

Closed Strings in the Misner Universe

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LPTHE and LPTENS, Paris

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based on hep-th/0307280 w/ M. Berkooz
and work in progress w/ M. Berkooz, B. Durin, D. Reichmann, M. Rozali

slides available from

<http://www.lpthe.jussieu.fr/~pioline/seminars.html>

Motivational string cosmology

- With LHC still far in the future, **Observational Cosmology** is now challenging string theory with high-precision data:

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- Time dependence is presumably incompatible with **target space supersymmetry**.
- Perturbative string theory requires an Euclidean worldsheet, hence Euclidean target space. The **analytic continuation** may be ambiguous or ill-defined, **Lorentzian observables** may be very different from their Euclidean counterparts.
- String theory is not content on a finite time interval, and one is frequently forced into **Big Bang / Big Crunch singularities, CTC** in the process of maximally extending the geometry.

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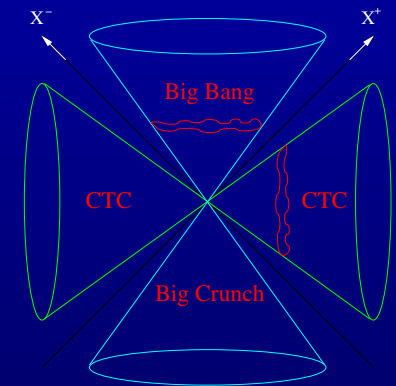
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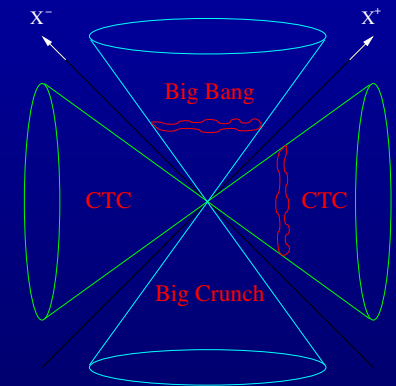
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- We shall focus in particular on the **extended perturbative string states** which wind around the collapsing dimension.

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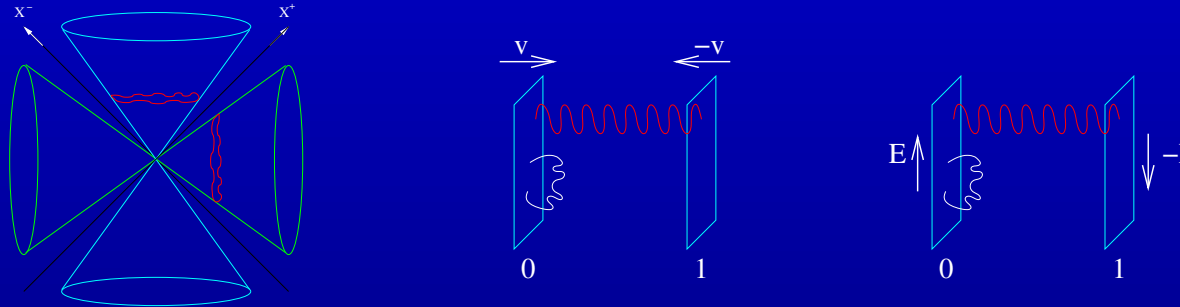
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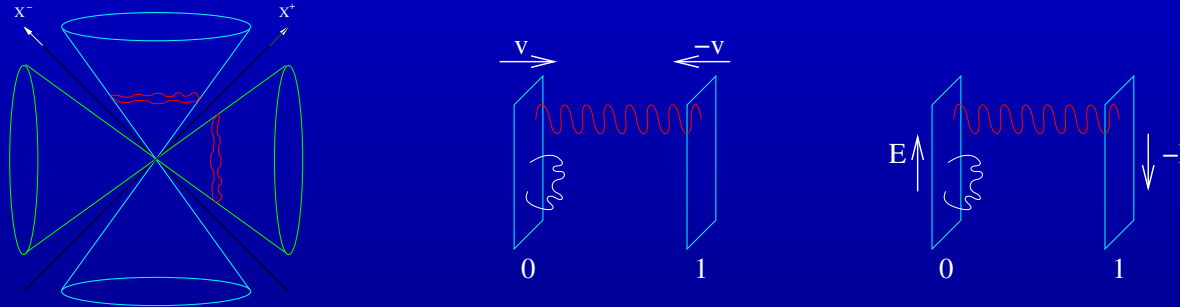
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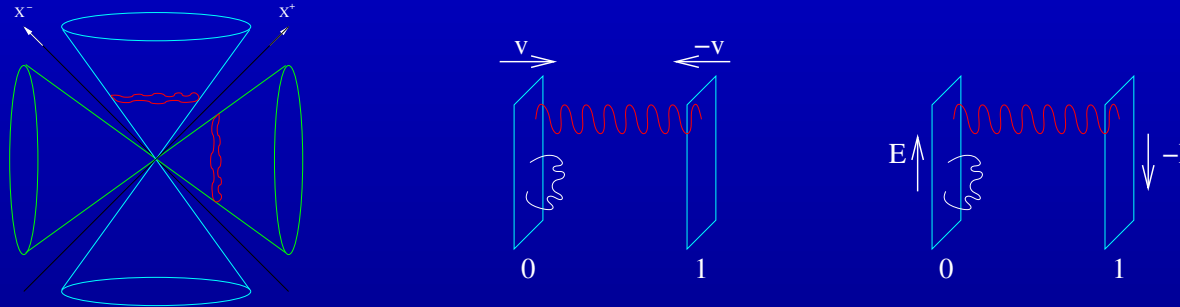
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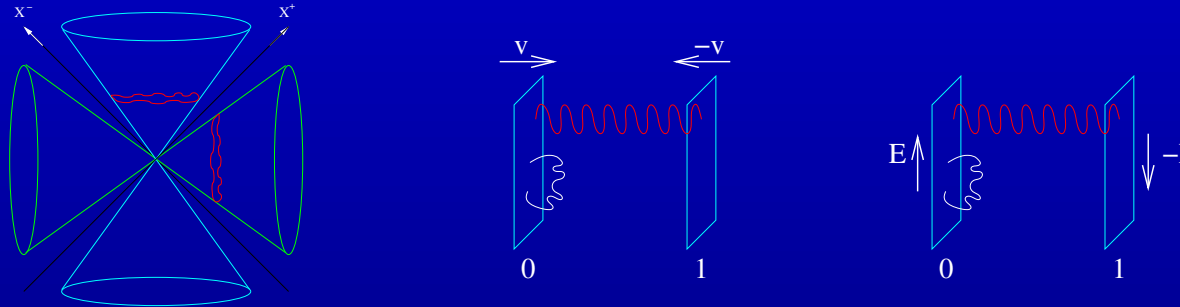


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- Can Schwinger production of twisted closed strings resolve the cosmological singularity of the Lorentzian orbifold ?

Outline of the talk

1. Introduction
2. The Lorentzian orbifold and its avatars
3. Closed strings in Misner space: first pass
3. Open strings in electric fields, revisited
4. Closed strings in Misner space: second pass
5. Conclusions, speculations

Misner, Taub-NUT, Grant...

