

Closed Strings in the Misner Universe

aka the Lorentzian orbifold

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Talk based on

hep-th/0307280 w/ M. Berkooz

hep-th/0405126 w/ M. Berkooz, and M. Rozali

hep-th/0406xxx w/ M. Berkooz, B. Durin and D. Reichmann

slides available from

<http://www.lpthe.jussieu.fr/pioline/seminars.html>

Motivational string cosmology

- **Observational Cosmology** is now challenging string theory with high-precision data:

$$\Omega_{baryon} = 0.047, \quad \Omega_{darkm} = 0.243, \quad \Omega_{\Lambda} = 0.71, \quad w = -0.98 \pm .12, \dots$$

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- With LHC still far in the future, understanding **StringY Cosmology** may bring us to closer contact with reality...

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In addition, String theory is not content on a finite time interval, and one is frequently forced into **Big Bang / Big Crunch singularities, CTC** in the process of maximally extending the geometry.

String theory and cosmological singularities

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- Usual Euclidean orbifolds, with a time-like singularity, can be resolved by **condensation of twisted strings**. **How about non-static orbifolds ?**

- In this talk, we shall study the “Lorentzian” orbifold of flat Minkowski space by a discrete boost, as a toy model of a singular cosmological universe where string theory can in principle be solved explicitly.

Outline of the talk

1. Euclidean and Lorentzian orbifolds, and their avatars

Misner, Taub-NUT, Grant...

2. Untwisted strings in Misner space

Hiscock, Konkowski; Berkooz Craps Kutasov Rajesh, ...

3. Twisted strings in Misner space: first pass

Nekrasov

3. A detour: Open strings in electric fields

Bachas Porrati; Berkooz BP

4. Twisted strings in Misner space: second pass

Berkooz BP Rozali

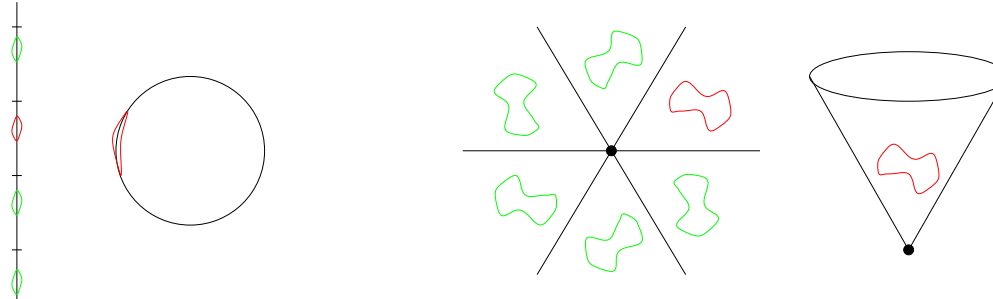
5. Comments on backreaction from winding strings

Strings on Euclidean orbifolds - untwisted states

- Well-known examples of orbifolds are the **circle**, R/Z , and the **rotation orbifold** R^2/Z_k .

Dixon Harvey Vafa Witten

- The spectrum of the quotient theory contains closed string states of the parent theory which are invariant under G : **untwisted states**.

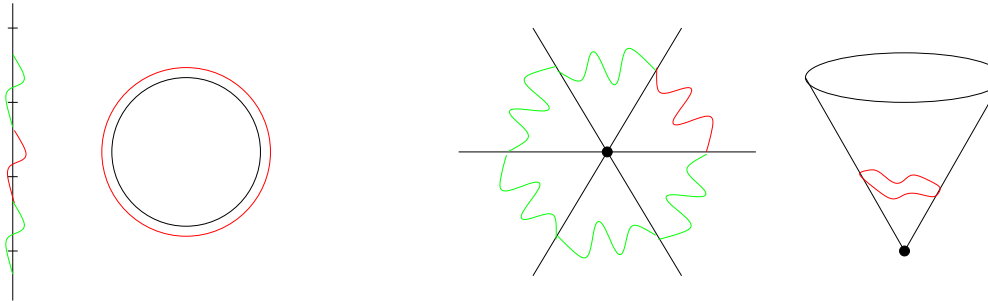


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- Modular invariance** requires that the spectrum should also include closed strings in the quotient theory which **close up to the action of G** in the parent theory: **twisted states**.



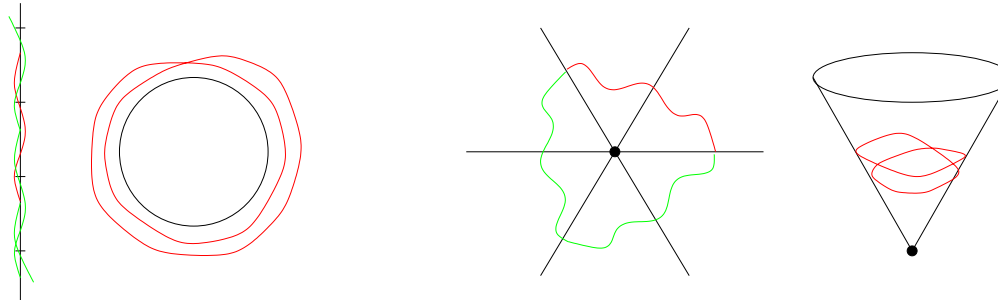
- When G acts non-freely, the twisted sector states are **localized at the fixed points**. They yield new localized degrees of freedom, which ensure the consistency of the background: anomaly free, divergence free...

Strings on Euclidean orbifolds - twisted sectors (cont.)

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- Twisted sectors are labelled by **conjugacy classes** of G . Higher twisted sectors correspond to multiply wound states.

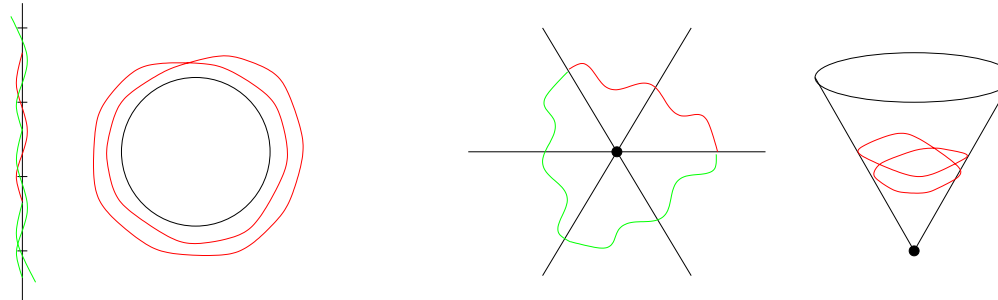


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- Additionally, each twisted sector admits excited levels. The ground state can be thought of as a **Gaussian wave function** centered at the origin.

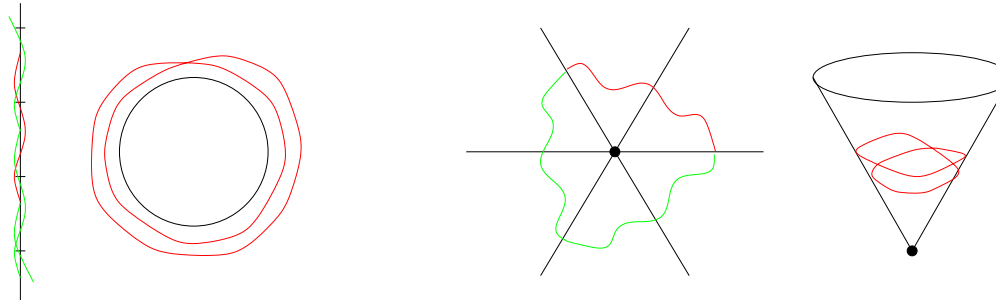
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- The **condensation** of these twisted states changes the vacuum, and effectively **resolves the singularity**: $R^2/Z_k \rightarrow R^2/Z_{k-1} \rightarrow \dots$ (tachyon), $R^4/Z_k \rightarrow$ multi-centered

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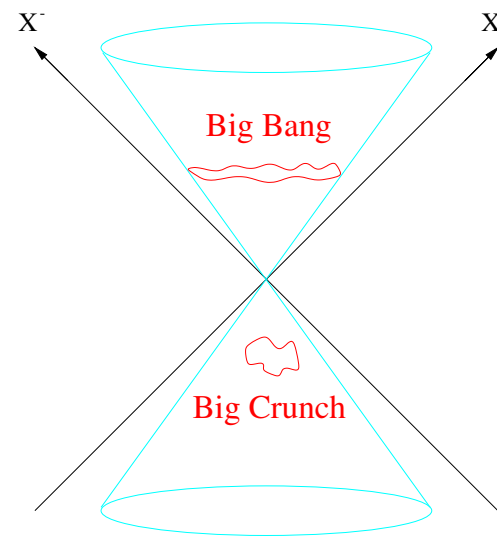
- The Lorentzian orbifold shares features with both examples: an infinite number of winding sectors, and a, non compact, fixed locus.

