

Closed Strings in the Misner Universe

a toy model of a cosmological singularity

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LPTHE and LPTENS, Paris

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Talk based on

hep-th/0307280 w/ M. Berkooz

hep-th/0405126 w/ M. Berkooz, and M. Rozali

hep-th/0407216 w/ M. Berkooz, B. Durin and D. Reichmann

slides available from

<http://www.lpthe.jussieu.fr/~pioline/seminars.html>

Motivational string cosmology

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- Most importantly, inflation does not get rid of the **initial singularity**. Can string theory evade the usual divergences of perturbative gravity and “no-bounce theorems” ?

Gasperini Veneziano

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- **Closed string field theory** would seem to be the natural framework to address these questions, unfortunately it remains untractable to this day, and possibly may not exist in principle . To what extent can the **first-quantized, on-shell formalism** be pushed to describe **particle production and backreaction** ?

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- Perturbative string theory requires an Euclidean worldsheet, hence Euclidean target space. Even if a Lorentzian target space may be obtained by analytic continuation, **Lorentzian observables** may be quite different from their Euclidean counterparts.

Cosmological backgrounds in string theory

- Even before **quantum** (g_s) corrections, string theory backgrounds undergo **classical** (α') corrections. Very few examples of cosmological solutions of tree-level string theory are known.

Antoniadis Bachas Ellis Nanopoulos, Kounnas Lüst, Nappi Witten...

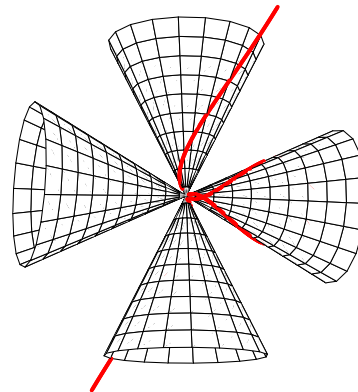
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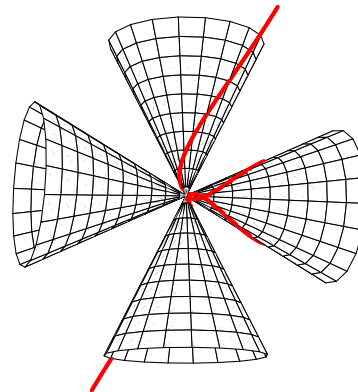
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- Our aim will be to understand **classical aspects of string propagation in this singular background**, and compute **tree-level particle/string production rates**. A much more ambitious task is to incorporate **gravitational backreaction**, and determine whether or not the cosmological singularity is resolved.

Outline of the talk

1. Euclidean and Lorentzian orbifolds, and their avatars

Misner, Taub-NUT, Grant...

2. Untwisted strings in Misner space

Hiscock, Konkowski; Berkooz Craps Kutasov Rajesh, ...

3. Twisted strings in Misner space: first pass

Nekrasov

4. A detour: Open strings in electric fields

Bachas Porrati; Berkooz BP

5. Twisted strings in Misner space: second pass

Berkooz BP Rozali

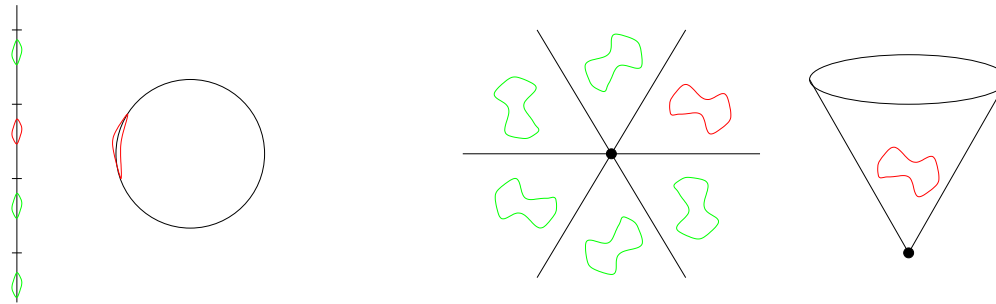
6. Comments on backreaction from winding strings

Strings on Euclidean orbifolds - untwisted states

- Well-known examples of orbifolds are the **circle**, R/Z , and the **rotation orbifold** R^2/Z_k .

Dixon Harvey Vafa Witten

- The spectrum of the quotient theory contains closed string states of the parent theory which are invariant under G : **untwisted states**.

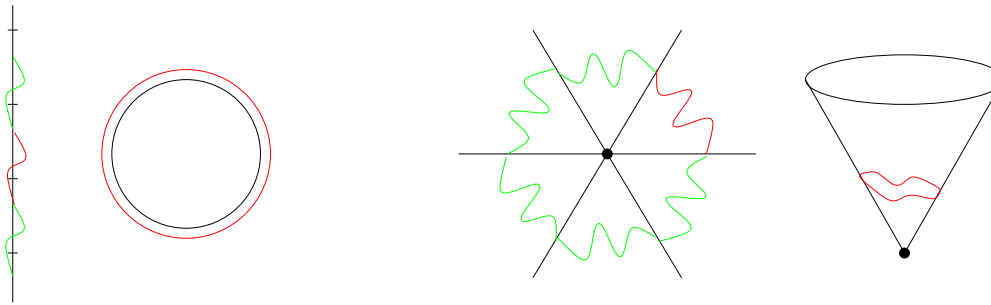


Strings on Euclidean orbifolds - twisted states

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- Modular invariance** requires that the spectrum should also include closed strings in the quotient theory which **close up to the action of G** in the parent theory: **twisted states**.



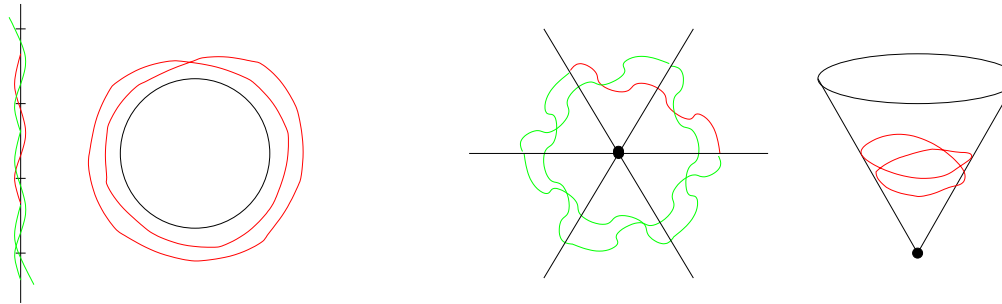
- When G acts non-freely, the twisted sector states are **localized at the fixed points**. They yield new localized degrees of freedom, which ensure the consistency of the background: anomaly free, divergence free, modular invariance...

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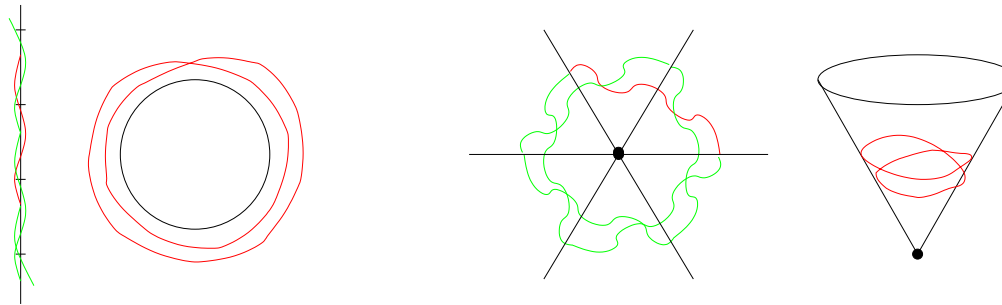


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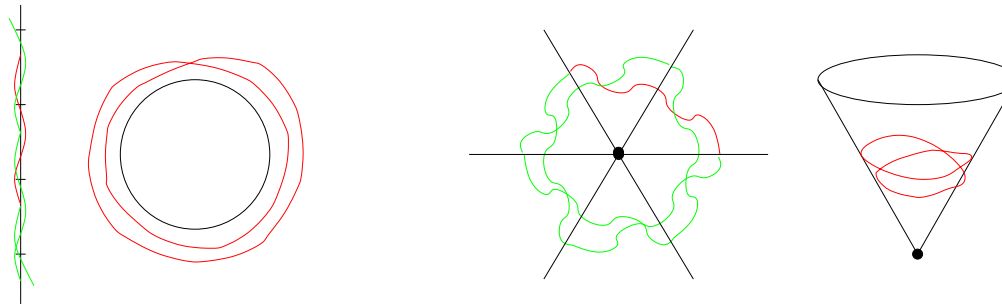
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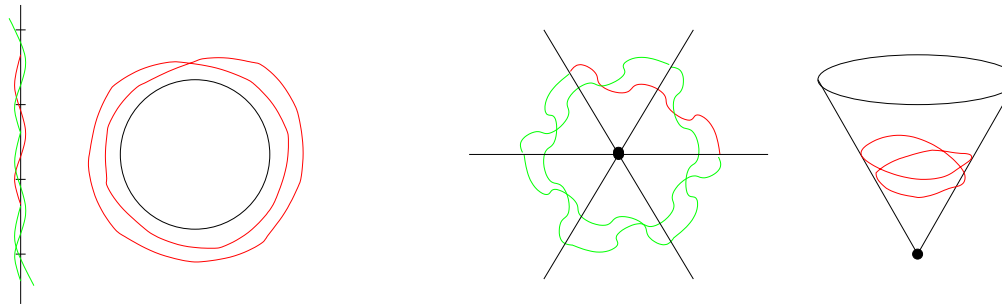
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- The **condensation** of these twisted states changes the vacuum, and effectively **resolves the singularity**: $R^2/Z_k \rightarrow R^2/Z_{k-1} \rightarrow \dots$ (tachyon), $R^4/Z_k \rightarrow$ multi-centered Eguchi-Hanson (massless mode).