Nekrasov

Berkooz BP

Berkooz BP; Berkooz Durin BP Reichmann Rozali

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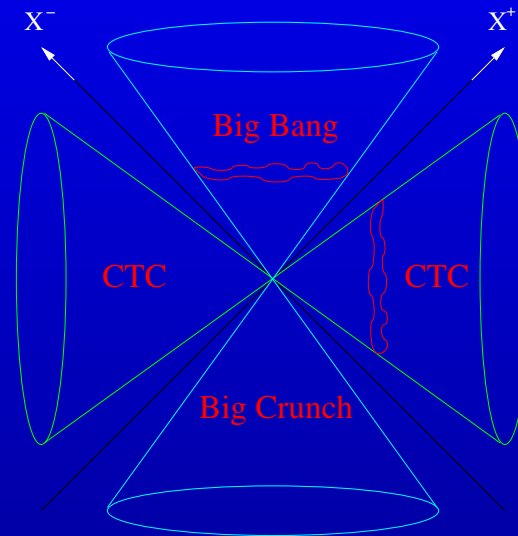
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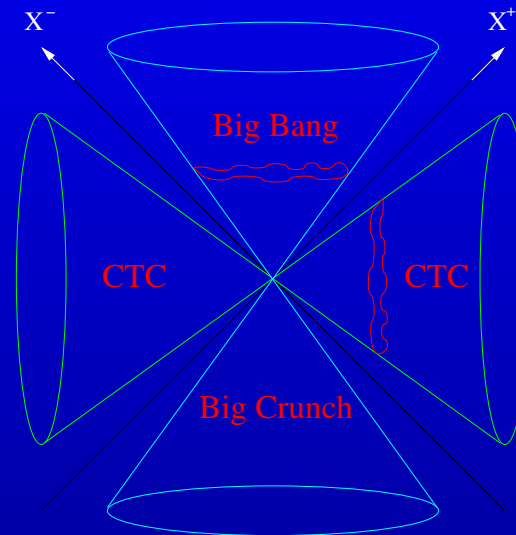
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- Finally, the **lightcone** $X^+X^- = 0$ gives rise to **non-Hausdorff** null lines, attached to the singularities.

The Misner/Milne Universe



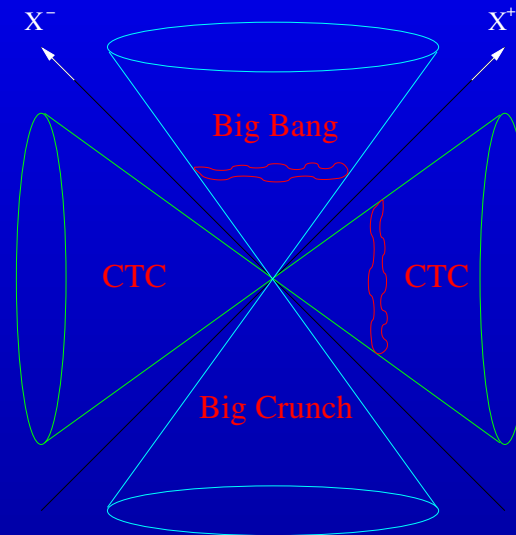
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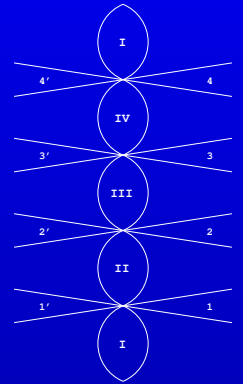
- Misner space was first introduced as a local model of **Lorentzian Taub-NUT** space:

$$ds^2 = 4l^2 U(t) \sigma_3^2 + 4l \sigma_3 dt + (t^2 + l^2)(\sigma_1^2 + \sigma_2^2), \quad U(t) = -1 + \frac{2mt + l^2}{t^2 + l^2}$$

A **bouncing** universe, isomorphic to $R^{1,1}/boost \times S^2$ around each singularity.

Stringy Misner-type Universes

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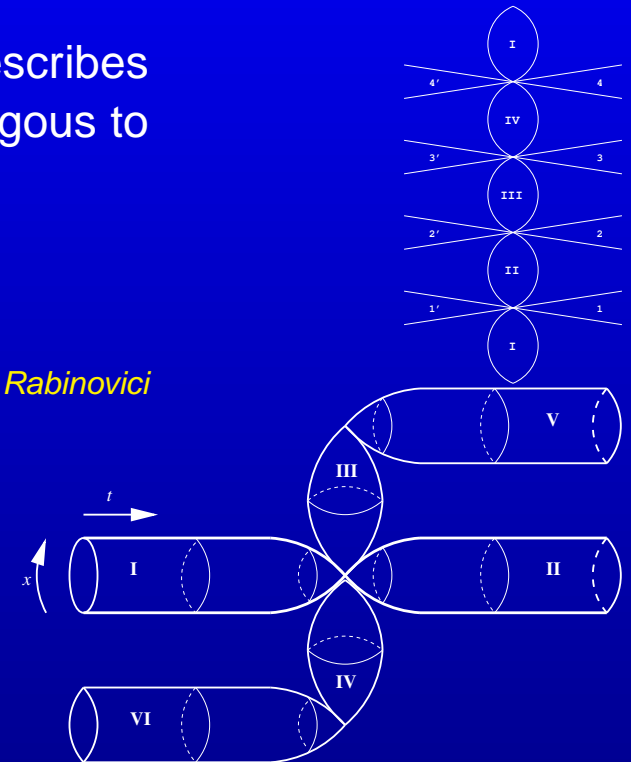
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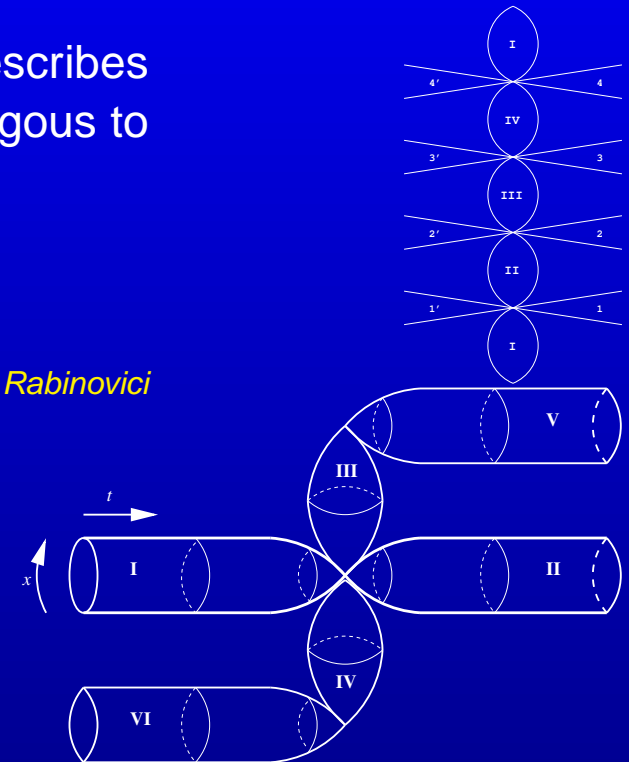
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- The **Lorentzian orientifold** $IIB/[(-)^F boost]/[\Omega(-)^{FL}]$ was also recently argued to describe orientifolds of non-supersymmetric strings with non-vanishing Neveu-Schwarz tadpoles.

Dudas Mourad Timirgaziu



The Grant space

- A close variant of Misner space is the quotient of flat space by the combination of a discrete boost and a translation on an extra direction, often known as the Grant space:

$$ds^2 = -2dX^+dX^- + dX^2 + (dX^i)^2, \quad (X^\pm, X) \sim (e^{\pm 2\pi\beta} X^\pm, X + 2\pi R)$$

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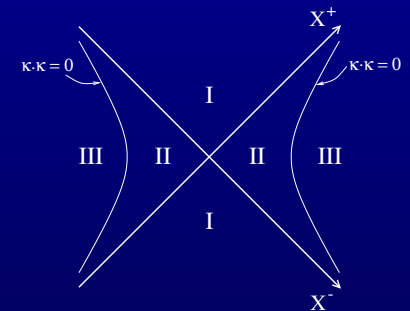
Gott 91, Grant 93

- Defining $Z^\pm = X^\pm e^{\mp\beta X/R}$, the metric can be written in the Kaluza-Klein form

$$ds^2 = R^2(dX + A)^2 - 2dZ^+dZ^- - \frac{E^2}{2R^2}(Z^+dZ^- - Z^-dZ^+)^2, \quad X \equiv X + 2\pi$$

with radius R and KK electric field

$$R^2 = 1 + 2EZ^+Z^-, \quad dA = \frac{E}{R^4}dZ^+dZ^-, \quad E = \beta/R$$



Cornalba Costa

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- All CTC have to pass into $X^+X^- < -1/(2E)$, hence may be suppressed by excising this region: *orientifold boundary conditions ?*

Closed strings in Misner space - untwisted states

- As usual in orbifold constructions, part of the spectrum involves string which close on the (flat) covering space, and are invariant under the orbifold projection:

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- Equivalently, after Poisson resummation over n , their wave function has integer boost momentum $j = x^+ \partial_+ - x^- \partial_-$,

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- The resulting eigenfunctions describe strings with momentum j along the Milne circle. For $k^+ > 0, k^- > 0$, they are exponentially decreasing in the Rindler wedges. j is now the (quantized) Rindler energy.

