Closed Strings in the Misner Universe

a toy model of a cosmological singularity

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LPTHE and LPTENS, Paris

Durham University

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Talk based on

hep-th/0307280 w/ M. Berkooz

hep-th/0405126 w/ M. Berkooz, and M. Rozali

hep-th/0407216 w/ M. Berkooz, B. Durin and D. Reichmann

slides available from

`http://www.lpthe.jussieu.fr/˜pioline/seminars.html`

Motivational string cosmology

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- Most importantly, inflation does not get rid of the initial singularity. Can string theory evade the usual divergences of perturbative gravity and "no-bounce theorems" ?

Gasperini Veneziano

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- Perturbative string theory requires an Euclidean worldsheet, hence Euclidean target space. Even if a Lorentzian target space may be obtained by analytic continuation, Lorentzian observables may be quite different from their Euclidean counterparts.

Cosmological backgrounds in string theory

- Even before quantum (g_s) corrections, string theory backgrounds undergo classical (α') corrections. Very few examples of cosmological solutions of tree-level string theory are known.

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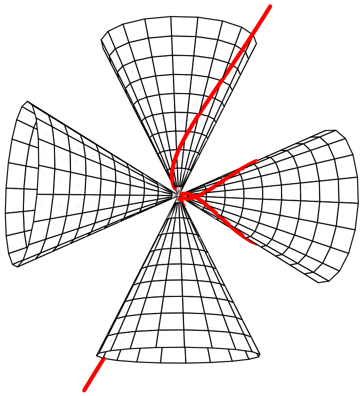

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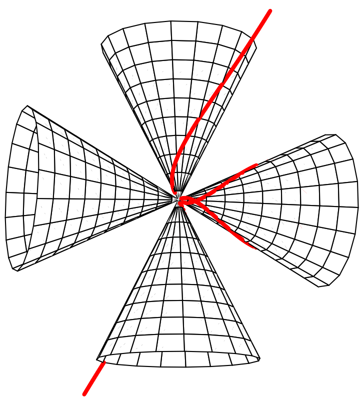


- Our aim will be to understand classical aspects of string propagation in this singular background, and compute tree-level particle/string production rates. A much more ambitious task is to incorporate gravitational backreaction, and determine whether or not the cosmological singularity is resolved.

Outline of the talk

1. Euclidean and Lorentzian orbifolds, and their avatars

   Misner, Taub-NUT, Grant...

2. Untwisted strings in Misner space

   Hiscock, Konkowski; Berkooz Craps Kutasov Rajesh, . . .

3. Twisted strings in Misner space: first pass

   Nekrasov

4. A detour: Open strings in electric fields

   Bachas Porrati; Berkooz BP

5. Twisted strings in Misner space: second pass

   Berkooz BP Rozali

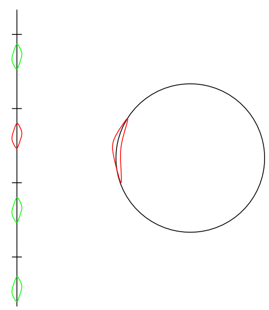
6. Comments on backreaction from winding strings

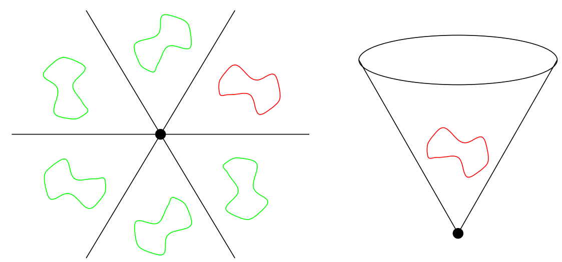
Strings on Euclidean orbifolds - untwisted states

- Well-known examples of orbifolds are the circle, R/Z, and the rotation orbifold R^2/Z_k.

Dixon Harvey Vafa Witten

- The spectrum of the quotient theory contains closed string states of the parent theory which are invariant under G: untwisted states.





Strings on Euclidean orbifolds - twisted states

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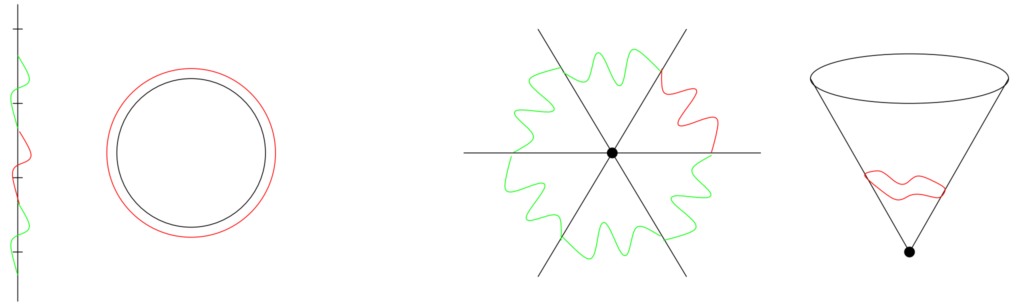
- Modular invariance requires that the spectrum should also include closed strings in the quotient theory which close up to the action of G in the parent theory: twisted states.



- When G acts non-freely, the twisted sector states are localized at the fixed points. They yield new localized degrees of freedom, which ensure the consistency of the background: anomaly free, divergence free, modular invariance...

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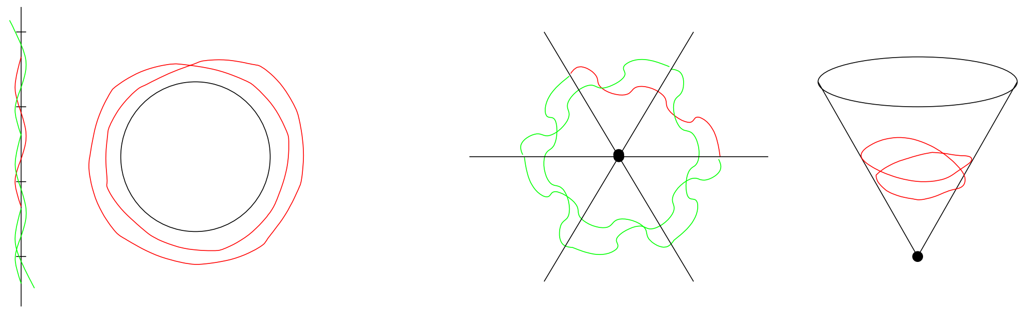

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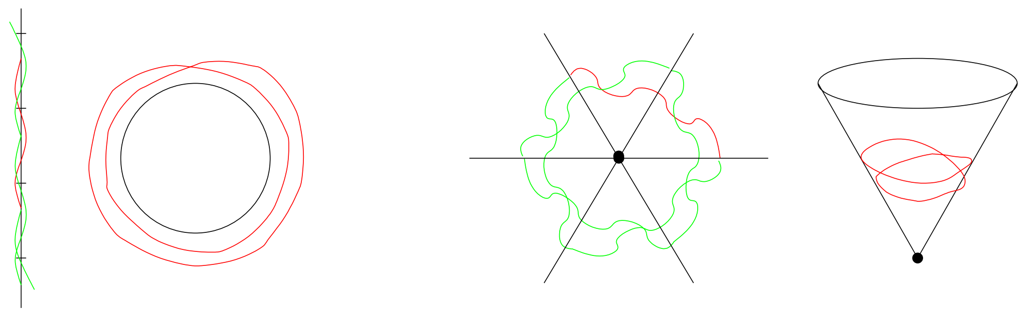

- Additionally, each twisted sector admits excited levels. The ground state can be thought of as a Gaussian wave function centered at the origin.

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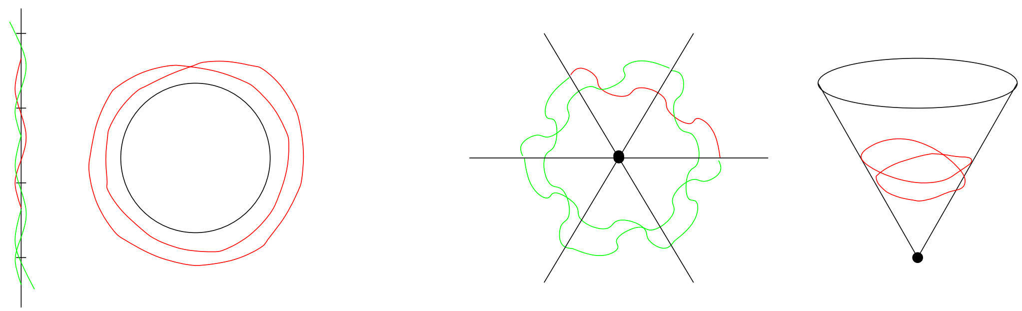

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- The condensation of these twisted states changes the vacuum, and effectively resolves the singularity: $R^2/Z_k \longrightarrow R^2/Z_{k-1} \longrightarrow \ldots$ (tachyon), $R^4/Z_k \longrightarrow$ multi-centered Eguchi-Hanson (massless mode).