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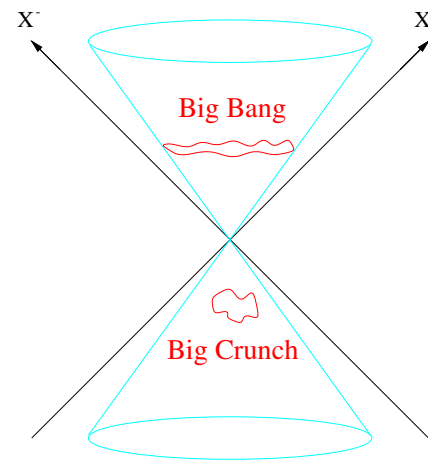
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- The Lorentzian orbifold shares features with both examples: an **infinite number of winding sectors**, and a, non compact, **fixed locus**.

The Lorentzian orbifold

- One of the (superficially) simplest time-dependent solution in string theory is the **quotient of flat Minkowski space by a discrete boost**, also known as **Misner space** (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^\pm \sim e^{\pm 2\pi\beta} X^\pm$$

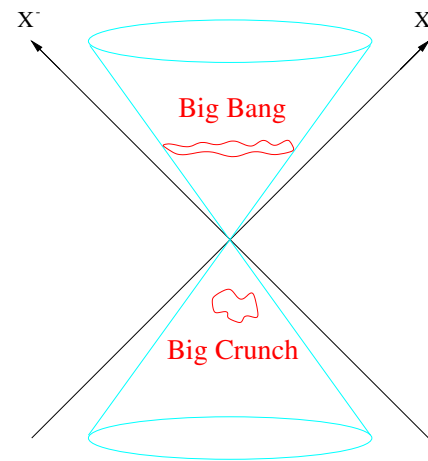


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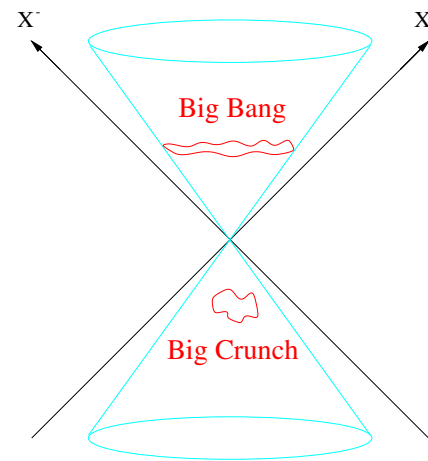
$$ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + (dX^i)^2, \quad \theta \equiv \theta + 2\pi, \quad X^\pm = T e^{\pm\beta\theta} / \sqrt{2}$$

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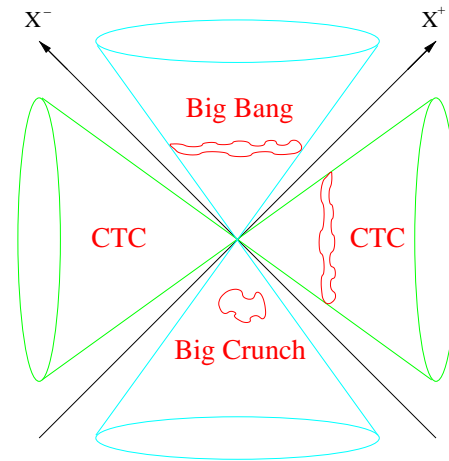
This is a **Kasner-type singularity** with zero curvature except at $T = 0$.

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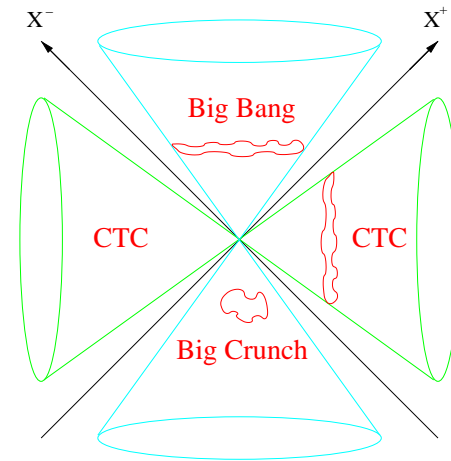


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- In addition, the **spacelike** regions $X^+X^- < 0$ describe two **Rindler wedges** with compact time, often known as **whiskers**, leading to **closed time-like curves**:

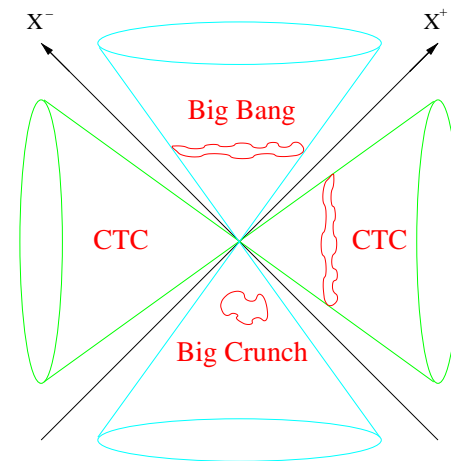
$$ds^2 = dr^2 - \beta^2 r^2 d\eta^2 + (dX^i)^2 \quad , \quad \eta \equiv \eta + 2\pi \quad , \quad X^\pm = \pm r e^{\pm\beta\eta} / \sqrt{2}$$

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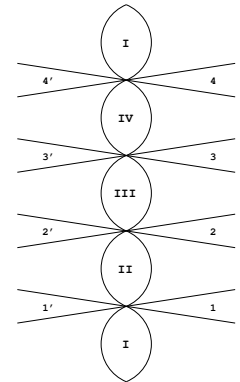
- Finally, the **lightcone** $X^+X^- = 0$ gives rise to a **null, non-Hausdorff** locus attached to the singularity.

Close relatives of the Misner Universe

- Misner space was first introduced as a local model of **Lorentzian Taub-NUT** space:

$$ds^2 = 4l^2 U(t) \sigma_3^2 + 4l \sigma_3 dt + (t^2 + l^2) (\sigma_1^2 + \sigma_2^2), \quad U(t) = -1 + \frac{2mt + l^2}{t^2 + l^2}$$

A **bouncing** universe, isomorphic to $R^{1,1}/boost \times S^2$ around each singularity.

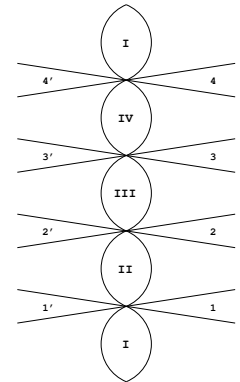


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- A close variant of Misner space is the quotient of flat space by the **combination of a discrete boost and a translation** on an extra direction, often known as the **Grant space**:

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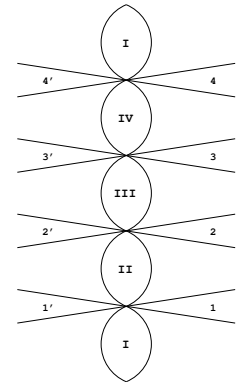
This describes the space away from two **moving cosmic strings**. The cosmological singularity is smoothed out, but regions with CTC remain.

Gott 91, Grant 93; Cornalba, Costa, Kounnas

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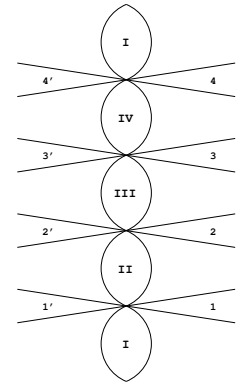
- The Misner geometry arose again more recently as the **M-theory** lift of a simple (**ekpyrotic**) cosmological solution of Einstein-dilaton gravity with no potential.

