 1 - q_1$, $q_4 = q_3 - 1$, by charge conservation.

Global Charged Unruh Modes

- Global modes may be defined by patching together Rindler modes, ie by analytic continuation across the horizons. Unruh modes are those which are superposition of positive energy Minkowski modes,

$$\Omega^j_{in,+} = \mathcal{V}^j_{in,P} = (-i\nu X^+X^-)[X^+/X^-]^{-ij/2} W_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}$$

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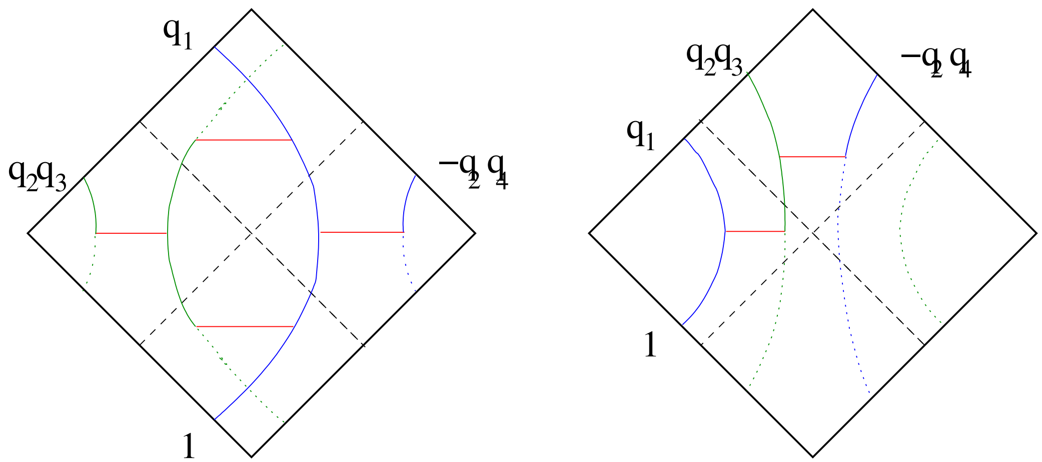
- There are two types of Unruh modes, involving 2 or 4 tunelling events:



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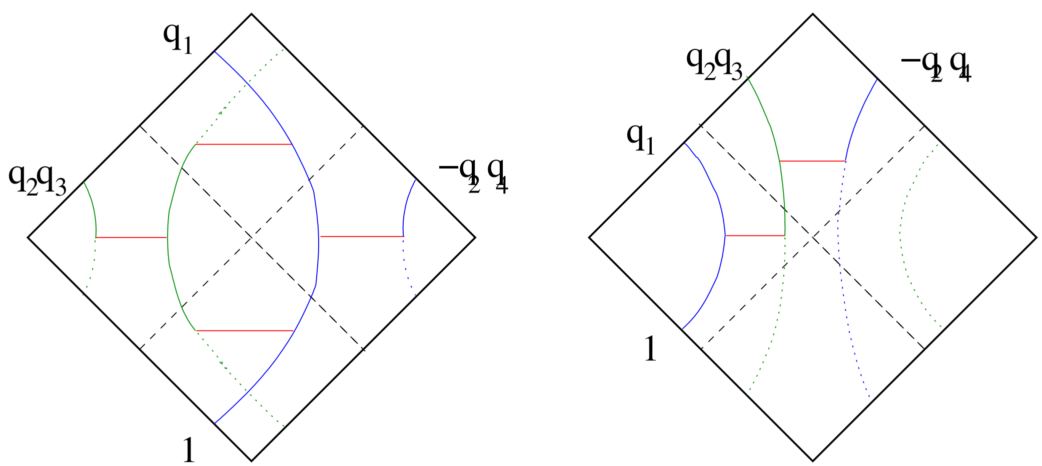

- Any state in Minkowski space can be represented as a state in the tensor product of the Hilbert spaces of the left and right Rindler patches. In contrast to neutral fields in Rindler space, Boulware-Fulling modes that vanish in L or R have positive Minkowski energy.

Closed string zero-modes

- Let us reanalyze the classical solutions for the closed string zero modes

$$X^{\pm}(\tau,\sigma) = e^{\mp\nu\sigma}\left[\pm\frac{1}{2\nu}\alpha_0^{\pm}e^{\pm\nu\tau} \mp \frac{1}{2\nu}\tilde{\alpha}_0^{\pm}e^{\mp\nu\tau}\right] \ , \quad \alpha_0^{\pm},\tilde{\alpha}_0^{\pm} \in R$$

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- The Milne time, or Rindler radius, is independent of σ:

$$4\nu^2 X^+ X^- = \alpha_0^+\tilde{\alpha}_0^- e^{2\nu\tau} + \alpha_0^-\tilde{\alpha}_0^+ e^{-2\nu\tau} - \alpha_0^+\alpha_0^- - \tilde{\alpha}_0^+\tilde{\alpha}_0^-$$

  We may thus follow the motion of a single point $\sigma = \sigma_0$ and obtain the rest of the worldsheet by smearing under the action of the boost.

