Closed Strings in the Misner Universe

aka the Lorentzian orbifold

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Talk based on

hep-th/0307280 w/ M. Berkooz
hep-th/0405126 w/ M. Berkooz, and M. Rozali
hep-th/0406xxx w/ M. Berkooz, B. Durin and D. Reichmann

slides available from
`http://www.lpthe.jussieu.fr/˜pioline/seminars.html`

Motivational string cosmology

- Observational Cosmology is now challenging string theory with high-precision data:

$$\Omega_{baryon} = 0.047\ , \quad \Omega_{darkm} = 0.243\ , \quad \Omega_{\Lambda} = 0.71\ , \quad w = -0.98 \pm .12\ , \ldots$$

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- With LHC still far in the future, understanding StringY Cosmology may bring us to closer contact with reality...

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In addition, String theory is not content on a finite time interval, and one is frequently forced into Big Bang / Big Crunch singularities, CTC in the process of maximally extending the geometry.

String theory and cosmological singularities

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- Usual Euclidean orbifolds, with a time-like singularity, can be resolved by condensation of twisted strings. How about non-static orbifolds ?

- In this talk, we shall study the “Lorentzian” orbifold of flat Minkowski space by a discrete boost, as a toy model of a singular cosmological universe where string theory can in principle be solved explicitely.

Outline of the talk

1. Euclidean and Lorentzian orbifolds, and their avatars

   Misner, Taub-NUT, Grant...

2. Untwisted strings in Misner space

   Hiscock, Konkowski; Berkooz Craps Kutasov Rajesh, . . .

3. Twisted strings in Misner space: first pass

   Nekrasov

3. A detour: Open strings in electric fields

   Bachas Porrati; Berkooz BP

4. Twisted strings in Misner space: second pass

   Berkooz BP Rozali

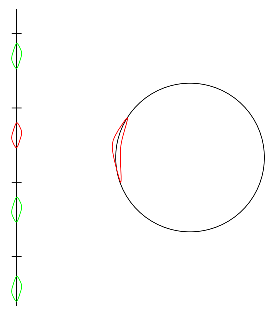
5. Comments on backreaction from winding strings

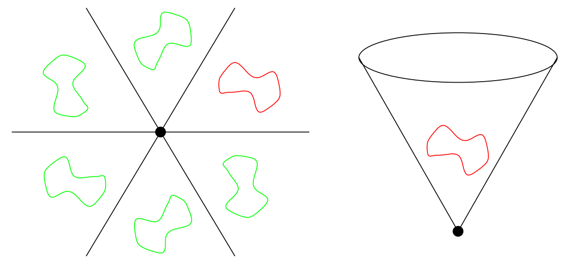
Strings on Euclidean orbifolds - untwisted states

- Well-known examples of orbifolds are the circle, R/Z, and the rotation orbifold R^2/Z_k.

Dixon Harvey Vafa Witten

- The spectrum of the quotient theory contains closed string states of the parent theory which are invariant under G: untwisted states.





Strings on Euclidean orbifolds - twisted states

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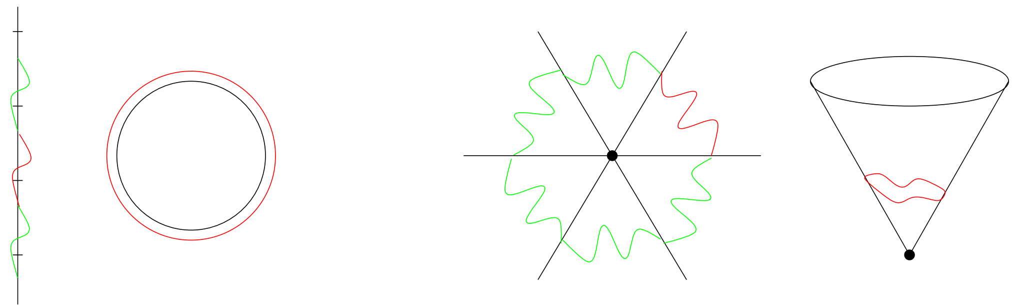
- Modular invariance requires that the spectrum should also include closed strings in the quotient theory which close up to the action of G in the parent theory: twisted states.



- When G acts non-freely, the twisted sector states are localized at the fixed points. They yield new localized degrees of freedom, which ensure the consistency of the background: anomaly free, divergence free...

Strings on Euclidean orbifolds - twisted sectors (cont.)

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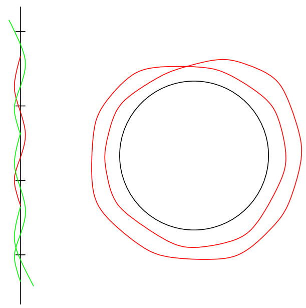
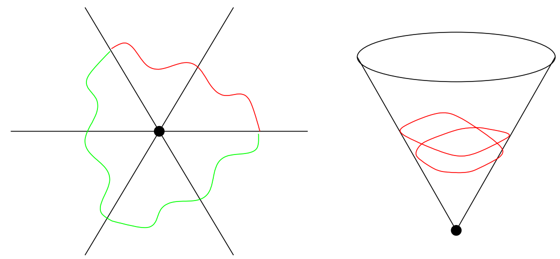
- Twisted sectors are labelled by conjugacy classes of G. Higher twisted sectors correspond to multiply wound states.





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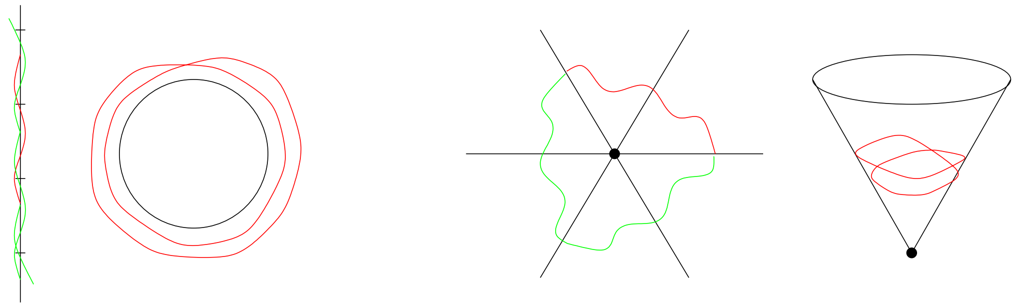

- Additionally, each twisted sector admits excited levels. The ground state can be thought of as a Gaussian wave function centered at the origin.

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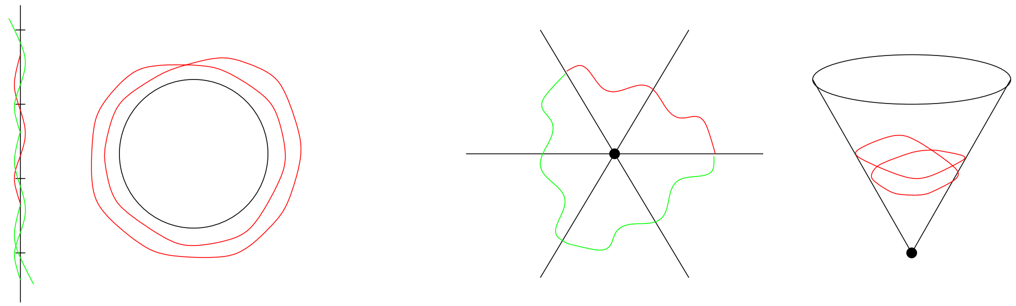

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- The condensation of these twisted states changes the vacuum, and effectively resolves the singularity: $R^2/Z_k \to R^2/Z_{k-1} \to \ldots$ (tachyon), $R^4/Z_k \to$ multi-centered

Eguchi-Hanson (massless mode).

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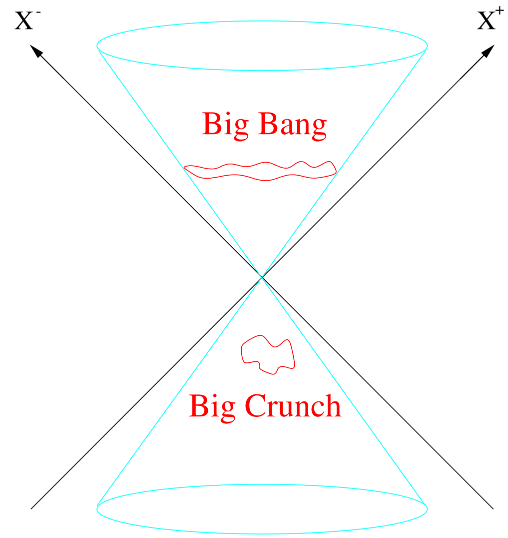
- The Lorentzian orbifold shares features with both examples: an infinite number of winding sectors, and a, non compact, fixed locus.