The Lorentzian orbifold

- One of the simplest examples of space-like singularities is the **quotient of flat Minkowski space by a discrete boost**, also known as **Misner space** (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^\pm \sim e^{\pm 2\pi\beta} X^\pm$$

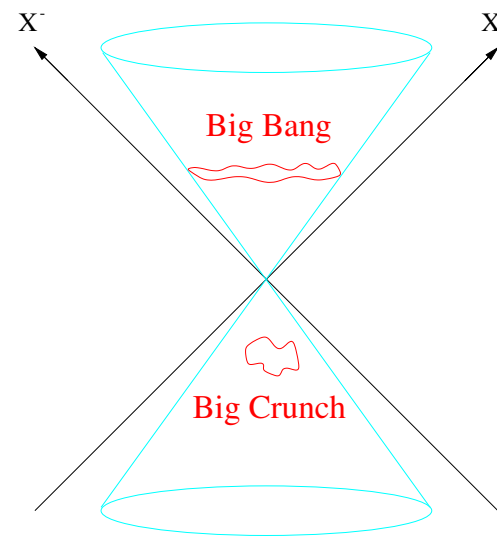


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- The **future** (past) regions $X^+X^- > 0$ describes a cosmological universe often known as the **Milne universe** (1932), **linearly expanding** away from a **Big Bang singularity** (or contracting into a Big Crunch singularity):

$$ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + (dX^i)^2, \quad \theta \equiv \theta + 2\pi, \quad X^\pm = T e^{\pm\beta\theta} / \sqrt{2}$$

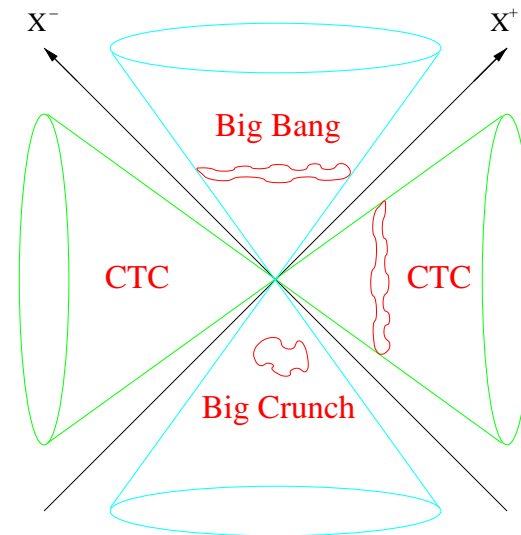
This is a (degenerate) **Kasner singularity**, everywhere **flat**, except for a **delta-function curvature** at $T = 0$.

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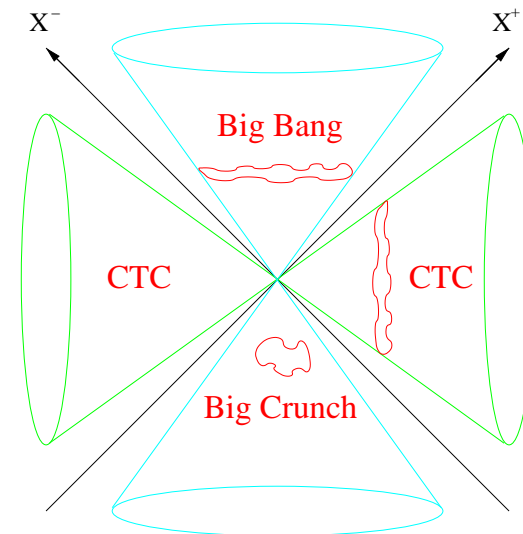


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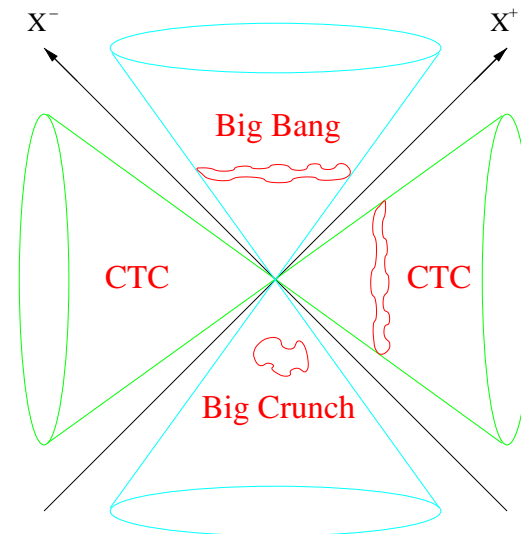
$$ds^2 = dr^2 - \beta^2 r^2 d\eta^2 + (dX^i)^2 \quad , \eta \equiv \eta + 2\pi \quad , \quad X^\pm = \pm r e^{\pm\beta\eta} / \sqrt{2}$$

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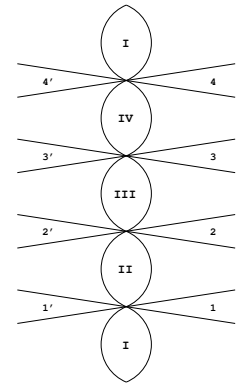
- Finally, the **lightcone** $X^+ X^- = 0$ gives rise to a **null, non-Hausdorff** locus attached to the singularity.

Close relatives of the Misner Universe

- Misner space was first introduced as a local model of **Lorentzian Taub-NUT** space:

$$ds^2 = 4l^2 U(t) \sigma_3^2 + 4l \sigma_3 dt + (t^2 + l^2) (\sigma_1^2 + \sigma_2^2), \quad U(t) = -1 + \frac{2mt + l^2}{t^2 + l^2}$$

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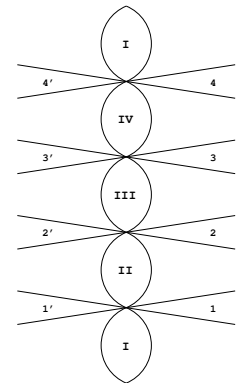
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- A close variant of Misner space is the quotient of flat space by the **combination of a discrete boost and a translation** on an extra direction, often known as the **Grant space**:

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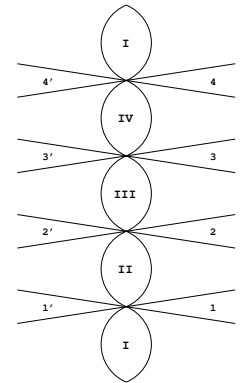
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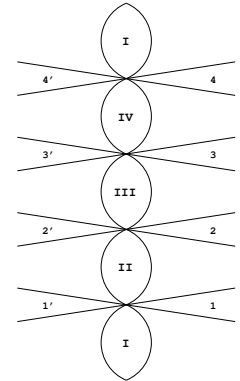
- The Misner geometry arose again more recently as the **M-theory** lift of a simple (**ekpyrotic**) cosmological solution of Einstein-dilaton gravity with no potential.