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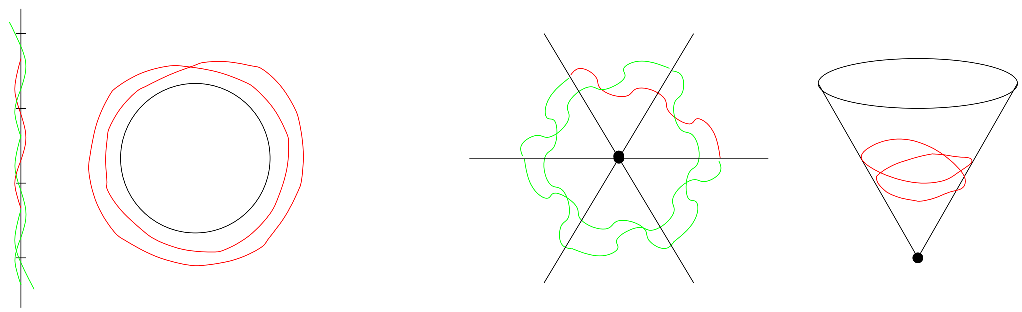

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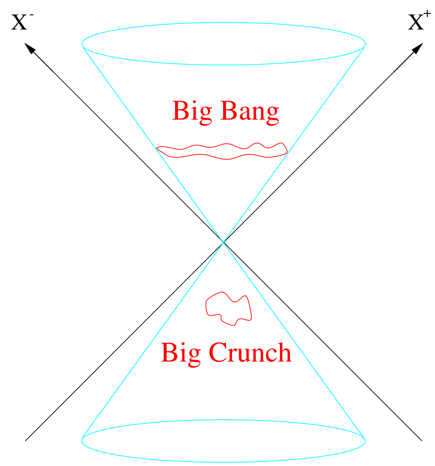
- The Lorentzian orbifold shares features with both examples: an infinite number of winding sectors, and a, non compact, fixed locus.

The Lorentzian orbifold

- One of the (superficially) simplest time-dependent solution in string theory is the quotient of flat Minkowski space by a discrete boost, also known as Misner space (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^\pm \sim e^{\pm 2\pi\beta} X^\pm$$



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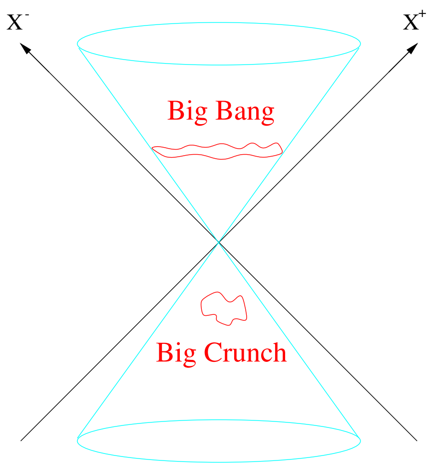

- The future (past) regions $X^+X^- > 0$ describes a cosmological universe often known as the Milne universe (1932), linearly expanding away from a Big Bang singularity (or contracting into a Big Crunch singularity):

$$ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + (dX^i)^2 \ , \quad \theta \equiv \theta + 2\pi \ , \quad X^\pm = T e^{\pm\beta\theta}/\sqrt{2}$$

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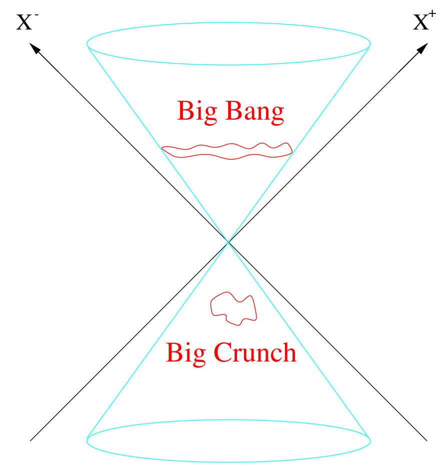

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This is a Kasner-type singularity with zero curvature except at $T = 0$.

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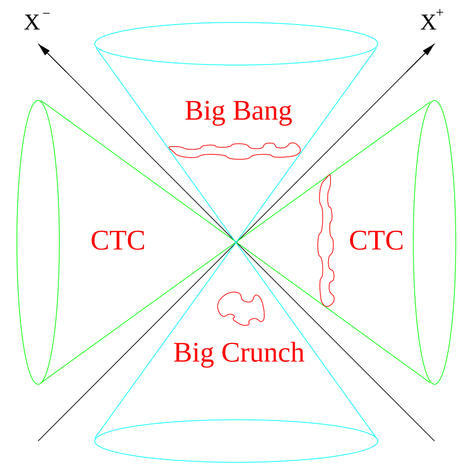

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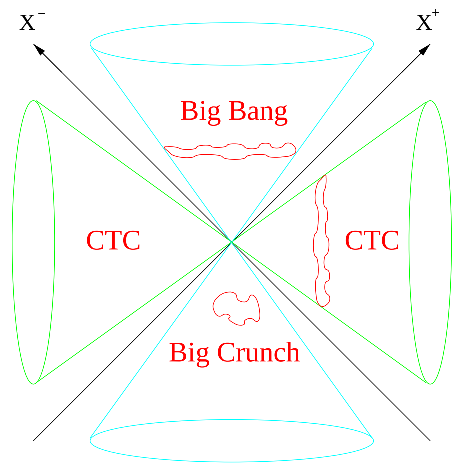

- In addition, the spacelike regions $X^+X^- < 0$ describe two Rindler wedges with compact time, often known as whiskers, leading to closed time-like curves:

$$ds^2 = dr^2 - \beta^2 r^2 d\eta^2 + (dX^i)^2 \quad , \eta \equiv \eta + 2\pi \; , \quad X^\pm = \pm r e^{\pm\beta\eta}/\sqrt{2}$$

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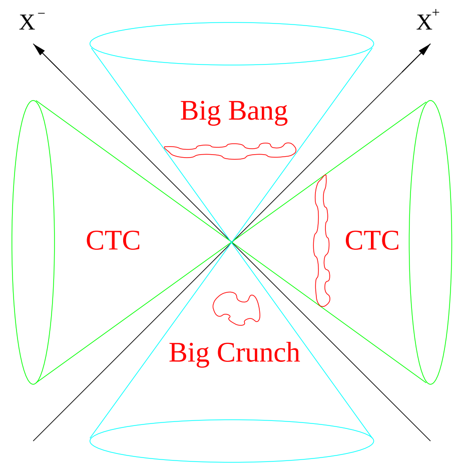

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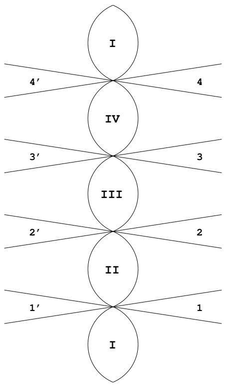
- Finally, the lightcone $X^+X^- = 0$ gives rise to a null, non-Hausdorff locus attached to the singularity.

Close relatives of the Misner Universe

- Misner space was first introduced as a local model of Lorentzian Taub-NUT space:



$$ds^2 = 4l^2 U(t)\sigma_3^2 + 4l\sigma_3 dt + (t^2 + l^2)(\sigma_1^2 + \sigma_2^2)\,, \quad U(t) = -1 + \frac{2mt + l^2}{t^2 + l^2}$$

A bouncing universe, isomorphic to $R^{1,1}/boost \times S^2$ around each singularity.

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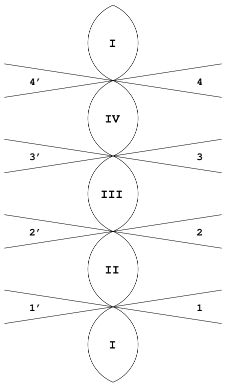

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- A close variant of Misner space is the quotient of flat space by the combination of a discrete boost and a translation on an extra direction, often known as the Grant space:

$$ds^2 = -2dX^+dX^- + dX^2 + (dX^i)^2\,, \quad (X^\pm, X) \sim (e^{\pm 2\pi\beta}X^\pm, X + 2\pi R)$$

This describes the space away from two moving cosmic strings. The cosmological singularity is smoothed out, but regions with CTC remain.

Gott 91, Grant 93; Cornalba, Costa, Kounnas

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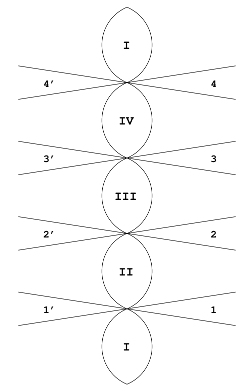

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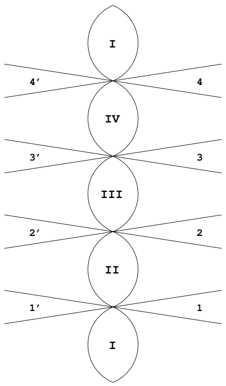
- The Misner geometry arose again more recently as the M-theory lift of a simple (ekpyrotic) cosmological solution of Einstein-dilaton gravity with no potential.

