

Closed Strings in the Misner Universe

aka the Lorentzian orbifold

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based on hep-th/0307280 w/ M. Berkooz
and work in progress w/ M. Berkooz, B. Durin, D. Reichmann, M. Rozali

slides available from

<http://www.lpthe.jussieu.fr/~pioline/seminars.html>

Motivational string cosmology

- **Observational Cosmology** is now challenging string theory with high-precision data:

$$\Omega_{baryon} = 0.047, \quad \Omega_{darkm} = 0.243, \quad \Omega_{\Lambda} = 0.71, \quad w = -0.98 \pm .12, \dots$$

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- With LHC still far in the future, understanding **StringY Cosmology** may be the only way to make contact with reality...

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- Perturbative string theory requires an Euclidean worldsheet, hence Euclidean target space. The **analytic continuation** may be ambiguous or ill-defined, **Lorentzian observables** may be very different from their Euclidean counterparts.
- String theory is not content on a finite time interval, and one is frequently forced into **Big Bang / Big Crunch singularities, CTC** in the process of maximally extending the geometry.

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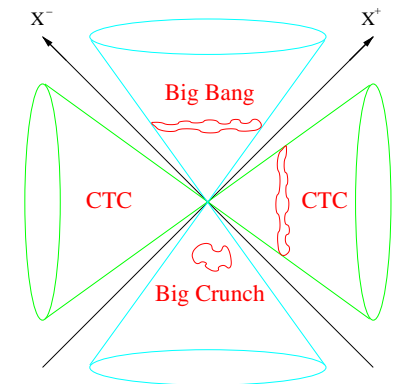
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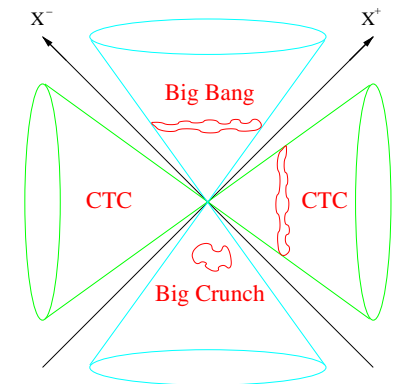
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- Our main focus will be on the **topological excitations** which wind around the collapsing dimension: can the production of winding states resolve the singularity ?

Outline of the talk

1. Introduction
2. The Lorentzian orbifold and its avatars
3. Closed strings in Misner space: first pass
3. A detour: Open strings in electric fields
4. Closed strings in Misner space: second pass
5. Conclusions, speculations

Misner, Taub-NUT, Grant...

Nekrasov

Berkooz BP

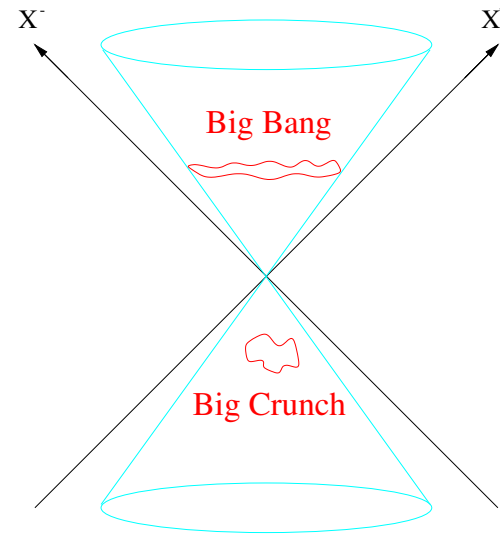
Berkooz BP; Berkooz Durin BP Reichmann Rozali

The Lorentzian orbifold

- One of the simplest examples of space-like singularities is the **quotient of flat Minkowski space by a discrete boost**, also known as **Misner space** (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^\pm \sim e^{\pm 2\pi\beta} X^\pm$$

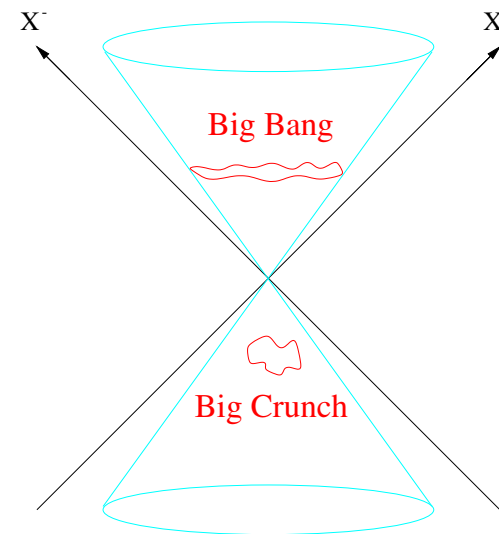


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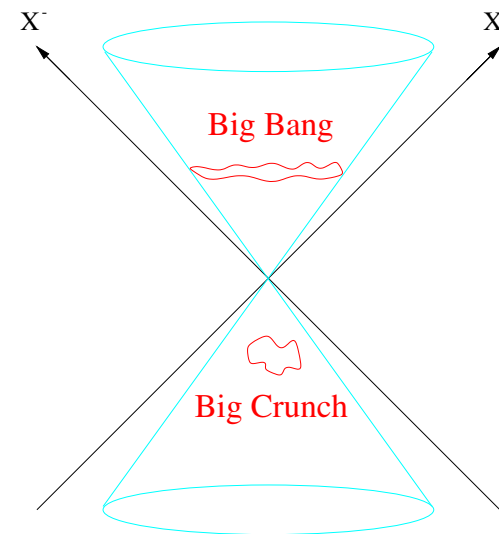
$$ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + (dX^i)^2, \quad \theta \equiv \theta + 2\pi, \quad X^\pm = T e^{\pm\beta\theta} / \sqrt{2}$$

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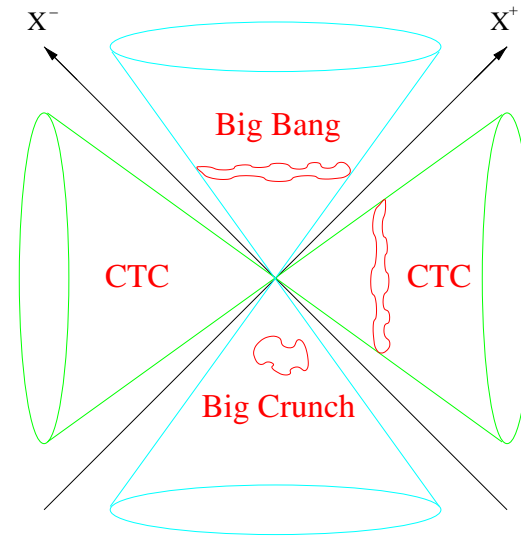
This is a (degenerate) **Kasner singularity**, everywhere **flat**, except for a **delta-function curvature** at $T = 0$.