Khoury Ovrut Seiberg Steinhard Turok

Close relatives of the Misner Universe (cont)

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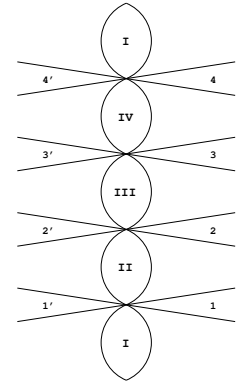
Nappi Witten; Elitzur Giveon Kutasov Rabinovici



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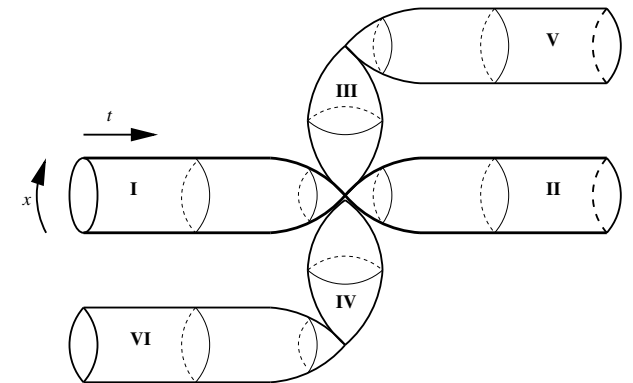
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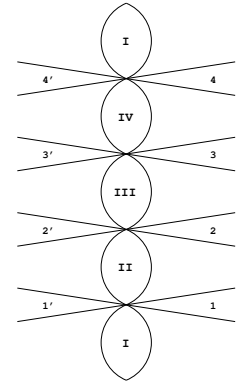
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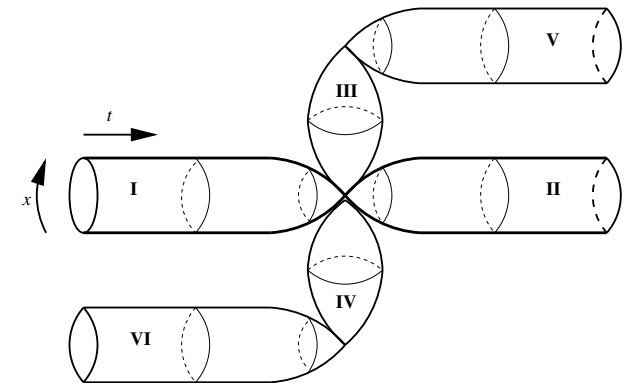
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- The **Lorentzian orientifold** $IIB/[(-)^F boost]/[\Omega(-)^{FL}]$ was also recently argued to

describe orientifolds of non-supersymmetric strings with non-vanishing Neveu-Schwarz tadpoles.

Dudas Mourad Timirgaziu

Closed strings in Misner space - untwisted states

- As usual in standard orbifolds, part of the spectrum involves closed strings on Minkowski covering space, which are **invariant under the orbifold projection**. In conformal gauge,

$$X^\pm(\sigma + 2\pi, \tau) = X^\pm(\sigma, \tau), \quad (\partial_\tau^2 - \partial_\sigma^2)X^\pm = 0$$

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- Vertex operators (or states) can be obtained by (infinite) **sum over images**, e.g.

$$\sum_{l=-\infty}^{\infty} \partial X^+ \bar{\partial} X^- \exp \left(ik^+ X^- e^{-2\pi\beta l} + ik^- X^+ e^{2\pi\beta l} + ik_i X^i \right)$$

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- Equivalently, after **Poisson resummation over l** , this is a superposition of states with **integer boost momentum $j = i(x^+ \partial_+ - x^- \partial_-)$** ,

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- The resulting eigenfunctions describe (topologically trivial) closed strings traveling around the Milne circle with integer momentum j .

Quantum fluctuations in field theory

- In the **Minkowski vacuum** (inherited from the covering space), the renormalized propagator can be obtained as a sum over images,

$$G(x; x') = \sum_{l=-\infty, l \neq 0}^{\infty} \int_0^{\infty} d\tau \int dp^{\mu} \exp \left(-ip^{-} (x^{+} - e^{2\pi\beta l} x^{+'}) - ip^{+} (x^{-} - e^{2\pi\beta l} x^{-'}) - ip^i (x^i - x^{i'}) \right)$$

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- The one-loop stress-energy tensor follows from $G(x, x)$, e.g for a conformally coupled scalar,

$$\langle T_{ab} \rangle = \lim_{x \rightarrow x'} \left[(1 - 2\xi) \nabla_a \nabla'_b - 2\xi \nabla_a \nabla_b + (2\xi - \frac{1}{2}) g_{ab} \nabla_c \nabla'^c \right] G(x, x')$$

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This leads to a **divergent quantum backreaction** (worse if the spin $|s| > 1$):

$$\langle T_{\mu}^{\nu} \rangle = \frac{K}{12\pi^2} T^{-4} \text{diag}(1, -3, 1, 1), \quad K = \sum_{l=1}^{\infty} \cosh(2\pi\beta l s) \frac{2 + \cosh 2\pi l \beta}{[\cosh 2\pi l \beta - 1]^2}$$

One-loop vacuum amplitude in field and string theory

- On the other hand, in string theory $\langle T_{\mu}^{\nu} \rangle(x)$ is an **off-shell** quantity, and only its integral over space-time is well defined:

$$\int dx^+ dx^- G(x, x) = \sum_{l=-\infty}^{+\infty} \int_0^{\infty} \frac{d\rho}{\rho^{D/2}} \frac{e^{-m^2 \rho}}{\sinh^2(\pi \beta l)}$$

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- This reproduces the zero-mode contribution to the string one-loop vacuum amplitude in the untwisted sector:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l, w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2 \rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho) \theta_1(i\beta(l + w\rho); \rho)|^2}$$

$$\theta_1(v; \rho) = 2q^{1/8} \sin \pi v \prod_{n=1}^{\infty} (1 - e^{2\pi i v} q^n)(1 - q^n)(1 - e^{-2\pi i v} q^n), \quad q = e^{2\pi i \rho}$$

Nekrasov, Cornalba Costa

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- The existence of **Regge trajectories** with arbitrary high spin implies new (log) **divergences in the bulk of the moduli space** which resemble long string poles in AdS_3 .

Scattering of untwisted states

- Tree-level scattering amplitudes of untwisted sector states can be computed from those in flat space by the inheritance principle,

$$\langle V(j_1, k_1) \dots V(j_n, k_n) \rangle_{Misner} = \int dv_1 \dots dv_n e^{i(j_1 v_1 + \dots + j_n v_n)}$$

$$\langle V(e^{\beta v_1} k_1^+, e^{-\beta v_1} k_1^-, k_1^i) \dots V(e^{\beta v_n} k_n^+, e^{-\beta v_n} k_n^-, k_n^i) \rangle_{Minkowski}$$

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- The integral diverges due to Regge behavior in the large momentum, fixed angle regime. E.g, the four-tachyon scattering amplitude in bosonic string leads to

$$\int dv v^{-\frac{1}{2}(k_1^i - k_3^i)^2 + i(j_2 - j_4)}$$

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- Could higher order corrections, e.g. resummation of ladder diagrams, lead to a finite amplitude ?

Deser McCarthy Steif; Cornalba Costa

Closed string in Misner space - twisted sectors

- In addition, there is **an infinite set of twisted sectors**, corresponding to strings on the covering space that close **up to the action of the orbifold group**:

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau), \quad \nu = 2\pi w\beta$$

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$$X_R^\pm(\tau - \sigma) = \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha_n^\pm e^{-i(n \pm i\nu)(\tau - \sigma)}$$

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- We will focus on the **quasi zero-mode** sector, which consists of two commuting pairs of real (i.e. hermitian) canonically conjugate operators,

$$[\alpha_0^+, \alpha_0^-] = -i\nu, \quad [\tilde{\alpha}_0^+, \tilde{\alpha}_0^-] = i\nu$$

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated by half of them, e.g.

$$\alpha_{n>0}^{\pm}, \quad \tilde{\alpha}_{n>0}^{\pm}, \quad \alpha_0^{-}, \quad \tilde{\alpha}_0^{+}$$

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- This is the familiar result for the vacuum energy $\frac{1}{2}\theta(1 - \theta)$ in the **Euclidean rotation orbifold**, after analytically continuing $\theta \rightarrow i\nu$.
- Due to the $i\nu/2$ term in the ground state energy, all states obtained by acting on $|0\rangle$ by creation operators $\alpha_{n<0}^{\pm}$ and by α_0^+ will have **imaginary energy**, hence **the physical state conditions** $L_0 = \tilde{L}_0 = 0$ seem to have no solutions.