Khoury Ovrut Seiberg Steinhard Turok

Close relatives of the Misner Universe (cont)

- The gauged WZW model $Sl(2) \times Sl(2)/U(1) \times U(1)$ describes a bouncing 4-dimensional Universe, with singularities analogous to the Lorentzian orbifold.

Nappi Witten; Elitzur Giveon Kutasov Rabinovici



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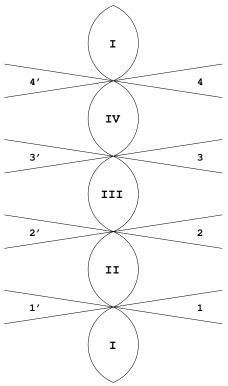

- The gauged WZW model $Sl(2)/U(1)$ at negative level orbifolded by a boost J describes two parallel Universes with a curvature and a Milne singularity, and compact whiskers.

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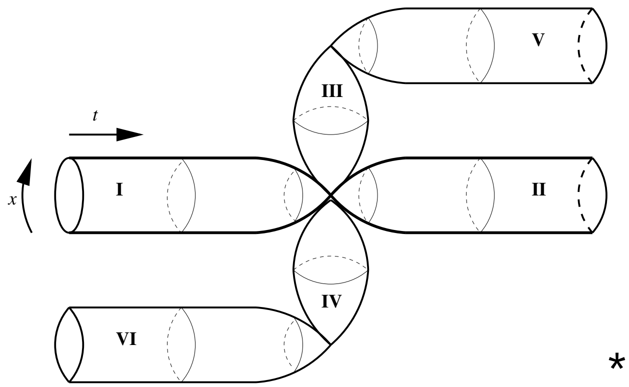

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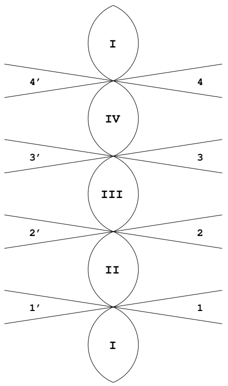

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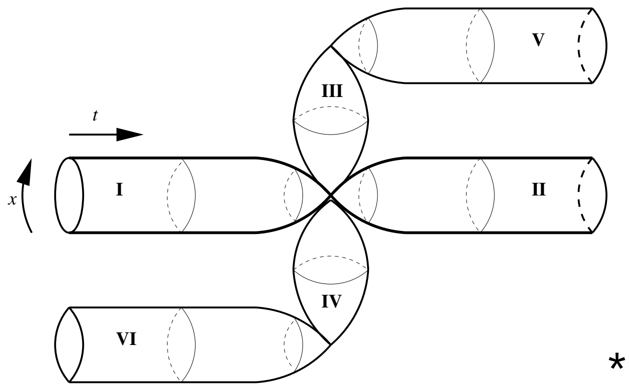

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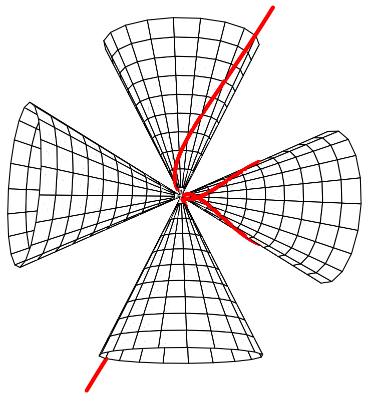
- The Lorentzian orientifold $IIB/[(-)^F boost]/[\Omega(-)^{F_L}]$ was also recently argued to describe orientifolds of non-supersymmetric strings with non-vanishing Neveu-Schwarz tadpoles.

Dudas Mourad Timirgaziu

Classical particles in the Misner Universe

- Classical particles propagate along straight lines on the covering space:

$$\begin{aligned} X^{\pm} &= x_0^{\pm} + p^{\pm}\tau \\ 2p^+p^- &= M^2 \\ j &= p^+x_0^- - p^-x_0^+ \end{aligned}$$



- As the particle approaches the singularity from the past, it starts spinning faster and faster, $\theta \sim \log |T|$, implying large gravitational backreaction.
- In the Rindler wedges, the particle winds infinitely many times around the time direction: at any fixed Rindler time, there is an infinity of copies of the particle, each with energy j: the total Rindler energy is infinite.

Quantum particles in the Misner Universe

- Quantum mechanically, the radial motion, for fixed boost momentum j, is governed by a Liouville-type potential:

$$\begin{aligned} \frac{1}{r}\partial_r r \partial_r + \frac{j^2}{r^2} &= M^2 , \qquad r = e^y , \qquad V(y) = -j^2 + M^2 e^{2y} \equiv 0 \\ -\frac{1}{T}\partial_T \partial_T - \frac{j^2}{T^2} &= M^2 , \qquad T = e^x , \qquad V(x) = -j^2 - M^2 e^{2x} \equiv 0 \end{aligned}$$

  The singularity is at infinite distance in the canonically normalized x or y coordinate.
- Wave functions of boost momentum j and spin s can be expressed as superpositions of plane waves on the covering space ($s =$spin)

$$f_{j,M^2,s}(x^+, x^-) = \int_{-\infty}^{\infty} dv \exp\left(ik^+ X^- e^{-2\pi\beta v} + ik^- X^+ e^{2\pi\beta v} + ik_i X^i + ivj + vs \right)$$

- They can be defined globally by continuing across the horizons. The *in* and *out* states defined at $T = -\infty$ and $T = +\infty$ are identical, hence no overall particle production.

Tree-level scattering of untwisted states

- As in standard orbifold constructions, part of the spectrum consists of closed strings of the parent theory, invariant under the orbifold projection. These topologically trivial states behave at low energy just like ordinary point particles.
- Tree-level scattering amplitudes of untwisted sector states can be computed from those in flat space by the inheritance principle,

$$\langle V(j_1,k_1)\dots V(j_n,k_n)\rangle_{Misner} = \int dv_1\dots dv_n\; e^{i(j_1 v_1+\dots+j_n v_n)}$$
$$\langle V(e^{\beta v_1}k_1^+, e^{-\beta v_1}k_1^-, k_1^i)\dots V(e^{\beta v_n}k_n^+, e^{-\beta v_n}k_n^-, k_n^i)\rangle_{Minkowski}$$

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$$\langle V(e^{\beta v_1} k_1^+, e^{-\beta v_1} k_1^-, k_1^i) \ldots V(e^{\beta v_n} k_n^+, e^{-\beta v_n} k_n^-, k_n^i)\rangle_{Minkowski}$$

- String amplitudes are suppressed in the high energy regime (fixed s/t, s/u). However, in the deep inelastic regime, ($s \to \infty$, t fixed), they exhibit Regge behavior $A \sim s^t$, as if strings acquired a size $\sqrt{\ln s}$:

$$langle V(j_1, k_1) \ldots V(j_n, k_n)\rangle_{Misner} \propto \int dv \; v^{-\frac{1}{2}(k_1^i - k_3^i)^2 + i(j_2 - j_4)}$$

which diverges if $(k_1^i - k_3^i)^2 \leq 2$, as a result of large graviton exchange near the cosmological singularity.

Berkooz Craps Rajesh Kutasov

Quantum fluctuations in field theory

- In the Minkowski vacuum (inherited from the covering space), the renormalized propagator can be obtained as a sum over images,

$$
\begin{aligned}
G(x;x') \quad = \quad & \sum_{l=-\infty, l\neq 0}^{\infty} \int_0^{\infty} d\tau \int dp^{\mu} \\
& \exp\left(-ip^{-}(x^{+} - e^{2\pi\beta l} x^{+'}) - ip^{+}(x^{-} - e^{2\pi\beta l} x^{-'}) - ip^{i}(x^{i} - x^{i'})\right)
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- The one-loop stress-energy tensor follows from the propagator at coinciding points $G(x,x)$, e.g for a free scalar field in 4D,

$$\langle T_{ab}\rangle = \lim_{x\to x'}\left[(1-2\xi)\nabla_a\nabla'_b - 2\xi\nabla_a\nabla_b + (2\xi - \frac{1}{2})g_{ab}\nabla_c\nabla'^c\right]G(x,x')$$

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This leads to a divergent quantum backreaction (worse if the spin $|s| > 1$):

$$\langle T^\nu_\mu\rangle = \frac{K}{12\pi^2}T^{-4}\text{diag}(1,-3,1,1)\ , \quad K = \sum_{l=1}^{\infty}\cosh(2\pi\beta l s)\frac{2+\cosh 2\pi l\beta}{[\cosh 2\pi l\beta - 1]^2}$$

Hiscock Konkowski 82

One-loop vacuum amplitude in field and string theory

- On the other hand, in string theory $\langle T_\mu^\nu \rangle(x)$ is an off-shell quantity, and only its integral over space-time is well defined:

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$$\theta_1(v;\rho) = 2q^{1/8} \sin \pi v \prod_{n=1}^{\infty} (1-e^{2\pi i v} q^n)(1-q^n)(1-e^{-2\pi i v} q^n)\ , \quad q = e^{2\pi i \rho}$$

Nekrasov; Cornalba Costa; Berkooz BP Rozali

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- The existence of Regge trajectories with arbitrary high spin implies new (log) divergences in the bulk of the moduli space which resemble long string poles in AdS_3.