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Quantum fluctuations and backreaction

- In the Minkowski vacuum (inherited from the covering space), the renormalized propagator can be obtained as a sum over images, e.g in D=4

$$G(x; x') = \sum_{n=-\infty, n \neq 0}^{\infty} [-2(X^+ - e^{2\pi\beta n} X^{+'})(X^- - e^{2\pi\beta n} X^{-'}) + (X^i - X^{i'})^2]^{-1}$$

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$$\langle T_{ab} \rangle = \lim_{x \rightarrow x'} \left[(1 - 2\xi) \nabla_a \nabla'_b - 2\xi \nabla_a \nabla_b + (2\xi - \frac{1}{2}) g_{ab} \nabla_c \nabla'^c \right] G(x, x')$$

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$$\langle T_{\mu}^{\nu} \rangle = \frac{K}{12\pi^2} T^{-4} \text{diag}(1, -3, 1, 1), \quad K = \sum_{n=1}^{\infty} \frac{2 + \cosh 2\pi n\beta}{[\cosh 2\pi n\beta - 1]^2}$$

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Hiscock Konkowski 82

- In the case of the Grant space, the one-loop energy momentum tensor diverges as $1/(R^2 T^2)$ on the chronological horizon, and $1/(T - T_n)^3$ on the polarized hypersurfaces. This is at the basis of Hawking's **chronology protection conjecture**. The divergence seems to have more to do with the CTC than with the singularity...

Scattering of untwisted states

- Scattering amplitudes of untwisted sector states can be computed from those in flat space by the inheritance principle,

$$\langle V(j_1, k_1) \dots V(j_n, k_n) \rangle_{Misner} = \int dv_1 \dots dv_n e^{ij_1 v_1 + \dots + j_n v_n}$$

$$\langle V(e^{\beta v_1} k_1^+, e^{-\beta v_1} k_1^-, k_1^i) \dots V(e^{\beta v_n} k_n^+, e^{-\beta v_n} k_n^-, k_n^i) \rangle_{Minkowski}$$

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- The integral diverges due to Regge behavior in the large momentum, fixed angle regime. E.g, the four-tachyon scattering amplitude in bosonic string leads to

$$\int dv v^{-\frac{1}{2}(k_1^i - k_3^i)^2 + i(j_2 - j_4)}$$

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Berkooz Craps Rajesh Kutasov

- The divergence mostly disappears for the Grant space, except for a localized contribution at $k_1^i = k_3^i$. The amplitude is also finite for transverse gravitons in type II superstring on Misner space, but reappears for longitudinal gravitons.

Berkooz Durin Pioline Reichmann, unpublished

Closed string in Misner space - twisted sectors

- In addition, there are closed string modes on the orbifold that descend from strings on the covering space that close up to the action of the orbifold group:

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau) , \quad \nu = 2\pi\omega\beta$$

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- They have a normal mode expansion:

$$X_R^\pm(\tau - \sigma) = \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha_n^\pm e^{-i(n \pm i\nu)(\tau - \sigma)}$$

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with canonical commutation relations

$$\begin{aligned} [\alpha_m^+, \alpha_n^-] &= -(m + i\nu)\delta_{m+n}, & [\tilde{\alpha}_m^+, \tilde{\alpha}_n^-] &= -(m - i\nu)\delta_{m+n} \\ (\alpha_m^\pm)^* &= \alpha_{-m}^\pm, & (\tilde{\alpha}_m^\pm)^* &= \tilde{\alpha}_{-m}^\pm \end{aligned}$$

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- There are no translational zero-modes, instead two pairs of quasi zero-modes which are canonically conjugate hermitian operators:

$$[\alpha_0^+, \alpha_0^-] = -i\nu, \quad [\tilde{\alpha}_0^+, \tilde{\alpha}_0^-] = i\nu$$

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated, e.g., by

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- This is the analytic continuation of the familiar result for a rotation orbifold $\frac{1}{2}\theta(1 - \theta)$ under $\theta \rightarrow i\nu \dots$
- Due to the $i\nu/2$ term in the ground state energy, all states obtained by acting on $|0\rangle$ by creation operators $\alpha_{n<0}^{\pm}$ and by α_0^+ will have imaginary energy, hence the physical state condition $L_0 = 0$ has no solutions.

One-loop amplitude

- Independently of this fact, one may compute the one-loop (Euclidean ws, Minkowskian target) free energy using path integral methods:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=0}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2 \rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho) x \theta_1(i\beta(l + w\rho); \rho)|^2}$$

where θ_1 is the Jacobi theta function,

$$\theta_1(v; \rho) = 2q^{1/8} \sin \pi v \prod_{n=1}^{\infty} (1 - e^{2\pi i v} q^n)(1 - q^n)(1 - e^{-2\pi i v} q^n), \quad q = e^{2\pi i \rho}$$

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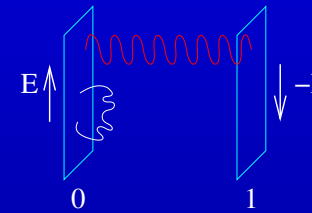
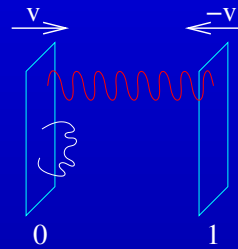
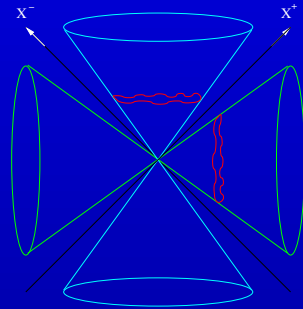
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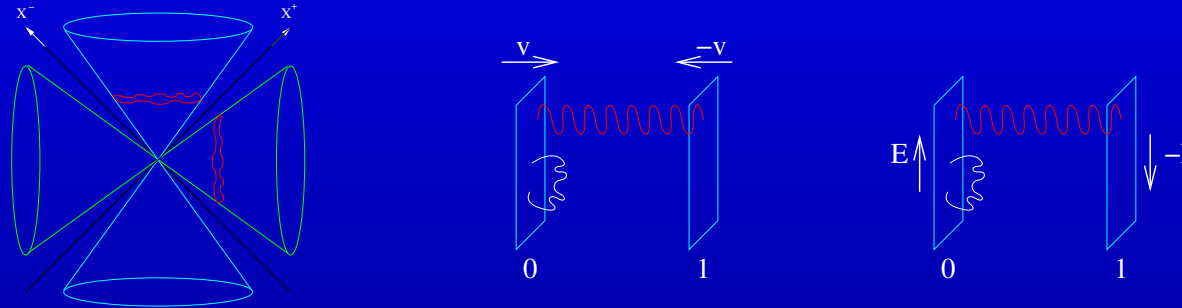
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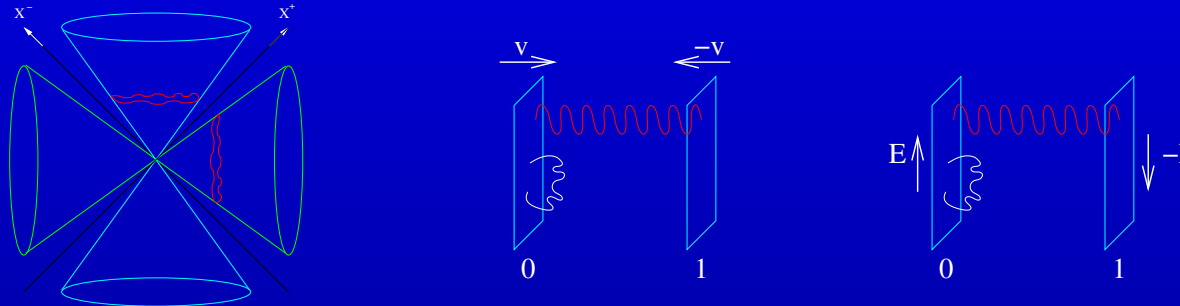