One-loop amplitude, twisted sector

- Independently of this fact, one may compute the one-loop path integral on an **Euclidean worldsheet and Minkowskian target-space**:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2 \rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho) x \theta_1(i\beta(l + w\rho); \rho)|^2}$$

where θ_1 is the Jacobi theta function,

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$$\frac{1}{2 \sinh(\beta w \rho)} = \sum_{n=1}^{\infty} q^{i(n+\frac{1}{2})\beta w}$$

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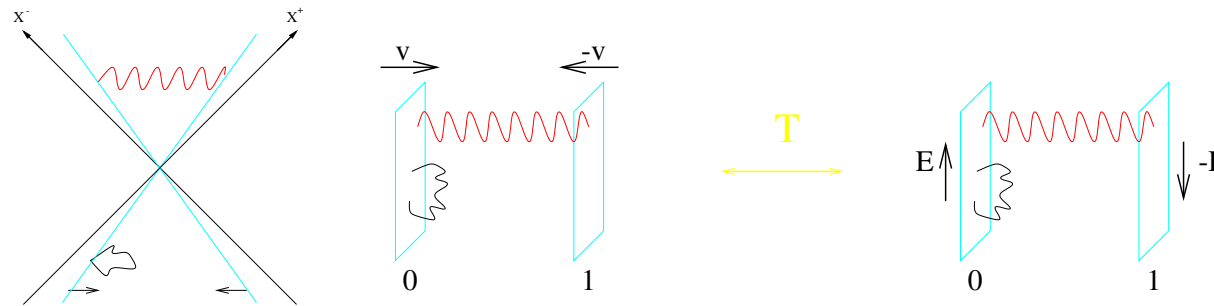
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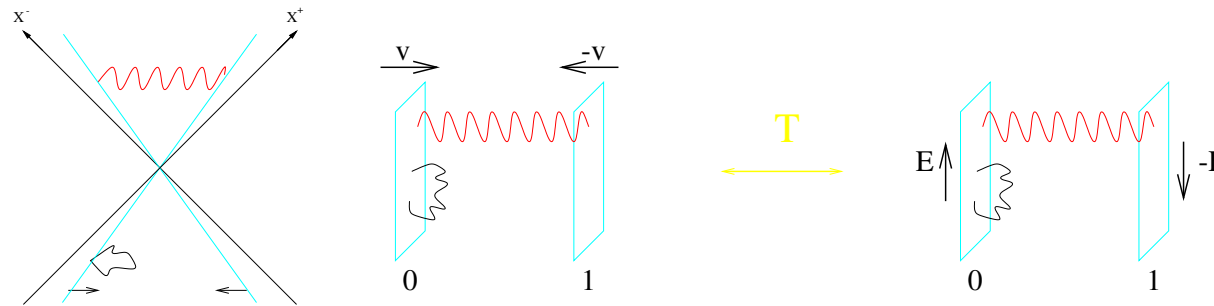
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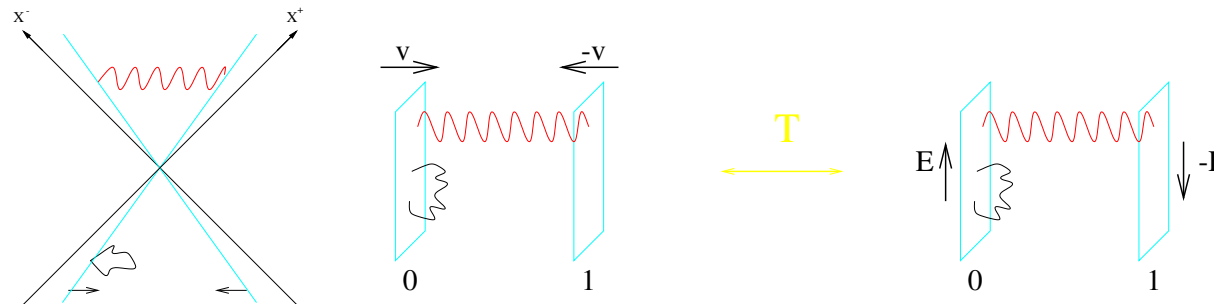


- Open strings stretched between two D-branes** with electric fields E_0 and E_1 have a spectrum

$$\omega_n = n + i\nu, \quad \nu := \text{Arctanh}E_1 - \text{Arctanh}E_0$$

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- This reproduces (one half of) the spectrum of **Closed strings in Misner space** upon identifying $\nu = w\beta$. The large winding number limit $w \rightarrow \infty$ amounts to a **near critical electric field** $E \rightarrow 1$.
- In particular, the **open string zero-modes** describe the motion of a **charged particle in an electric field**, and have a structure **isomorphic** to the closed string case.

Charged particle and open string zero-modes

- Recall the first quantized **charged particle in an electric field**:

$$L = \frac{1}{2}m \left(-2\partial_\tau X^+ \partial_\tau X^- + (\partial_\tau X^i)^2 \right) + \frac{\nu}{2} \left(X^+ \partial_\tau X^- - X^- \partial_\tau X^+ \right)$$

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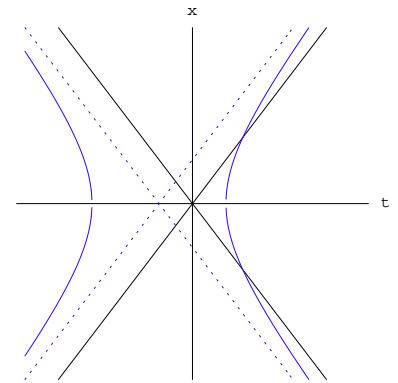
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- Classical trajectories are hyperbolas centered at an arbitrary point,

$$X^\pm = x_0^\pm \pm \frac{1}{\nu} a_0^\pm e^{\pm\nu\tau}$$

$P^\pm = \pm\nu x_0^\pm$ is the conserved **linear momentum**, and a_0^\pm the **velocity**.



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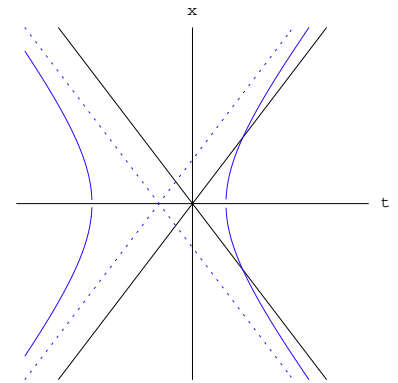
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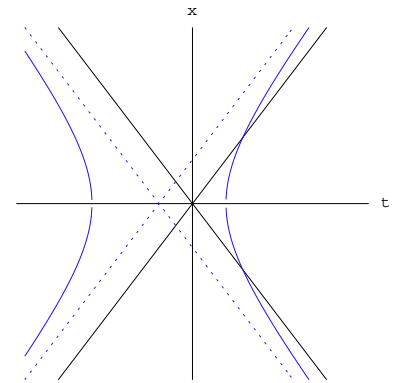
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- Upon quantizing a_0^\pm as creation/annihilation operators in a Fock space, **electrons and positrons would have no physical state...**

Charged particle and Klein-Gordon equation

- Quantum mechanically, one represents the canonical momenta as derivatives, $\pi^\pm = i\partial/\partial x^\mp$, hence a_0^\pm, x_0^\pm as **covariant derivatives**

$$a_0^\pm = i\partial_\mp \pm \frac{\nu}{2}x^\pm, \quad x_0^\pm = \mp \frac{1}{\nu} \left(i\partial_\mp \mp \frac{\nu}{2}x^\pm \right)$$

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- The zero-mode piece of L_0 , **including the bothersome** $\frac{i\nu}{2}$,

$$L_0^{(0)} = -a_0^+ a_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\nabla^+ \nabla^- + \nabla^- \nabla^+)$$

is just the **Klein-Gordon operator** of a particle of charge ν , and has well-behaved eigenmodes $L_0 = -m^2$ for any $m^2 > 0$.