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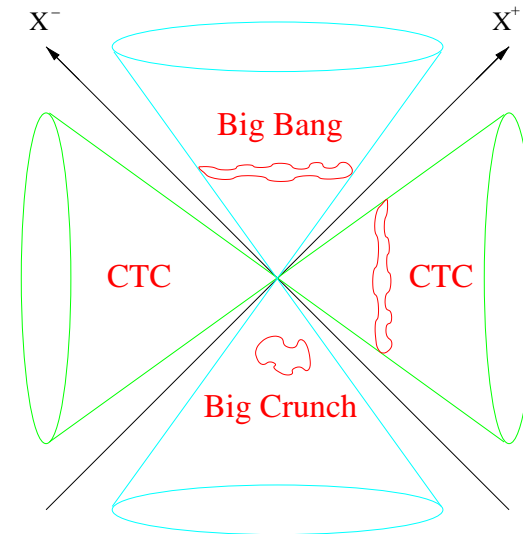


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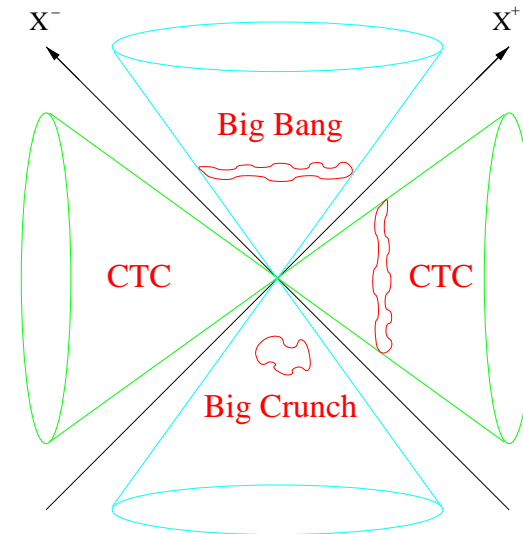
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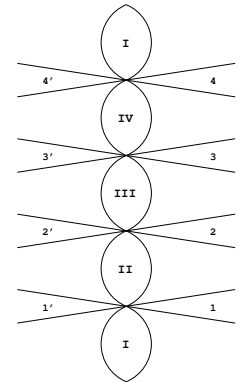
- Finally, the **lightcone** $X^+X^- = 0$ gives rise to a **null, non-Hausdorff** locus attached to the singularity.

Close relatives of the Misner Universe

- Misner space was first introduced as a local model of **Lorentzian Taub-NUT** space:

$$ds^2 = 4l^2 U(t) \sigma_3^2 + 4l \sigma_3 dt + (t^2 + l^2) (\sigma_1^2 + \sigma_2^2), \quad U(t) = -1 + \frac{2mt + l^2}{t^2 + l^2}$$

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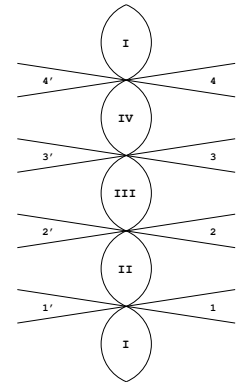
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- A close variant of Misner space is the quotient of flat space by the **combination of a discrete boost and a translation** on an extra direction, often known as the **Grant space**:

$$ds^2 = -2dX^+ dX^- + dX^2 + (dX^i)^2, \quad (X^\pm, X) \sim (e^{\pm 2\pi\beta} X^\pm, X + 2\pi R)$$

This describes the space away from two **moving cosmic strings**. The cosmological singularity is smoothed out, but regions with CTC remain.

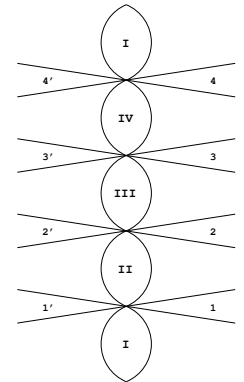


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Gott 91, Grant 93; Cornalba, Costa, Kounnas

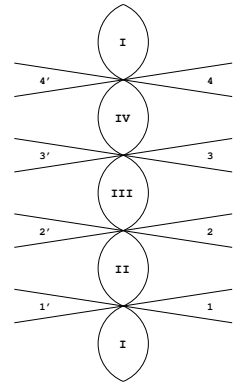
- The Misner geometry arose again more recently as the **M-theory** lift of a simple (**ekpyrotic**) cosmological solution of Einstein-dilaton gravity with no potential.

Khoury Ovrut Seiberg Steinhard Turok

Close relatives of the Misner Universe (cont)

- The **gauged WZW model** $Sl(2) \times Sl(2)/U(1) \times U(1)$ describes a **bouncing 4-dimensional Universe**, with singularities analogous to the Lorentzian orbifold.

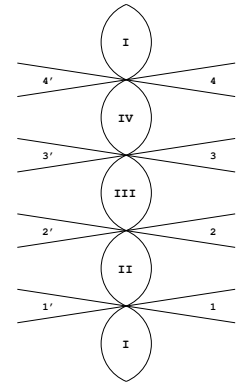
Nappi Witten; Elitzur Giveon Kutasov Rabinovici



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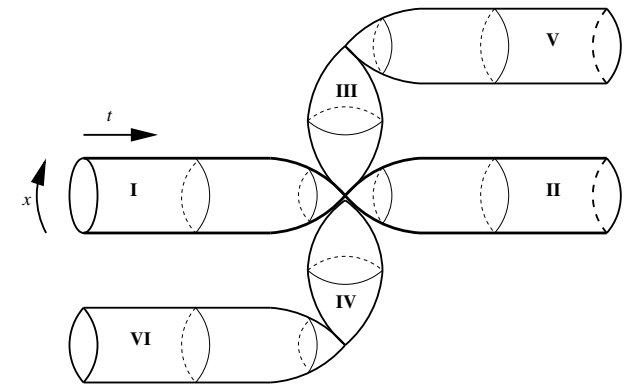
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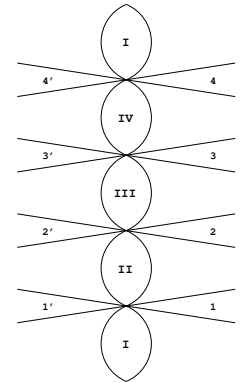
Tseytlin Vafa; Craps Kutasov Rajesh; Craps Ovrut