Khoury Ovrut Seiberg Steinhard Turok

Close relatives of the Misner Universe (cont)

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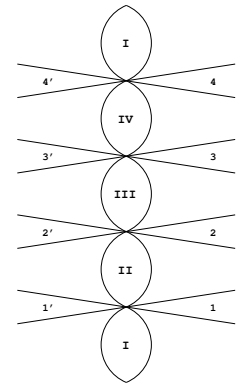
Nappi Witten; Elitzur Giveon Kutasov Rabinovici



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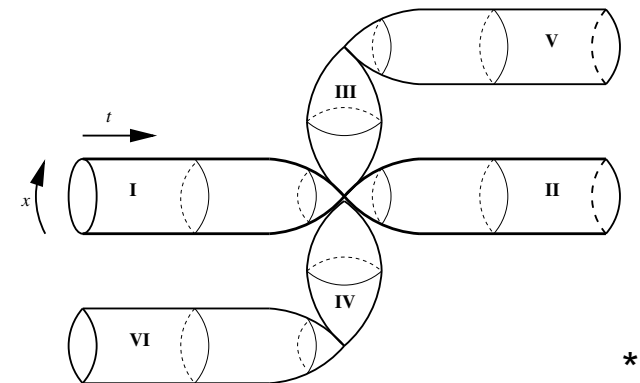
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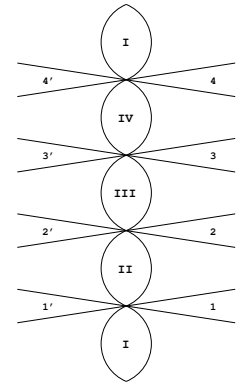
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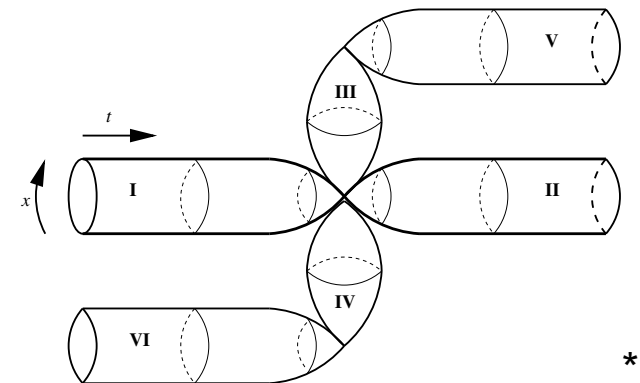
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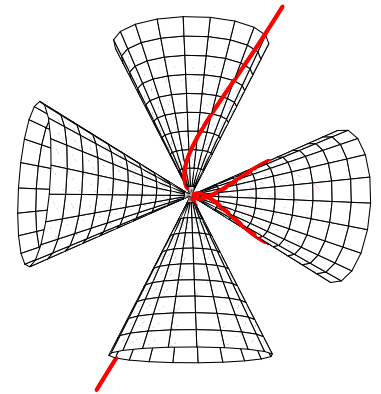
- The **Lorentzian orientifold** $IIB/[(-)^F boost]/[\Omega(-)^{FL}]$ was also recently argued to describe orientifolds of non-supersymmetric strings with non-vanishing Neveu-Schwarz tadpoles.

Dudas Mourad Timirgaziu

Classical particles in the Misner Universe

- Classical particles propagate along straight lines on the covering space:

$$\begin{aligned}
 X^\pm &= x_0^\pm + p^\pm \tau \\
 2p^+ p^- &= M^2 \\
 j &= p^+ x_0^- - p^- x_0^+
 \end{aligned}$$



- As the particle approaches the singularity from the past, it starts spinning faster and faster, $\theta \sim \log |T|$, implying **large gravitational backreaction**.
- In the Rindler wedges, the particle winds infinitely many times around the time direction: at any fixed Rindler time, there is an infinity of copies of the particle, each with energy j : **the total Rindler energy is infinite**.

Quantum particles in the Misner Universe

- Quantum mechanically, the radial motion, for fixed **boost momentum** j , is governed by a **Liouville-type** potential:

$$\frac{1}{r} \partial_r r \partial_r + \frac{j^2}{r^2} = M^2, \quad r = e^y, \quad V(y) = -j^2 + M^2 e^{2y} \equiv 0$$

$$-\frac{1}{T} \partial_T \partial_T - \frac{j^2}{T^2} = M^2, \quad T = e^x, \quad V(x) = -j^2 - M^2 e^{2x} \equiv 0$$

The singularity is at **infinite distance** in the canonically normalized x or y coordinate.

- Wave functions of boost momentum j and spin s can be expressed as superpositions of plane waves on the covering space ($s = \text{spin}$)

$$f_{j,M^2,s}(x^+, x^-) = \int_{-\infty}^{\infty} dv \exp \left(ik^+ X^- e^{-2\pi\beta v} + ik^- X^+ e^{2\pi\beta v} + ik_i X^i + ivj + vs \right)$$

- They can be defined globally by **continuing across the horizons**. The *in* and *out* states defined at $T = -\infty$ and $T = +\infty$ are identical, hence **no overall particle production**.

Tree-level scattering of untwisted states

- As in standard orbifold constructions, part of the spectrum consists of closed strings of the parent theory, **invariant under the orbifold projection**. These topologically trivial states behave at low energy just like **ordinary point particles**.
- **Tree-level scattering amplitudes of untwisted sector states** can be computed from those in flat space by the **inheritance principle**,

$$\langle V(j_1, k_1) \dots V(j_n, k_n) \rangle_{Misner} = \int dv_1 \dots dv_n e^{i(j_1 v_1 + \dots + j_n v_n)}$$

$$\langle V(e^{\beta v_1} k_1^+, e^{-\beta v_1} k_1^-, k_1^i) \dots V(e^{\beta v_n} k_n^+, e^{-\beta v_n} k_n^-, k_n^i) \rangle_{Minkowski}$$

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- String amplitudes are suppressed in the **high energy regime** (fixed $s/t, s/u$). However, in the deep inelastic regime, ($s \rightarrow \infty, t$ fixed), they exhibit Regge behavior $A \sim s^t$, as if strings acquired a size $\sqrt{\ln s}$:

$$\langle V(j_1, k_1) \dots V(j_n, k_n) \rangle_{\text{Misner}} \propto \int dv v^{-\frac{1}{2}(k_1^i - k_3^i)^2 + i(j_2 - j_4)}$$

which diverges if $(k_1^i - k_3^i)^2 \leq 2$, as a result of **large graviton exchange near the cosmological singularity**.

Quantum fluctuations in field theory

- In the **Minkowski vacuum** (inherited from the covering space), the renormalized propagator can be obtained as a sum over images,

$$G(x; x') = \sum_{l=-\infty, l \neq 0}^{\infty} \int_0^{\infty} d\tau \int dp^{\mu} \exp \left(-ip^{-} (x^{+} - e^{2\pi\beta l} x^{+'}) - ip^{+} (x^{-} - e^{2\pi\beta l} x^{-'}) - ip^i (x^i - x^{i'}) \right)$$

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- The one-loop stress-energy tensor follows from the propagator at coinciding points $G(x, x)$, e.g for a free scalar field in 4D,

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This leads to a **divergent quantum backreaction** (worse if the spin $|s| > 1$):

$$\langle T_{\mu}^{\nu} \rangle = \frac{K}{12\pi^2} T^{-4} \text{diag}(1, -3, 1, 1), \quad K = \sum_{l=1}^{\infty} \cosh(2\pi\beta l s) \frac{2 + \cosh 2\pi l \beta}{[\cosh 2\pi l \beta - 1]^2}$$

One-loop vacuum amplitude in field and string theory

- On the other hand, in string theory $\langle T_{\mu}^{\nu} \rangle(x)$ is an **off-shell** quantity, and only its integral over space-time is well defined:

$$\int dx^+ dx^- G(x, x) = \sum_{l=-\infty}^{+\infty} \int_0^{\infty} \frac{d\rho}{\rho^{D/2}} \frac{e^{-m^2 \rho}}{\sinh^2(\pi \beta l)}$$

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- As usual, the ultraviolet divergence at $\rho \rightarrow 0$ is regularized in string theory by modular invariance:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l, w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2 \rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho) \theta_1(i\beta(l + w\rho); \rho)|^2}$$

$$\theta_1(v; \rho) = 2q^{1/8} \sin \pi v \prod_{n=1}^{\infty} (1 - e^{2\pi i v} q^n)(1 - q^n)(1 - e^{-2\pi i v} q^n), \quad q = e^{2\pi i \rho}$$

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- The existence of **Regge trajectories** with arbitrary high spin implies new (log) **divergences in the bulk of the moduli space** which resemble long string poles in AdS_3 .