Closed string in Misner space - twisted sectors

- In addition, there is an infinite set of twisted sectors, corresponding to strings on the covering space that close up to the action of the orbifold group:

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau) \ , \quad \nu = 2\pi w \beta$$

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$$X^{\pm} = \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha_n^{\pm} \, e^{-i(n \pm i\nu)(\tau - \sigma)} + \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \mp i\nu)^{-1} \tilde{\alpha}_n^{\pm} \, e^{-i(n \mp i\nu)(\tau + \sigma)}$$

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- We will focus on the quasi zero-mode sector, which consists of two commuting pairs of real (i.e. hermitian) canonically conjugate operators,

$$[\alpha_0^+, \alpha_0^-] = -i\nu \ , \quad [\tilde{\alpha}_0^+, \tilde{\alpha}_0^-] = i\nu$$

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated by half of them, say $\alpha^{\pm}_{n>0}\ ,\ \tilde{\alpha}^{\pm}_{n>0}\ ,\quad \alpha^{-}_{0}\ ,\ \tilde{\alpha}^{+}_{0}$

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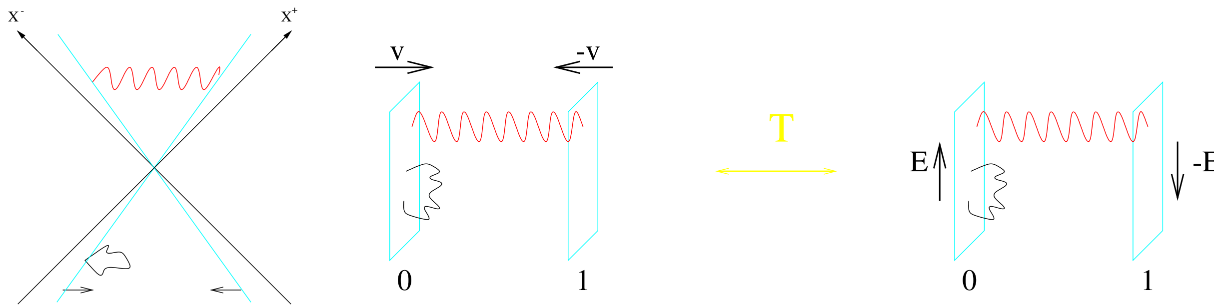
However, this scheme overlooks the fact that α^{+}_{0} and α^{-}_{0} are self-hermitian !

A detour via Open strings in electric field

- A very similar puzzle is faced in the case of colliding D-branes, or in the T-dual process of charged open strings in a constant electric field:



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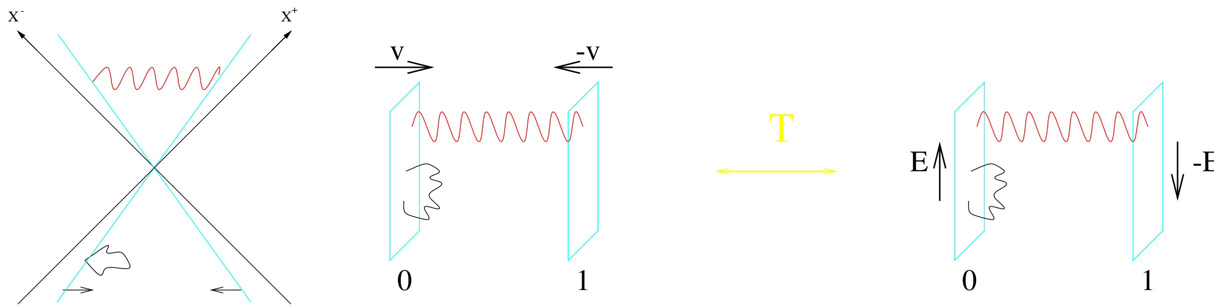

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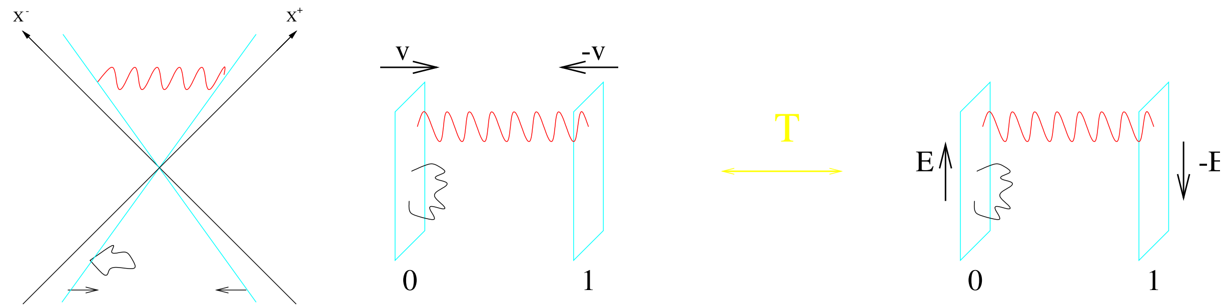

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- This reproduces (one half of) the spectrum of Closed strings in Misner space upon identifying $\nu = w\beta$. The large winding number limit $w \to \infty$ amounts to a near critical electric field $E \to 1/\pi$.
- In particular, the open string zero-modes describe the motion of a charged particle in an electric field, and have a structure isomorphic to the closed string case.

Charged particle and open string zero-modes

- Recall the first quantized charged particle in an electric field:

$$L = \frac{1}{2}m\left(-2\partial_\tau X^+\partial_\tau X^- + (\partial_\tau X^i)^2\right) + \frac{\nu}{2}\left(X^+\partial_\tau X^- - X^-\partial_\tau X^+\right)$$

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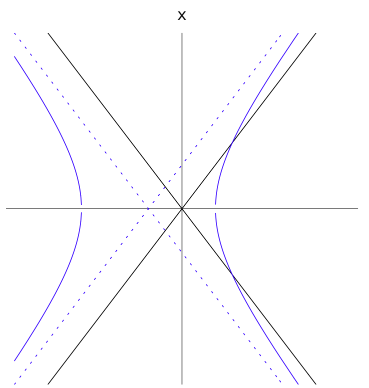
- Classical trajectories are hyperbolas centered at an arbitrary point,

$$X^\pm = x_0^\pm \pm \frac{1}{\nu}a_0^\pm e^{\pm\nu\tau}$$

$P^\pm = \pm\nu x_0^\pm$ is the conserved linear momentum, and $a_0^\pm$ the velocity.



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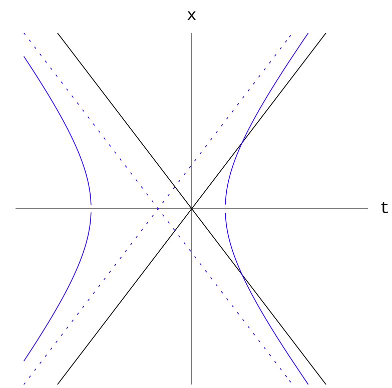

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$$[a_0^+, a_0^-] = -i\nu \ , \quad [x_0^+, x_0^-] = -\frac{i}{\nu} \ , \quad L_0 = -a_0^+ a_0^- + \frac{i\nu}{2} + \text{excited.}$$

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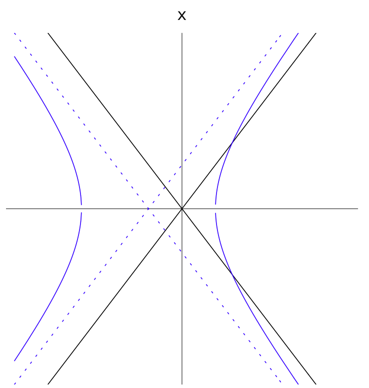

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- Upon quantizing $a_0^\pm$ as creation/annihilation operators in a Fock space, electrons and positrons would have no physical state...

Charged particle and Klein-Gordon equation

- Quantum mechanically, one represents the canonical momenta as derivatives, $\pi^{\pm} = i\partial/\partial x^{\mp}$, hence $a_0^{\pm}, x_0^{\pm}$ as covariant derivatives

$$a_0^{\pm} = i\partial_{\mp} \pm \frac{\nu}{2} x^{\pm} \,, \quad x_0^{\pm} = \mp \frac{1}{\nu} \left(i\partial_{\mp} \mp \frac{\nu}{2} x^{\pm} \right)$$

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$$\langle f|g\rangle = \int dx^+ dx^- f^*(x^+, x^-)\; g(x^+, x^-)$$

- The zero-mode piece of L_0, including the bothersome $\frac{i\nu}{2}$,

$$L_0^{(0)} = -a_0^+ a_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\nabla^+\nabla^- + \nabla^-\nabla^+)$$

  is just the Klein-Gordon operator of a particle of charge ν, and has well-behaved eigenmodes $L_0 = -m^2$ for any $m^2 > 0$.