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$$X^{\pm}(\tau,\sigma) = e^{\mp\nu\sigma}\left[\pm\frac{1}{2\nu}\alpha_0^{\pm}e^{\pm\nu\tau} \mp \frac{1}{2\nu}\tilde{\alpha}_0^{\pm}e^{\mp\nu\tau}\right] , \quad \alpha_0^{\pm}, \tilde{\alpha}_0^{\pm} \in R$$

- The Milne time, or Rindler radius, is independent of σ:

$$4\nu^2 X^+ X^- = \alpha_0^+\tilde{\alpha}_0^- e^{2\nu\tau} + \alpha_0^-\tilde{\alpha}_0^+ e^{-2\nu\tau} - \alpha_0^+\alpha_0^- - \tilde{\alpha}_0^+\tilde{\alpha}_0^-$$

  We may thus follow the motion of a single point $\sigma = \sigma_0$ and obtain the rest of the worldsheet by smearing under the action of the boost.

- Up to a shift of τ and σ, the physical state conditions require

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- The behavior at early/late proper time now depends on $\epsilon\tilde{\epsilon}$: For $\epsilon\tilde{\epsilon} = 1$, the string begin/ends in the Milne regions. For $\epsilon\tilde{\epsilon} = -1$, the string begin/ends in the Rindler regions.

Short and long strings ($j = 0$)

Choosing $j = 0$ for simplicity, we have two very different types of solutions:

- $\epsilon = 1$, $\tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma, \tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma} \,, \quad T = \frac{M}{\nu} \sinh(\nu\tau) \,, \quad \theta = \nu\sigma$$

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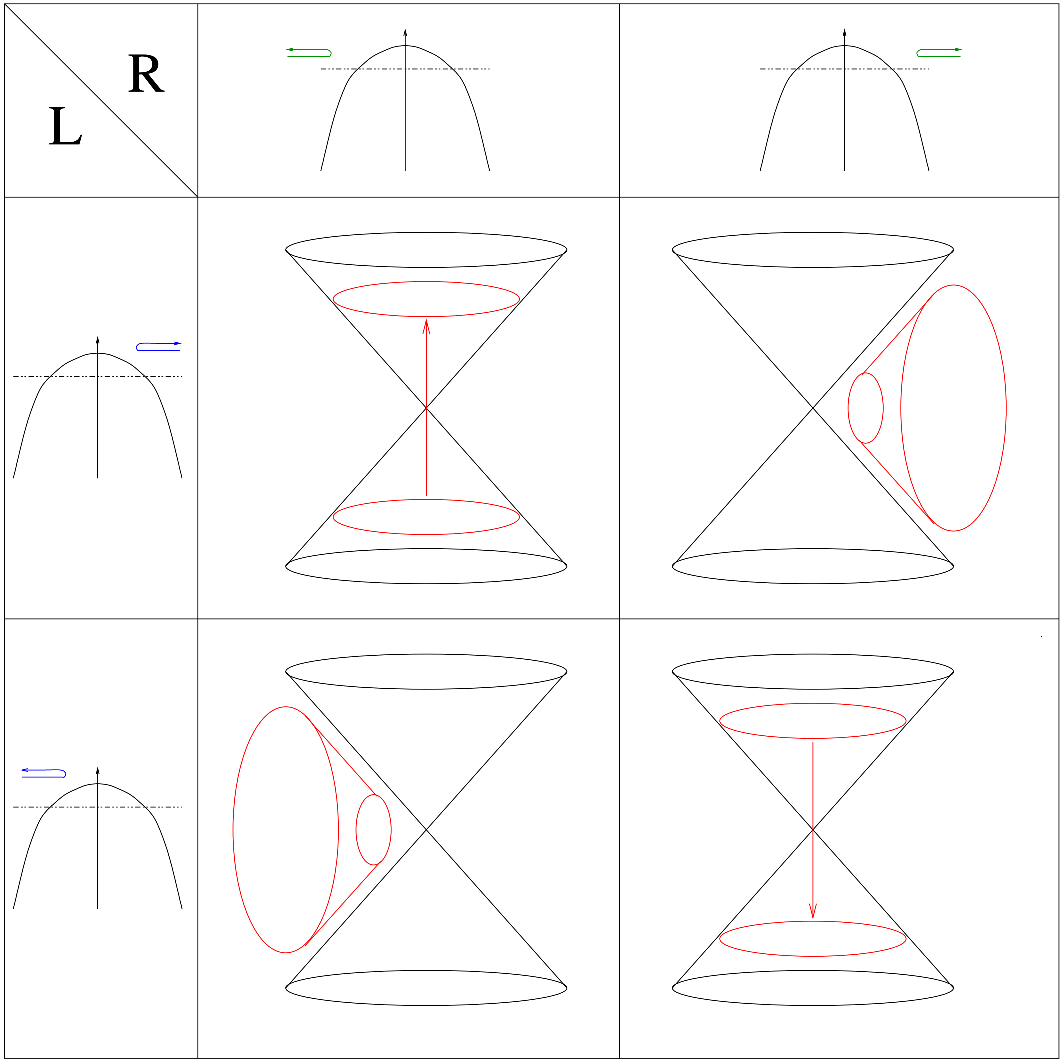
$\epsilon = -1$, $\tilde{\epsilon} = 1$ is the analogue in the left Rindler patch.