Closed string in Misner space - twisted sectors

- In addition, there is **an infinite set of twisted sectors**, corresponding to strings on the covering space that close **up to the action of the orbifold group**:

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau), \quad \nu = 2\pi\omega\beta$$

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- They have a normal mode expansion:

$$X^\pm = \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha_n^\pm e^{-i(n \pm i\nu)(\tau - \sigma)} + \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \mp i\nu)^{-1} \tilde{\alpha}_n^\pm e^{-i(n \mp i\nu)(\tau + \sigma)}$$

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$$\begin{aligned} [\alpha_m^+, \alpha_n^-] &= -(m + i\nu)\delta_{m+n} & , & & [\tilde{\alpha}_m^+, \tilde{\alpha}_n^-] &= -(m - i\nu)\delta_{m+n} \\ (\alpha_m^\pm)^* &= \alpha_{-m}^\pm & , & & (\tilde{\alpha}_m^\pm)^* &= \tilde{\alpha}_{-m}^\pm \end{aligned}$$

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- We will focus on the **quasi zero-mode** sector, which consists of two commuting pairs of real (i.e. hermitian) canonically conjugate operators,

$$[\alpha_0^+, \alpha_0^-] = -i\nu, \quad [\tilde{\alpha}_0^+, \tilde{\alpha}_0^-] = i\nu$$

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated by half of them, say $\alpha_{n>0}^{\pm}$, $\tilde{\alpha}_{n>0}^{\pm}$, α_0^- , $\tilde{\alpha}_0^+$

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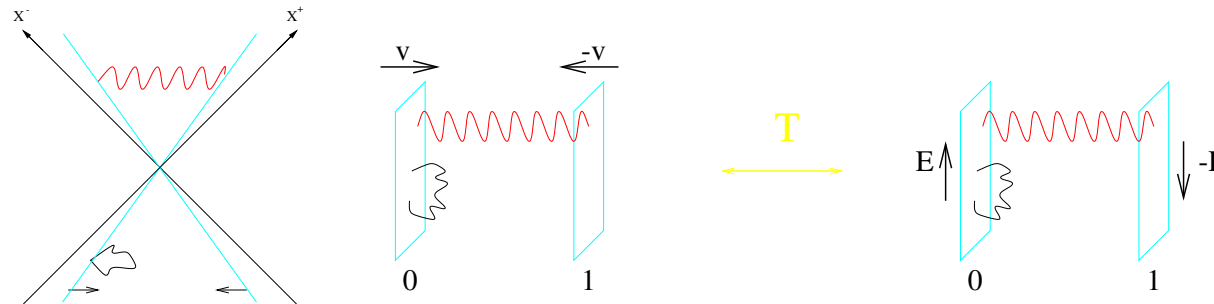
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However, this scheme overlooks the fact that α_0^+ and α_0^- are **self-hermitian** !

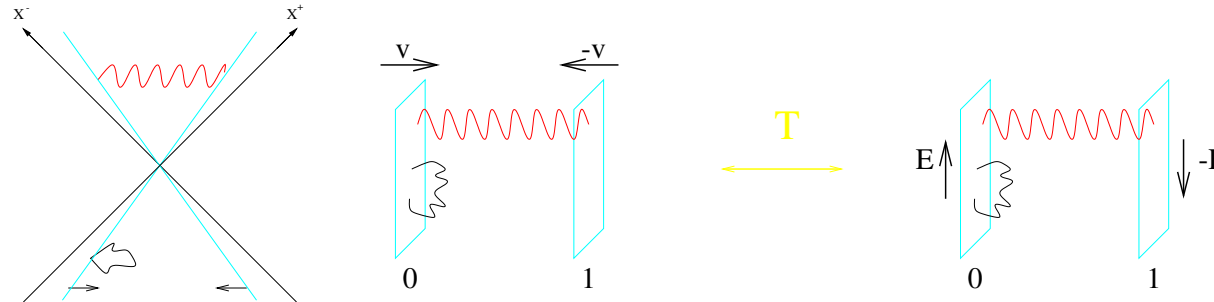
A detour via Open strings in electric field

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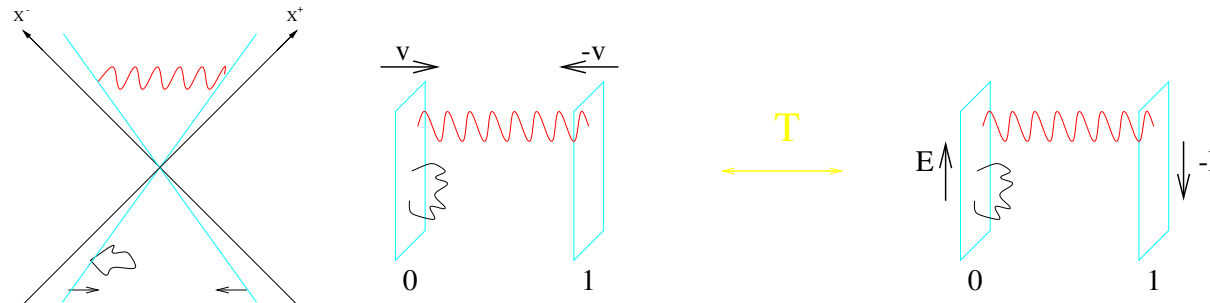


- **Open strings stretched between two D-branes** with electric fields E_0 and E_1 have a spectrum

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- This reproduces (one half of) the spectrum of **Closed strings in Misner space** upon identifying $\nu = w\beta$. The large winding number limit $w \rightarrow \infty$ amounts to a **near critical electric field** $E \rightarrow 1/\pi$.
- In particular, the **open string zero-modes** describe the motion of a **charged particle in an electric field**, and have a structure **isomorphic** to the closed string case.

Charged particle and open string zero-modes

- Recall the first quantized **charged particle in an electric field**:

$$L = \frac{1}{2}m \left(-2\partial_\tau X^+ \partial_\tau X^- + (\partial_\tau X^i)^2 \right) + \frac{\nu}{2} \left(X^+ \partial_\tau X^- - X^- \partial_\tau X^+ \right)$$

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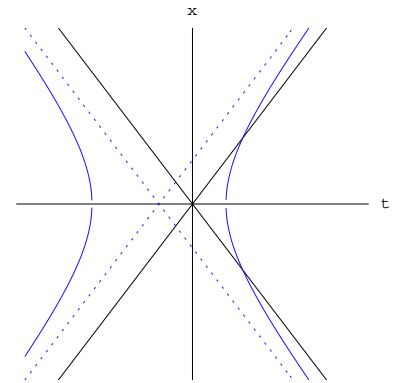
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- Classical trajectories are hyperbolas centered at an arbitrary point,

$$X^\pm = x_0^\pm \pm \frac{1}{\nu} a_0^\pm e^{\pm\nu\tau}$$

$P^\pm = \pm\nu x_0^\pm$ is the conserved **linear momentum**, and a_0^\pm the **velocity**.



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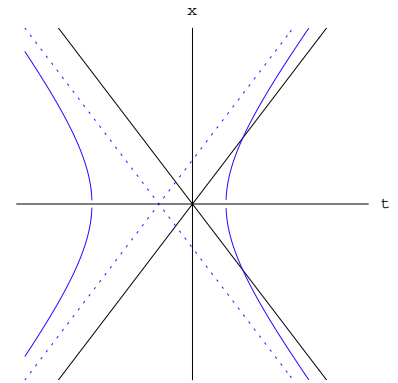
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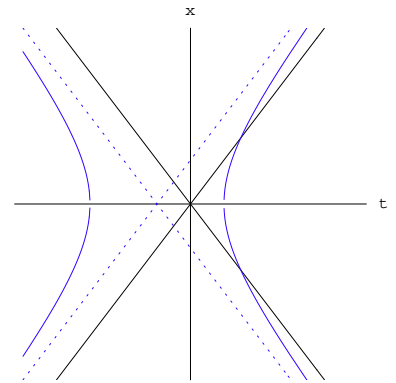
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- Upon quantizing a_0^\pm as creation/annihilation operators in a Fock space, **electrons and positrons would have no physical state...**

Charged particle and Klein-Gordon equation

- Quantum mechanically, one represents the canonical momenta as derivatives, $\pi^\pm = i\partial/\partial x^\mp$, hence a_0^\pm, x_0^\pm as **covariant derivatives**

$$a_0^\pm = i\partial_\mp \pm \frac{\nu}{2}x^\pm, \quad x_0^\pm = \mp\frac{1}{\nu} \left(i\partial_\mp \mp \frac{\nu}{2}x^\pm \right)$$

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- The zero-mode piece of L_0 , **including the bothersome $\frac{i\nu}{2}$** ,

$$L_0^{(0)} = -a_0^+ a_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\nabla^+ \nabla^- + \nabla^- \nabla^+)$$

is just the **Klein-Gordon operator** of a particle of charge ν , and has well-behaved eigenmodes $L_0 = -m^2$ for any $m^2 > 0$.