- Recall that for open strings stretched between two D-branes with electromagnetic fields F_0 and F_1 (resp. twisted closed strings on an orbifold $X^\mu \equiv R^\mu_\nu X^\nu$), proper frequencies satisfy

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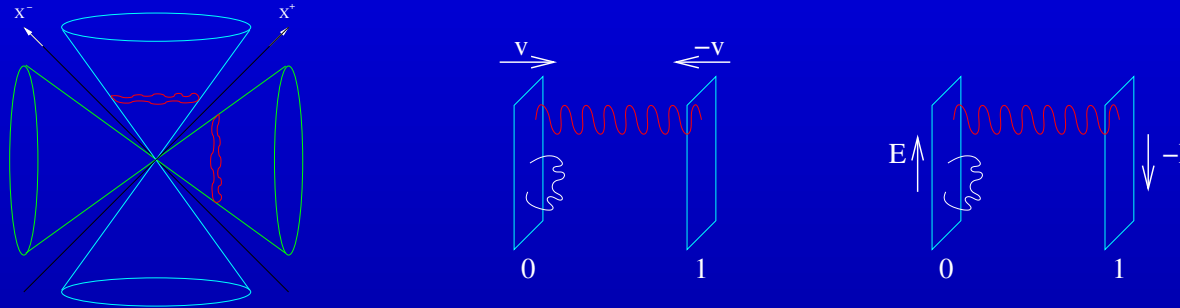
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- More precisely, the charged open string has half the excited modes of the twisted closed strings, and **isomorphic quasi-zero modes**.

Open string mode expansion

- The light-cone embedding coordinates have the normal mode expansion

$$X^\pm = x_0^\pm + i \sum_{n=-\infty}^{+\infty} (-)^n (n \pm i\nu)^{-1} a_n^\pm e^{-i(n \pm i\nu)\tau} \cos[(n \pm i\nu)\sigma]$$

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- The world-sheet Hamiltonian, normal ordered with respect to the vacuum annihilated by $a_{n>0}^+$, $a_{n>0}^-$ and a_0^+ , takes the form

$$L_0^{l.c.} = - \sum_{m=0}^{\infty} a_{-m}^+ a_m^- - \sum_{m=1}^{\infty} a_{-m}^- a_m^+ + \frac{i\nu}{2}(1 - i\nu) - \frac{1}{12}$$

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- By the same token, charged open strings should have no physical states...

One-loop amplitude and Schwinger pair production

- Using this quantization scheme, the one-loop (Euclidean worldsheet, Minkowskian target) vacuum free energy reads

$$A_{bos} = \frac{i\pi V_{26}(e_0 + e_1)}{2} \int_0^\infty \frac{dt}{(4\pi^2 t)^{13}} \frac{e^{-\pi\nu^2 t/2}}{\eta^{21}(it/2) \theta_1(t\nu/2; it/2)}$$

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- Each of the poles at $t = 2k/\nu$ contributes to the imaginary part, yielding the rate for charged string pair production,

$$\mathcal{W} = \frac{1}{2(2\pi)^{25}} \frac{(e_0 + e_1)}{\nu} \sum_{k=1}^{\infty} (-)^{k+1} \left(\frac{|\nu|}{k}\right)^{13} \sum_{N=-1}^{\infty} c_b(N) \exp\left(-2\pi k \frac{N}{|\nu|} - 2\pi k |\nu|\right)$$

Bachas Porrati

where $\eta^{-24}(q) = \sum_{N=-1}^{\infty} c_b(N) q^N$. This can be viewed as the sum of the Schwinger production rates for each state in the spectrum, of mass $m^2 = 2N + \nu^2$.

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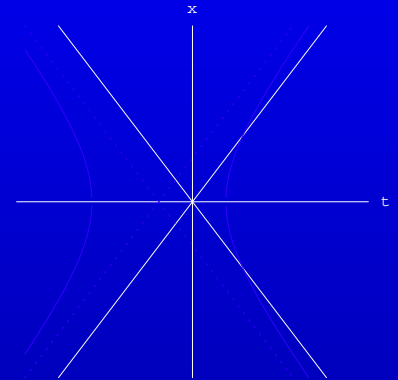
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- This seems to support the quantization scheme based on a vacuum, hence the absence of physical states. But physical states do exist classically, how could quantization make them disappear altogether ?

Open string zero-modes

- Let us reconsider the quantization of the open string zero-mode

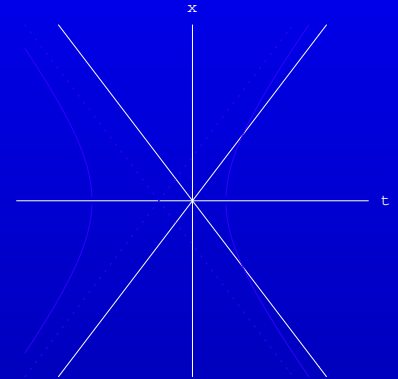
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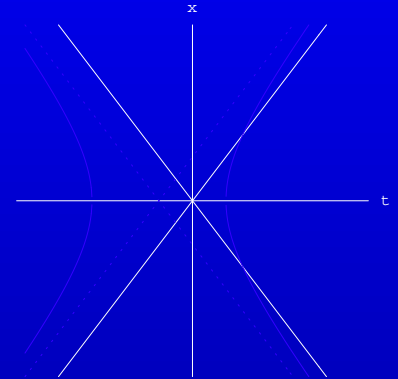
- For $\sigma = 0$, this is just the trajectory of a charged particle in an electric field,

$$L = \frac{1}{2}m \left(-2\partial_\tau X^+ \partial_\tau X^- + (\partial_\tau X^i)^2 \right) + \frac{e}{2} \left(X^+ \partial_\tau X^- - X^- \partial_\tau X^+ \right)$$

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- The canonical momenta

$$\pi^\pm = m \partial_\tau X^\pm \mp \frac{e}{2} X^\pm = \mp \frac{e}{2} x_0^\pm + \frac{1}{2} a_0^\pm e^{\pm e \tau / m}, \quad \pi^i = m \partial_\tau X^i = p^i$$

satisfy the usual equal-time commutation rules

$$[\pi^+, x^-] = [\pi^-, x^+] = i, \quad [\pi^i, x^j] = i \delta_{ij}$$

Quantizing the open string zero-modes

- At $\tau = 0$, one can thus express

$$a_0^\pm = \pi^\pm \pm \frac{\nu}{2} x^\pm, \quad x_0^\pm = \mp \frac{1}{\nu} \left(\pi^\pm \mp \frac{\nu}{2} x^\pm \right)$$

hence recovering the canonical commutation relations of the open string zero-mode:

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- Quantum mechanically, one may represent $\pi^\pm = i\partial/\partial x^\mp$ so that a_0^\pm become **covariant derivatives** in the electric field ν .
- The zero-mode piece of L_0 , **including the evil** $\frac{i\nu}{2}$,

$$L_0^{(0)} = -a_0^+ a_0^- + \frac{i\nu}{2} = -\frac{1}{2}(a_0^+ a_0^- + a_0^- a_0^+)$$

is just the **Klein-Gordon operator** of a particle of 2D mass $M^2 = -2L_0^{(0)}$ and charge ν .