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^\pm = (P \pm Q)/\sqrt{2}$, the Klein-Gordon operator can be rewritten as an inverted harmonic oscillator:

$$M^2 = -\frac{1}{2}(P^2 - Q^2) \; , \qquad [P, Q] = i$$

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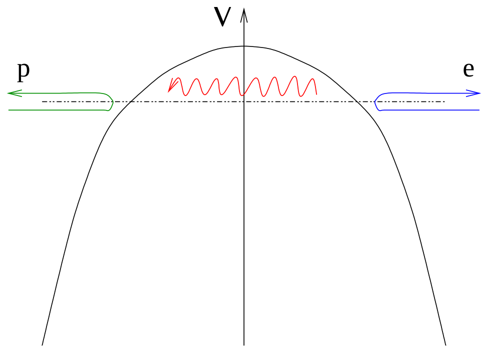
$$e^- \to (1 + \eta)\, e^- + \eta\, e^+$$



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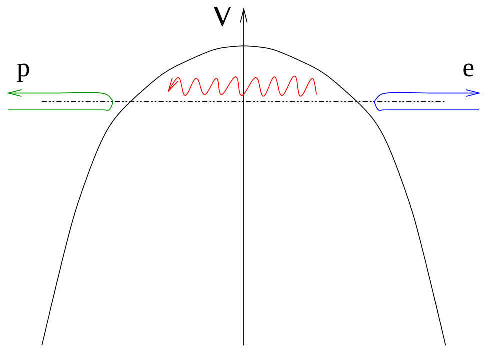

- The tunneling rate can be computed semiclassically, $\eta \sim \exp\left(-2\oint PdQ\right) = e^{-\pi M^2/\nu}$ which reproduces the Schwinger pair creation rate.

Brezin Itzykson; Brout Massar Parentani Spindel

Lorentzian vs Euclidean states

- The Wick rotation $X^0 \to -iX^0$, $\nu \to i\nu$ turns the electric field (Schwinger) problem in $R^{1,1}$ into the magnetic field (Landau) problem in R^2.

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- The correct Lorentzian spectrum of the electric problem consists of a continuous delta-normalizable spectrum with real mass squared.
- The zero-mode contribution to the one-loop amplitude can be interpreted either way,

$$\frac{1}{2i\sin(\nu t/2)} = \sum_{n=1}^{\infty} e^{-i(n+\frac{1}{2})\nu t} = \int dM^2 \rho(M^2) e^{-M^2 t/2}$$

The density of states is semi-classically from the reflection phase,

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Lorentzian vs Euclidean states

- The Wick rotation $X^0 \to -iX^0$, $\nu \to i\nu$ turns the electric field (Schwinger) problem in $R^{1,1}$ into the magnetic field (Landau) problem in R^2.
- The real discrete normalizable spectrum of the Landau problem rotates to a with discrete spectrum with imaginary energy in the Schwinger problem.
- The correct Lorentzian spectrum of the electric problem consists of a continuous delta-normalizable spectrum with real mass squared.
- The zero-mode contribution to the one-loop amplitude can be interpreted either way,

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- The physical spectrum of the charged open string can be explicitely worked out, and is free of ghosts: a tachyon at level 0, a transverse gauge boson at level 1, . . .

Charged particle in Rindler space

- For applications to the Milne universe, one should diagonalize the boost momentum J, ie consider an accelerated observer.

Gabriel Spindel; Mottola Cooper

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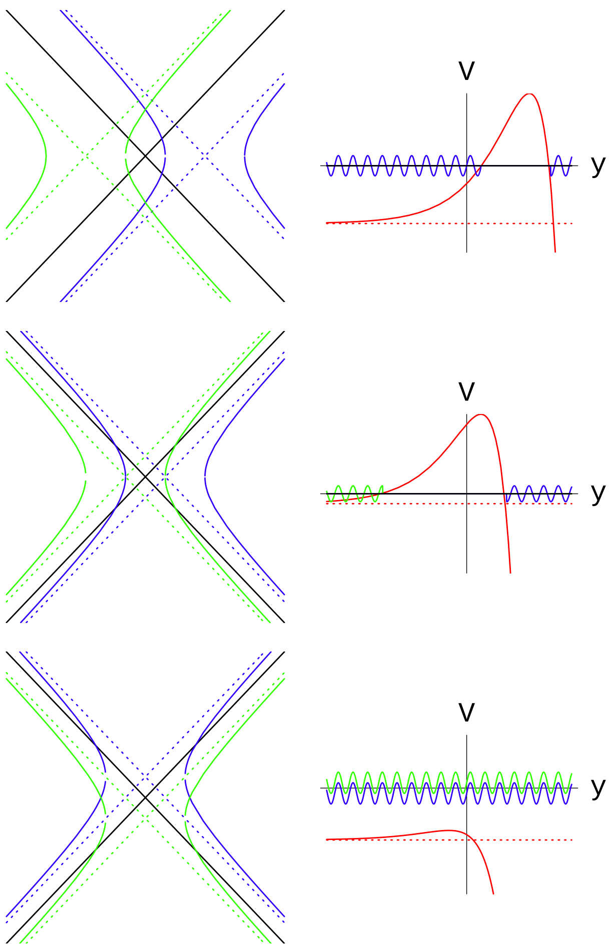

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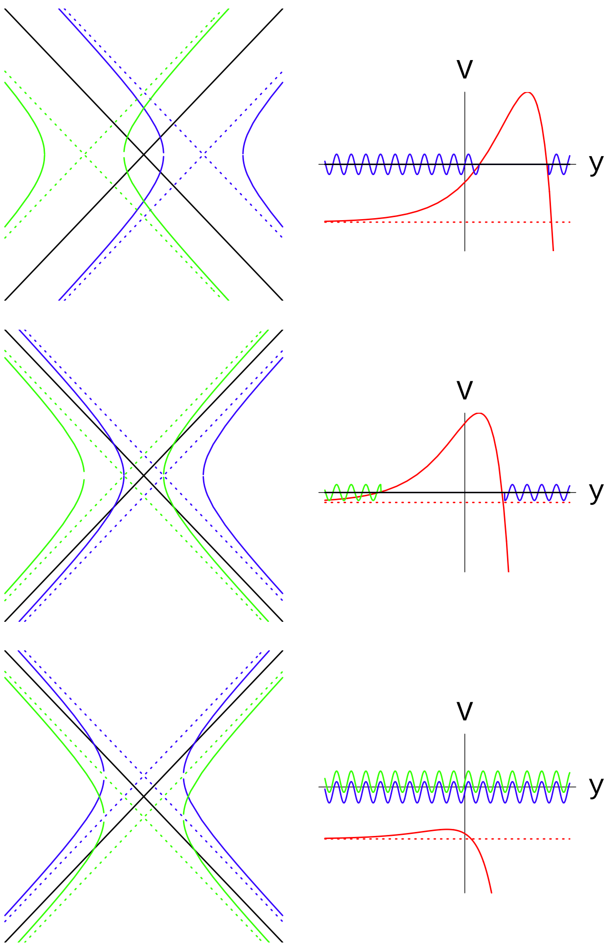

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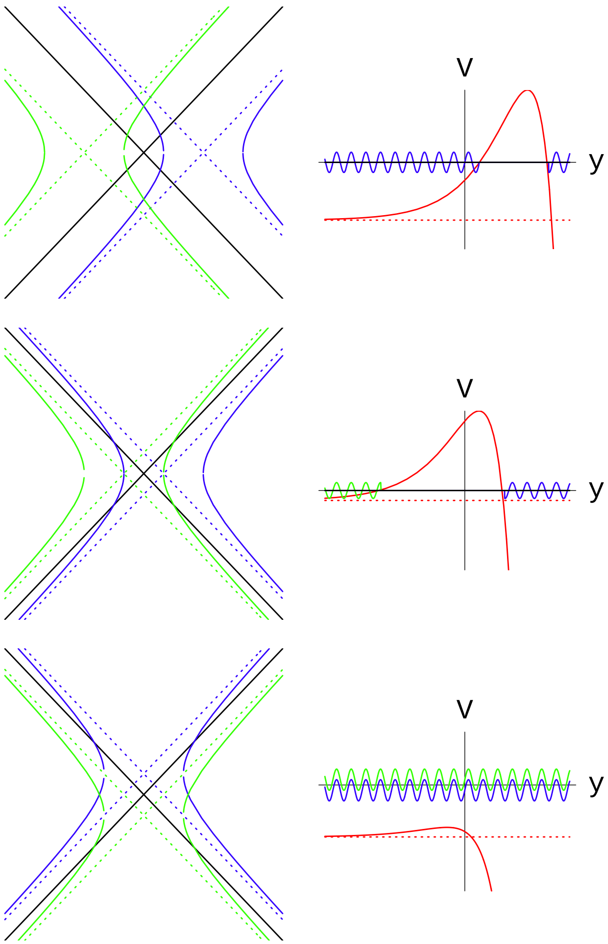