The Lorentzian orbifold

- One of the simplest examples of space-like singularities is the quotient of flat Minkowski space by a discrete boost, also known as Misner space (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^\pm \sim e^{\pm 2\pi\beta} X^\pm$$



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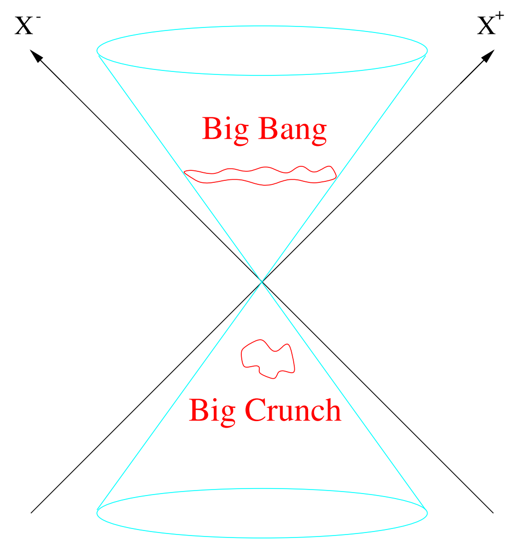

- The future (past) regions $X^+X^- > 0$ describes a cosmological universe often known as the Milne universe (1932), linearly expanding away from a Big Bang singularity (or contracting into a Big Crunch singularity):

$$ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + (dX^i)^2 \,, \quad \theta \equiv \theta + 2\pi \,, \quad X^\pm = T e^{\pm\beta\theta}/\sqrt{2}$$

This is a (degenerate) Kasner singularity, everywhere flat, except for a delta-function curvature at $T = 0$.

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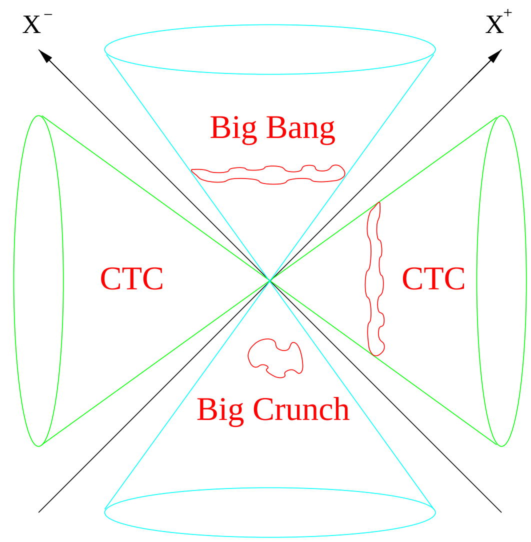

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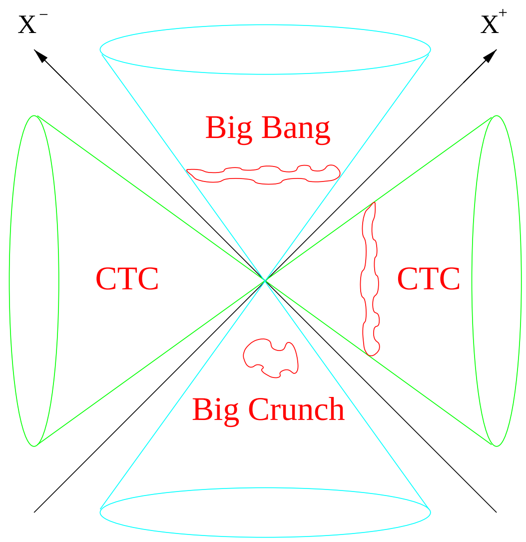

- In addition, the spacelike regions $X^+X^- < 0$ describe two Rindler wedges with compact time, often known as whiskers, leading to closed time-like curves:

$$ds^2 = dr^2 - \beta^2 r^2 d\eta^2 + (dX^i)^2 \quad , \eta \equiv \eta + 2\pi \ , \quad X^\pm = \pm r e^{\pm\beta\eta}/\sqrt{2}$$

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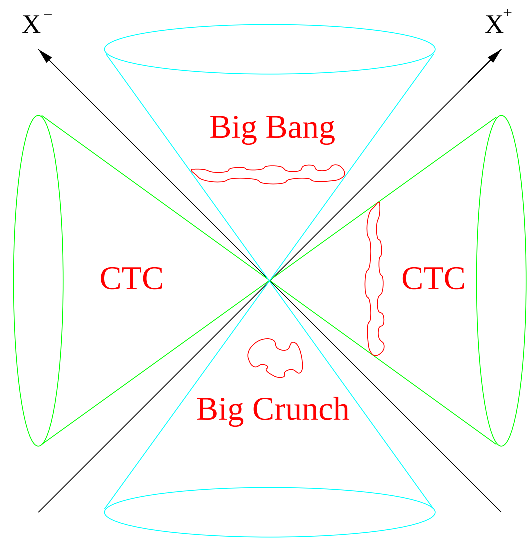

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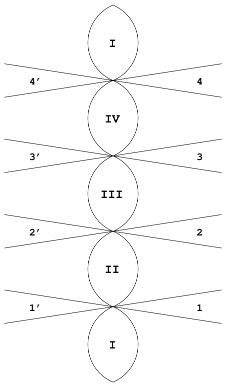
- Finally, the lightcone $X^+X^- = 0$ gives rise to a null, non-Hausdorff locus attached to the singularity.

Close relatives of the Misner Universe

- Misner space was first introduced as a local model of Lorentzian Taub-NUT space:



$$ds^2 = 4l^2 U(t)\sigma_3^2 + 4l\sigma_3 dt + (t^2+l^2)(\sigma_1^2+\sigma_2^2)\,, \quad U(t) = -1 + \frac{2mt + l^2}{t^2+l^2}$$

A bouncing universe, isomorphic to $R^{1,1}/boost \times S^2$ around each singularity.

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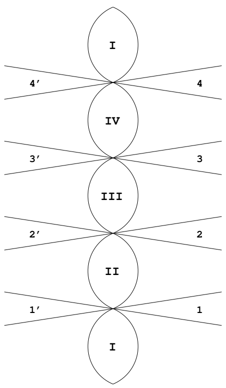

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- A close variant of Misner space is the quotient of flat space by the combination of a discrete boost and a translation on an extra direction, often known as the Grant space:

$$ds^2 = -2dX^+dX^- + dX^2 + (dX^i)^2\,, \quad (X^\pm, X) \sim (e^{\pm 2\pi\beta}X^\pm, X + 2\pi R)$$

This describes the space away from two moving cosmic strings. The cosmological singularity is smoothed out, but regions with CTC remain.

Gott 91, Grant 93; Cornalba, Costa, Kounnas

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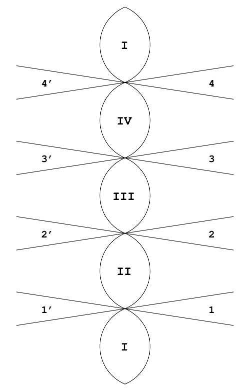

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- The Misner geometry arose again more recently as the M-theory lift of a simple (ekpyrotic) cosmological solution of Einstein-dilaton gravity with no potential.

Khoury Ovrut Seiberg Steinhard Turok

Close relatives of the Misner Universe (cont)

- The gauged WZW model $Sl(2) \times Sl(2)/U(1) \times U(1)$ describes a bouncing 4-dimensional Universe, with singularities analogous to the Lorentzian orbifold.