Short and long strings

Closed string trajectories are thus generated by the motion of two decoupled particles in inverted harmonic oscillators:



Relation to open string modes

- Instead of following the motion of a point at fixed σ, one may consider instead a point at fixed $\sigma+\tau$: this is precisely the trajectory of the open string zero-mode.
- Using the covariant derivative representation

$$\alpha_0^{\pm} = i\partial_{\mp} \pm \frac{\nu}{2} x^{\pm}\,, \quad \tilde{\alpha}_0^{\pm} = i\partial_{\mp} \mp \frac{\nu}{2} x^{\pm}$$

we observe that $x^{\pm}$ is the Heisenberg operator corresponding to the location of the closed string (at $\sigma=0$):

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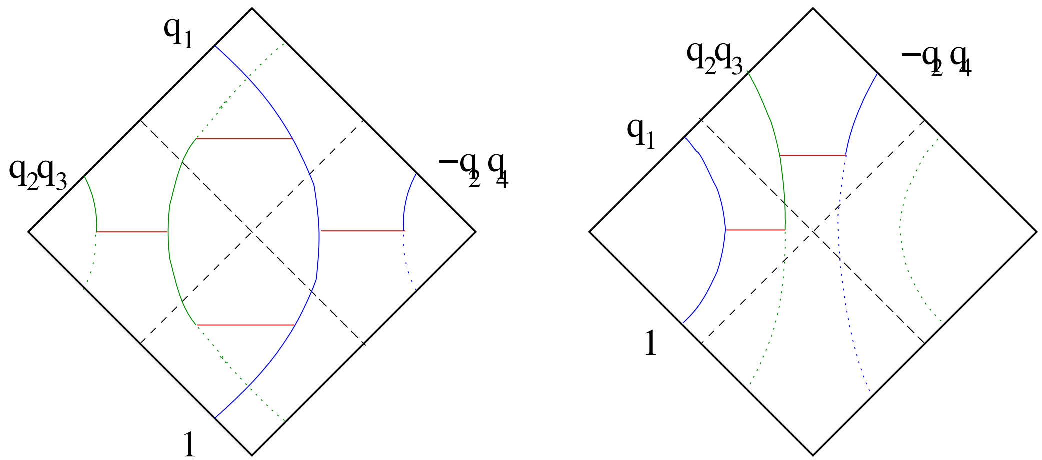
- The open string global wave functions. . .



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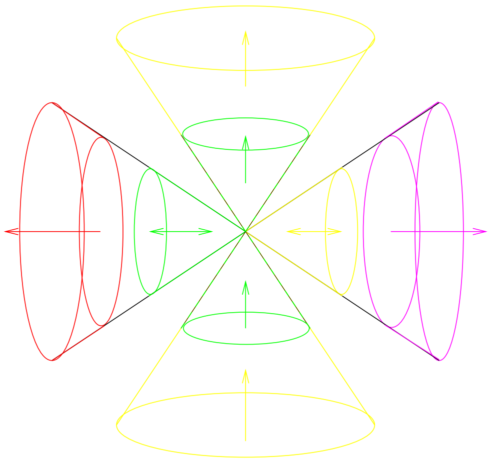

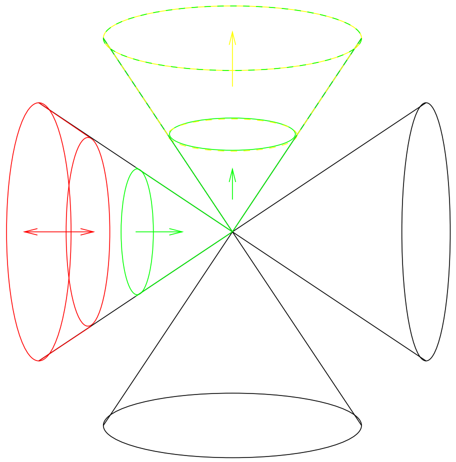

Comments on winding string production

- The production rate of winding strings can be evaluated by WKB methods: for the Misner Universe,

$$R \sim \exp\left(-\pi \frac{M^2}{\beta w}\right) \quad \to 1 \text{ as } w \to \infty$$

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The total production rate appears to be infinite.

- One expects that the backreaction due to particle production c an be described in a mean field approach by a deformed geometry, e.g. in the Milne regions,

$$ds^2 = -dT^2 + a^2(T)d\theta^2$$

or, in the Rindler regions,

$$ds^2 = dr^2 - b^2(r)d\eta^2$$

with $b(r) = ia(ir)$.