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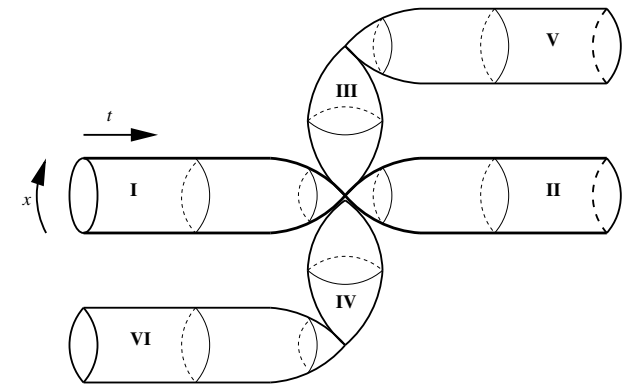
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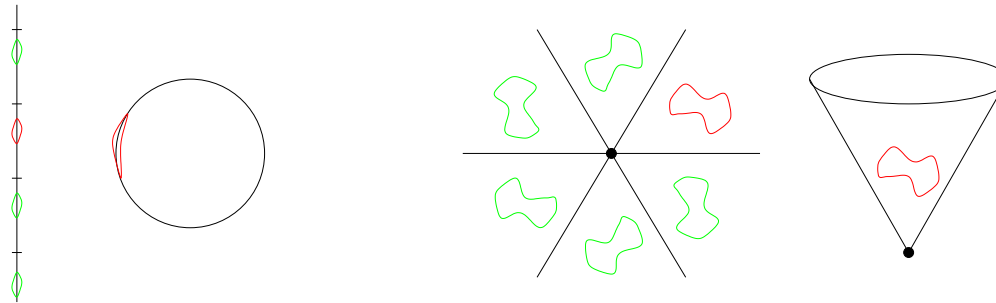


- The **Lorentzian orientifold** $IIB/[(-)^F boost]/[\Omega(-)^{FL}]$ was also recently argued to describe orientifolds of non-supersymmetric strings with non-vanishing Neveu-Schwarz tadpoles.

Dudas Mourad Timirgaziu

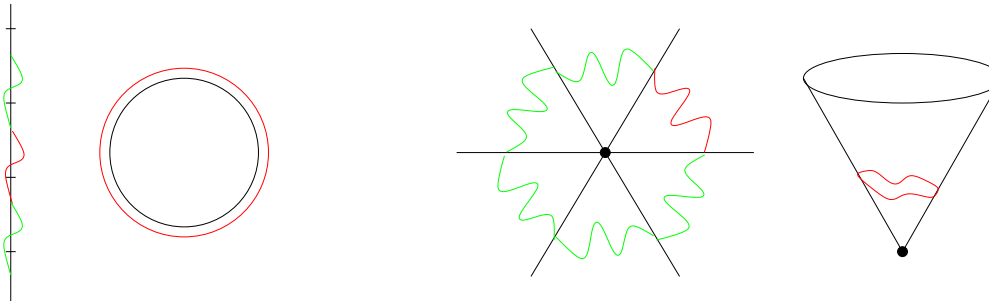
Strings on Euclidean orbifolds - untwisted states

- Well-known examples of orbifolds are the **circle**, R/Z , and the **rotation orbifold** R^2/Z_k .
- The spectrum of the quotient theory contains closed string states of the parent theory which are invariant under G : **untwisted states**.



Strings on Euclidean orbifolds - twisted states

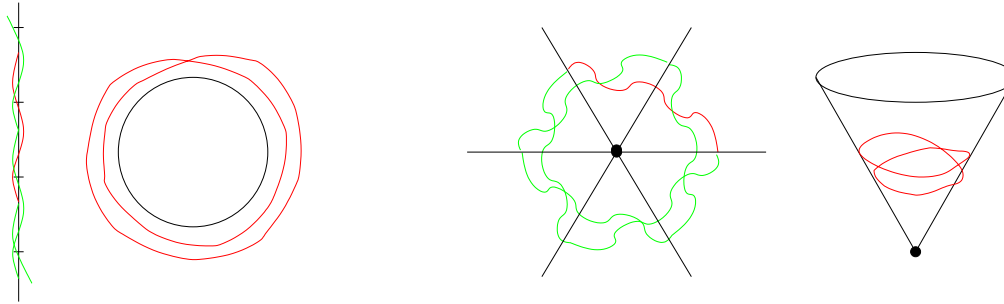
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- Modular invariance** requires that the spectrum should also include closed strings in the quotient theory which **close up to the action of G** in the parent theory: **twisted states**.



- When G acts non-freely, the twisted sector states are **localized at the fixed points**. They yield new localized degrees of freedom, which ensure the consistency of the background: anomaly free, divergence free...

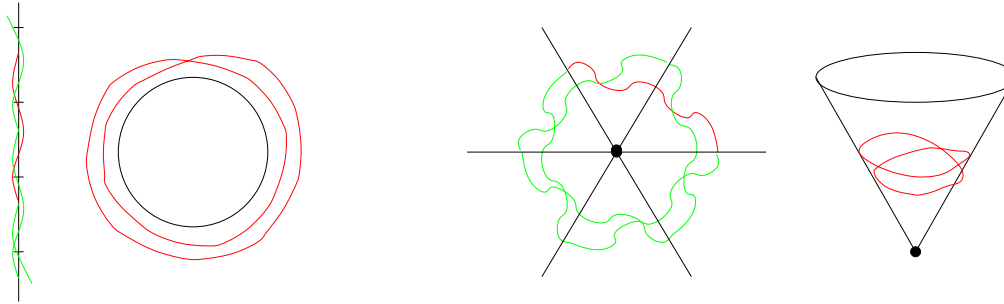
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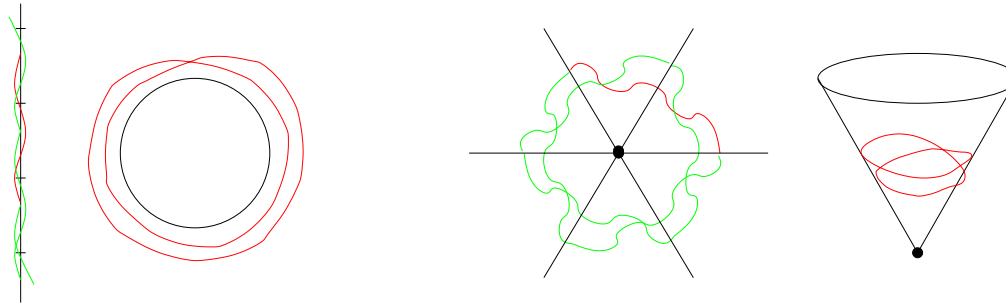
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- Additionally, each twisted sector admits excited levels. The ground state can be thought as a **Gaussian wave function** centered at the origin.