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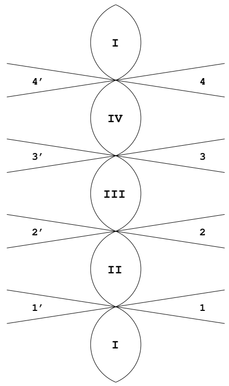

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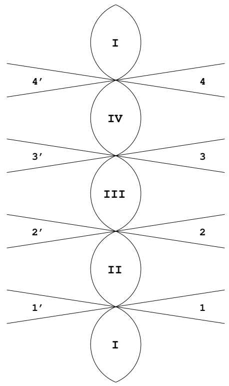

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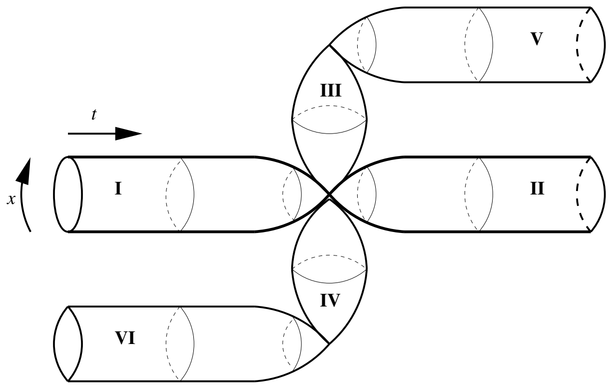

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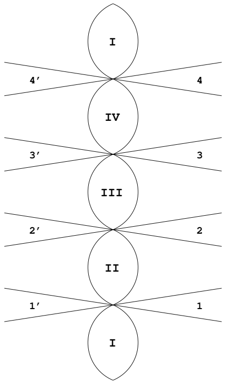

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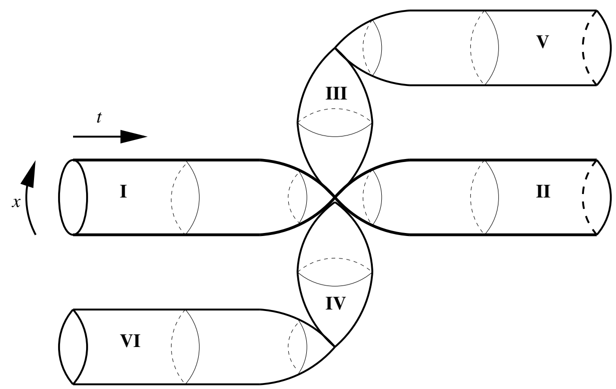

- The Lorentzian orientifold $IIB/[(-)^F boost]/[\Omega(-)^{F_L}]$ was also recently argued to describe orientifolds of non-supersymmetric strings with non-vanishing Neveu-Schwarz tadpoles.

Closed strings in Misner space - untwisted states

- As usual in standard orbifolds, part of the spectrum involves closed strings on Minkowski covering space, which are invariant under the orbifold projection. In conformal gauge,

$$X^{\pm}(\sigma + 2\pi, \tau) = X^{\pm}(\sigma, \tau) \, , \quad (\partial_\tau^2 - \partial_\sigma^2) X^{\pm} = 0$$

satisfying the Virasoro (physical state) condition $(\dot{X}^{\mu} \pm X'^{\mu})^2 = 0$.

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- Vertex operators (or states) can be obtained by (infinite) sum over images, e.g.

$$\sum_{l=-\infty}^{\infty} \partial X^+ \bar{\partial} X^- \exp\left(i k^+ X^- e^{-2\pi\beta l} + i k^- X^+ e^{2\pi\beta l} + i k_i X^i \right)$$

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- Equivalently, after Poisson resummation over l, this is a superposition of states with integer boost momentum $j = i(x^+\partial_+ - x^-\partial_-)$,

$$(\sum_{j=-\infty}^{\infty}) \partial X^+ \bar{\partial} X^- \int_{-\infty}^{\infty} dv \exp\left(ik^+ X^- e^{-2\pi\beta v} + ik^- X^+ e^{2\pi\beta v} + ik_i X^i + ivj \right)$$

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- Equivalently, after Poisson resummation over l, this is a superposition of states with integer boost momentum $j=i(x^+\partial_+-x^-\partial_-)$,

$$(\sum_{j=-\infty}^{\infty})\partial X^+\bar\partial X^-\int_{-\infty}^{\infty}dv\exp\left(ik^+X^-e^{-2\pi\beta v}+ik^-X^+e^{2\pi\beta v}+ik_iX^i+ivj\right)$$

- The resulting eigenfunctions describe (topologically trivial) closed strings traveling around the Milne circle with integer momentum j.

Quantum fluctuations in field theory

- In the Minkowski vacuum (inherited from the covering space), the renormalized propagator can be obtained as a sum over images,

$$
\begin{aligned}
G(x;x') \quad = \quad & \sum_{l=-\infty, l\neq 0}^{\infty} \int_0^{\infty} d\tau \int dp^{\mu} \\
& \exp\left(-ip^-(x^+ - e^{2\pi\beta l}x^{+\prime}) - ip^+(x^- - e^{2\pi\beta l}x^{-\prime}) - ip^i(x^i - x^{i\prime})\right)
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- The one-loop stress-energy tensor follows from $G(x,x)$, e.g for a conformally coupled scalar,

$$\langle T_{ab}\rangle = \lim_{x\to x'}\left[(1-2\xi)\nabla_a\nabla'_b - 2\xi\nabla_a\nabla_b + (2\xi - \frac{1}{2})g_{ab}\nabla_c\nabla'^{c}\right]G(x,x')$$

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This leads to a divergent quantum backreaction (worse if the spin $|s| > 1$):

$$\langle T_\mu^\nu\rangle = \frac{K}{12\pi^2}T^{-4}\text{diag}(1,-3,1,1)\ , \quad K = \sum_{l=1}^{\infty}\cosh(2\pi\beta l s)\frac{2+\cosh 2\pi l\beta}{[\cosh 2\pi l\beta - 1]^2}$$

Hiscock Konkowski 82

One-loop vacuum amplitude in field and string theory

- On the other hand, in string theory $\langle T_\mu^\nu \rangle(x)$ is an off-shell quantity, and only its integral over space-time is well defined:

$$\int dx^+ dx^- G(x,x) = \sum_{l=-\infty}^{+\infty} \int_0^\infty \frac{d\rho}{\rho^{D/2}} \frac{e^{-m^2\rho}}{\sinh^2(\pi\beta l)}$$

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- This reproduces the zero-mode contribution to the string one-loop vacuum amplitude in the untwisted sector:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{\left|\eta^{21}(\rho)\ \theta_1(i\beta(l+w\rho);\rho)\right|^2}$$

$$\theta_1(v;\rho) = 2q^{1/8} \sin \pi v \prod_{n=1}^{\infty} (1-e^{2\pi i v} q^n)(1-q^n)(1-e^{-2\pi i v} q^n)\ , \quad q = e^{2\pi i \rho}$$

Nekrasov, Cornalba Costa

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- The existence of Regge trajectories with arbitrary high spin implies new (log) divergences in the bulk of the moduli space which resemble long string poles in AdS_3.

Scattering of untwisted states

- Tree-level scattering amplitudes of untwisted sector states can be computed from those in flat space by the inheritance principle,

$$\langle V(j_1,k_1)\dots V(j_n,k_n)\rangle_{Misner} = \int dv_1 \dots dv_n \, e^{i(j_1 v_1+\dots+j_n v_n)}$$
$$\langle V(e^{\beta v_1}k_1^+, e^{-\beta v_1}k_1^-, k_1^i)\dots V(e^{\beta v_n}k_n^+, e^{-\beta v_n}k_n^-, k_n^i)\rangle_{Minkowski}$$

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- The integral diverges due to Regge behavior in the large momentum, fixed angle regime. E.g, the four-tachyon scattering amplitude in bosonic string leads to

$$\int dv \; v^{-\frac{1}{2}(k_1^i - k_3^i)^2 + i(j_2 - j_4)}$$

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Berkooz Craps Rajesh Kutasov

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Berkooz Craps Rajesh Kutasov

- Could higher order corrections,e.g. resummation of ladder diagrams, lead to a finite amplitude ?

Deser McCarthy Steif; Cornalba Costa

Closed string in Misner space - twisted sectors

- In addition, there is an infinite set of twisted sectors, corresponding to strings on the covering space that close up to the action of the orbifold group:

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau) \ , \quad \nu = 2\pi w \beta$$

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- They have a normal mode expansion:

$$\begin{aligned} X^{\pm}_R(\tau - \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha^{\pm}_n \, e^{-i(n \pm i\nu)(\tau - \sigma)} \\ X^{\pm}_L(\tau + \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \mp i\nu)^{-1} \tilde{\alpha}^{\pm}_n \, e^{-i(n \mp i\nu)(\tau + \sigma)} \end{aligned}$$

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with canonical commutation relations

$$[\alpha_m^+, \alpha_n^-] = -(m + i\nu)\delta_{m+n} \quad , \quad [\tilde{\alpha}_m^+, \tilde{\alpha}_n^-] = -(m - i\nu)\delta_{m+n}$$
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- We will focus on the quasi zero-mode sector, which consists of two commuting pairs of real (i.e. hermitian) canonically conjugate operators,

$$[\alpha_0^+, \alpha_0^-] = -i\nu \ , \quad [\tilde{\alpha}_0^+, \tilde{\alpha}_0^-] = i\nu$$

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated by half of them, e.g.

$$\alpha^{\pm}_{n>0}\ , \quad \tilde{\alpha}^{\pm}_{n>0}\ , \qquad \alpha^{-}_{0}\ , \quad \tilde{\alpha}^{+}_{0}$$

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- The worldsheet Hamiltonian, normal-ordered wrt to this vacuum, reads

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  with a similar answer for $\tilde L_0$.
- This is the familiar result for the vacuum energy $\frac{1}{2}\theta(1-\theta)$ in the Euclidean rotation orbifold, after analytically continuing $\theta \to i\nu$.
- Due to the $i\nu/2$ term in the ground state energy, all states obtained by acting on $|0\rangle$ by creation operators $\alpha^{\pm}_{n<0}$ and by α_0^+ will have imaginary energy, hence the physical state conditions $L_0 = \tilde L_0 = 0$ seem to have no solutions.