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^\pm = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator just becomes an **inverted harmonic oscillator**:

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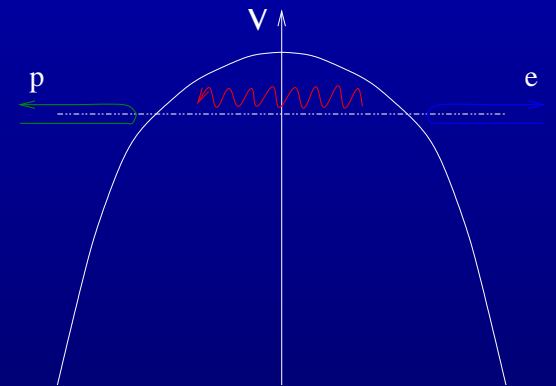
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- The latter admits a respectable **delta-normalizable spectrum of scattering states**, in terms of **parabolic cylinder functions**, e.g:

$$\phi_{in}^+ = D_{-\frac{1}{2} + i\frac{M^2}{2\nu}} \left(e^{-\frac{3i\pi}{4}} u \right) e^{-i\tilde{p}t} e^{i\nu x t/2}$$



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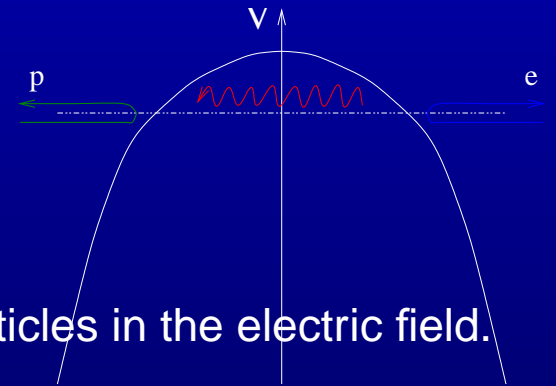
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- These correspond to **non-compact trajectories** of charged particles in the electric field. **Tunnelling** is just (stimulated) **Schwinger pair creation**,

$$e^- \rightarrow (1 + \eta) e^- + \eta e^+, \quad \eta \sim e^{-\pi M^2/\nu}$$



Lorentzian vs Euclidean states

- Analytic continuation $X^0 \rightarrow -iX^0$, $\nu \rightarrow i\nu$ takes us from an electric field in $R^{1,1}$ to a magnetic field in R^2 . At the same time, one should Wick rotate the worldsheet time.

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- The contribution of the zero-modes to the one-loop amplitude can be interpreted either way,

$$\frac{1}{2i \sin(\nu t/2)} = \sum_{n=1}^{\infty} e^{-i(n+\frac{1}{2})\nu t} = \int dM^2 \rho(M^2) e^{-M^2 t/2}$$

where the density of states is obtained from the reflection phase shift,

$$\rho(M^2) = \frac{1}{\nu} \log \Lambda - \frac{1}{2\pi i} \frac{d}{dM^2} \log \frac{\Gamma\left(\frac{1}{2} + i\frac{M^2}{2\nu}\right)}{\Gamma\left(\frac{1}{2} - i\frac{M^2}{2\nu}\right)}$$

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- The physical spectrum can be explicitly worked out at low levels, and is **free of ghosts**: a tachyon at level 0, a **transverse gauge boson** at level 1, ...

Physical spectrum at low level

- The ground state **tachyon**

$$|T\rangle = \phi(x^+, x^-) |0_{ex}, k\rangle$$

should satisfy the Virasoro constraint

$$L_0 |T\rangle = \left[-\frac{1}{2} (\alpha_0^+ \alpha_0^- + \alpha_0^- \alpha_0^+) + \frac{1}{2} \nu^2 - 1 + \frac{1}{2} k_i^2 \right] |T\rangle$$

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- The L_1 **Virasoro constraint** eliminates one polarization. Despite the non-vanishing two-dimensional mass $k_i^2 - \nu^2$, the **spurious state** $L_{-1}\phi|0\rangle$ is still physical, eliminating an extra polarization.
- One thus has $D - 2$ **transverse** degrees of freedom, ie a **massless gauge boson** in D dimensions.

Charged particle in Rindler space

- For applications to the Milne universe, one should diagonalize the boost momentum J , ie consider an accelerated observer.

Gabriel Spindel; Mottola Cooper

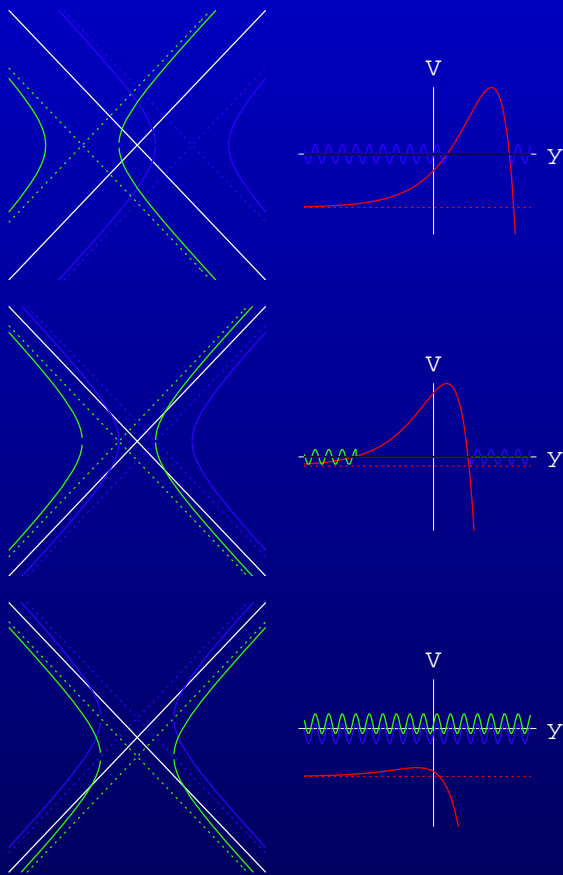
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- In the **Rindler** patch R , letting $f(r, \eta) = e^{-iJ\eta} f_J(r)$ and $r = e^y$, one gets a Schrodinger equation for a particle in a potential

$$V(y) = M^2 e^{2y} - \left(J + \frac{1}{2}\nu e^{2y}\right)^2$$



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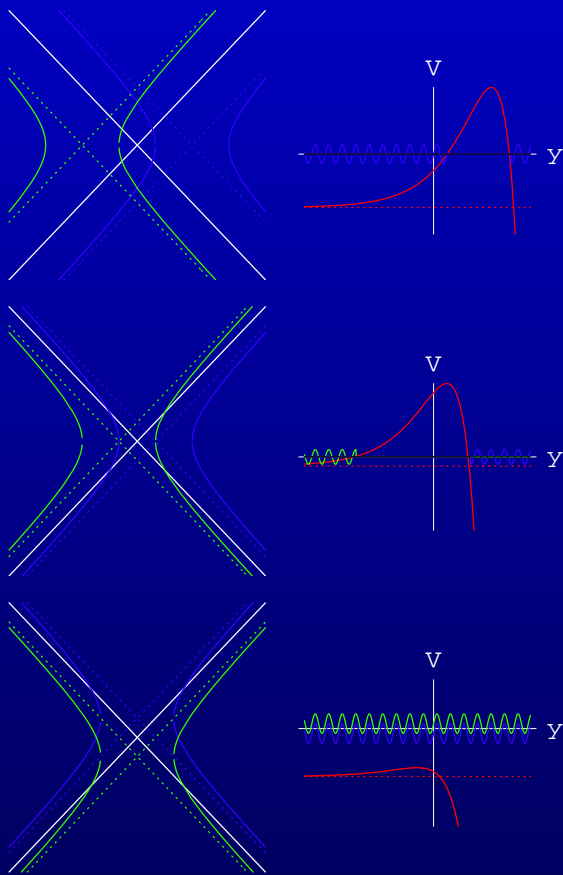
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Charged particle in Rindler space