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^\pm = (P \pm Q)/\sqrt{2}$, the Klein-Gordon operator can be rewritten as an **inverted harmonic oscillator**:

$$M^2 = -\frac{1}{2}(P^2 - Q^2), \quad [P, Q] = i$$

- The latter admits a respectable **delta-normalizable spectrum of scattering states**, in terms of **parabolic cylinder functions**.

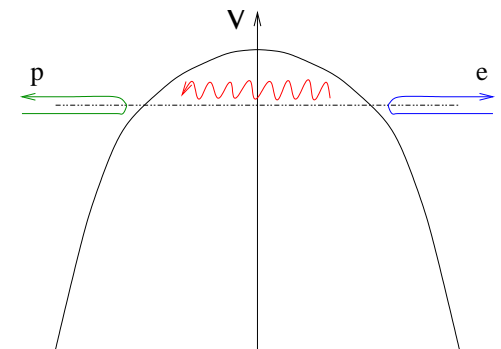
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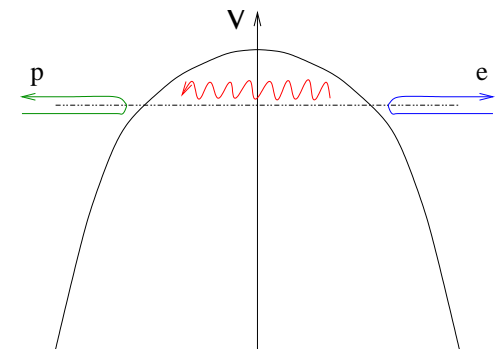
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- The tunneling rate can be computed semiclassically, $\eta \sim \exp(-2 \oint P dQ) = e^{-\pi M^2/\nu}$ which reproduces the Schwinger pair creation rate.

Brezin Itzykson; Brout Massar Parentani Spindel

Lorentzian vs Euclidean states

- The Wick rotation $X^0 \rightarrow -iX^0$, $\nu \rightarrow i\nu$ turns the electric field (Schwinger) problem in $R^{1,1}$ into the magnetic field (Landau) problem in R^2 .

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The density of states is semi-classically from the reflection phase,

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- The physical spectrum of the charged open string can be explicitly worked out, and is free of ghosts: a tachyon at level 0, a transverse gauge boson at level 1, ...

Charged particle in Rindler space

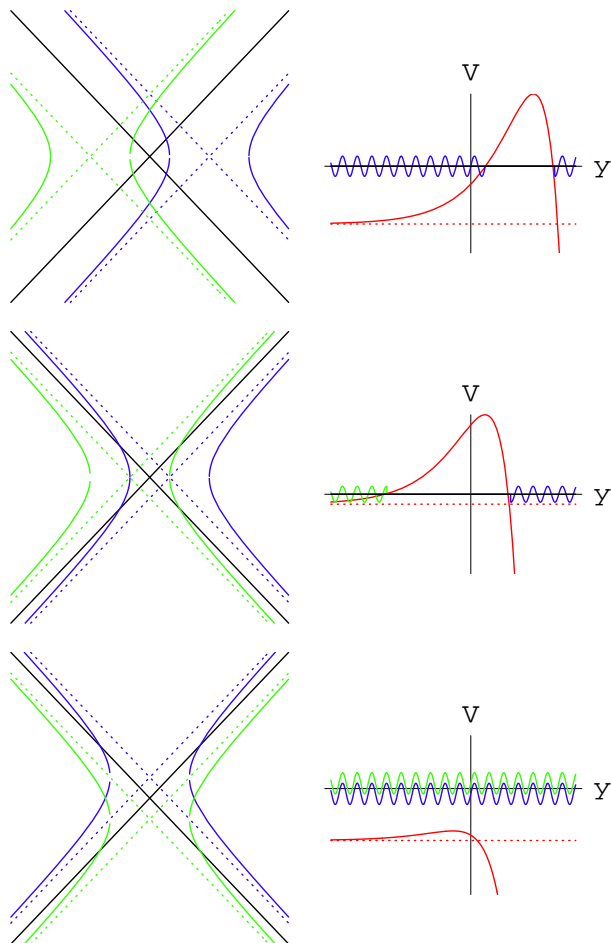
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Gabriel Spindel; Mottola Cooper

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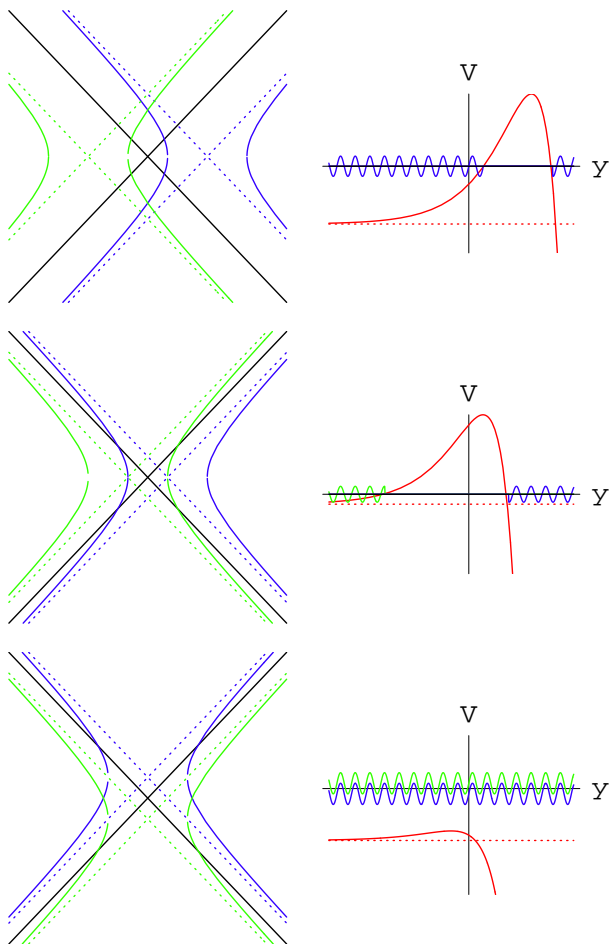
- In the **Rindler** patch R , letting $f(r, \eta) = e^{-ij\eta} f_j(r)$ and $r = e^y$, one gets a **Schrödinger equation** for a particle in a potential