One-loop amplitude, twisted sector

- Independently of this fact, one may compute the one-loop path integral on an Euclidean worldsheet and Minkowskian target-space:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho)x\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

where θ_1 is the Jacobi theta function,

$$\theta_1(v;\rho) = 2q^{1/8}\sin\pi v \prod_{n=1}^{\infty}(1-e^{2\pi i v}q^n)(1-q^n)(1-e^{-2\pi i v}q^n)\ , \quad q = e^{2\pi i \rho}$$

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- In the twisted sector, the left-moving zero-modes contribute

$$\frac{1}{2\sinh(\beta w\rho)]} = \sum_{n=1}^{\infty} q^{i(n+\frac{1}{2})\beta w}$$

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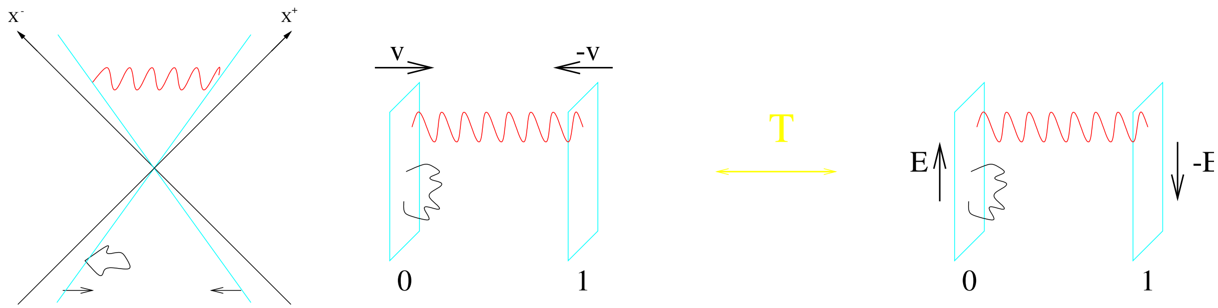
A detour via Open strings in electric field

- A very similar puzzle is faced in the case of colliding D-branes, or in the T-dual process of charged open strings in a constant electric field:



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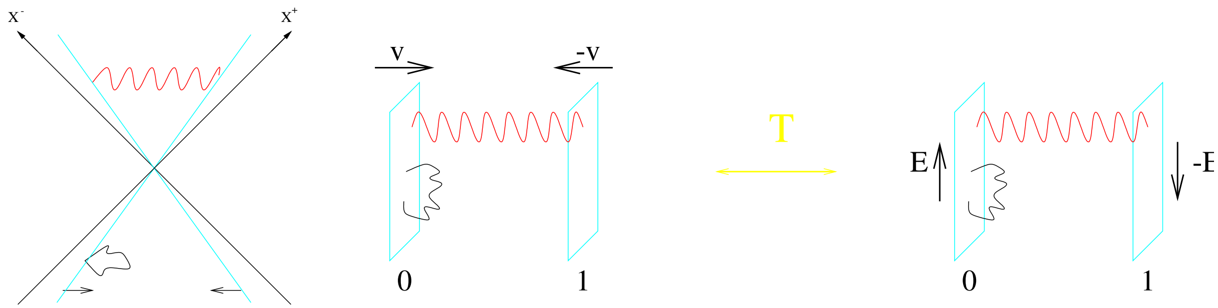

- Open strings stretched between two D-branes with electric fields E_0 and E_1 have a spectrum

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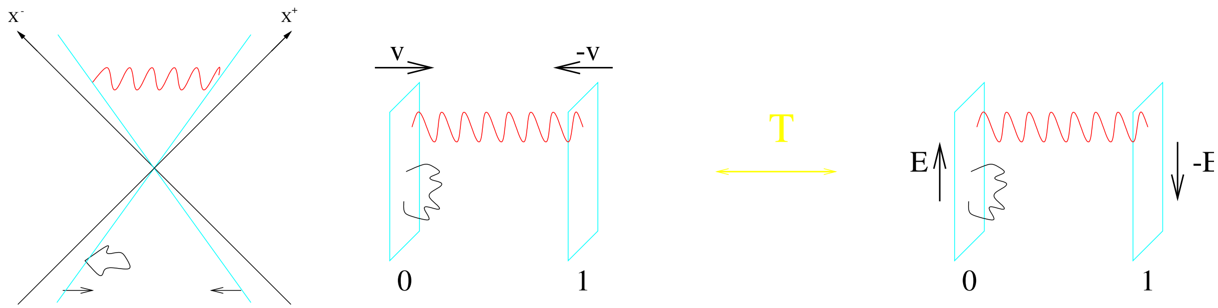

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- This reproduces (one half of) the spectrum of Closed strings in Misner space upon identifying $\nu = w\beta$. The large winding number limit $w \to \infty$ amounts to a near critical electric field $E \to 1$.
- In particular, the open string zero-modes describe the motion of a charged particle in an electric field, and have a structure isomorphic to the closed string case.

Charged particle and open string zero-modes

- Recall the first quantized charged particle in an electric field:

$$L = \frac{1}{2}m\left(-2\partial_\tau X^+\partial_\tau X^- + (\partial_\tau X^i)^2\right) + \frac{\nu}{2}\left(X^+\partial_\tau X^- - X^-\partial_\tau X^+\right)$$

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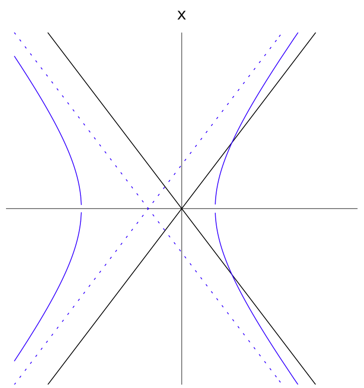
- Classical trajectories are hyperbolas centered at an arbitrary point,

$$X^\pm = x_0^\pm \pm \frac{1}{\nu}a_0^\pm e^{\pm\nu\tau}$$

$P^\pm = \pm\nu x_0^\pm$ is the conserved linear momentum, and $a_0^\pm$ the velocity.



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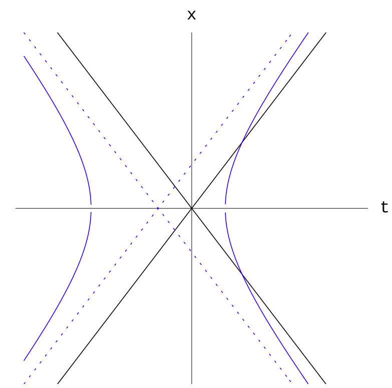

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- Canonical quantization imply the open string zero-mode commutation relations

$$[a_0^+, a_0^-] = -i\nu \ , \quad [x_0^+, x_0^-] = -\frac{i}{\nu} \ , \quad L_0 = -a_0^+ a_0^- + \frac{i\nu}{2} + \text{excited.}$$

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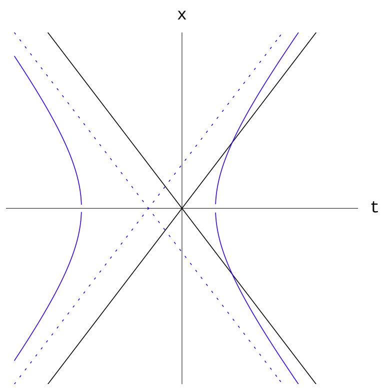

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- Upon quantizing $a_0^\pm$ as creation/annihilation operators in a Fock space, electrons and positrons would have no physical state...

Charged particle and Klein-Gordon equation

- Quantum mechanically, one represents the canonical momenta as derivatives, $\pi^{\pm} = i\partial/\partial x^{\mp}$, hence $a_0^{\pm}, x_0^{\pm}$ as covariant derivatives

$$a_0^{\pm} = i\partial_{\mp} \pm \frac{\nu}{2} x^{\pm}\,, \quad x_0^{\pm} = \mp\frac{1}{\nu}\left(i\partial_{\mp} \mp \frac{\nu}{2} x^{\pm}\right)$$

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- The zero-mode piece of L_0, including the bothersome $\frac{i\nu}{2}$,

$$L_0^{(0)} = -a_0^+ a_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\nabla^+ \nabla^- + \nabla^- \nabla^+)$$

  is just the Klein-Gordon operator of a particle of charge ν, and has well-behaved eigenmodes $L_0 = -m^2$ for any $m^2 > 0$.