- For example, $a(T) = \sqrt{\beta^2 T^2 + \epsilon^2}$ leads to a smooth cosmological region with a neck, and two disconnected Rindler regions with singularities at the two tips (and an Euclidean region in between).

Winding strings in deformed Milne space

- The propagation of winding strings can be studied in a semi-classical fashion as before:

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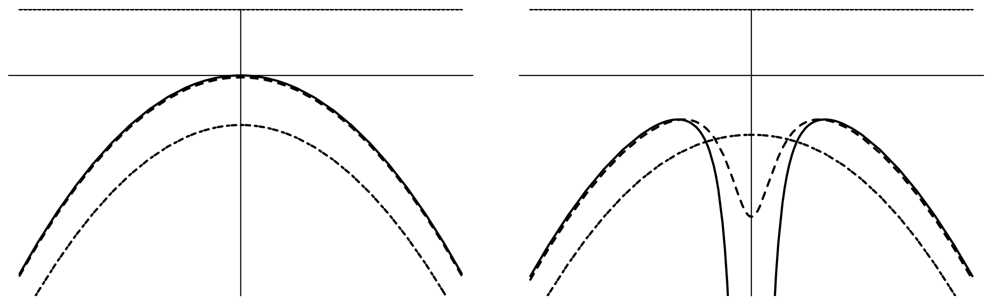

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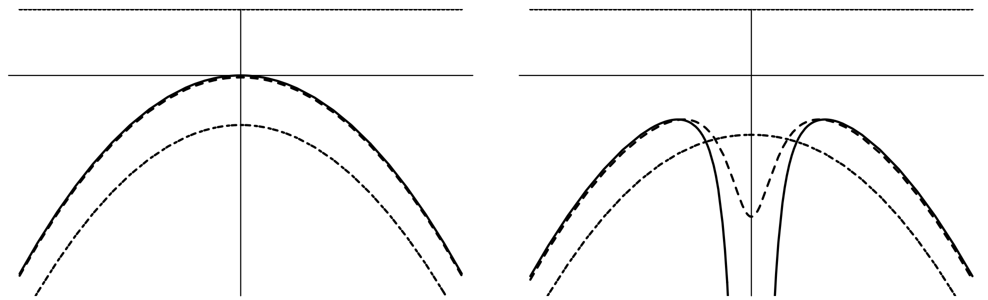

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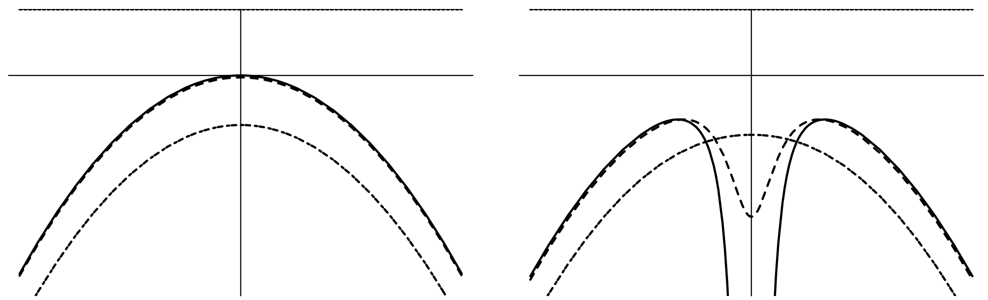

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- For $a(T) = \sqrt{\beta^2 T^2 + \epsilon^2}$, the singularity at 0 is resolved. For small enough w, there is a meta-stable (non-physical) state around $T = 0$, which may be excited by incoming strings...

Winding strings in deformed Rindler space

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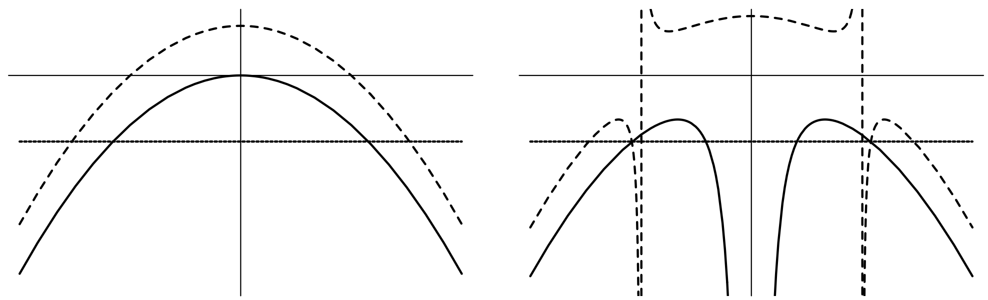