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^\pm = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator can be rewritten as an **inverted harmonic oscillator**:

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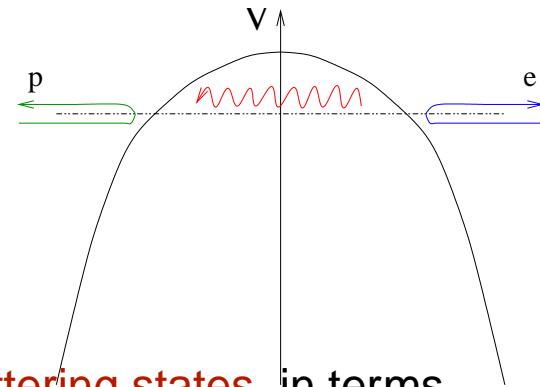
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- More explicitly, in terms of $u = (\tilde{p} + \nu x)\sqrt{2/\nu}$,

$$\left(-\partial_u^2 - \frac{1}{4}u^2 + \frac{M^2}{2\nu} \right) \psi_{\tilde{p}}(u) = 0$$

- The latter admits a respectable **delta-normalizable spectrum of scattering states**, in terms of **parabolic cylinder functions**, e.g:

$$\phi_{in}^+(x, t) = D_{-\frac{1}{2} + i\frac{M^2}{2\nu}} \left(e^{-\frac{3i\pi}{4}} u \right) e^{-i\tilde{p}t} e^{i\nu x t/2}$$



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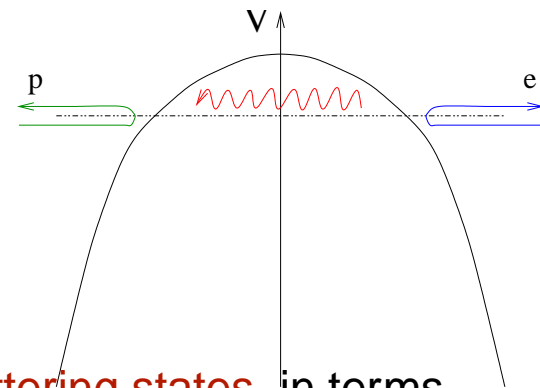
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$$\left(-\partial_u^2 - \frac{1}{4}u^2 + \frac{M^2}{2\nu} \right) \psi_{\tilde{p}}(u) = 0$$

- The latter admits a respectable **delta-normalizable spectrum of scattering states**, in terms of **parabolic cylinder functions**, e.g:

$$\phi_{in}^+(x, t) = D_{-\frac{1}{2} + i\frac{M^2}{2\nu}} \left(e^{-\frac{3i\pi}{4}} u \right) e^{-i\tilde{p}t} e^{i\nu xt/2}$$



- These correspond to **non-compact** trajectories of charged particles in the electric field. **Tunnelling** is just (stimulated) **Schwinger pair creation**,

$$e^- \rightarrow (1 + \eta) e^- + \eta e^+, \quad \eta \sim e^{-\pi M^2/\nu}$$

Brezin Itzykson; Brout Massar Parentani Spindel

Lorentzian vs Euclidean states

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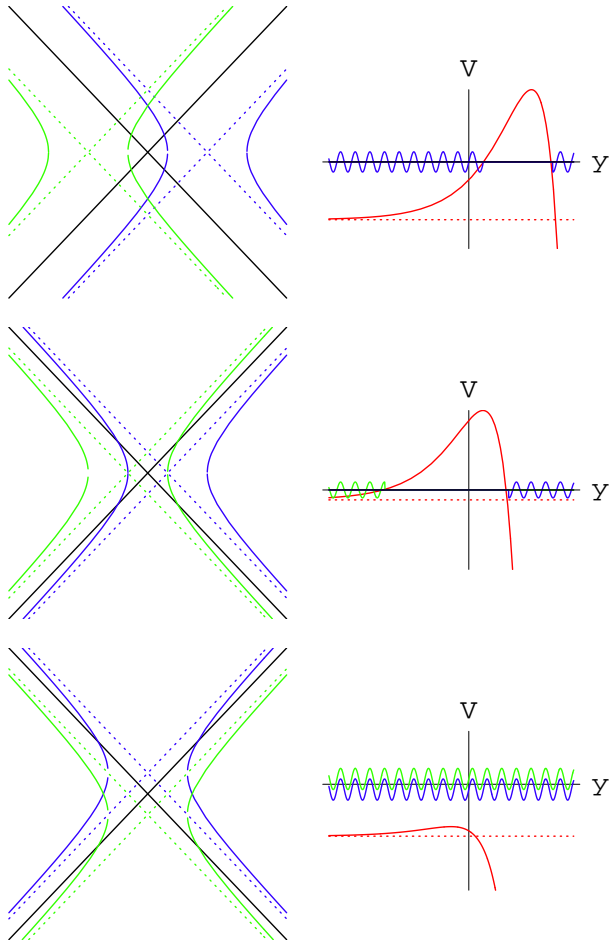
- The physical spectrum of the charged open string can be explicitly worked out, and is **free of ghosts**: a tachyon at level 0, a **transverse gauge boson** at level 1, ...

Charged particle in Rindler space

- For applications to the Milne universe, one should diagonalize the **boost momentum** J , ie consider an **accelerated observer**.