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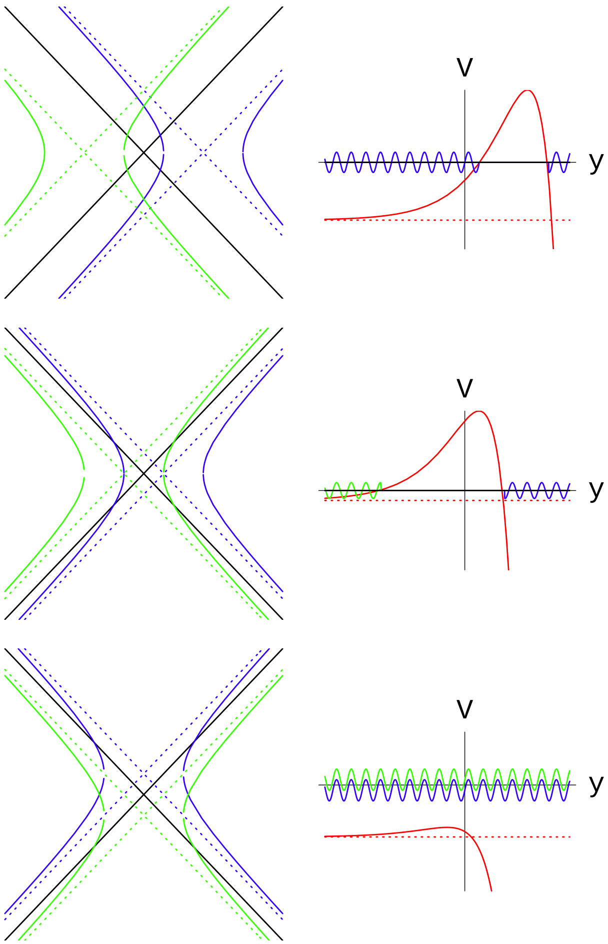

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- If $j > M^2/(2\nu)$, the electron branches cross the horizons. regions. There is no tunneling, but partial reflection amounts to a combination of Schwinger and Hawking emission.

Rindler modes

- Incoming modes from Rindler infinity I_R^- read, in terms of parabolic cylinder functions:

$$\mathcal{V}^j_{in,R} = e^{-ij\eta} r^{-1} M_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),-\frac{ij}{2}}(i\nu r^2/2)$$

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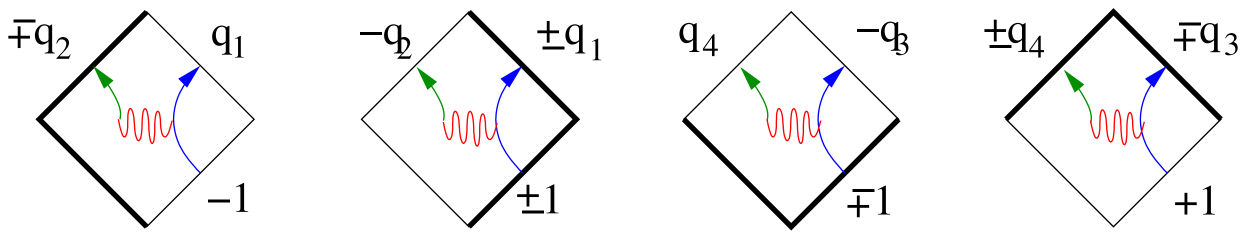

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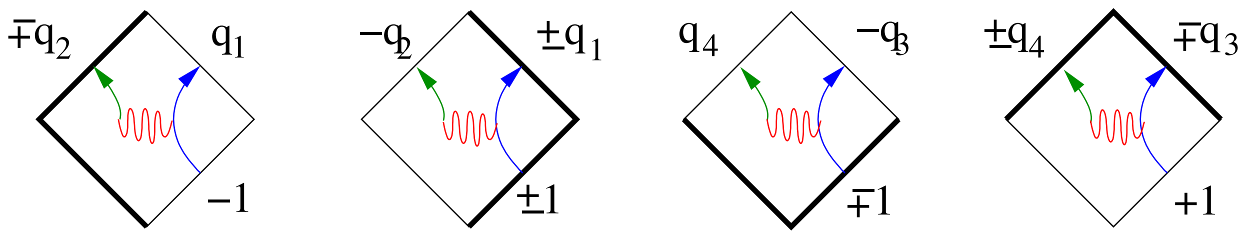

- The reflection coefficients can be computed ($q_1 = 1 - q_2$, $q_3 = q_4 + 1$):

$$q_2 = e^{-\frac{\pi M^2}{2\nu}} \frac{|\sinh \pi j|}{\cosh\left[\pi\left(j - \frac{M^2}{2\nu}\right)\right]}\,, \quad q_4 = e^{-\frac{\pi M^2}{2\nu}} \frac{\cosh\left[\pi\left(j - \frac{M^2}{2\nu}\right)\right]}{|\sinh \pi j|}$$

Global Charged Unruh Modes

- Global modes may be defined by patching together Rindler modes, ie by analytic continuation across the horizons. Unruh modes are those which are superposition of positive energy Minkowski modes,

$$\Omega^j_{in,+} = \mathcal{V}^j_{in,P} = (-i\nu X^+X^-)[X^+/X^-]^{-ij/2} W_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}$$

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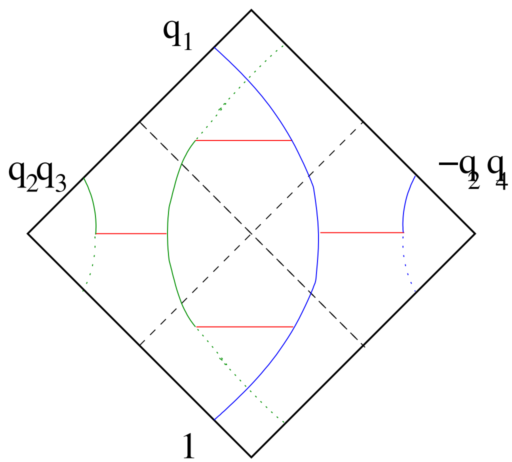
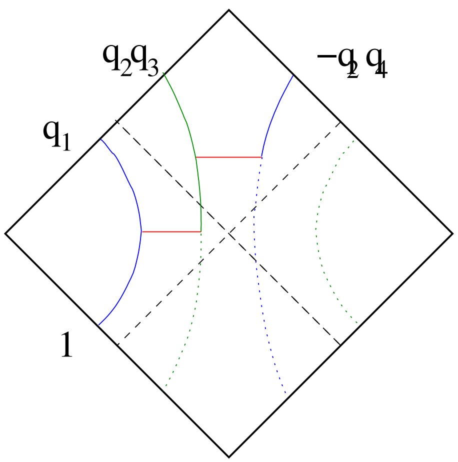
- There are two types of Unruh modes, involving 2 or 4 tunelling events:





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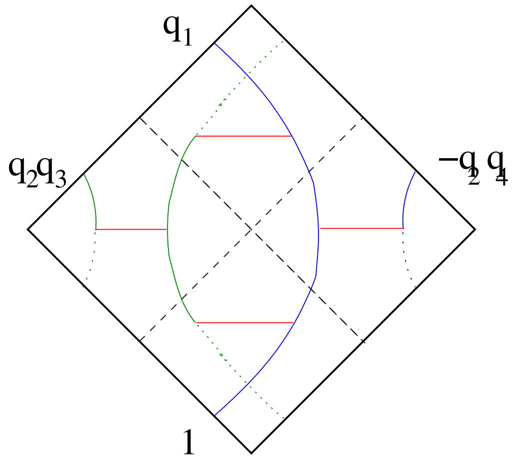

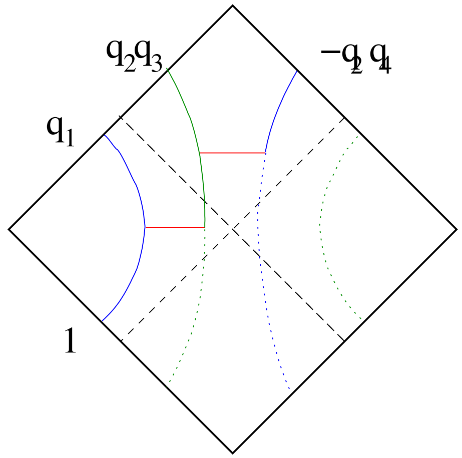

- Any state in Minkowski space can be represented as a state in the tensor product of the Hilbert spaces of the left and right Rindler patches.

Closed string zero-modes

- Let us reanalyze the classical solutions for the closed string zero modes

$$X^{\pm}(\tau,\sigma) = e^{\mp\nu\sigma}\left[\pm\frac{1}{2\nu}\alpha_0^{\pm}e^{\pm\nu\tau} \mp \frac{1}{2\nu}\tilde{\alpha}_0^{\pm}e^{\mp\nu\tau}\right] \ , \quad \alpha_0^{\pm}, \tilde{\alpha}_0^{\pm} \in R$$

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- Up to a shift of τ and σ, the physical state conditions require

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- The behavior at early/late proper time now depends on $\epsilon\tilde{\epsilon}$: For $\epsilon\tilde{\epsilon} = 1$, the string begins/ends in the Milne regions. For $\epsilon\tilde{\epsilon} = -1$, the string begins/ends in the Rindler regions.