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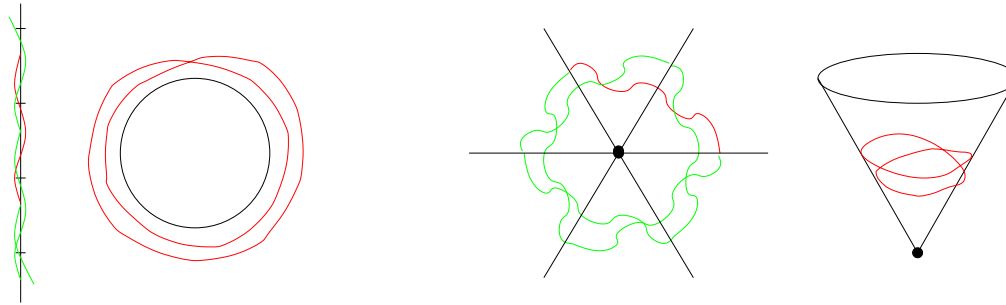
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- Additionally, each twisted sector admits excited levels. The ground state can be thought as a **Gaussian wave function** centered at the origin.
- The **condensation** of these twisted states changes the vacuum, and effectively **resolves the singularity**: $R^2/Z_k \rightarrow R^2/Z_{k-1} \rightarrow \dots$ (tachyon), $R^4/Z_k \rightarrow$ multi-centered Eguchi-Hanson (massless mode).

Strings on Euclidean orbifolds - twisted sectors (cont.)

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- The Lorentzian orbifold share features with both examples: an **infinite number of winding sectors**, and a, **non compact, fixed locus**.

Closed strings in Misner space - untwisted states

- As usual in standard orbifolds, part of the spectrum involves closed strings on Minkowski covering space, which are **invariant under the orbifold projection**. In conformal gauge,

$$X^\pm(\sigma + 2\pi, \tau) = X^\pm(\sigma, \tau), \quad (\partial_\tau^2 - \partial_\sigma^2)X^\pm = 0$$

satisfying the Virasoro (physical state) condition $(\dot{X}^\mu \pm X'^\mu)^2 = 0$.

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$$\sum_{n=-\infty}^{\infty} \partial X^+ \bar{\partial} X^- \exp\left(ik^+ X^- e^{-2\pi\beta n} + ik^- X^+ e^{2\pi\beta n} + ik_i x^i\right)$$

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- The resulting eigenfunctions describe **closed strings traveling around the Milne circle** with integer momentum j .

Quantum fluctuations in field theory

- In the **Minkowski vacuum** (inherited from the covering space), the renormalized propagator can be obtained as a sum over images,

$$G(x; x') = \sum_{n=-\infty, n \neq 0}^{\infty} \int_0^{\infty} d\tau \int dp^{\mu} \exp \left(-ip^{-}(x^{+} - e^{2\pi\beta n} x^{+'}) - ip^{+}(x^{-} - e^{2\pi\beta n} x^{-}') - ip^i(x^i - x^{i'}) \right)$$

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- The one-loop stress-energy tensor follows from $G(x, x)$, e.g for a conformally coupled scalar,

$$\langle T_{ab} \rangle = \lim_{x \rightarrow x'} \left[(1 - 2\xi) \nabla_a \nabla'_b - 2\xi \nabla_a \nabla_b + (2\xi - \frac{1}{2}) g_{ab} \nabla_c \nabla'^c \right] G(x, x')$$

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leading to a **divergent quantum backreaction**:

$$\langle T_{\mu}^{\nu} \rangle = \frac{K}{12\pi^2} T^{-4} \text{diag}(1, -3, 1, 1), \quad K = \sum_{n=1}^{\infty} \frac{2 + \cosh 2\pi n\beta}{[\cosh 2\pi n\beta - 1]^2}$$

One-loop vacuum amplitude in field and string theory

- On the other hand, in string theory $\langle T_{\mu}^{\nu} \rangle(x)$ is an **off-shell** quantity, and only its integral over space-time is well defined:

$$\int dx^+ dx^- G(x, x) = \sum_{l=-\infty}^{+\infty} \int_0^{\infty} \frac{d\rho}{\rho^{D/2}} \frac{e^{-m^2 \rho}}{\sinh^2(\pi \beta l)}$$

- This reproduces the zero-mode contribution to the string one-loop vacuum amplitude in the untwisted sector:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l, w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2 \rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho) x \theta_1(i\beta(l + w\rho); \rho)|^2}$$

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- The local divergence in $\langle T_\mu^\nu \rangle(x)$ is integrable and yields a finite free energy.
- The existence of **Regge trajectories** with arbitrary high spin implies new (log) **divergences in the bulk of the moduli space, not unlike long string poles in AdS_3 .**

Scattering of untwisted states

- Tree-level scattering amplitudes of untwisted sector states can be computed from those in flat space by the inheritance principle,

$$\langle V(j_1, k_1) \dots V(j_n, k_n) \rangle_{Misner} = \int dv_1 \dots dv_n e^{i(j_1 v_1 + \dots + j_n v_n)}$$

$$\langle V(e^{\beta v_1} k_1^+, e^{-\beta v_1} k_1^-, k_1^i) \dots V(e^{\beta v_n} k_n^+, e^{-\beta v_n} k_n^-, k_n^i) \rangle_{Minkowski}$$

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- The integral diverges due to Regge behavior in the large momentum, fixed angle regime. E.g, the four-tachyon scattering amplitude in bosonic string leads to

$$\int dv v^{-\frac{1}{2}(k_1^i - k_3^i)^2 + i(j_2 - j_4)}$$

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Berkooz Craps Rajesh Kutasov

- It could be that eikonal resummation of ladder diagrams may lead to a finite result, e.g.

$$A \sim -G \frac{s^2}{t} \rightarrow -G \frac{s^2}{t + (2\pi G s)^2} \quad (3\text{D gravity})$$

Deser McCarthy Steif; Cornalba Costa

Closed string in Misner space - twisted sectors

- In addition, there is **an infinite set of twisted sectors**, corresponding to strings on the covering space that close **up to the action of the orbifold group**:

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau) , \quad \nu = 2\pi\omega\beta$$

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$$X_R^\pm(\tau - \sigma) = \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha_n^\pm e^{-i(n \pm i\nu)(\tau - \sigma)}$$

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- There are no translational zero-modes, instead **two pairs of quasi zero-modes** which are canonically conjugate **real** operators:

$$[\alpha_0^+, \alpha_0^-] = -i\nu, \quad [\tilde{\alpha}_0^+, \tilde{\alpha}_0^-] = i\nu$$