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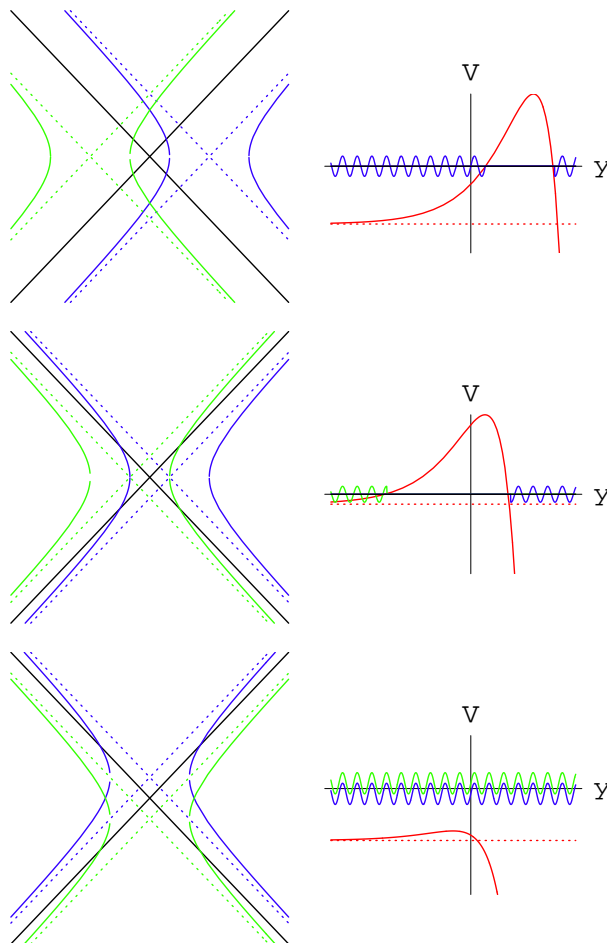
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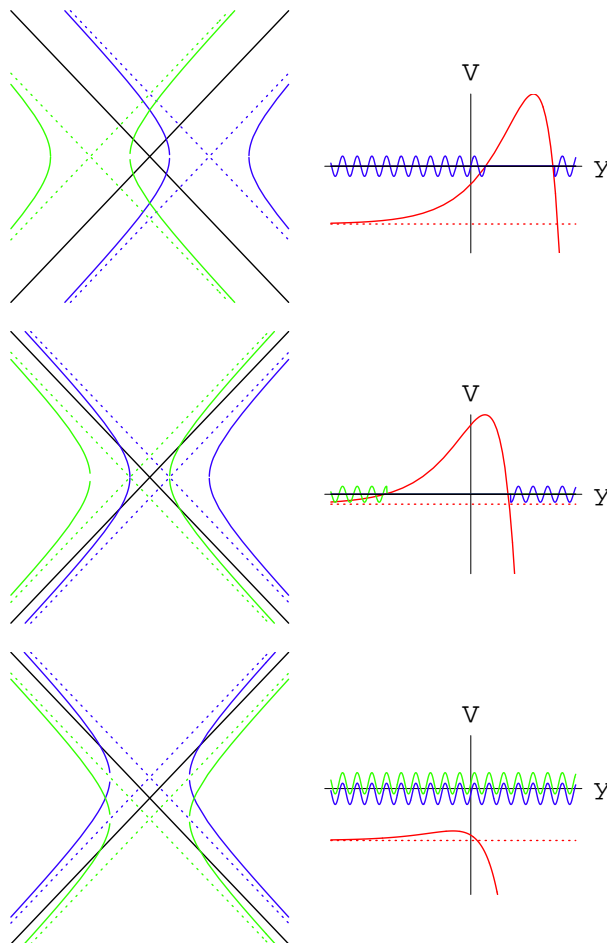
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- If $j > M^2/(2\nu)$, the electron branches cross the horizons. regions. There is **no tunneling**, but partial reflection amounts to a combination of **Schwinger** and **Hawking** emission.

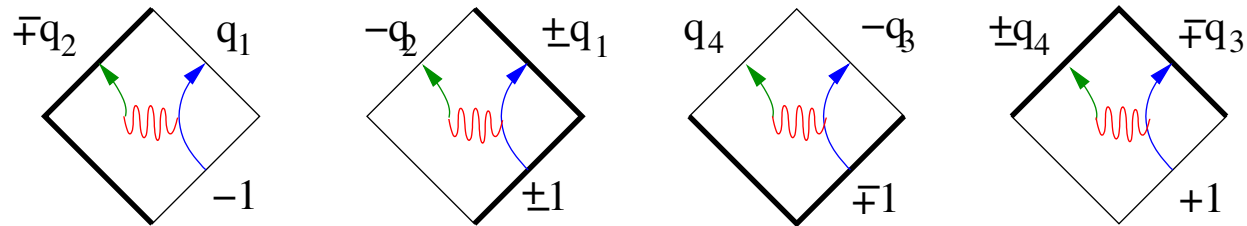
Rindler modes

- Incoming modes from Rindler infinity I_R^- read, in terms of parabolic cylinder functions:

$$\mathcal{V}_{in,R}^j = e^{-ij\eta} r^{-1} M_{-i(\frac{j}{2} - \frac{m^2}{2\nu}), -\frac{ij}{2}}(i\nu r^2/2)$$

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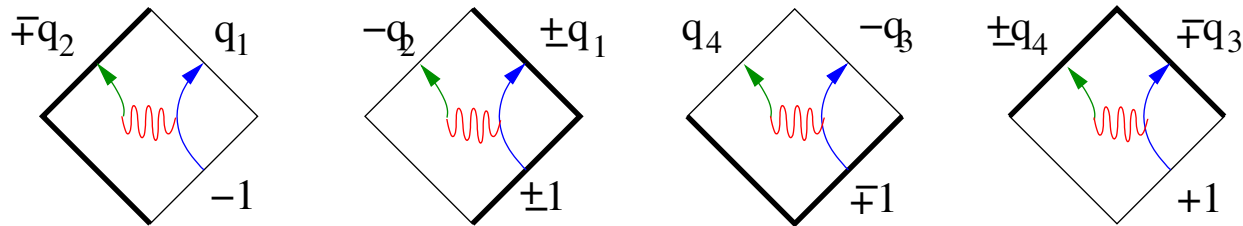
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- The reflection coefficients can be computed ($q_1 = 1 - q_2, q_3 = q_4 + 1$):

$$q_2 = e^{-\frac{\pi M^2}{2\nu}} \frac{|\sinh \pi j|}{\cosh \left[\pi \left(j - \frac{M^2}{2\nu} \right) \right]}, \quad q_4 = e^{-\frac{\pi M^2}{2\nu}} \frac{\cosh \left[\pi \left(j - \frac{M^2}{2\nu} \right) \right]}{|\sinh \pi j|}$$

Global Charged Unruh Modes

- Global modes may be defined by patching together Rindler modes, ie by **analytic continuation across the horizons**. **Unruh modes** are those which are superposition of **positive energy** Minkowski modes,

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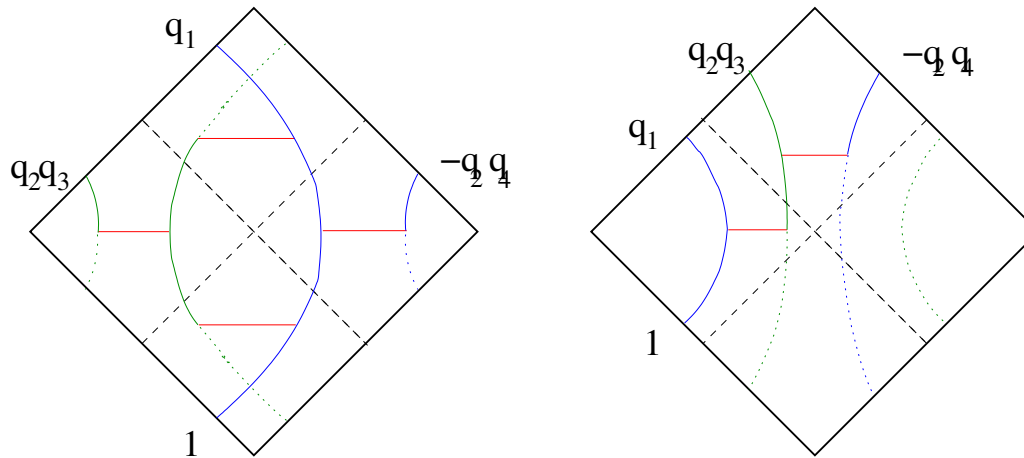
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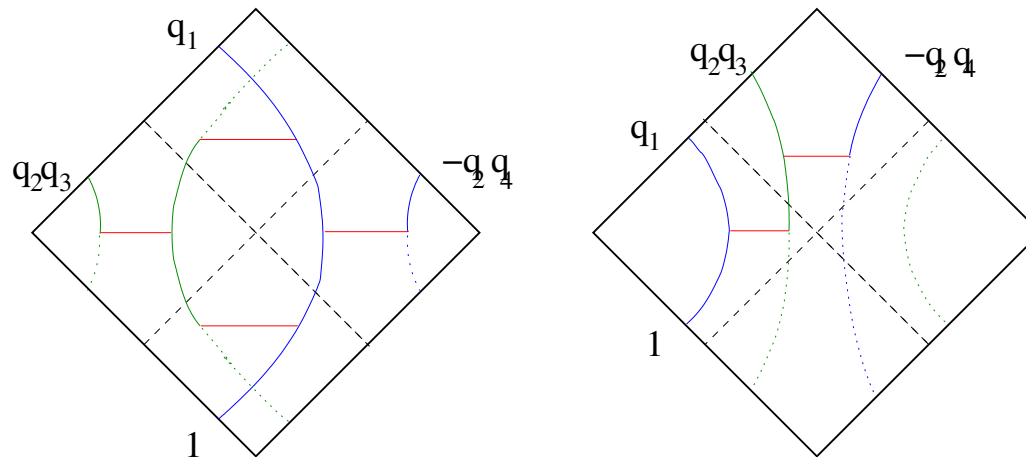
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- Any state in Minkowski space can be represented as a state in the **tensor product of the Hilbert spaces of the left and right Rindler patches**.