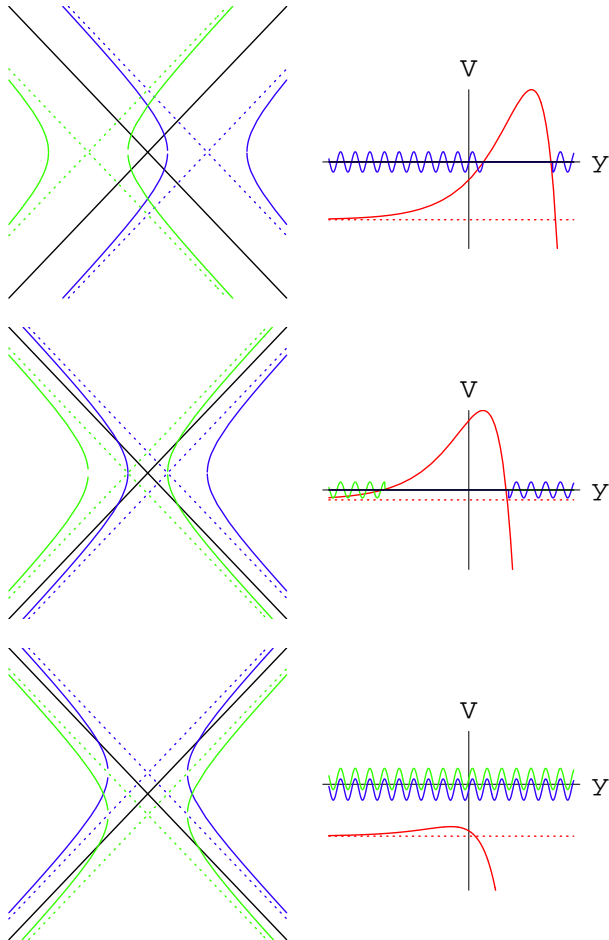
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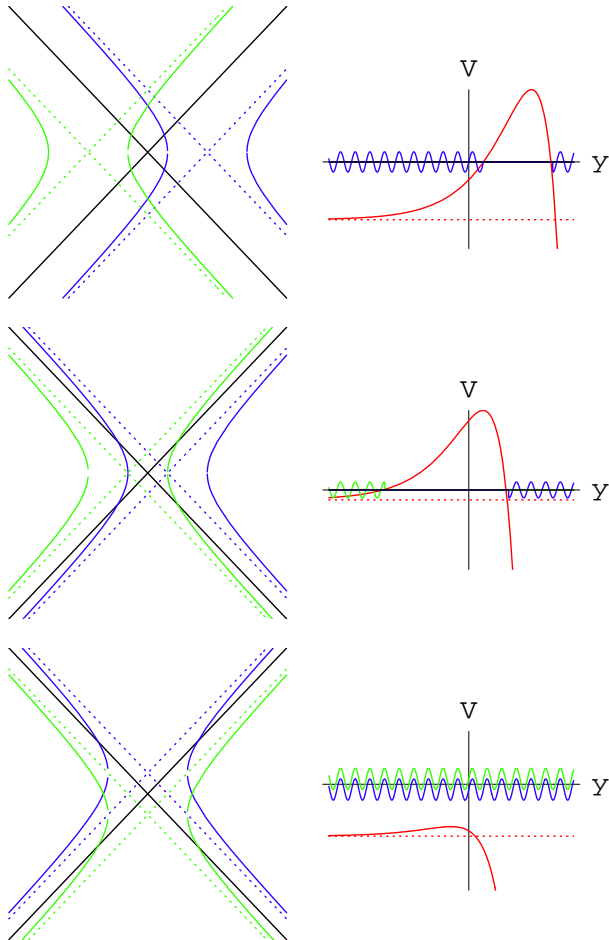
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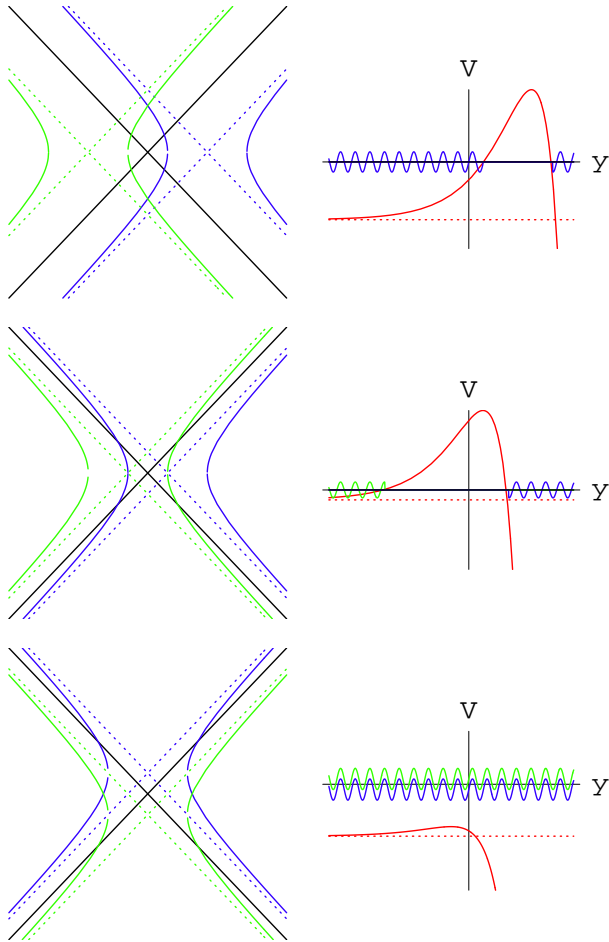
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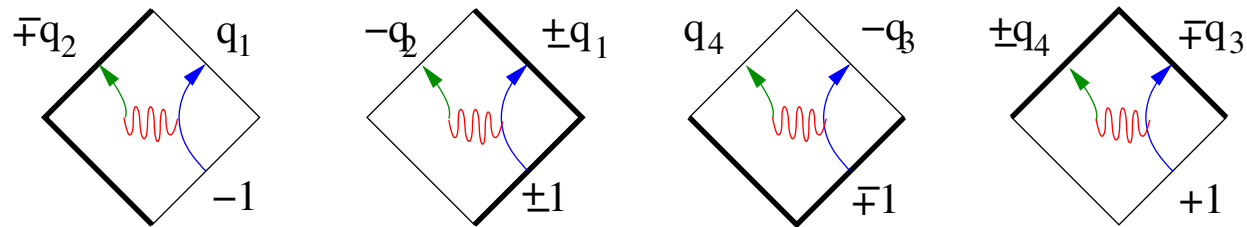
Rindler modes

- Incoming modes from Rindler infinity I_R^- read, in terms of parabolic cylinder functions:

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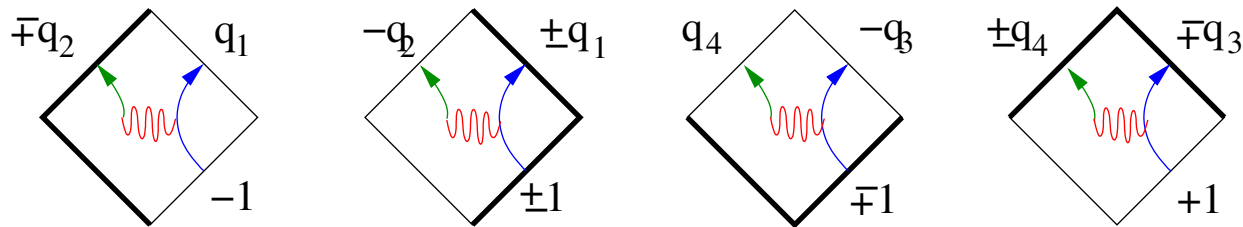
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- The reflection coefficients can be computed:

$$q_1 = e^{-\pi j} \frac{\cosh \left[\pi \frac{M^2}{2\nu} \right]}{\cosh \left[\pi \left(j - \frac{M^2}{2\nu} \right) \right]}, \quad q_3 = e^{\pi \left(j - \frac{M^2}{2\nu} \right)} \frac{\cosh \left[\pi \frac{M^2}{2\nu} \right]}{|\sinh \pi j|}$$

and $q_2 = 1 - q_1$, $q_4 = q_3 - 1$, by charge conservation.

Global Charged Unruh Modes

- Global modes may be defined by patching together Rindler modes, ie by **analytic continuation across the horizons**. **Unruh modes** are those which are superposition of **positive energy** Minkowski modes,

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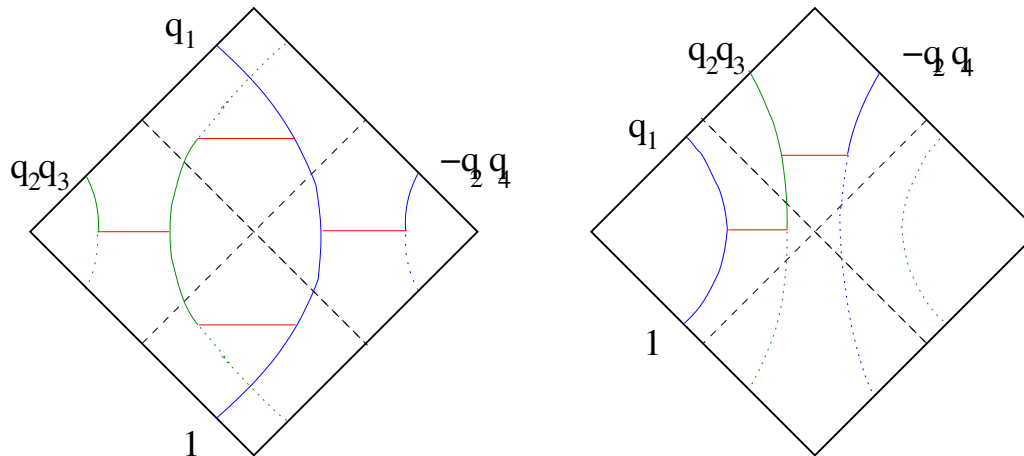
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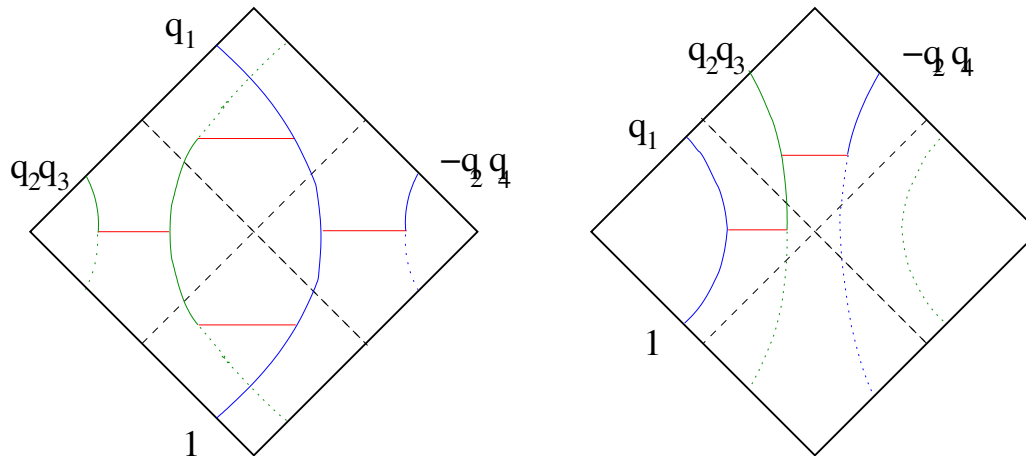
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- Any state in Minkowski space can be represented as a state in the **tensor product of the Hilbert spaces of the left and right Rindler patches**. In contrast to neutral fields in Rindler space, Boulware-Fulling modes that vanish in L or R have positive Minkowski energy.