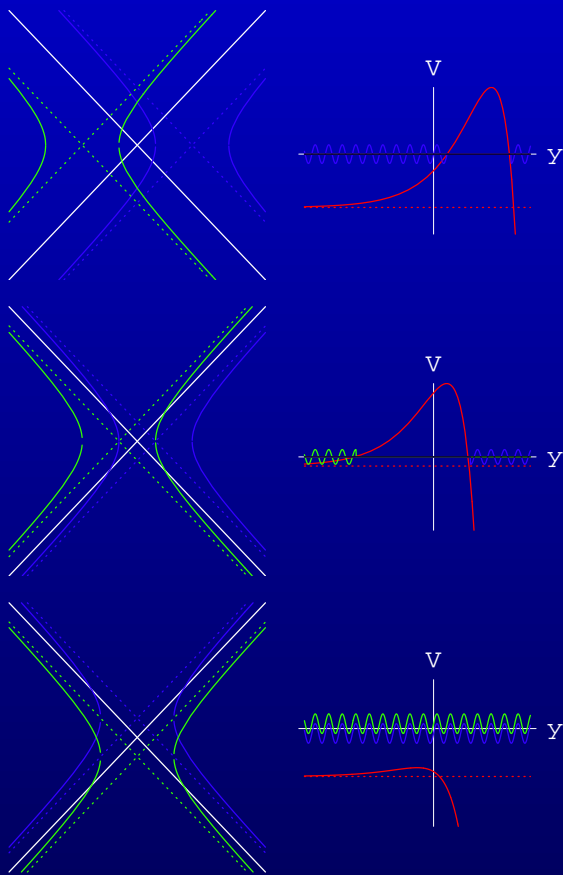
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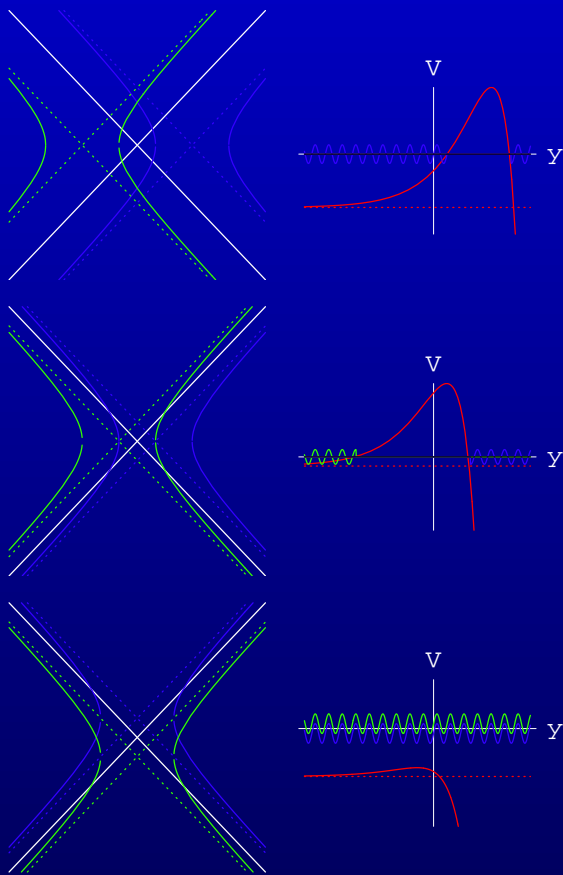
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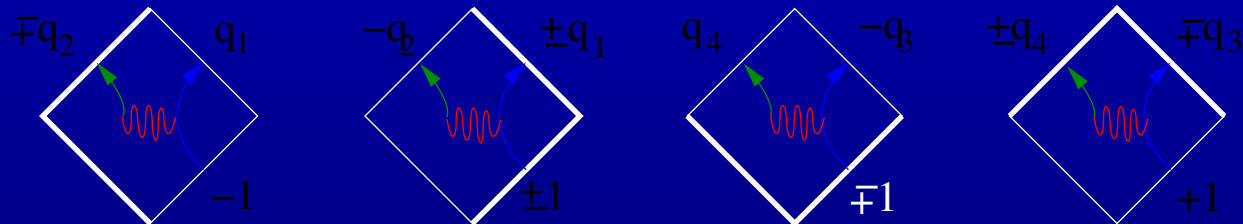
Rindler modes

- Solutions are expressable in terms of parabolic cylinder functions:
Incoming modes from Rindler infinity I_R^- read

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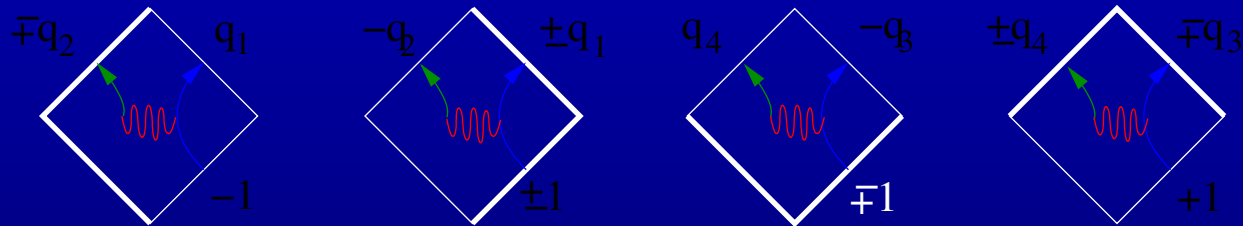
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- The reflection coefficients can be computed:

$$q_1 = e^{-\pi j} \frac{\cosh \left[\pi \frac{M^2}{2\nu} \right]}{\cosh \left[\pi \left(j - \frac{M^2}{2\nu} \right) \right]}, \quad q_3 = e^{\pi \left(j - \frac{M^2}{2\nu} \right)} \frac{\cosh \left[\pi \frac{M^2}{2\nu} \right]}{|\sinh \pi j|}$$

and $q_2 = 1 - q_1, q_4 = q_3 - 1$, by charge conservation.

Global Charged Unruh Modes

- Global Unruh modes may be defined by patching together Rindler modes, ie by **analytic continuation across the horizons**:

$$\Omega_{in,+}^j = \mathcal{V}_{in,P}^j = W_{-i(\frac{j}{2}-\frac{m^2}{2\nu}), \frac{ij}{2}}(-i\nu X^+ X^-)[X^+/X^-]^{-ij/2}$$

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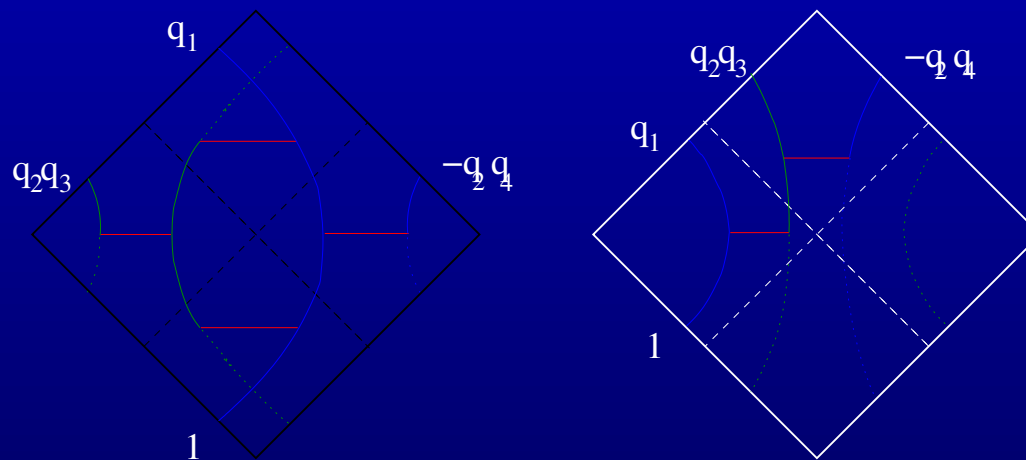
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- There are two types of modes, involving 2 or 4 tunnelling events:



Closed string zero-modes

- Let us analyze the classical solutions for the closed string zero modes

$$X^\pm(\tau, \sigma) = \pm \frac{1}{2\nu} \alpha_0^\pm e^{\pm\nu(\tau-\sigma)} \mp \frac{1}{2\nu} \tilde{\alpha}_0^\pm e^{\mp\nu(\tau+\sigma)}, \quad \alpha_0^\pm, \tilde{\alpha}_0^\pm \in R$$

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- The behavior at early/late proper time now depends on $\epsilon\tilde{\epsilon}$: For $\epsilon\tilde{\epsilon} = 1$, the string begin/ends in the Milne regions. For $\epsilon\tilde{\epsilon} = -1$, the string begin/ends in the Rindler regions.