Closed string zero-modes

- Let us reanalyze the classical solutions for the closed string zero modes

$$X^\pm(\tau, \sigma) = e^{\mp\nu\sigma} \left[\pm \frac{1}{2\nu} \alpha_0^\pm e^{\pm\nu\tau} \mp \frac{1}{2\nu} \tilde{\alpha}_0^\pm e^{\mp\nu\tau} \right], \quad \alpha_0^\pm, \tilde{\alpha}_0^\pm \in R$$

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$$4\nu^2 X^+ X^- = \alpha_0^+ \tilde{\alpha}_0^- e^{2\nu\tau} + \alpha_0^- \tilde{\alpha}_0^+ e^{-2\nu\tau} - \alpha_0^+ \alpha_0^- - \tilde{\alpha}_0^+ \tilde{\alpha}_0^-$$

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- The behavior at early/late proper time now depends on $\epsilon\tilde{\epsilon}$: For $\epsilon\tilde{\epsilon} = 1$, the string begins/ends in the **Milne** regions. For $\epsilon\tilde{\epsilon} = -1$, the string begins/ends in the **Rindler** regions.

Short and long strings

Choosing $j = 0$ for simplicity, we have two very different types of solutions:

- $\epsilon = 1, \tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma, \tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma}, \quad T = \frac{M}{\nu} \sinh(\nu\tau), \quad \theta = \nu\sigma$$

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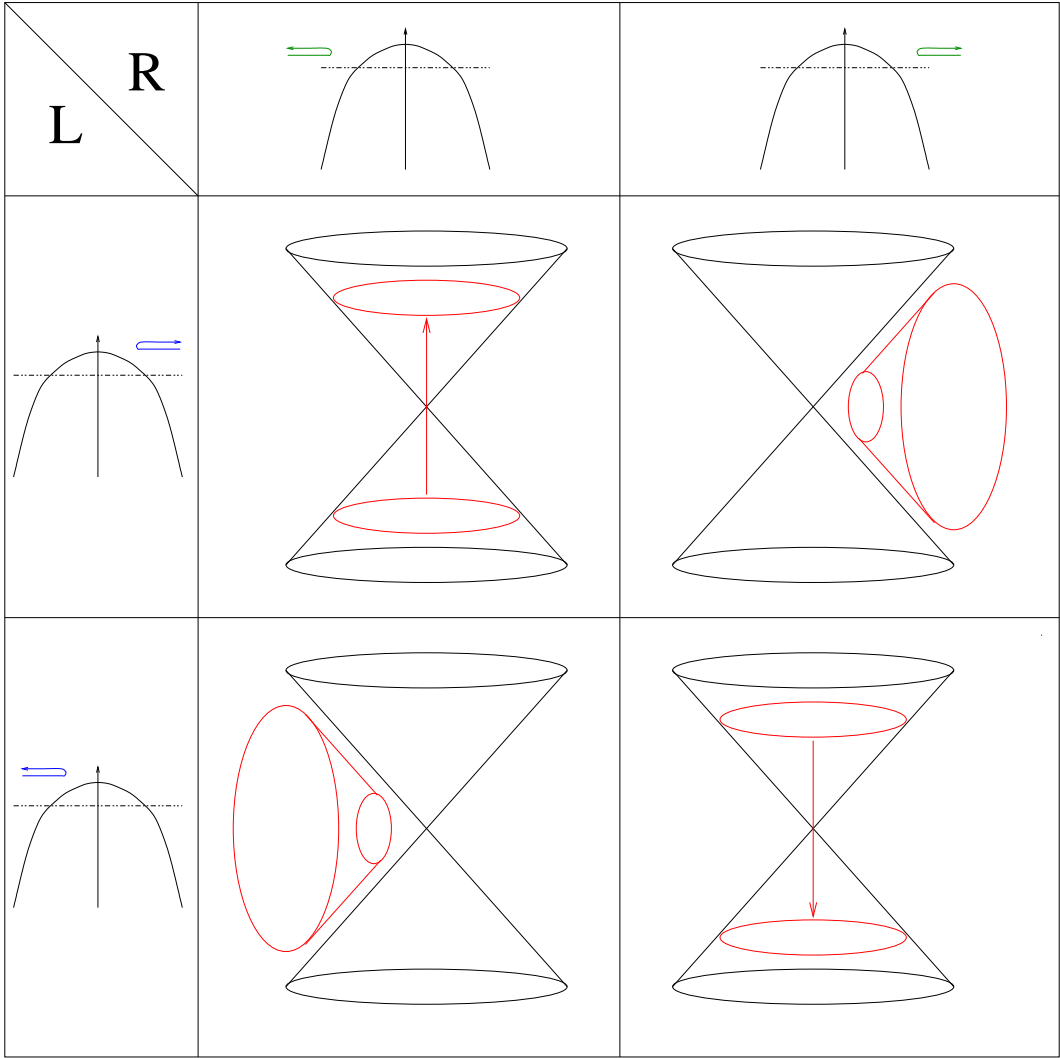
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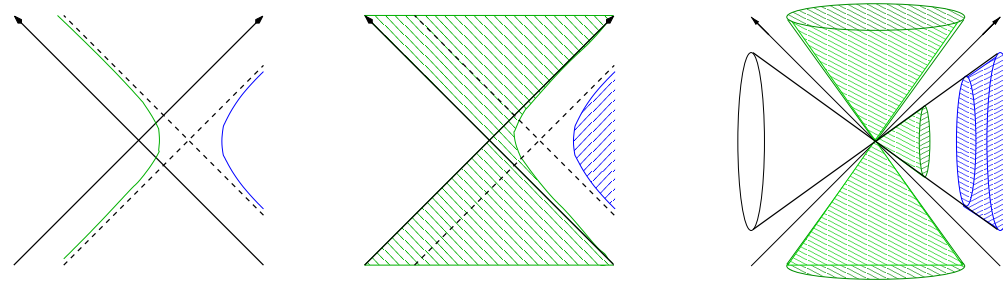
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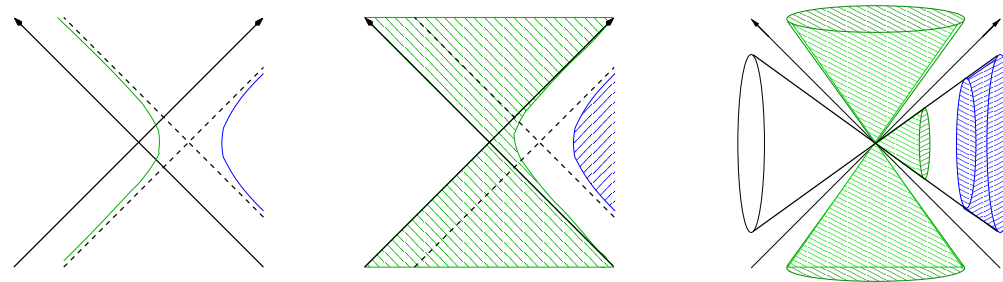
From open to closed strings

- Instead of following the motion of a point at fixed σ , one may consider instead a point at fixed $\sigma + \tau$: this is precisely the **trajectory of the open string zero-mode**. The rest of the worldsheet follows by smearing under the action of the boost:

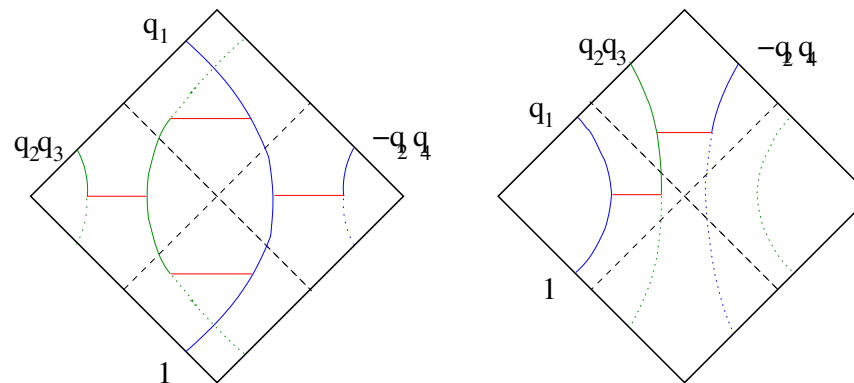


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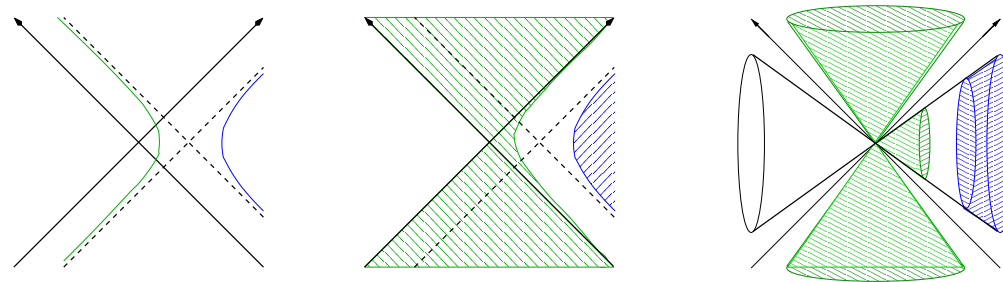