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated by half of them, e.g.

$$\alpha_{n>0}^{\pm}, \quad \tilde{\alpha}_{n>0}^{\pm}, \quad \alpha_0^-, \quad \tilde{\alpha}_0^+$$

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- The worldsheet Hamiltonian, **normal-ordered wrt to this vacuum**, reads

$$L_0^{l.c.} = - \sum_{n=0}^{\infty} (\alpha_n^+)^* \alpha_n^- - \sum_{n=1}^{\infty} (\alpha_n^-)^* \alpha_n^+ + \frac{1}{2} i\nu(1 - i\nu) - 1 + L_{int}$$

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- Due to the $i\nu/2$ term in the ground state energy, all states obtained by acting on $|0\rangle$ by creation operators $\alpha_{n<0}^{\pm}$ and by α_0^+ will have **imaginary energy**, hence **the physical state condition $L_0 = 0$ has no solutions.**

One-loop amplitude, twisted sector

- Independently of this fact, one may compute the one-loop path integral on an **Euclidean worldsheet and Minkowskian target-space**:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho)x \theta_1(i\beta(l+w\rho); \rho)|^2}$$

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- In the **twisted** sector, the left-moving zero-modes contribute

$$\frac{1}{2 \sinh(\beta w \rho)} = \sum_{n=1}^{\infty} q^{i(n+\frac{1}{2})\beta w}$$

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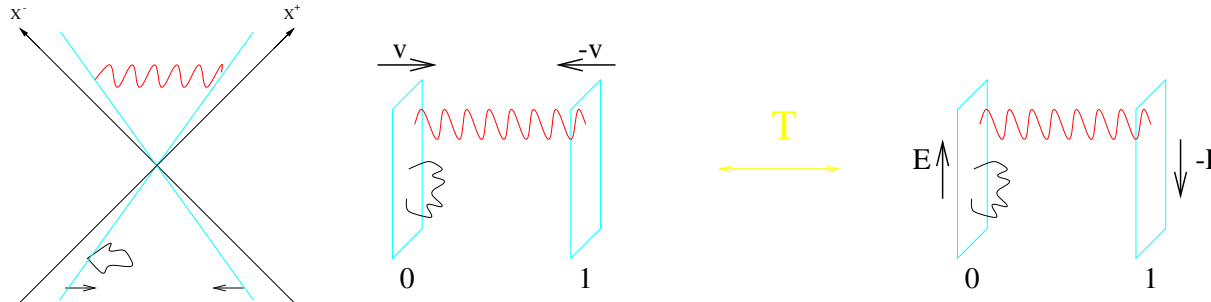
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- The absence of physical twisted states crushes our hopes for resolving the singularity... yet does not sound very sensible. An important point: α_0^+ and α_0^- are not hermitian conjugate to each other, but rather **self-hermitian**...

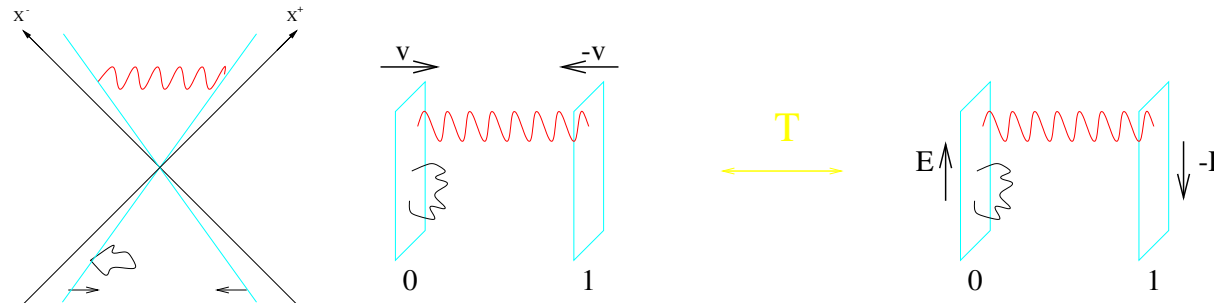
A detour via Open strings in electric field

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- Recall that for open strings stretched between two D-branes with electromagnetic fields F_0 and F_1 , proper frequencies satisfy

$$e^{-2\pi i \omega_n} = \frac{1 + F_0}{1 - F_0} \cdot \frac{1 - F_1}{1 + F_1}$$

For $F_0 \neq F_1$, the open string carries a net electric charge, and the motion of its center of motion is **that of a charged particle**.

- In the case of an **electric field** $F_1 = E dx^+ \wedge dx^-$, $F_0 = 0$, the resulting spectrum is

$$\omega_n = n + i\nu, \quad \nu := \text{Arctanh} E = w\beta$$

just as in the **Lorentzian orbifold** case. The large winding limit amounts to a **near critical electric field**.

Open string mode expansion

- The light-cone embedding coordinates have the normal mode expansion

$$X^\pm = x_0^\pm + i \sum_{n=-\infty}^{+\infty} (-)^n (n \pm i\nu)^{-1} a_n^\pm e^{-i(n \pm i\nu)\tau} \cos[(n \pm i\nu)\sigma]$$

with reality conditions $(a_n^\pm)^* = a_{-n}^\pm$, $(x_0^\pm)^* = x_0^\pm$

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- By the same token, charged open strings should have no physical states... yet electrons and positrons do exist.

Charged particle and open string zero-modes

- Let us recall the quantization of a **charged particle in an electric field**:

$$L = \frac{1}{2}m \left(-2\partial_\tau X^+ \partial_\tau X^- + (\partial_\tau X^i)^2 \right) + \frac{e}{2} \left(X^+ \partial_\tau X^- - X^- \partial_\tau X^+ \right)$$

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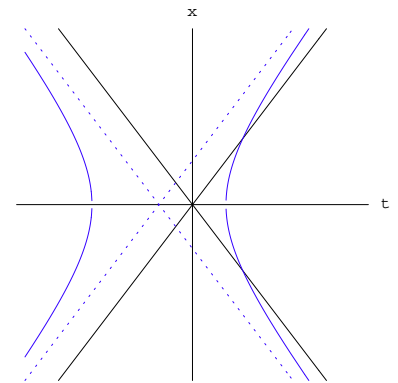
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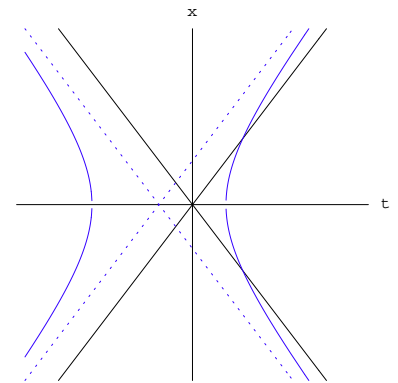
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- Starting from the canonical equal-time commutation rules

$$[\pi^+, x^-] = [\pi^-, x^+] = i, \quad [\pi^i, x^j] = i\delta_{ij}$$

one recovers the open string zero-mode commutation relations ($\nu = e$),

$$[a_0^+, a_0^-] = -i\nu, \quad [x_0^+, x_0^-] = -\frac{i}{\nu}$$



Charged particle and ppen string zero-modes

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$$a_0^\pm = i\partial_\mp \pm \frac{\nu}{2}x^\pm, \quad x_0^\pm = \mp \frac{1}{\nu} \left(i\partial_\mp \mp \frac{\nu}{2}x^\pm \right)$$