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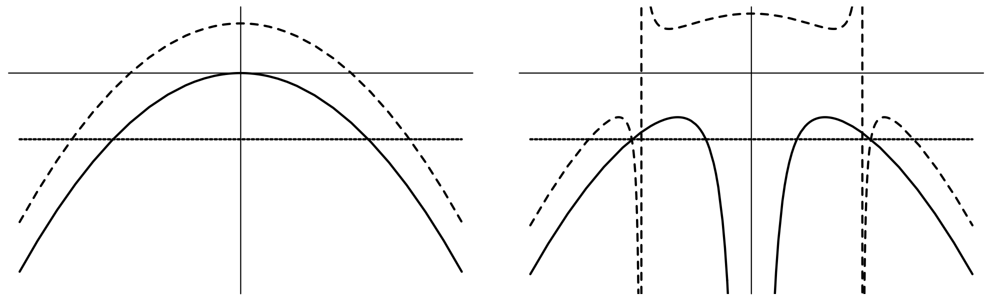

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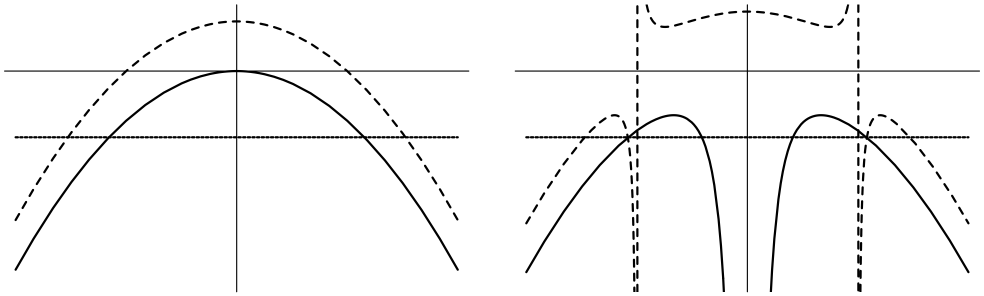

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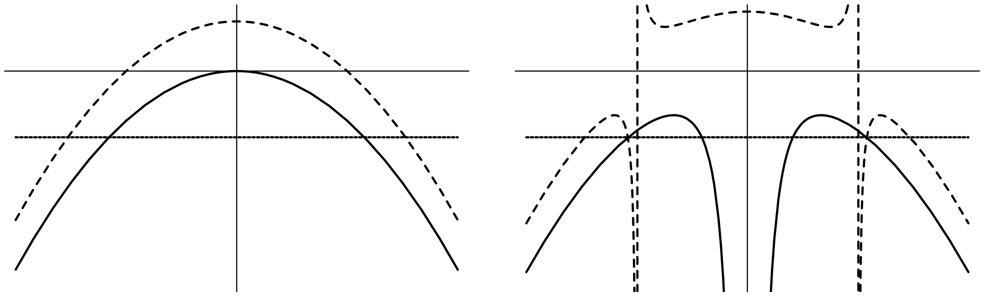

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- Of course, ϵ could also be imaginary, in which case the two cosmological regions would disconnect...

Tunelling and particle production

- Consider now the motion in the classically forbidden region: as always in quantum mechanics, one is instructed to rotate to Euclidean time, i.e. flip the sign of p_T^2. Equivalently, flip the sign of the potential:

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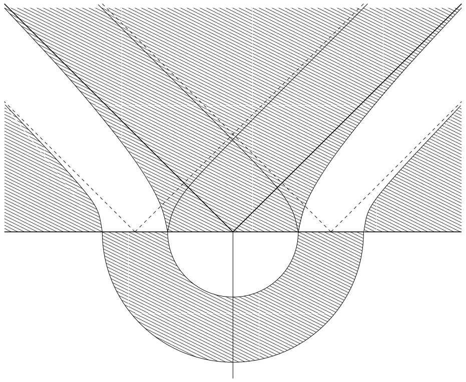

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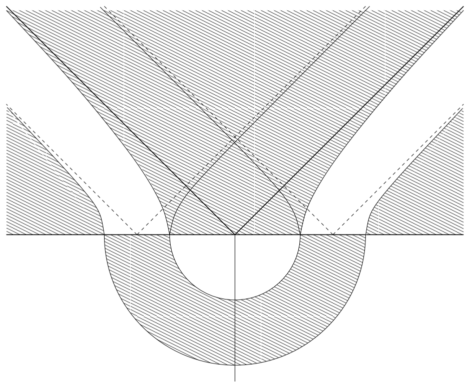

- Short and long strings are thus spontaneously produced in correlated pairs.

Effective gravity analysis

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- A bounce in dimension i requires $H_i' > 0$ when $H_i = 0$, hence

$$(D-2)p_i + \rho \geq \sum_{j \neq i} p_j$$

The most efficient solution is a gas of scalar momentum states, with $p = \rho$: provides enough pressure for the bounce.

Effective gravity analysis (cont.)