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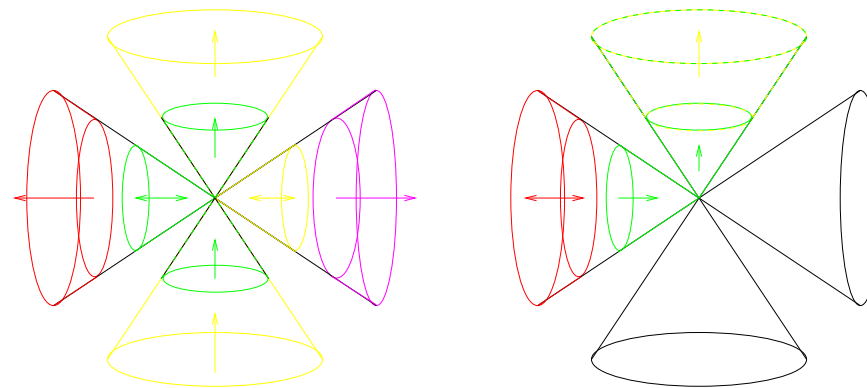


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- The open string global wave functions are also closed string wave functions. . .



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Spontaneous production of winding strings

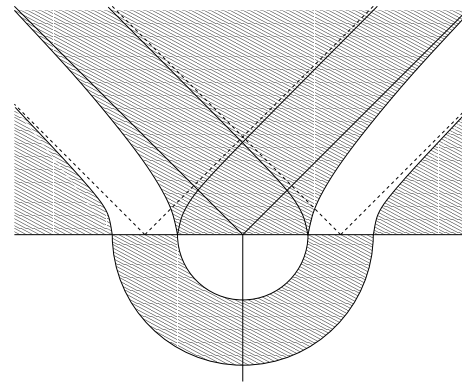
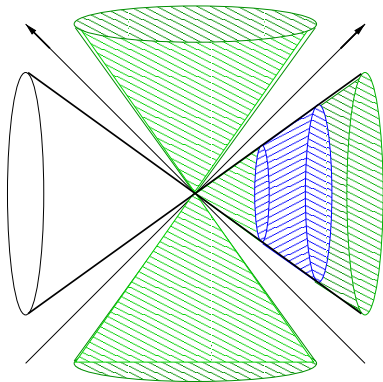
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- **Spontaneous pair production of winding strings** can be described by cutting open a periodic trajectory, either in **imaginary proper time**, or in the **Euclidean rotation orbifold**:



A word on Quantization in the Rindler patch

- For **long strings** in conformal gauge, the worldsheet coordinate τ is **spacelike** wrt to the induced metric. For **short strings**, the induced metric undergoes a **signature flip** as it wanders in the Rindler patch.

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- The spectrum is thus **unbounded from below** (and above). However, the periodicity of time may render this issue moot.

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- A bounce in dimension i requires $H'_i > 0$ at the point where $H_i = 0$, i.e.

$$(D-2)p_i + \rho \geq \sum_{j \neq i} p_j$$

The most efficient solution is a gas of scalar momentum states, with $p = \rho$: provides enough pressure for the bounce.

Effective gravity analysis (cont.)

- However, consider fundamental strings wrapped around dimension i ,

$$\rho = \frac{T}{V}, \quad p_i = -\rho, \quad p_{j \neq i} = 0, \quad V = \prod_{j \neq i} a_j \quad \Rightarrow \quad D \leq 3$$

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- Modelling the dilaton as the radius of the \sharp th direction, the strings become membranes wrapped around (i, \sharp) :

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The bounce is allowed when $D \leq 4$.

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- We assumed a constant number of wound strings: one should incorporate the dependence of the production rate on the Hubble parameters.

Effective gravity analysis (cont.)

- Einstein's equations imply that the quantity $\mu = \left(\frac{H_k}{H_i} - 1 \right) / \left(\frac{H_j}{H_i} - \frac{3}{4-D} \right)$ is constant.

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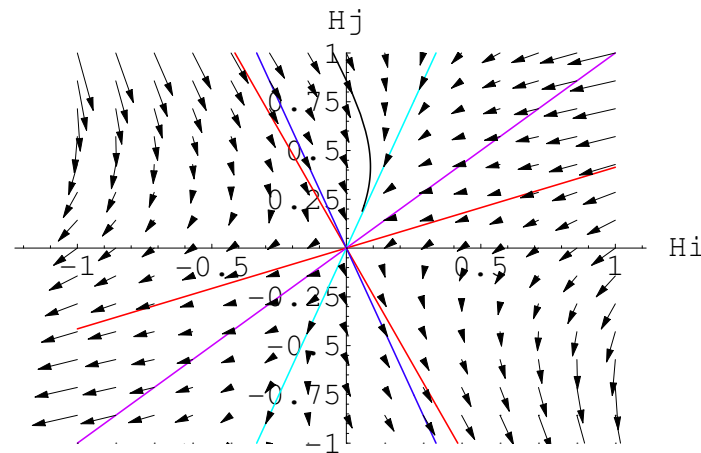
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A bounce for direction i in units of the eleven-dimensional frame therefore takes place for any initial condition such that $2\mu + D - 3 > 0$ and $2 < D < 4$.



Conformal perturbation

- Rather than computing the backreaction from quantum production of (squeezed) winding strings, one may consider deforming the orbifold CFT with a **marginal twist field**, i.e. adding a **coherent superposition** of winding strings:

$$S_\lambda = \int d^2\sigma \partial X^+ \bar{\partial} X^- + \lambda_{-w} V_{+w} + \lambda_{+w} V_{-w}$$

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$$\langle e^{ikX} \rangle_\lambda \sim \lambda_w \lambda_{-w} \langle w | e^{ikX} | -w \rangle ,$$

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- In addition, the winding string also sources twisted states whose winding number is a multiple of w :

$$\langle V_{-2w} \rangle_\lambda \sim \lambda_w \lambda_w \langle w | V_{-2w} | w \rangle ,$$

The size of twisted states

- The scattering amplitude of N untwisted states off one winding string can be computed by **Hamiltonian quantization on the cylinder with twisted boundary conditions**.
- As in flat space, the **off-shell form factor** is formally zero due to infinite zero-point fluctuations,

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- At large ν , $\Delta(\nu) \sim 2 \log \nu$, which indicates that the winding string grows to a size $\sqrt{\log w}$, T-dual to the Regge growth a high energy.

Classical backreaction

- The **untwisted fields sourced by a twisted state** with wave function $f(x^+, x^-)$ are then given by the zero-mode overlap

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- Three-point functions of three twisted fields cannot be computed in Hamiltonian formalism, but can be obtained by analytic continuation of the amplitude for $p^+ \neq 0$ strings in the Nappi-Witten pp-wave:

$$\int dx_1^\pm dx_2^\pm [f_1(x_1^\pm) f_2(x_2^\pm)]^* \exp\left[(x_1^+ - x_2^+)(x_1^- - x_2^-) \Xi(\nu_1, \nu_2)\right] f_3\left(\frac{\nu_1 x_1^\pm + \nu_2 x_2^\pm}{\nu_1 + \nu_2}\right)$$

D'Appollonio, Kiritsis; Cheung Freidel Savvidy

where the size of the non-locality is given by the (real) ratio

$$\Xi(\nu_1, \nu_2) = -i \frac{1 - \frac{i\nu_3}{\nu_1\nu_2} \frac{\gamma(i\nu_3)}{\gamma(i\nu_1)\gamma(i\nu_2)}}{1 + \frac{i\nu_3}{\nu_1\nu_2} \frac{\gamma(i\nu_3)}{\gamma(i\nu_1)\gamma(i\nu_2)}}, \quad \gamma(x) = \frac{\Gamma(x)}{\Gamma(1-x)}$$

Note that the non-locality scale $1/\sqrt{\Xi}$ diverges when $\nu_1\nu_2\gamma(i\nu_1)\gamma(i\nu_2) = i\nu_3\gamma(i\nu_3)$.

Conclusions

We discussed closed strings in a toy model of a cosmological singularity. However, some of the features we uncovered should carry over to more general geometries:

- Winding string production can be understood semi-classically as **tunneling under the barrier** in regions with compact time, or **scattering over the barrier** in cosmological regions. In general, it can be computed as a **tree-level two-point function** in an appropriate basis depending on the choice of vacuum.

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- Finally, Misner space is very finely tuned wrt to initial conditions. How about string theory on the (BKL) **Mixmaster** Misner Universe ?