Closed string zero-modes

- Let us reanalyze the classical solutions for the closed string zero modes

$$X^\pm(\tau, \sigma) = e^{\mp\nu\sigma} \left[\pm \frac{1}{2\nu} \alpha_0^\pm e^{\pm\nu\tau} \mp \frac{1}{2\nu} \tilde{\alpha}_0^\pm e^{\mp\nu\tau} \right], \quad \alpha_0^\pm, \tilde{\alpha}_0^\pm \in \mathbb{R}$$

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- The behavior at early/late proper time now depends on $\epsilon\tilde{\epsilon}$: For $\epsilon\tilde{\epsilon} = 1$, the string begins/ends in the **Milne** regions. For $\epsilon\tilde{\epsilon} = -1$, the string begins/ends in the **Rindler** regions.

Short and long strings

Choosing $j = 0$ for simplicity, we have two very different types of solutions:

- $\epsilon = 1, \tilde{\epsilon} = 1$:

$$X^\pm(\sigma, \tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma}, \quad T = \frac{M}{\nu} \sinh(\nu\tau), \quad \theta = \nu\sigma$$

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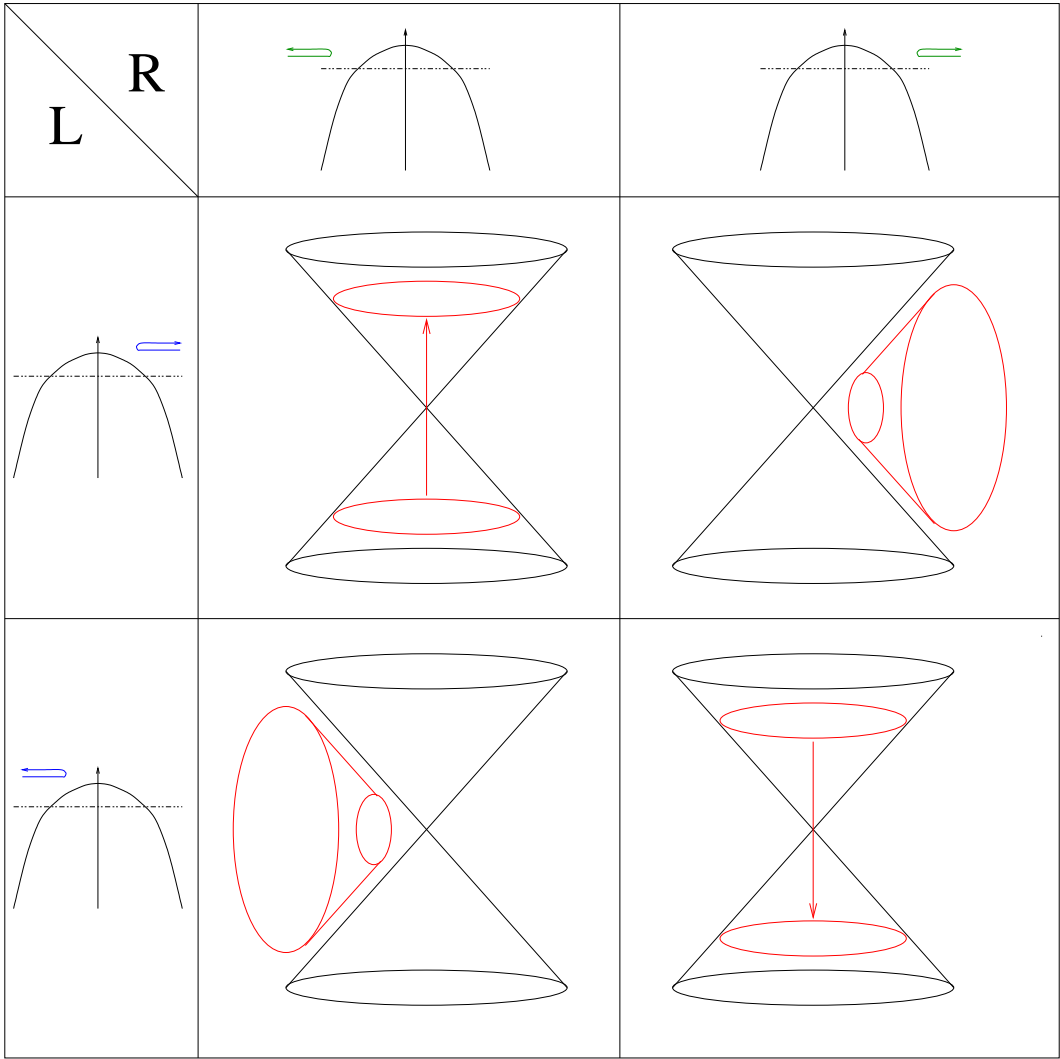
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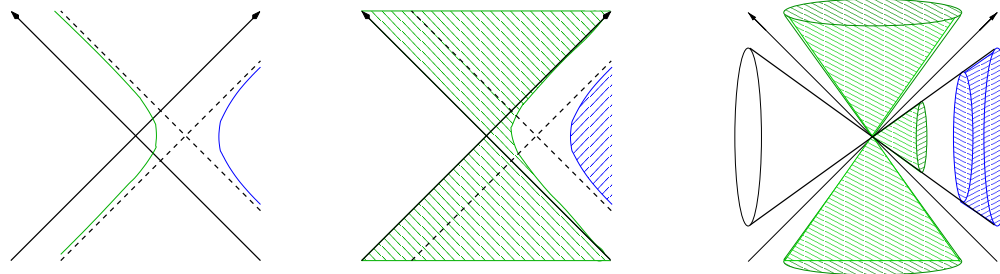
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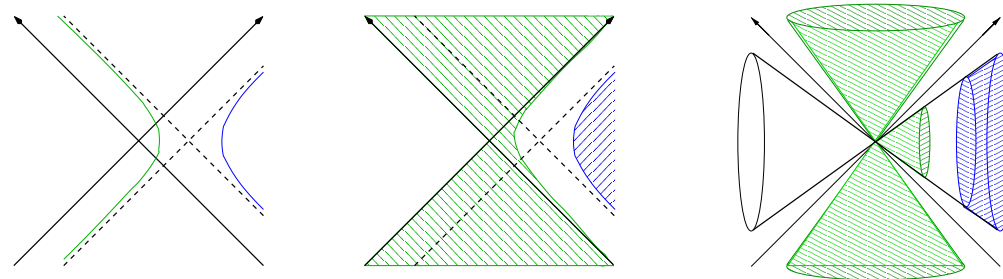
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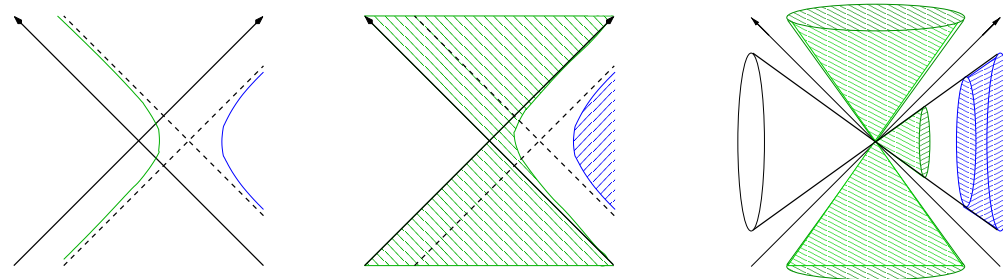


- Using the covariant derivative representation, we observe that x^\pm is the **Heisenberg operator** corresponding to the location of the closed string (at $\sigma = 0$):