- Nevertheless, consider fundamental strings wrapped around dimension i,

$$\rho = \frac{T}{V}\ , \quad p_i = -\rho\ , \quad p_{j\neq i} = 0\ , \quad V = \prod_{j\neq i} a_j \quad \Rightarrow D \leq 3$$

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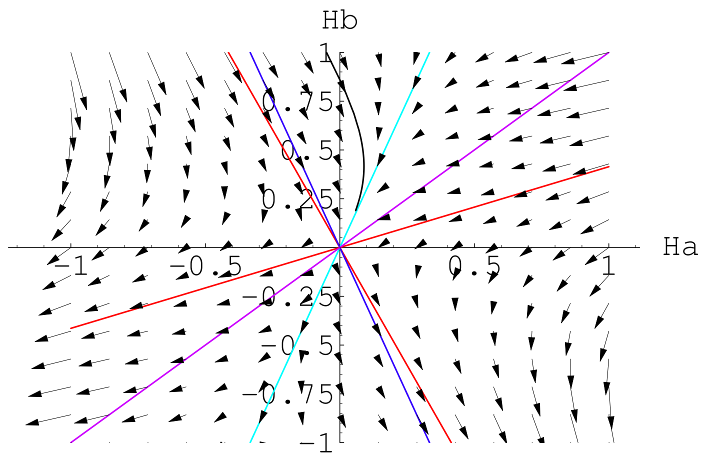

- This result seems to go opposite to the fact that winding states prevent infinite expansion. Non-isotropy is an important ingredient.

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- We assumed a constant number of wound strings: one should incorporate the dependence of the production rate on the Hubble parameters.

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$$E = -\int_{-\infty}^{\infty} d\tau \left(X^+ \partial_\sigma X^- - X^- \partial_\sigma X^+ \right) = \int_{-\infty}^{\infty} d\tau \, r^2 \partial_\sigma \eta$$

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- Introducing a cut-off $-T \leq \tau < T$, the Rindler energy

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- The Rindler energy spectrum is unbounded both above and below:

$$E_{short} < -e^{2\nu T} \frac{M \tilde{M}}{4\nu^2} < e^{2\nu T} \frac{M \tilde{M}}{4\nu^2} < E_{long}$$

How can one prevent the decay into short strings ?

Conclusions - speculations

- Winding states in the Milne Universe behave in close analogy with open strings in an electric field. Using intuition from open strings, we have found that physical states do exist in the twisted sector of the Lorentzian orbifold, and can be pair produced.

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- If one manages to make sense of the winding production rate, and if the singularity gets resolved, what happens to the whiskers ? Can they provide some time-independent dual description of the cosmological evolution ?

Conclusions - speculations (cont.)

- To demonstrate that the singularity is resolved, one should in principle take into account the production of (an infinite number) of twisted sector states in correlated pairs, i.e. squeezed states: non-local deformations of the worldsheet ? closed string field theory ?

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- As a less ambitious goal, can one compute scattering amplitudes of twisted states, and check if they are better behaved than untwisted states. This can be found by careful analytic continuation from the Nappi-Witten plane wave.

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- More generally, we still lack a framework to compute the production of closed strings in cosmological backgrounds. Those however are likely to lead to large departures from FRW cosmology, and possibly spectacular effects: cosmological bounce, Hagedorn phase transition...

  Lawrence Martinec, Gubser

Appendices (not shown during talk)

Vacua of Misner space

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$$H^{(1)}_{-ij}(mT)e^{-ij\theta} \sim e^{-ij\theta - imT}/\sqrt{T}$$

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- As $T \to 0$, solutions become independent of the mass, and define the conformal vacuum basis,

$$J_{ij}(mT) = H^{(1)}_{-ij}(mT) + H^{(2)}_{-ij}(mT) \sim e^{-ij\theta + ij\log(mT)}$$

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Closed string one-loop vacuum amplitude

- Independently of this fact, one may compute the one-loop (Euclidean ws, Minkowskian target) free energy using path integral methods:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho)x\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

where θ_1 is the Jacobi theta function,

$$\theta_1(v;\rho) = 2q^{1/8} \sin \pi v \prod_{n=1}^{\infty} (1-e^{2\pi i v}q^n)(1-q^n)(1-e^{-2\pi i v}q^n)\ , \quad q = e^{2\pi i \rho}$$

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- In the untwisted sector, this reproduces the integrated vacuum free energy found by the method of images:

$$\int dx^+ dx^- G(x,x) = \sum_{l=-\infty}^{+\infty} \int_0^\infty \frac{d\rho}{\rho^{D/2}} \frac{e^{-m^2\rho}}{\sinh^2(\pi\beta l)}$$

One-loop amplitude and Schwinger pair production

- Using this quantization scheme, the one-loop (Euclidean worldsheet, Minkowskian target) vacuum free energy reads

$$A_{bos} = \frac{i\pi V_{26}(e_0 + e_1)}{2} \int_0^\infty \frac{dt}{(4\pi^2 t)^{13}} \frac{e^{-\pi\nu^2 t/2}}{\eta^{21}(it/2)\ \theta_1(t\nu/2; it/2)}$$