Short and long strings

Choosing $j = 0$ for simplicity, we have two very different types of solutions:

- $\epsilon = 1$, $\tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma, \tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma} \ , \quad T = \frac{M}{\nu} \sinh(\nu\tau) \ , \quad \theta = \nu\sigma$$

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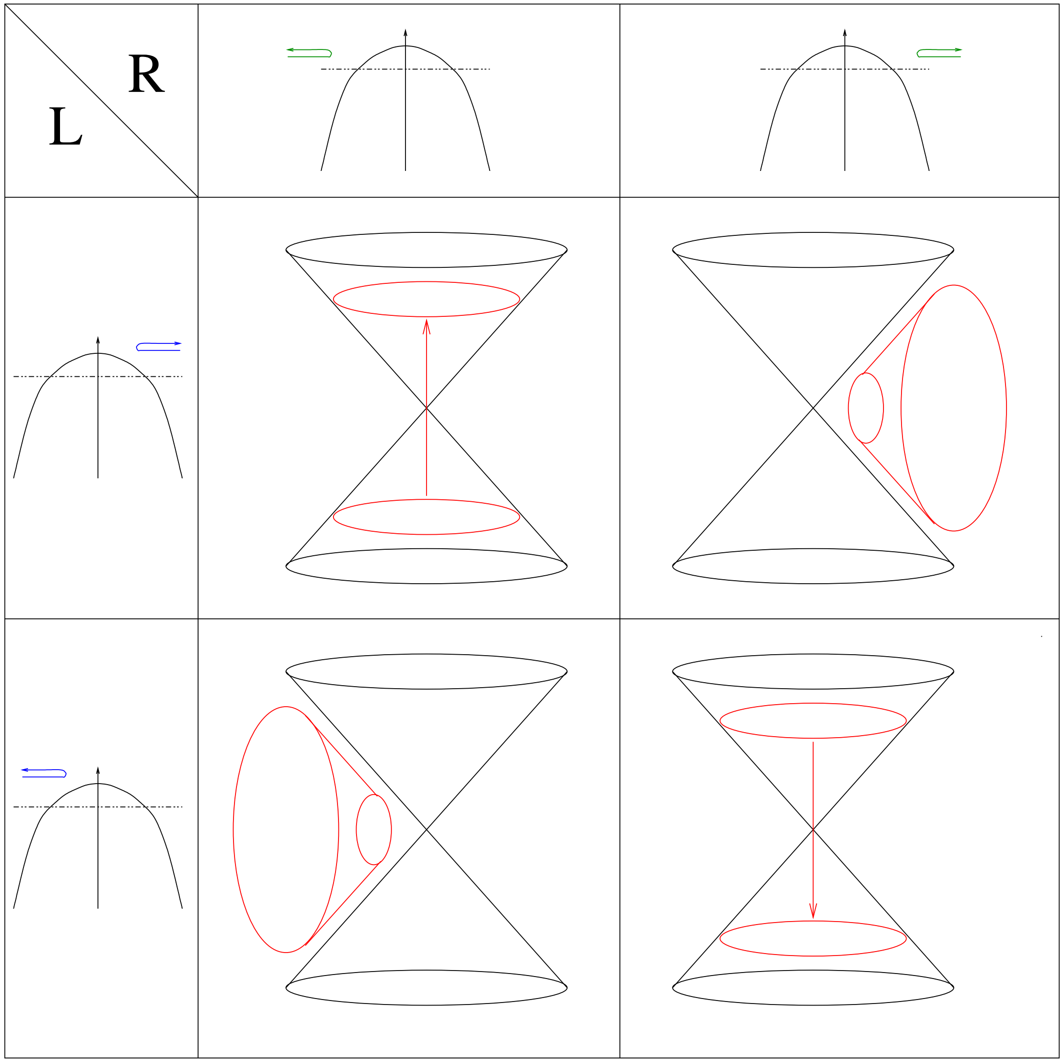
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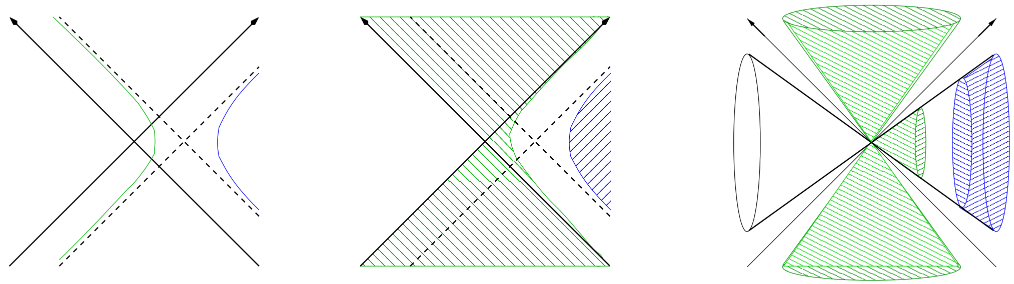

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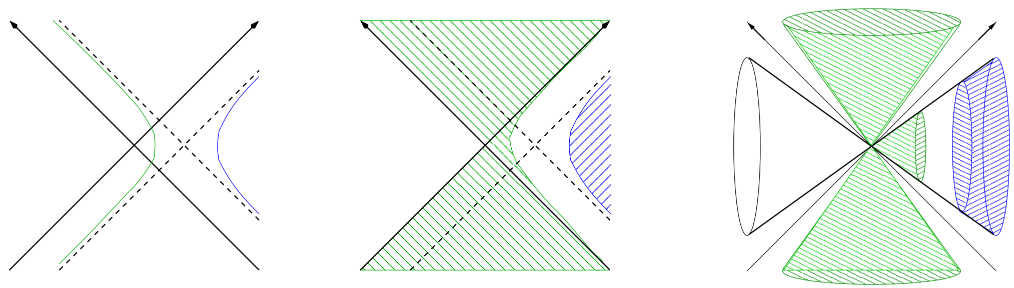

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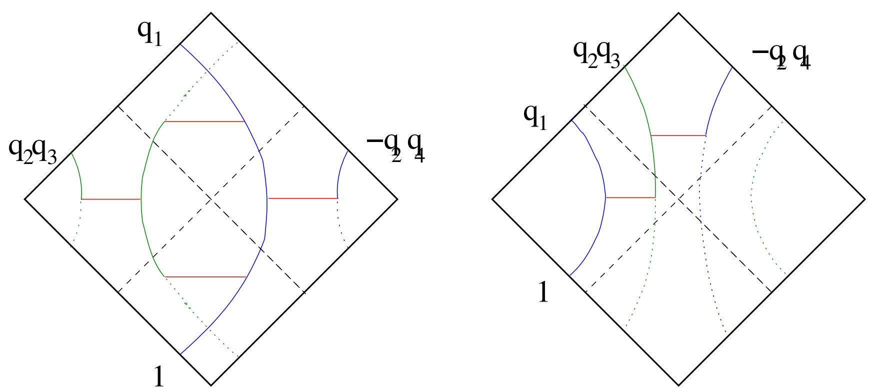

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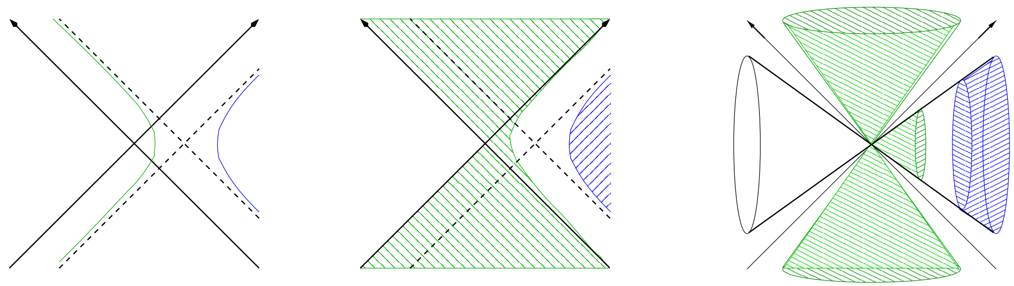
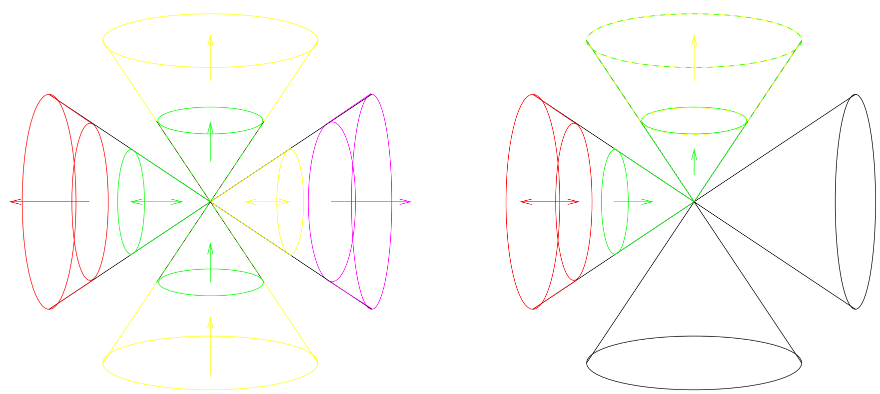


- The open string global wave functions are also closed string wave functions. . .