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^{\pm} = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator can be rewritten as an inverted harmonic oscillator:

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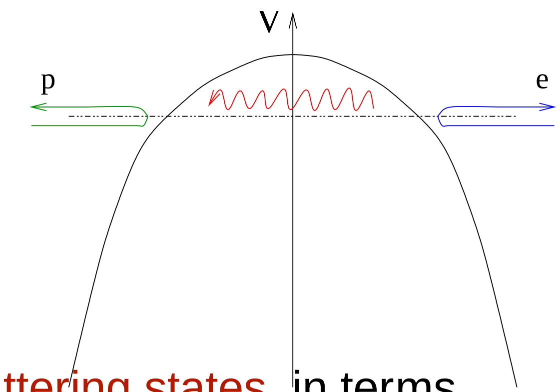
- More explicitely, in terms of $u = (\tilde{p} + \nu x)\sqrt{2/\nu}$,

$$\left(-\partial_u^2 - \frac{1}{4}u^2 + \frac{M^2}{2\nu} \right) \psi_{\tilde{p}}(u) = 0$$



- The latter admits a respectable delta-normalizable spectrum of scattering states, in terms of parabolic cylinder functions, e.g:

$$\phi_{in}^{+}(x,t) = D_{-\frac{1}{2}+i\frac{M^2}{2\nu}}(e^{-\frac{3i\pi}{4}}u)e^{-i\tilde{p}t}e^{i\nu xt/2}$$

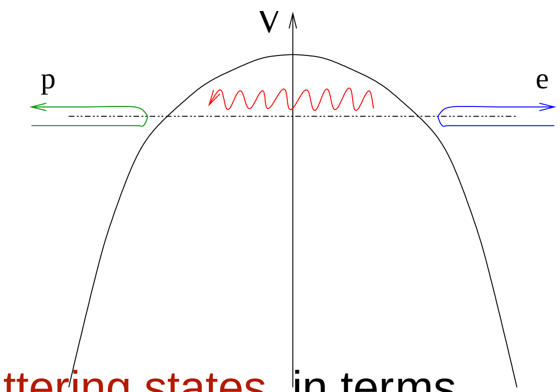
Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^{\pm} = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator can be rewritten as an inverted harmonic oscillator:

$$M^2 = a_0^+ a_0^- + a_0^- a_0^+ = -\frac{1}{2}(P^2 - Q^2)$$

- More explicitely, in terms of $u = (\tilde{p} + \nu x)\sqrt{2/\nu}$,

$$\left(-\partial_u^2 - \frac{1}{4}u^2 + \frac{M^2}{2\nu} \right) \psi_{\tilde{p}}(u) = 0$$



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- These correspond to non-compact trajectories of charged particles in the electric field. Tunnelling is just (stimulated) Schwinger pair creation,

$$e^- \longrightarrow (1+\eta)\, e^- + \eta\, e^+ \,, \quad \eta \sim e^{-\pi M^2/\nu}$$

Brezin Itzykson; Brout Massar Parentani Spindel

Lorentzian vs Euclidean states

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$$\frac{1}{2i\sin(\nu t/2)} = \sum_{n=1}^{\infty} e^{-i(n+\frac{1}{2})\nu t} = \int dM^2 \rho(M^2) e^{-M^2 t/2}$$

The density of states is obtained from the reflection phase shift,

$$\rho(M^2) = \frac{1}{\nu}\log\Lambda - \frac{1}{2\pi i}\frac{d}{dM^2}\log\frac{\Gamma\left(\frac{1}{2}+i\frac{M^2}{2\nu}\right)}{\Gamma\left(\frac{1}{2}-i\frac{M^2}{2\nu}\right)}$$

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- The physical spectrum of the charged open string can be explicitely worked out, and is free of ghosts: a tachyon at level 0, a transverse gauge boson at level 1, . . .

Charged particle in Rindler space

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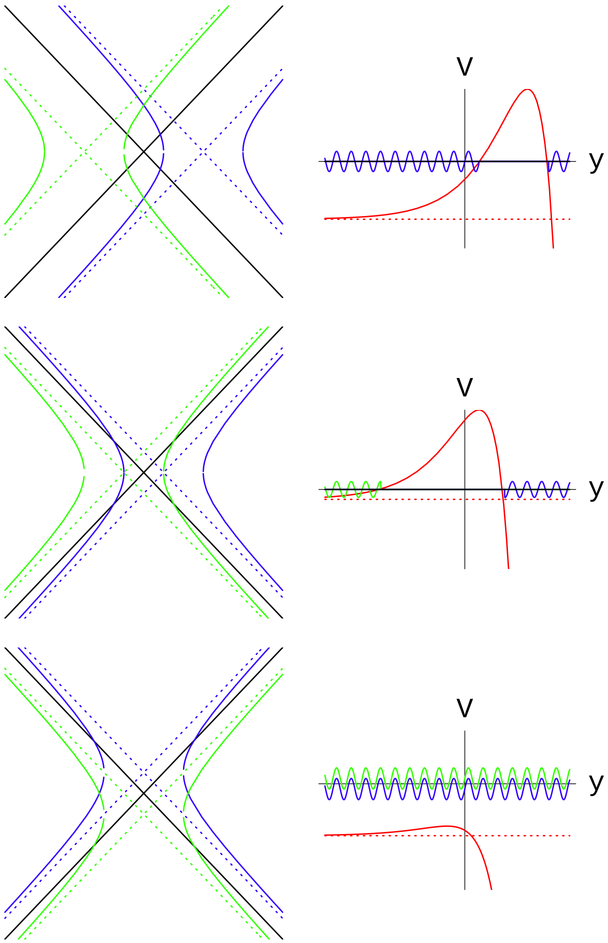
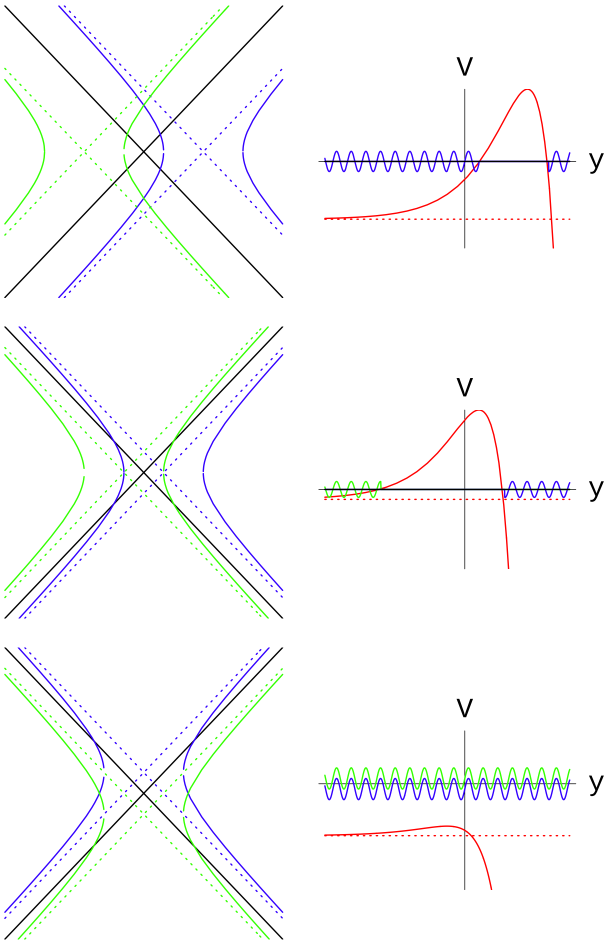