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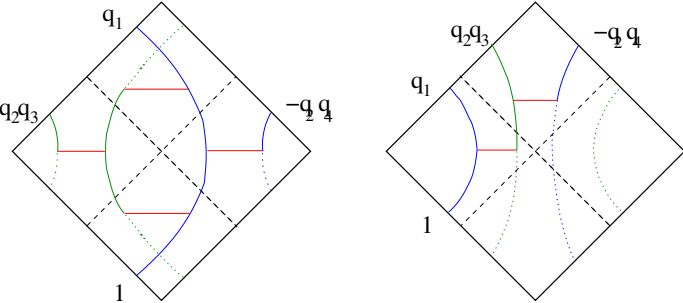
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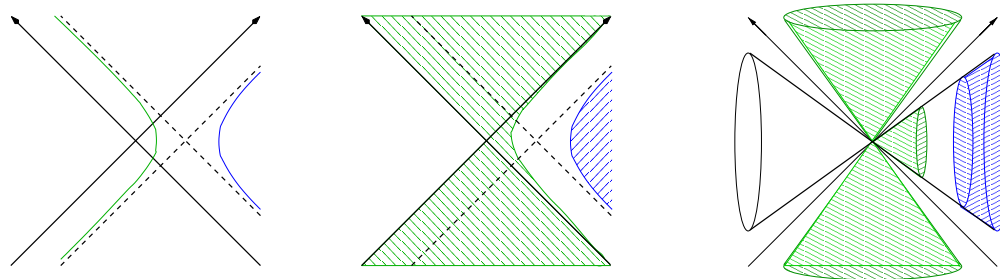
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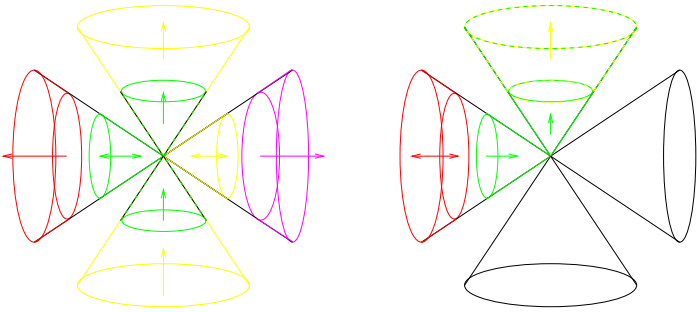
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- The open string global wave functions are also closed string wave functions...



Spontaneous production of winding strings

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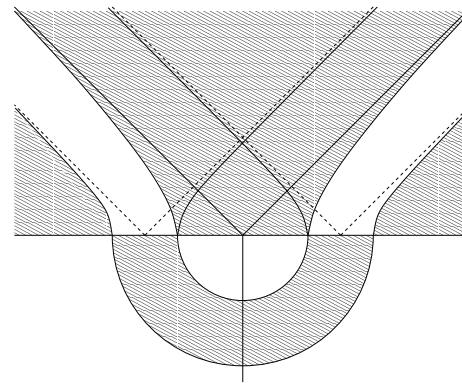
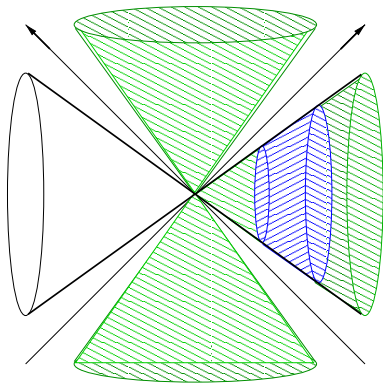
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- Similarly, in the closed string problem, **tunneling under the barrier** corresponds to **induced pair production of winding strings**.
- **Spontaneous pair production of winding strings** can be described by cutting open a periodic trajectory, either in **imaginary proper time**, or in the **Euclidean rotation orbifold**:



A few words on second quantization

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- In contrast, **long strings** are mixed particle \otimes anti-particle states. Creation and annihilation can be discriminated according to the sign of w , depending on the Rindler patch.

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- In this scheme, **short strings** correspond to particle \otimes particle (going forward in time), or anti-particle \otimes anti-particle (going backward in time): can be quantized as **creation or annihilation** operators.
- In contrast, **long strings** are mixed particle \otimes anti-particle states. Creation and annihilation can be discriminated according to the sign of w , depending on the Rindler patch.
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- Finally, motivated by holography, one may try to quantize with respect to the radial evolution in Rindler space. Short and long strings would be analogous to normalizable / non-normalizable modes.

Quantization in the Rindler patch

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- If so we should quantize the string with respect to the “time” coordinate σ rather than τ . The Rindler energy is given by the canonical generator associated to boosts,

$$W = - \int_{-\infty}^{\infty} d\tau \left(X^+ \partial_{\sigma} X^- - X^- \partial_{\sigma} X^+ \right) = \int_{-\infty}^{\infty} d\tau r^2 \partial_{\sigma} \eta$$

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- The total Rindler energy of a long string is **infinite**, due to its extension towards $r \rightarrow \infty$. The **energy density** by unit of radial distance

$$w(r) = \frac{4\nu^2 r^3 \operatorname{sgn}(\nu)}{\sqrt{(M^2 + \tilde{M}^2 - 4\nu^2 r^2)^2 - 4M^2 \tilde{M}^2}}$$

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- The spectrum is thus unbounded from below (and above): Can CTC prevent the vacuum to decay ?

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$$H_i' = -H_i \left(\sum_{j=1}^d H_j \right) + p_i + \frac{1}{D-1} \left(\rho - \sum_{j=1}^d p_j \right)$$

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- A bounce in dimension i requires $H_i' > 0$ at the point where $H_i = 0$, i.e.

$$(D-2)p_i + \rho \geq \sum_{j \neq i} p_j$$

The most efficient solution is a gas of scalar momentum states, with $p = \rho$: provides enough pressure for the bounce.

Effective gravity analysis (cont.)

- However, consider fundamental strings wrapped around dimension i ,

$$\rho = \frac{T}{V}, \quad p_i = -\rho, \quad p_{j \neq i} = 0, \quad V = \prod_{j \neq i} a_j \quad \Rightarrow \quad D \leq 3$$

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- We assumed a constant number of wound strings: one should incorporate the dependence of the production rate on the Hubble parameters.

Effective gravity analysis (cont.)

- Einstein's equations imply that the quantity

$$\mu = \left(\frac{H_k}{H_i} - 1 \right) / \left(\frac{H_j}{H_i} - \frac{3}{4 - D} \right)$$

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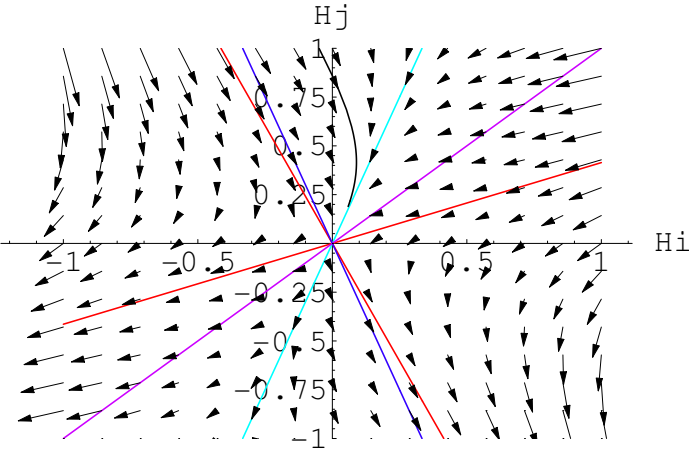
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A bounce for direction i in units of the eleven-dimensional frame therefore takes place for any initial condition such that $2\mu + D - 3 > 0$ and $2 < D < 4$.



Conclusions

We discussed closed strings in a toy model of a cosmological singularities. However, some of the features we uncovered should carry over to more general geometries:

- Winding string production can be understood semi-classically as **tunneling under the barrier** in regions with compact time, or **scattering over the barrier** in cosmological regions. In general, it can be computed as a **tree-level two-point function** in an appropriate basis depending on the choice of vacuum.

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- The production rate for winding strings in a singular geometry diverges at large winding number. **Can the resolved geometry be determined self-consistently so that the**

divergences at one-loop cancel those at tree-level ?