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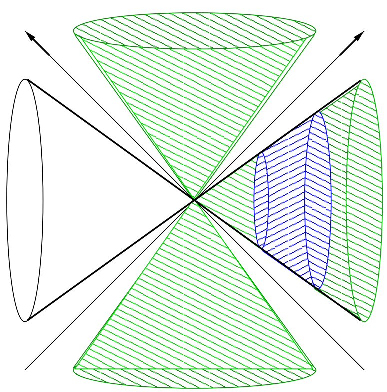
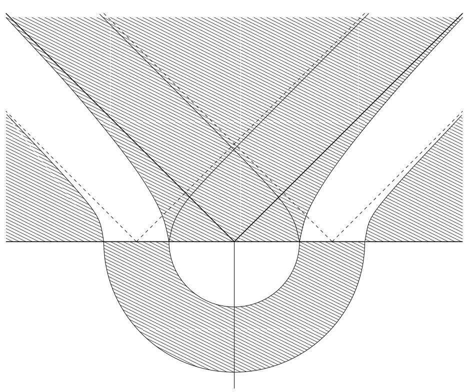
- Similarly, in the closed string problem, tunneling under the barrier corresponds to induced pair production of winding strings.
- Spontaneous pair production of winding strings can be described by cutting open a periodic trajectory, either in imaginary proper time, or in the Euclidean rotation orbifold:





A word on Quantization in the Rindler patch

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$$w(r) = \frac{4\nu^2 r^3 \text{sgn}\,(\nu)}{\sqrt{(M^2 + \tilde{M}^2 - 4\nu^2 r^2)^2 - 4M^2\tilde{M}^2}}$$

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- The spectrum is thus unbounded from below (and above). However, the periodicity of time may render this issue moot.

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- Consider a general Kasner ansatz $ds^2 = -dt^2 + \sum_{i=1}^{D} a_i^2(t) dx_i^2$, $T_\mu^\nu = \text{diag}(-\rho, p_i)$
- Einstein's equations can be written in terms of $H_i = \dot{a}_i / a_i$ as

$$H_i' = -H_i \left(\sum_{j=1}^{d} H_j \right) + p_i + \frac{1}{D-1} \left(\rho - \sum_{j=1}^{d} p_i \right)$$

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- A bounce in dimension i requires $H_i' > 0$ at the point where $H_i = 0$, i.e.

$$(D-2)p_i + \rho \geq \sum_{j \neq i} p_j$$

The most efficient solution is a gas of scalar momentum states, with $p = \rho$: provides enough pressure for the bounce.

Effective gravity analysis (cont.)

- However, consider fundamental strings wrapped around dimension i,

$$\rho = \frac{T}{V}\,, \quad p_i = -\rho\,, \quad p_{j\neq i} = 0\,, \quad V = \prod_{j\neq i} a_j \quad \Rightarrow D \leq 3$$

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Brandenberger Vafa; Tseytlin Vafa

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- We assumed a constant number of wound strings: one should incorporate the dependence of the production rate on the Hubble parameters.

Effective gravity analysis (cont.)

- Einstein's equations imply that the quantity $\mu = \left(\frac{H_k}{H_i} - 1\right) / \left(\frac{H_j}{H_i} - \frac{3}{4-D}\right)$ is constant.

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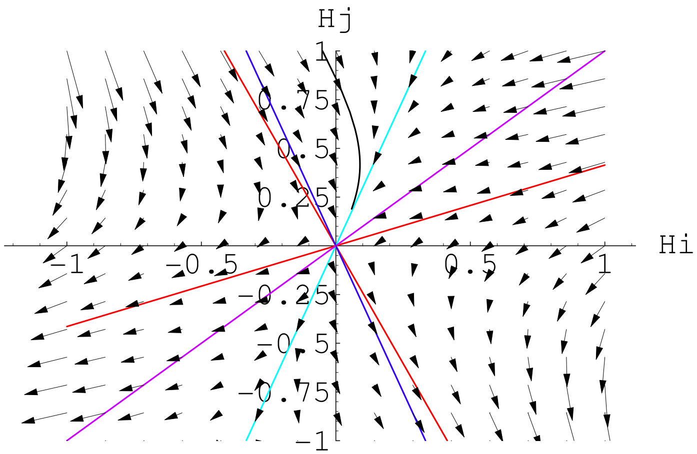
A bounce for direction i in units of the eleven-dimensional frame therefore takes place for any initial condition such that $2\mu + D - 3 > 0$ and $2 < D < 4$.



Conformal perturbation

- Rather than computing the backreaction from quantum production of (squeezed) winding strings, one may consider deforming the orbifold CFT with a marginal twist field, i.e. adding a coherent superposition of winding strings:

$$S_\lambda = \int d^2\sigma \, \partial X^+ \bar{\partial} X^- + \lambda_{-w} V_{+w} + \lambda_{+w} V_{-w}$$

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- While this deformation is marginal at leading order, it implies a one-point function for untwisted fields

$$\langle e^{ikX} \rangle_\lambda \sim \lambda_w \lambda_{-w} \langle w | e^{ikX} | - w \rangle \ ,$$

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  which can be cancelled by deformating S at order λ^2 by an untwisted field: this is the untwisted field classically sourced by the winding string $V_{\pm w}$.
- In addition, the winding string also sources twisted states whose winding number is a multiple of w:

$$\langle V_{-2w} \rangle_\lambda \sim \lambda_w \lambda_w \langle w | V_{-2w} | w \rangle \ ,$$

The size of twisted states

- The scattering amplitude of N untwisted states off one winding string can be computed by Hamiltonian quantization on the cylinder with twisted boundary conditions.
- As in flat space, the off-shell form factor is formally zero due to infinite zero-point fluctuations,

$$\langle -w| e^{ikX}(z,\bar{z}) |w\rangle = \exp\left(-k^+ k^- \Delta\right) \ , \quad \Delta \equiv \infty$$

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- At large ν, $\Delta(\nu) \sim 2 \log \nu$, which indicates that the winding string grows to a size $\sqrt{\log w}$, T-dual to the Regge growth a high energy.

Berkooz Durin BP Reichmann

Classical backreaction

- The untwisted fields sourced by a twisted state with wave function $f_(x^+, x^-)$ are then given by the zero-mode overlap

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- Three-point functions of three twisted fields cannot be computed in Hamiltonian formalism, but can be obtained by analytic continuation of the amplitude for $p^+ \neq 0$ strings in the Nappi-Witten pp-wave:

D'Appollonio, Kiritsis; Cheung Freidel Savvidy

$$\int dx_1^\pm dx_2^\pm \left[f_1(x_1^\pm) f_2(x_2^\pm)\right]^* \exp\left[(x_1^+ - x_2^+)(x_1^- - x_2^-)\Xi(\nu_1,\nu_2)\right] \; f_3\left(\frac{\nu_1 x_1^\pm + \nu_2 x_2^\pm}{\nu_1+\nu_2}\right)$$

where the size of the non-locality is given by the (real) ratio

$$\Xi(\nu_1,\nu_2) = -i\frac{1 - \frac{i\nu_3}{\nu_1\nu_2}\frac{\gamma(i\nu_3)}{\gamma(i\nu_1)\gamma(i\nu_2)}}{1 + \frac{i\nu_3}{\nu_1\nu_2}\frac{\gamma(i\nu_3)}{\gamma(i\nu_1)\gamma(i\nu_2)}}\,, \qquad \gamma(x) = \frac{\Gamma(x)}{\Gamma(1-x)}$$

Note that the non-locality scale $1/\sqrt{\Xi}$ diverges when $\nu_1\nu_2\gamma(i\nu_1)\gamma(i\nu_2) = i\nu_3\gamma(i\nu_3)$.

Conclusions

We discussed closed strings in a toy model of a cosmological singularity. However, some of the features we uncovered should carry over to more general geometries:

- Winding string production can be understood semi-classically as tunneling under the barrier in regions with compact time, or scattering over the barrier in cosmological regions. In general, it can be computed as a tree-level two-point function in an appropriate basis depending on the choice of vacuum.

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- The production rate for winding strings in a singular geometry diverges when $j \to 0$. Can the resolved geometry be determined self-consistently à la Fischler-Susskind ?
- Finally, Misner space is very finely tuned wrt to initial conditions. How about string theory on the (BKL) Mixmaster Misner Universe ?