- In the Rindler patch R, letting $f(r,\eta) = e^{-ij\eta} f_j(r)$ and $r = e^y$, one gets a Schrödinger equation for a particle in a potential

$$V(y) = M^2 e^{2y} - (j + \frac{1}{2}\nu\, e^{2y})^2$$



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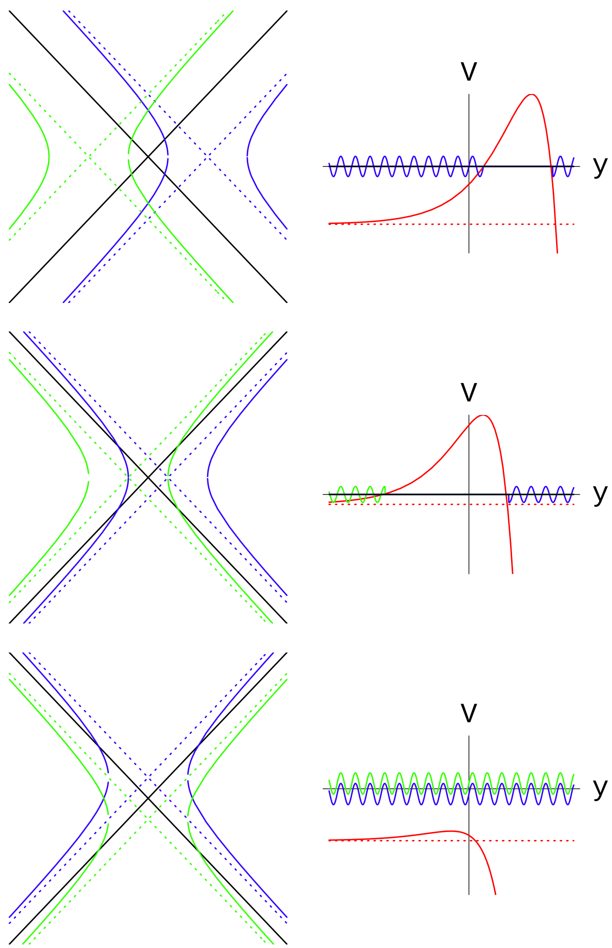

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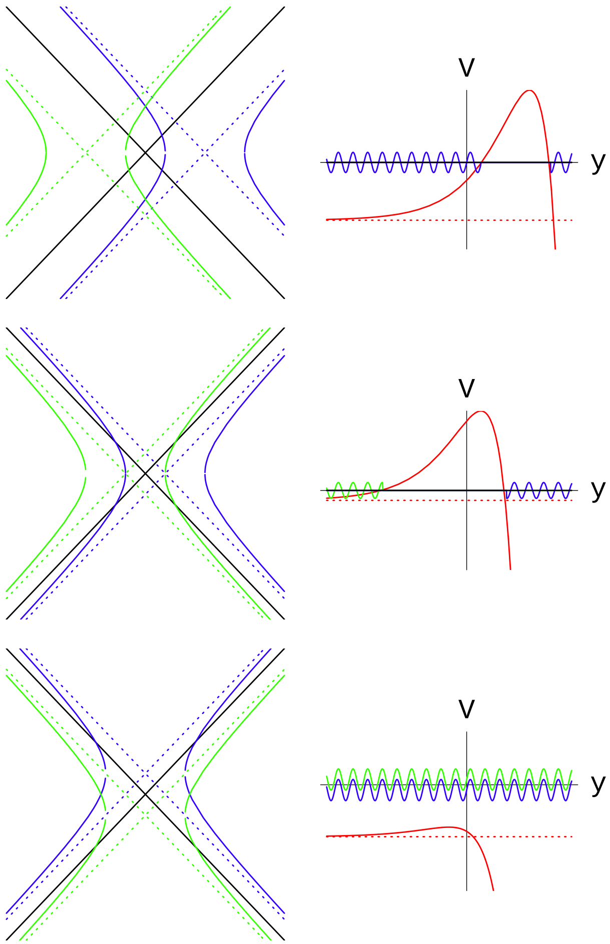

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- If $j > M^2/(2\nu)$, the electron branches cross the horizons. regions. There is no tunneling, but partial reflection amounts to a combination of Schwinger and Hawking emission.

Rindler modes

- Incoming modes from Rindler infinity I_R^- read, in terms of parabolic cylinder functions:

$$\mathcal{V}^j_{in,R} = e^{-ij\eta} r^{-1} M_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),-\frac{ij}{2}}(i\nu r^2/2)$$

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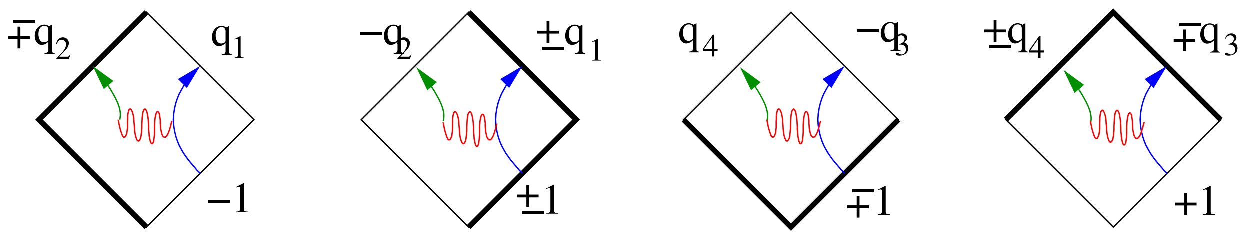

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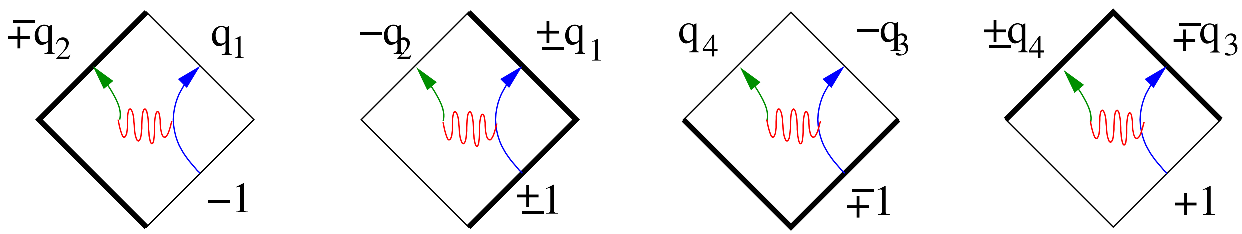

- The reflection coefficients can be computed:

$$q_1 = e^{-\pi j} \frac{\cosh\left[\pi \frac{M^2}{2\nu}\right]}{\cosh\left[\pi\left(j-\frac{M^2}{2\nu}\right)\right]}, \quad q_3 = e^{\pi\left(j-\frac{M^2}{2\nu}\right)} \frac{\cosh\left[\pi\frac{M^2}{2\nu}\right]}{|\sinh \pi j|}$$

and $q_2 = 1 - q_1$, $q_4 = q_3 - 1$, by charge conservation.

Global Charged Unruh Modes

- Global modes may be defined by patching together Rindler modes, ie by analytic continuation across the horizons. Unruh modes are those which are superposition of positive energy Minkowski modes,

$$\begin{aligned}\Omega^j_{in,+} &= \mathcal{V}^j_{in,P} = (-i\nu X^+X^-)[X^+/X^-]^{-ij/2}W_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}} \\ \omega^j_{in,-} &= \mathcal{U}^j_{in,P} = (i\nu X^+X^-)[X^+/X^-]^{-ij/2}M_{i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}\end{aligned}$$

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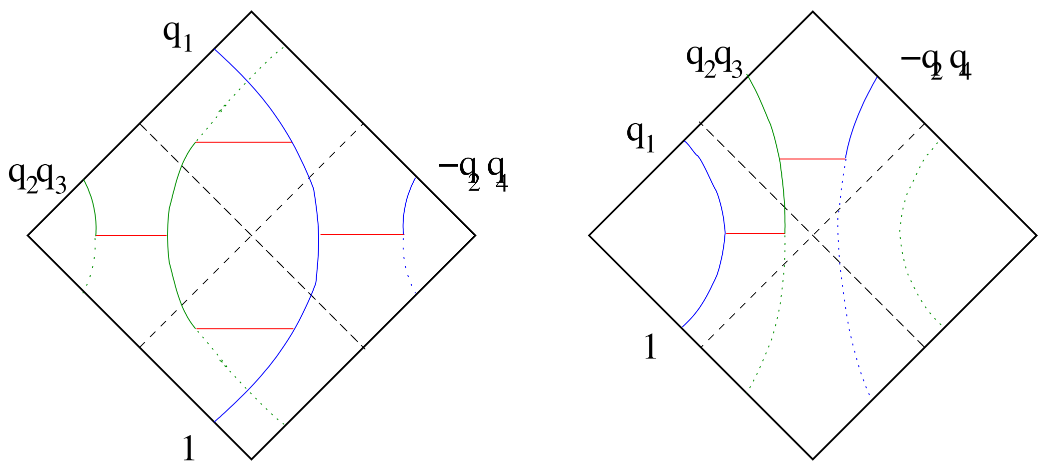