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- The zero-mode piece of L_0 , **including the evil** $\frac{i\nu}{2}$,

$$L_0^{(0)} = -a_0^+ a_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\nabla_0^+ \nabla_0^- + \nabla_0^- \nabla_0^+)$$

is just the **Klein-Gordon operator** of a particle of charge ν .

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^\pm = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator can be rewritten as an **inverted harmonic oscillator**:

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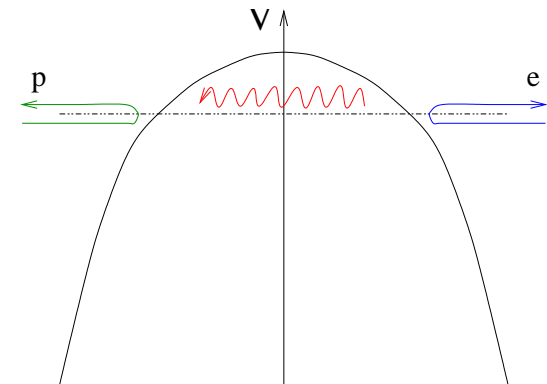
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- The latter admits a respectable **delta-normalizable spectrum of scattering states**, in terms of **parabolic cylinder functions**, e.g:

$$\phi_{in}^+ = D_{-\frac{1}{2} + i\frac{M^2}{2\nu}} \left(e^{-\frac{3i\pi}{4}u} \right) e^{-i\tilde{p}t} e^{i\nu x t/2}$$



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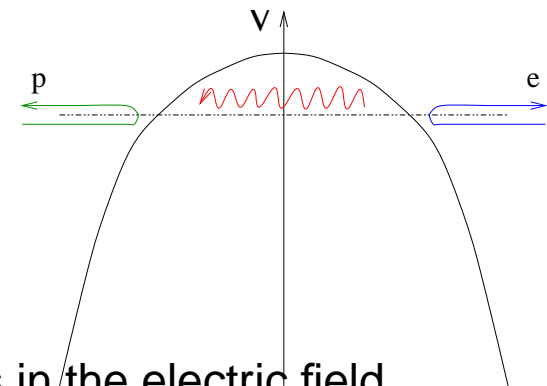
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- These correspond to **non-compact trajectories** of charged particles in the electric field. **Tunnelling** is just (stimulated) **Schwinger pair creation**,

$$e^- \rightarrow (1 + \eta) e^- + \eta e^+, \quad \eta \sim e^{-\pi M^2/\nu}$$



Lorentzian vs Euclidean states

- Analytic continuation $X^0 \rightarrow -iX^0$, $\nu \rightarrow i\nu$ turns an electric field in $R^{1,1}$ into a magnetic field in R^2 . At the same time, one should Wick rotate the worldsheet time.

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$$\frac{1}{2i \sin(\nu t/2)} = \sum_{n=1}^{\infty} e^{-i(n+\frac{1}{2})\nu t} = \int dM^2 \rho(M^2) e^{-M^2 t/2}$$

The density of states is obtained from the **reflection phase shift**,

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- The physical spectrum can be explicitly worked out at low levels, and is **free of ghosts**: a tachyon at level 0, a **transverse gauge boson** at level 1, ...

Charged particle in Rindler space

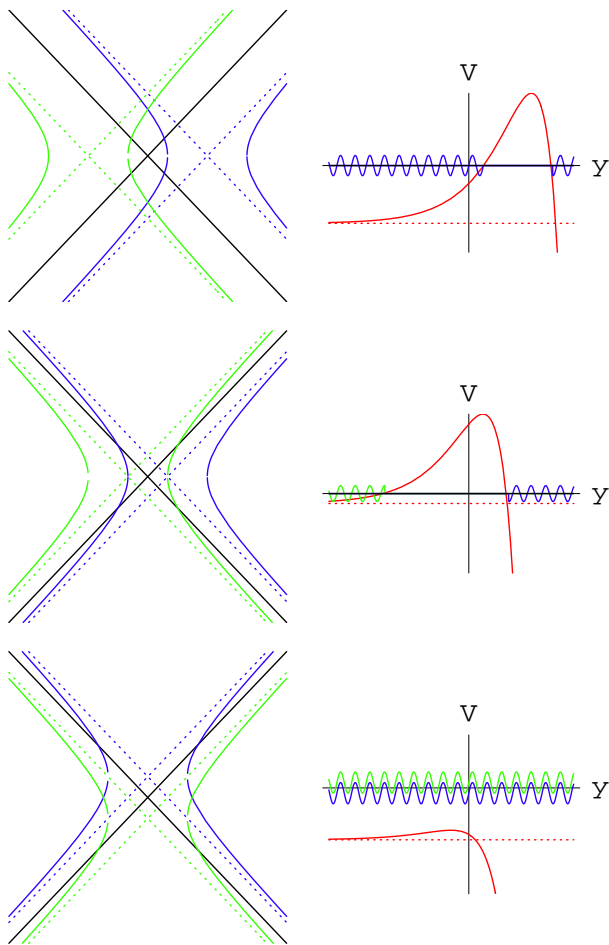
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Gabriel Spindel; Mottola Cooper