Short and long strings ($j = 0$)

- $\epsilon = 1, \tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma, \tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma}, \quad T = \frac{M}{\nu} \sinh(\nu\tau), \quad \theta = \nu\sigma$$

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- For $|j| > 0$, the short string in the Milne region attaches to a short string in the Rindler region stretching from $r = 0$ to $r_0 = |j|/(M + \tilde{M})$ and back. The induced worldsheet metric is of Misner type at the light-cone:

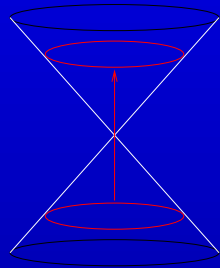
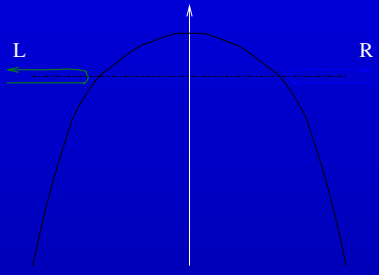
$$-2dX^+dX^- = -\nu j d\tau d\sigma + \nu |j| (\tau - \tau_0) d\sigma^2 - \frac{1}{2}(M^2 + \tilde{M}^2) d\tau^2$$

much like long strings or supertubes in Gödel Universe.

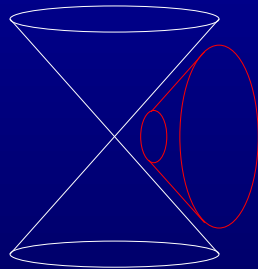
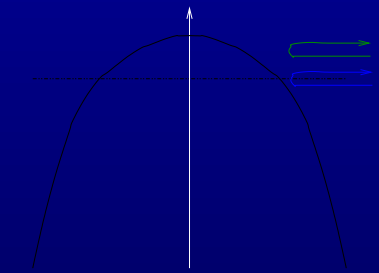
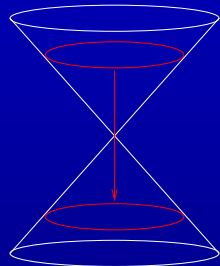
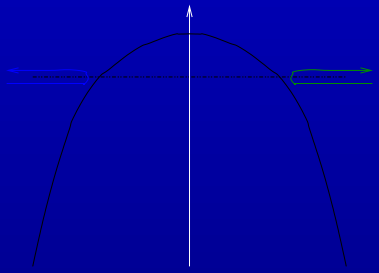
Drukker Fiol Simon; Israel

Short and long strings (static modes)

Just as in the open string case, we may now quantize the left and right-moving zero-modes separately as particles in inverted harmonic oscillator:

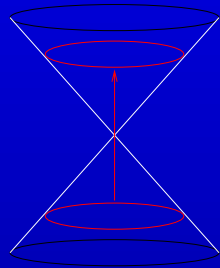
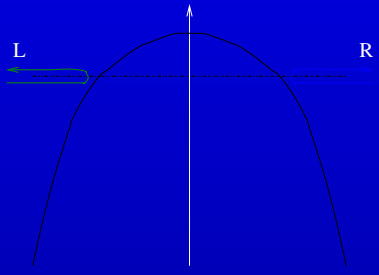


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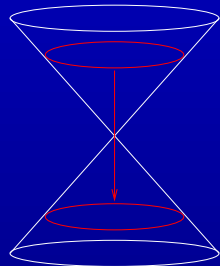
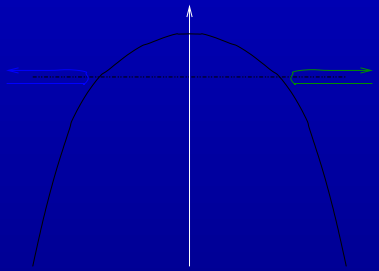


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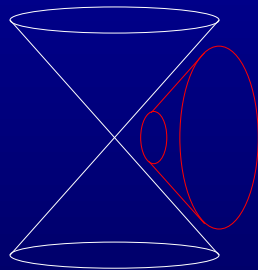
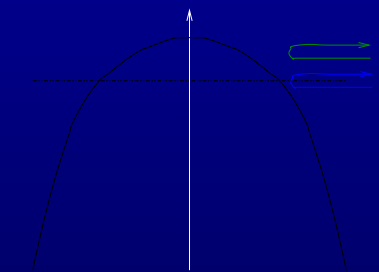
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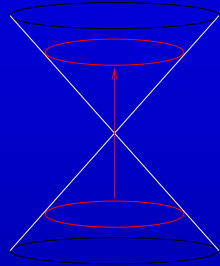
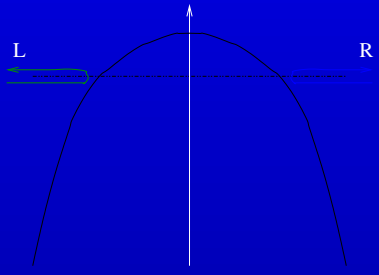


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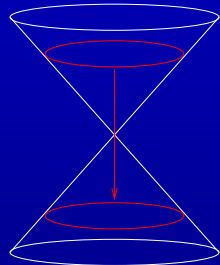
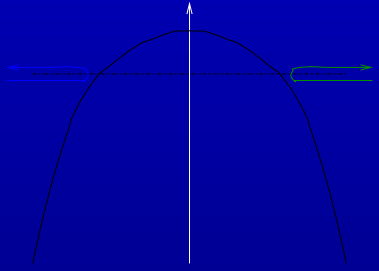