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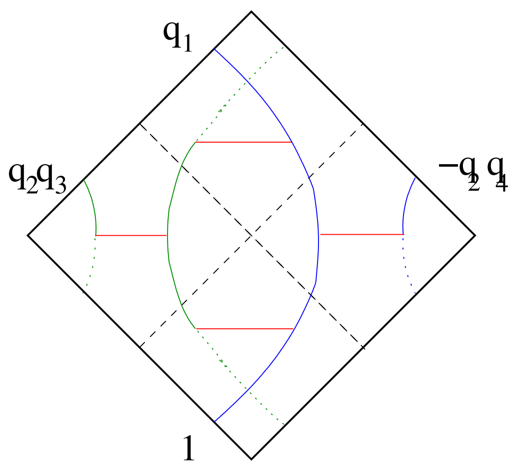

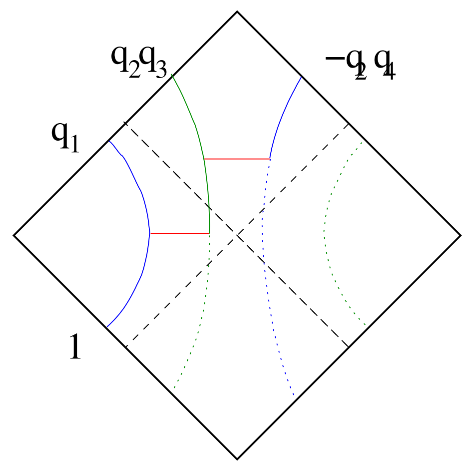

- Any state in Minkowski space can be represented as a state in the tensor product of the Hilbert spaces of the left and right Rindler patches. In contrast to neutral fields in Rindler space, Boulware-Fulling modes that vanish in L or R have positive Minkowski energy.

Closed string zero-modes

- Let us reanalyze the classical solutions for the closed string zero modes

$$X^{\pm}(\tau,\sigma) = e^{\mp\nu\sigma}\left[\pm\frac{1}{2\nu}\alpha_0^{\pm}e^{\pm\nu\tau} \mp \frac{1}{2\nu}\tilde{\alpha}_0^{\pm}e^{\mp\nu\tau}\right] \;, \quad \alpha_0^{\pm}, \tilde{\alpha}_0^{\pm} \in R$$

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- The behavior at early/late proper time now depends on $\epsilon\tilde{\epsilon}$: For $\epsilon\tilde{\epsilon} = 1$, the string begins/ends in the Milne regions. For $\epsilon\tilde{\epsilon} = -1$, the string begins/ends in the Rindler regions.

Short and long strings

Choosing $j = 0$ for simplicity, we have two very different types of solutions:

- $\epsilon = 1$, $\tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma,\tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma} \ , \quad T = \frac{M}{\nu} \sinh(\nu\tau) \ , \quad \theta = \nu\sigma$$

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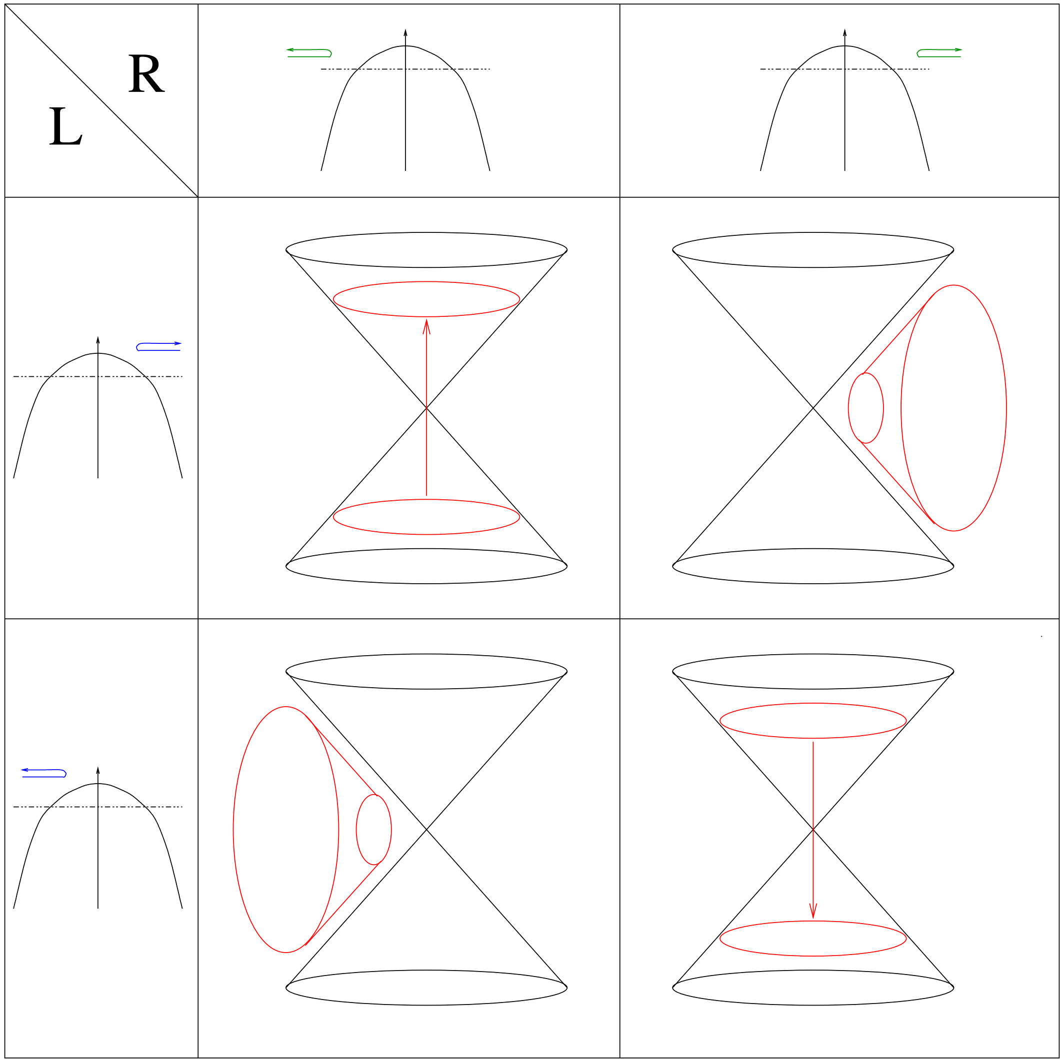
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From open to closed strings

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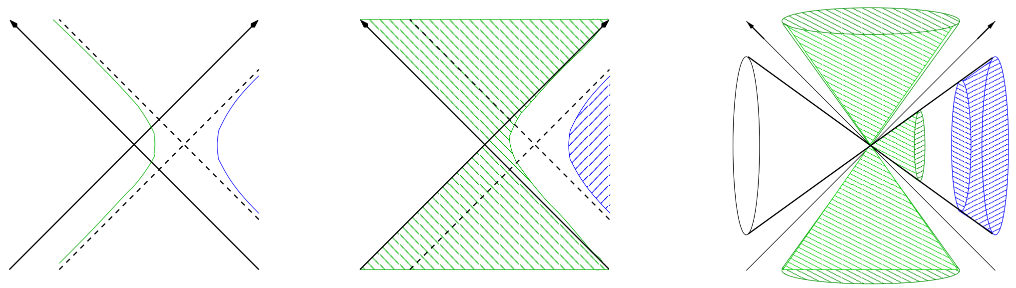

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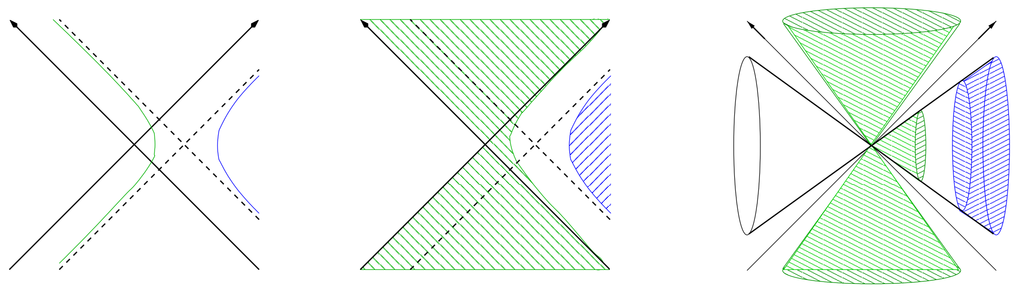