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- Each of the poles at $t = 2k/\nu$ contributes to the imaginary part, yielding the production rate of charged open strings,

$$\mathcal{W} = \frac{1}{2(2\pi)^{25}} \frac{(e_0+e_1)}{\nu} \sum_{k=1}^{\infty} (-)^{k+1} \left(\frac{|\nu|}{k}\right)^{13} \sum_{N=-1}^{\infty} c_b(N) \exp\left(-2\pi k \frac{N}{|\nu|} - 2\pi k |\nu|\right)$$

Bachas Porrati

where $\eta^{-24}(q) = \sum_{N=-1}^{\infty} c_b(N) q^N$. This can be viewed as the sum of the Schwinger production rates for each state in the spectrum, of mass $m^2 = 2N + \nu^2$.

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- This seems to support the quantization scheme based on a vacuum, hence the absence of physical states. But physical states do exist classically, how could quantization make them disappear altogether ?

Wick rotation to a rotation orbifold

- Note first that the (future) Milne region $ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + dx_i^2$ cannot be directly Wick-rotated to Euclidean.

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- Rotating $\beta = i\mu$, the Rindler region becomes get indeed an Euclidean metric,

$$ds^2 = dr^2 + \mu^2 r^2 d\eta^2 + (dX^i)^2 = 2\, dZ d\bar{Z} + (dX^i)^2$$

$$Z = X^+ = re^{i\mu\eta}\,, \quad \bar{Z} = -X^- = re^{-i\mu\eta} \qquad (r > 0)$$

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- By the same token, the left Rindler wedge rotates to another copy of the Euclidean plane with the origin removed: the complete analytic continuation of Misner space is therefore

$$\widetilde{R^2\backslash\{0\}}_L / e^{i\mu} \backslash \widetilde{R^2\backslash\{0\}}_R$$

and states of interest are non-normalizable !

D'Appolonio Kiritsis

The one-loop amplitude again

- Recall the (Euclidean ws, Minkowskian target) one-loop amplitude:

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- In addition, there are poles in the bulk of the moduli space, for

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leading to logarithmic divergences $\int d\rho d\bar{\rho}/|\rho-\rho_0|^2 \sim log\epsilon$, analogous to the long strings in AdS_3.

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- In contrast to the open string case, these poles do not yield an imaginary part: the overall cosmological particle production seems to vanish. *This is not to say that there is no particle production at intermediate stages !*

Physical spectrum at low level

- The ground state tachyon

$$|T\rangle = \phi(x^+, x^-)|0_{ex}, k\rangle$$

should satisfy the Virasoro constraint

$$L_0|T\rangle = \left[-\frac{1}{2}\left(a_0^+ a_0^- + a_0^- a_0^+\right) + \frac{1}{2}\nu^2 - 1 + \frac{1}{2}k_i^2\right]|T\rangle$$

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- Level 1 states consist of
$$|A\rangle = \left(-f^+\, a_{-1}^- - f^-\, a_{-1}^+ + f^i\, a_{-1}^i\right)|0_{ex}, k\rangle$$
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- The L_1 Virasoro constraint eliminates one polarization. Despite the non-vanishing two-dimensional mass $k_i^2 - \nu^2$, the spurious state $L_{-1}\phi|0\rangle$ is still physical, eliminating an extra polarization.
- One thus has $D-2$ transverse degrees of freedom, ie a massless gauge boson in D dimensions.

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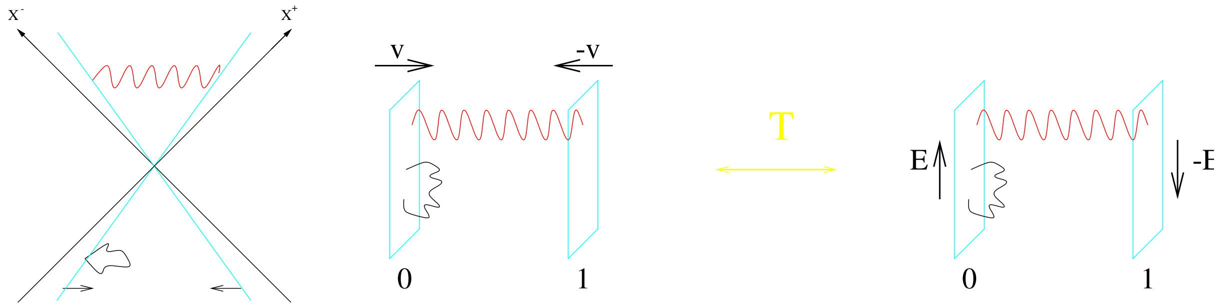
- In particular, the head-on collision of two D-branes has a strong analogy with the Lorentzian closed string orbifold:



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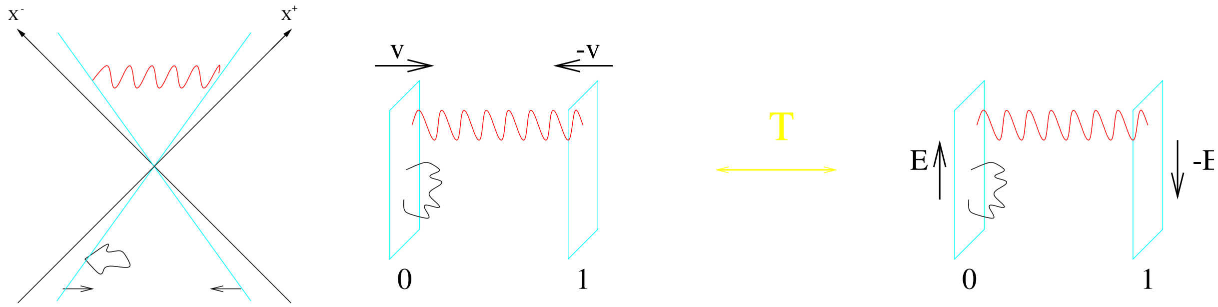

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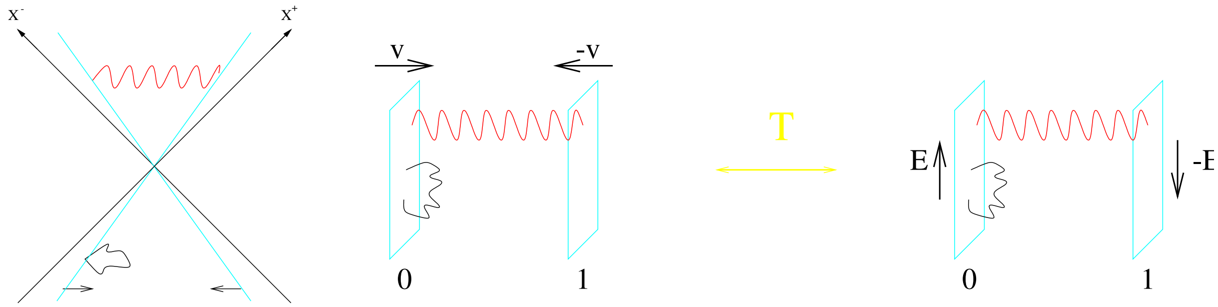

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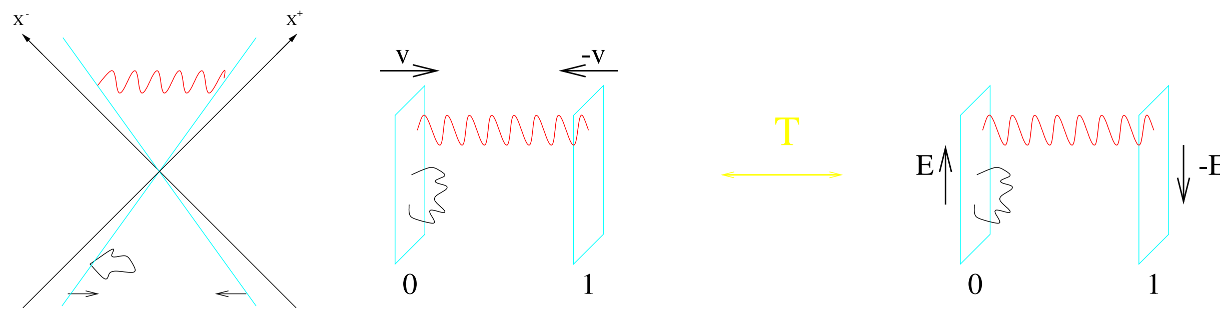

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- Can Schwinger production of twisted closed strings resolve the cosmological singularity of the Lorentzian orbifold ?

The Grant space

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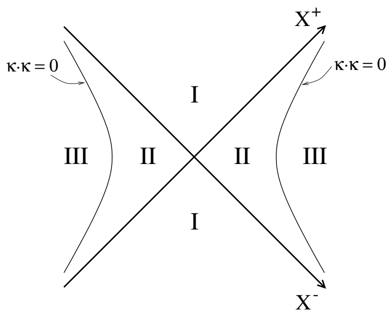
- Defining $Z^{\pm} = X^{\pm} e^{\mp \beta X/R}$, the metric can be written in the Kaluza-Klein form

$$ds^2 = R^2 (dX + A)^2 - 2dZ^+ dZ^- - \frac{E^2}{2R^2} (Z^+ dZ^- - Z^- dZ^+)^2 \,, \quad X \equiv X + 2\pi$$

with radius R and KK electric field

$$R^2 = 1 + 2EZ^+ Z^- \,, \quad dA = \frac{E}{R^4} dZ^+ dZ^- \,, \quad E = \beta / R$$



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- All CTC have to pass into $X^+X^- < -1/(2E)$, hence may be suppressed by excising this region: *orientifold boundary conditions ?*

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