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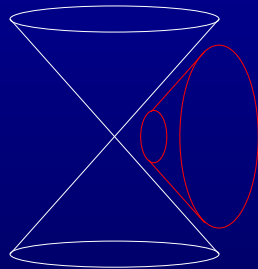
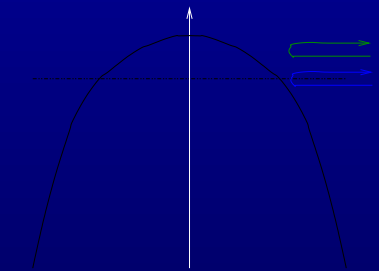
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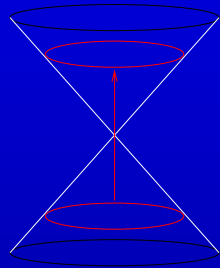
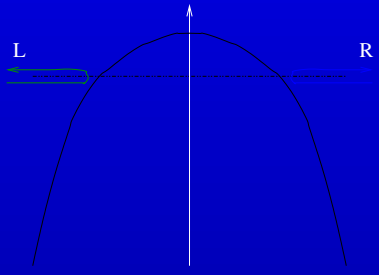
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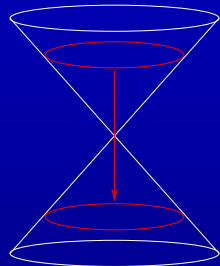
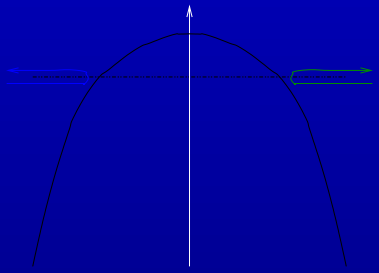
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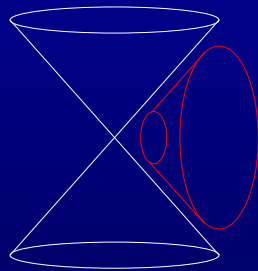
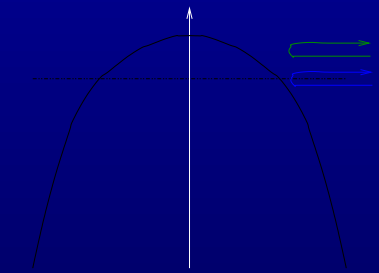
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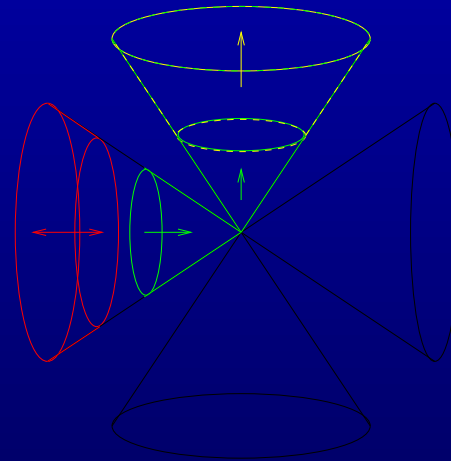
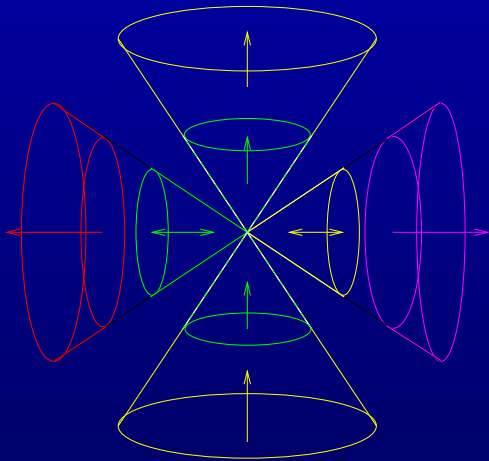
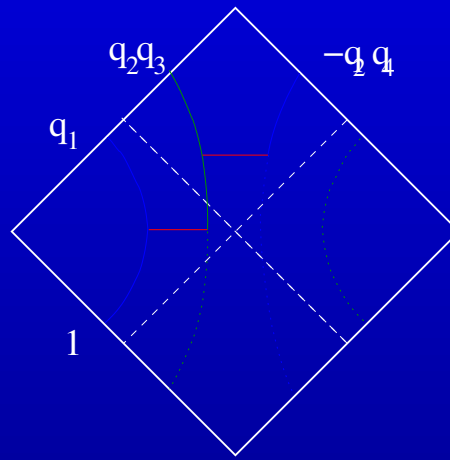
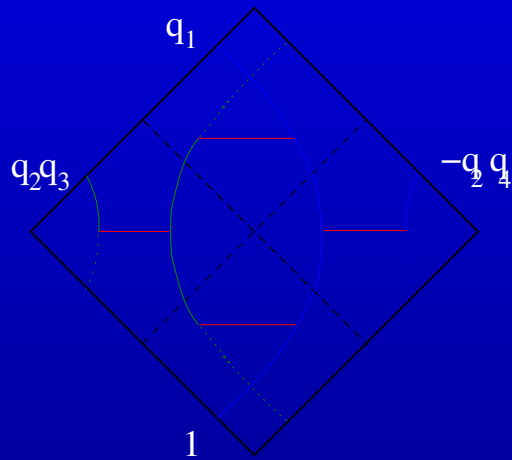


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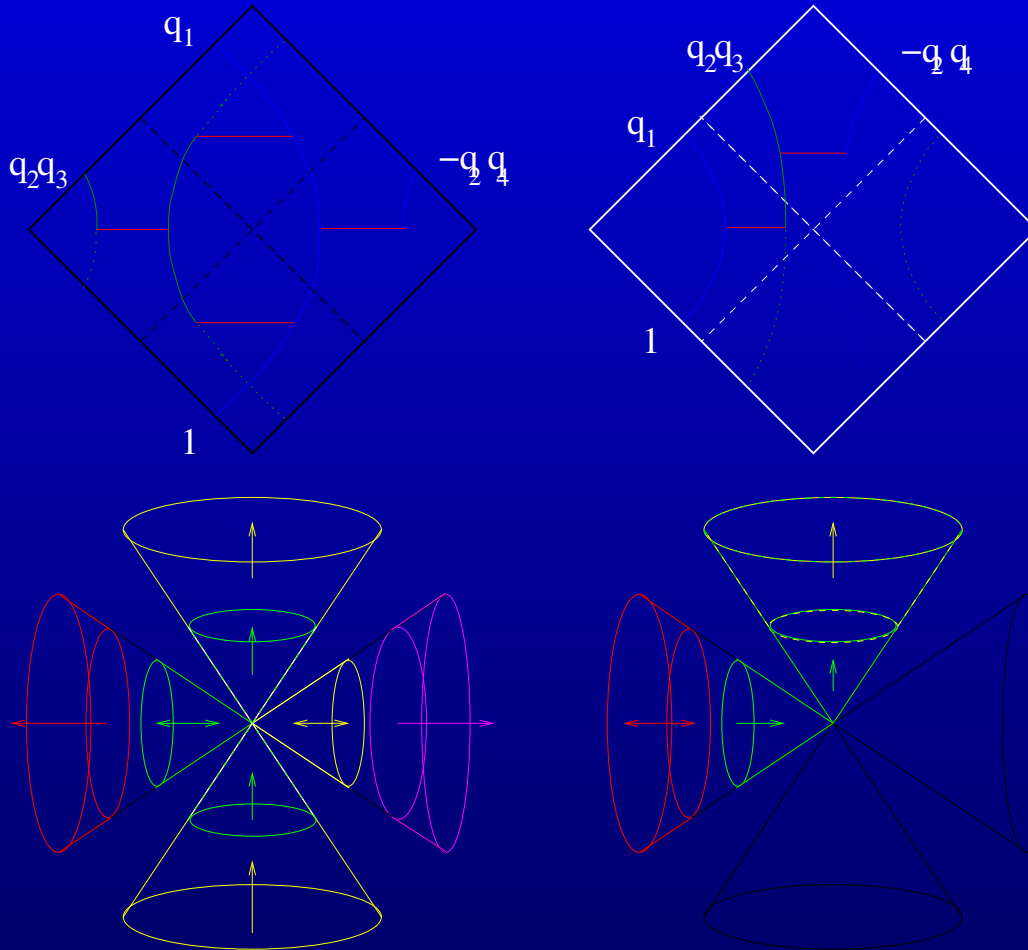
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- Instead of following the motion of a point at fixed σ , one may consider instead fixed $\sigma + \tau$: these are the trajectories of the open string zero-mode, in Rindler coordinates.



- The probability amplitude of winding strings at $T = +\infty$, assuming that there are no stretched pairs in the whiskers, is q_1 times the incoming amplitude at $T = -\infty$.

$$q_1 = e^{-\pi j} \frac{\cosh \left[\pi \frac{M^2}{2\nu} \right]}{\cosh \left[\pi \left(j - \frac{M^2}{2\nu} \right) \right]},$$

hence $q_1 = 0$ if $j = 0$.

The one-loop amplitude again

- Recall the (Euclidean ws, Minkowskian target) one-loop amplitude:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=0}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2 \rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho) \theta_1(i\beta(l+w\rho); \rho)|^2}$$

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Maldacena Ooguri

- These can be traced to the existence of infinite families of periodic orbits, localized on the light-cone (currently under investigation)

Wick rotation to a rotation orbifold

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- Rotating $\beta = i\mu$, the Rindler region becomes get indeed an Euclidean metric,

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- By the same token, the left Rindler wedge rotates to another copy of the Euclidean plane with the origin removed: the complete analytic continuation of Misner space is therefore

$$\widetilde{R^2 \setminus \{0\}}_L / e^{i\mu} \setminus \widetilde{R^2 \setminus \{0\}}_R$$

and states of interest are **non-normalizable** !

Conclusions - speculations

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- To demonstrate this, one should take into account the production of (an infinite number) of twisted sector states are produced in correlated pairs, i.e. squeezed states: non-local deformations of the worldsheet ? string field theory ?

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- As a less ambitious goal, can one compute **scattering amplitudes of twisted states**, and check if they are better behaved than untwisted states. For this, the relation with **negative level $Sl(2)/U(1)$** and double analytic continuation of the **Nappi-Witten plane wave** may be useful.

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- The closed string orbifold we have discussed are highly **non-generic** trajectories on the **cosmological billiard**: Do whiskers feature also for more general Kasner-like singularities ?