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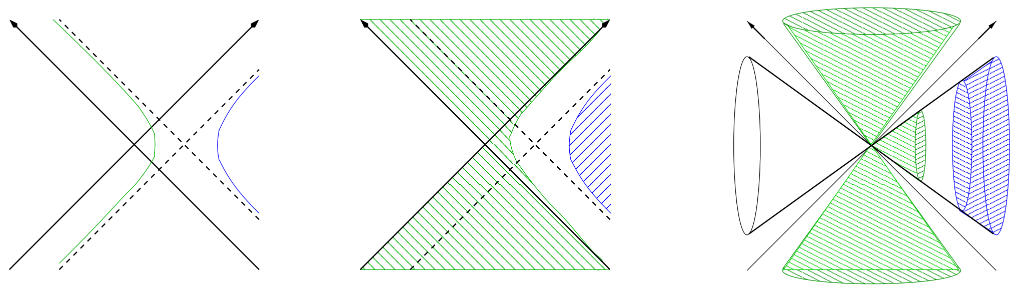

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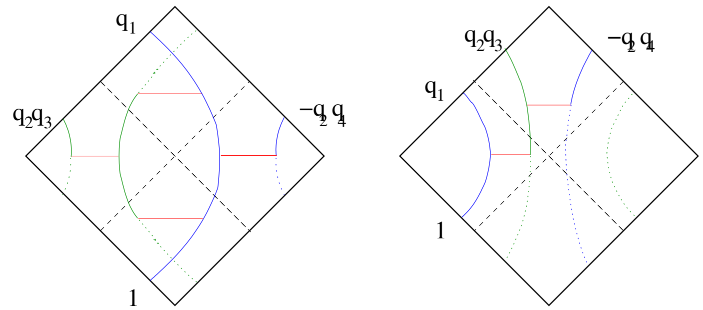
q1
q2q3
−q2 q4
1
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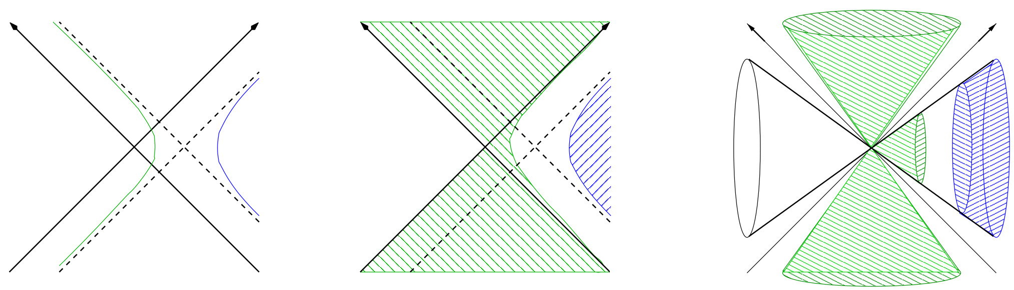

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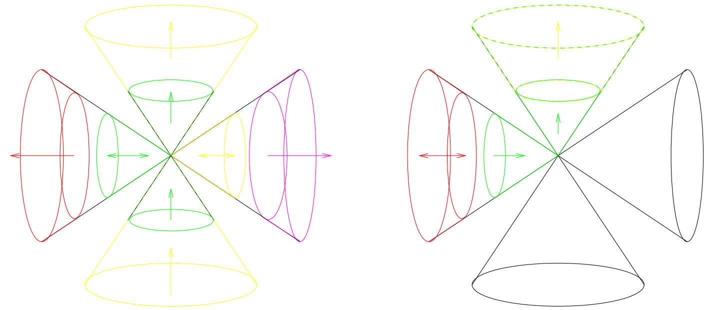

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- Spontaneous pair production on the other hand involves no incoming state. It can be described semi-classically by cutting open an Euclidean periodic trajectory at the turning point, and evolving in real time henceforth.
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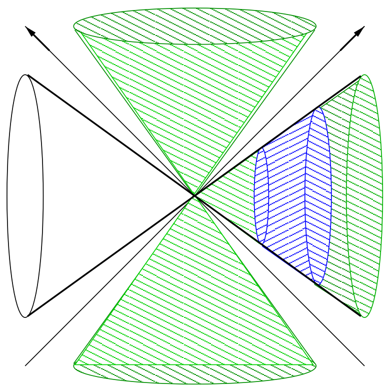
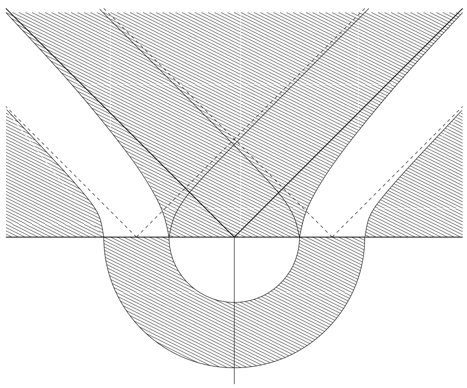
- Spontaneous pair production of winding strings can be described by cutting open a periodic trajectory, either in imaginary proper time, or in the Euclidean rotation orbifold:





A few words on second quantization

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- Instead, one may (in two different ways) identify the Hilbert space with that of a single charged particle, including its center of motion. The wave function is thus a state in the tensor product of the left and right Rindler patches. One can define in and out vacua, and find global pair production.

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- Finally, motivated by holography, one may try to quantize with respect to the radial evolution in Rindler space. Short and long strings would be analogous to normalizable / non-normalizable modes.

Quantization in the Rindler patch

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- If so we should quantize the string with respect to the "time" coordinate σ rather than τ. The Rindler energy is given by the canonical generator associated to boosts,

$$W = -\int_{-\infty}^{\infty} d\tau \left(X^+ \partial_\sigma X^- - X^- \partial_\sigma X^+ \right) = \int_{-\infty}^{\infty} d\tau \; r^2 \partial_\sigma \eta$$

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- The total Rindler energy of a long string is infinite, due to its extension towards $r \to \infty$. The energy density by unit of radial distance

$$w(r) = \frac{4\nu^2 r^3 \text{sgn}\,(\nu)}{\sqrt{(M^2 + \tilde{M}^2 - 4\nu^2 r^2)^2 - 4M^2\tilde{M}^2}}$$

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- The spectrum is thus unbounded from below (and above): Can CTC prevent the vacuum to decay ?

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- A bounce in dimension i requires $H_i' > 0$ at the point where $H_i = 0$, i.e.

$$(D-2) p_i + \rho \geq \sum_{j \neq i} p_j$$

The most efficient solution is a gas of scalar momentum states, with $p = \rho$: provides enough pressure for the bounce.

Effective gravity analysis (cont.)

- However, consider fundamental strings wrapped around dimension i,

$$\rho = \frac{T}{V}\,, \quad p_i = -\rho\,, \quad p_{j\neq i} = 0\,, \quad V = \prod_{j\neq i} a_j \quad \Rightarrow D \leq 3$$

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- We assumed a constant number of wound strings: one should incorporate the dependence of the production rate on the Hubble parameters.

Effective gravity analysis (cont.)

- Einstein's equations imply that the quantity

$$\mu = \left(\frac{H_k}{H_i} - 1 \right) \Big/ \left(\frac{H_j}{H_i} - \frac{3}{4-D} \right)$$

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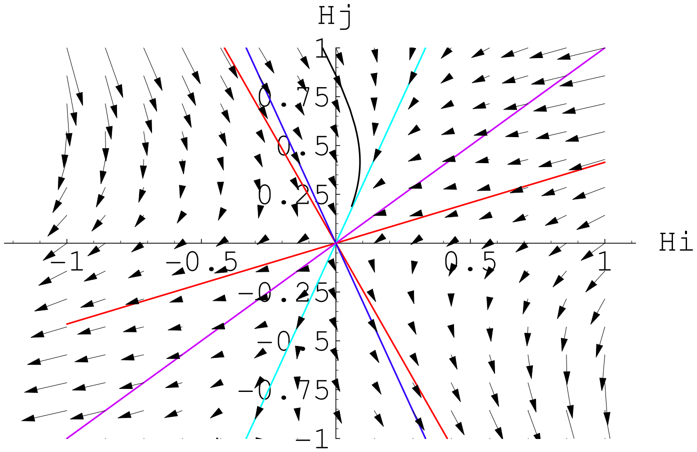
A bounce for direction i in units of the eleven-dimensional frame therefore takes place for any initial condition such that $2\mu + D - 3 > 0$ and $2 < D < 4$.


Hj
1
0.75
0.5
0.25
-0.25
-0.5
-0.75
-1
-1
-0.5
0.5
1
Hi

Conclusions

We discussed closed strings in a toy model of a cosmological singularities. However, some of the features we uncovered should carry over to more general geometries:

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- The production rate for winding strings in a singular geometry diverges at large winding number. Can the resolved geometry be determined self-consistently so that the

divergences at one-loop cancel those at tree-level ?