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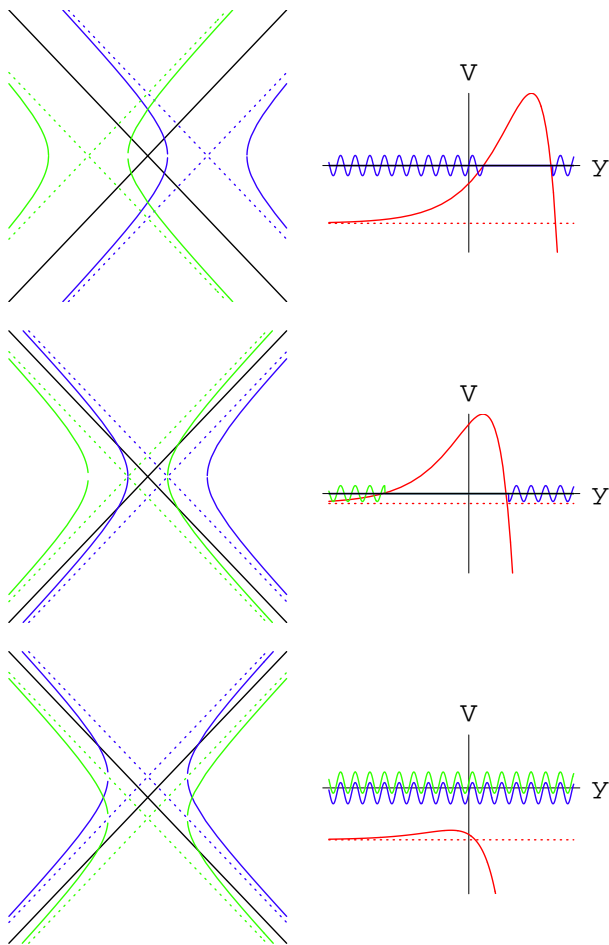
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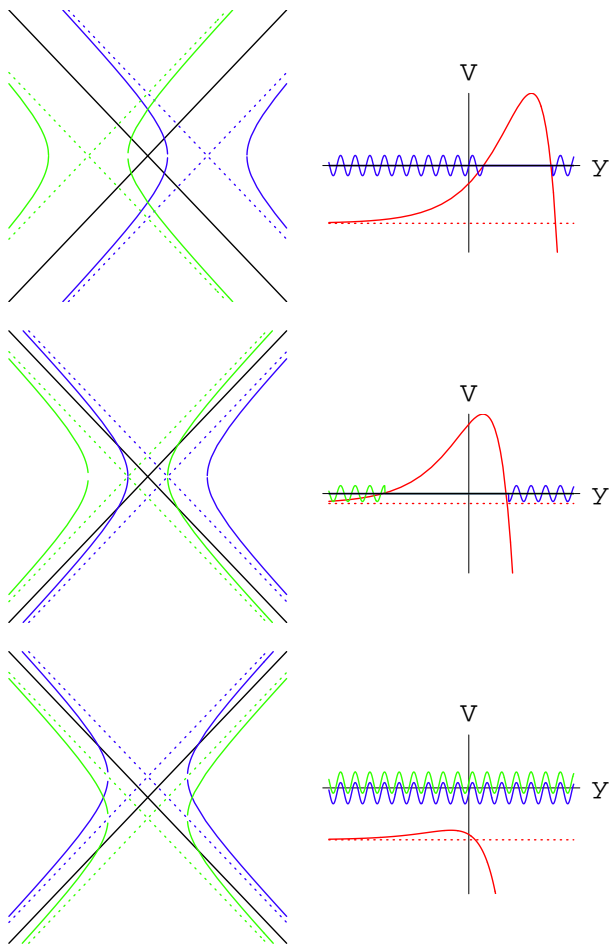
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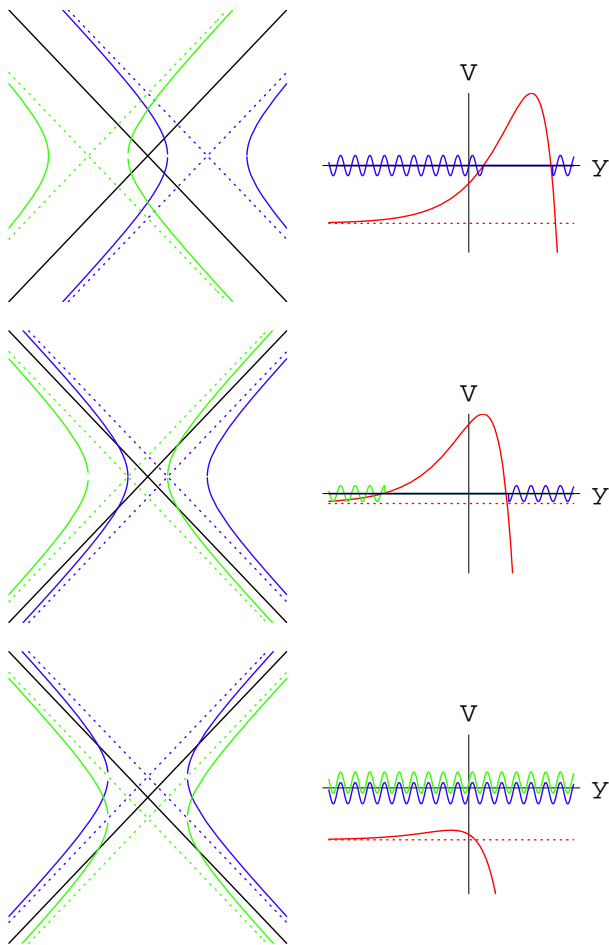
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- If $j > M^2/(2\nu)$, the electron branches extend in the Milne regions. There is **no tunneling**, but partial reflection amounts to a combination of **Schwinger** and **Hawking** emission.

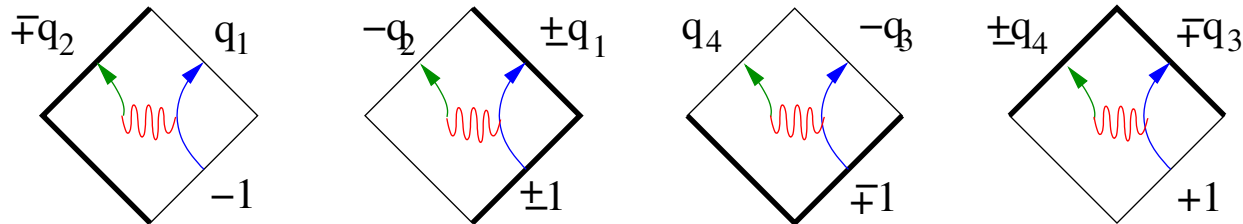
Rindler modes

- Incoming modes from Rindler infinity I_R^- read, in terms of parabolic cylinder functions:

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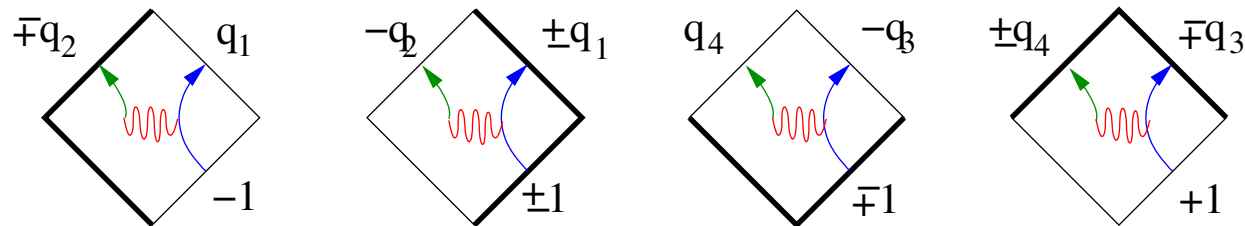
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- The reflection coefficients can be computed:

$$q_1 = e^{-\pi j} \frac{\cosh\left[\pi \frac{M^2}{2\nu}\right]}{\cosh\left[\pi \left(j - \frac{M^2}{2\nu}\right)\right]}, \quad q_3 = e^{\pi \left(j - \frac{M^2}{2\nu}\right)} \frac{\cosh\left[\pi \frac{M^2}{2\nu}\right]}{|\sinh \pi j|}$$

and $q_2 = 1 - q_1$, $q_4 = q_3 - 1$, by charge conservation.

Global Charged Unruh Modes

- Global modes may be defined by patching together Rindler modes, ie by **analytic continuation across the horizons**. **Unruh modes** are those which are superposition of **positive energy** Minkowski modes,

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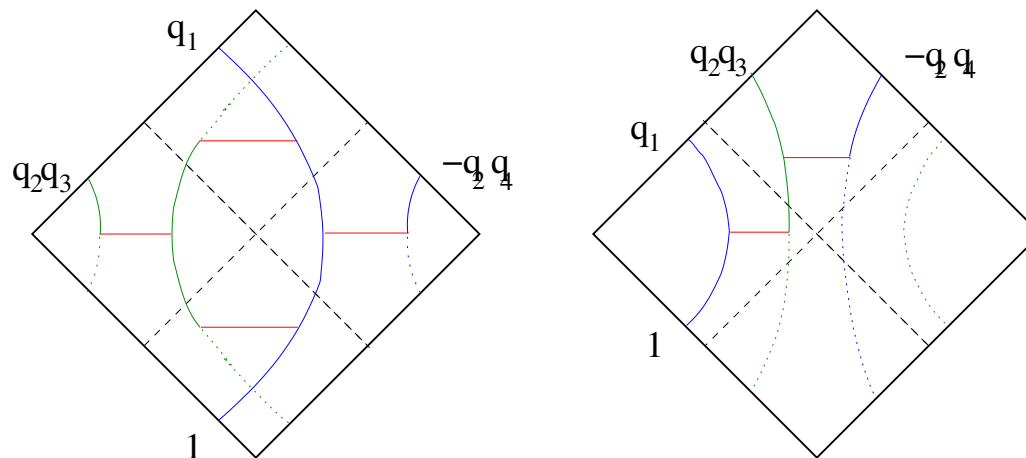
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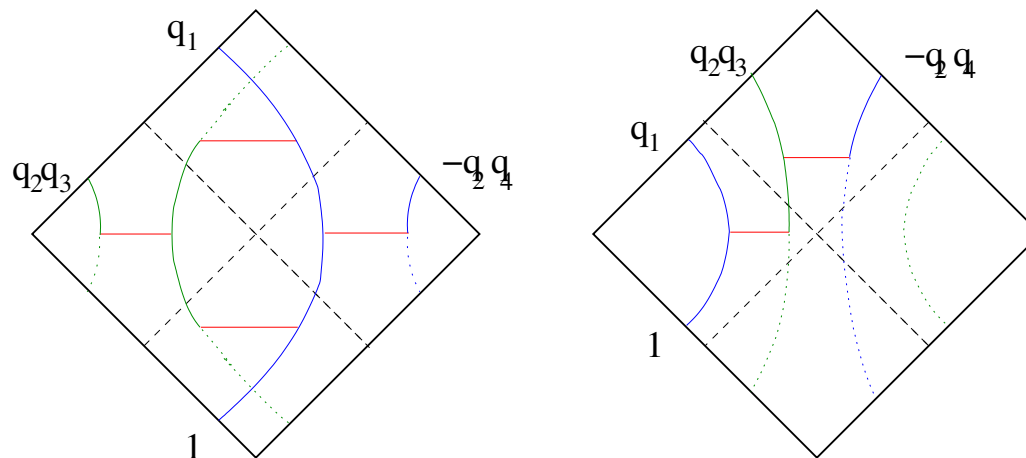
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- Any state in Minkowski space can be represented as a state in the **tensor product of the Hilbert spaces of the left and right Rindler patches**. In contrast to neutral fields in Rindler space, Boulware-Fulling modes that vanish in L or R have positive Minkowski energy.

Closed string zero-modes

- Let us reanalyze the classical solutions for the closed string zero modes

$$X^\pm(\tau, \sigma) = e^{\mp\nu\sigma} \left[\pm \frac{1}{2\nu} \alpha_0^\pm e^{\pm\nu\tau} \mp \frac{1}{2\nu} \tilde{\alpha}_0^\pm e^{\mp\nu\tau} \right], \quad \alpha_0^\pm, \tilde{\alpha}_0^\pm \in R$$

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- Up to a shift of τ and σ , the physical state conditions require

$$\alpha_0^+ = \alpha_0^- = \epsilon \frac{M}{\sqrt{2}}, \quad \tilde{\alpha}_0^+ = \tilde{\alpha}_0^- = \tilde{\epsilon} \frac{\tilde{M}}{\sqrt{2}}, \quad M^2 - \tilde{M}^2 = 2\nu j \in Z$$

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We may thus follow the motion of a single point $\sigma = \sigma_0$ and obtain the rest of the worldsheet by **smearing under the action of the boost**.

- Up to a shift of τ and σ , the physical state conditions require

$$\alpha_0^+ = \alpha_0^- = \epsilon \frac{M}{\sqrt{2}}, \quad \tilde{\alpha}_0^+ = \tilde{\alpha}_0^- = \tilde{\epsilon} \frac{\tilde{M}}{\sqrt{2}}, \quad M^2 - \tilde{M}^2 = 2\nu j \in \mathbb{Z}$$

- The behavior at early/late proper time now depends on $\epsilon\tilde{\epsilon}$: For $\epsilon\tilde{\epsilon} = 1$, the string begin/ends in the **Milne** regions. For $\epsilon\tilde{\epsilon} = -1$, the string begin/ends in the **Rindler** regions.

Short and long strings ($j = 0$)

Choosing $j = 0$ for simplicity, we have two very different types of solutions:

- $\epsilon = 1, \tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma, \tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma}, \quad T = \frac{M}{\nu} \sinh(\nu\tau), \quad \theta = \nu\sigma$$

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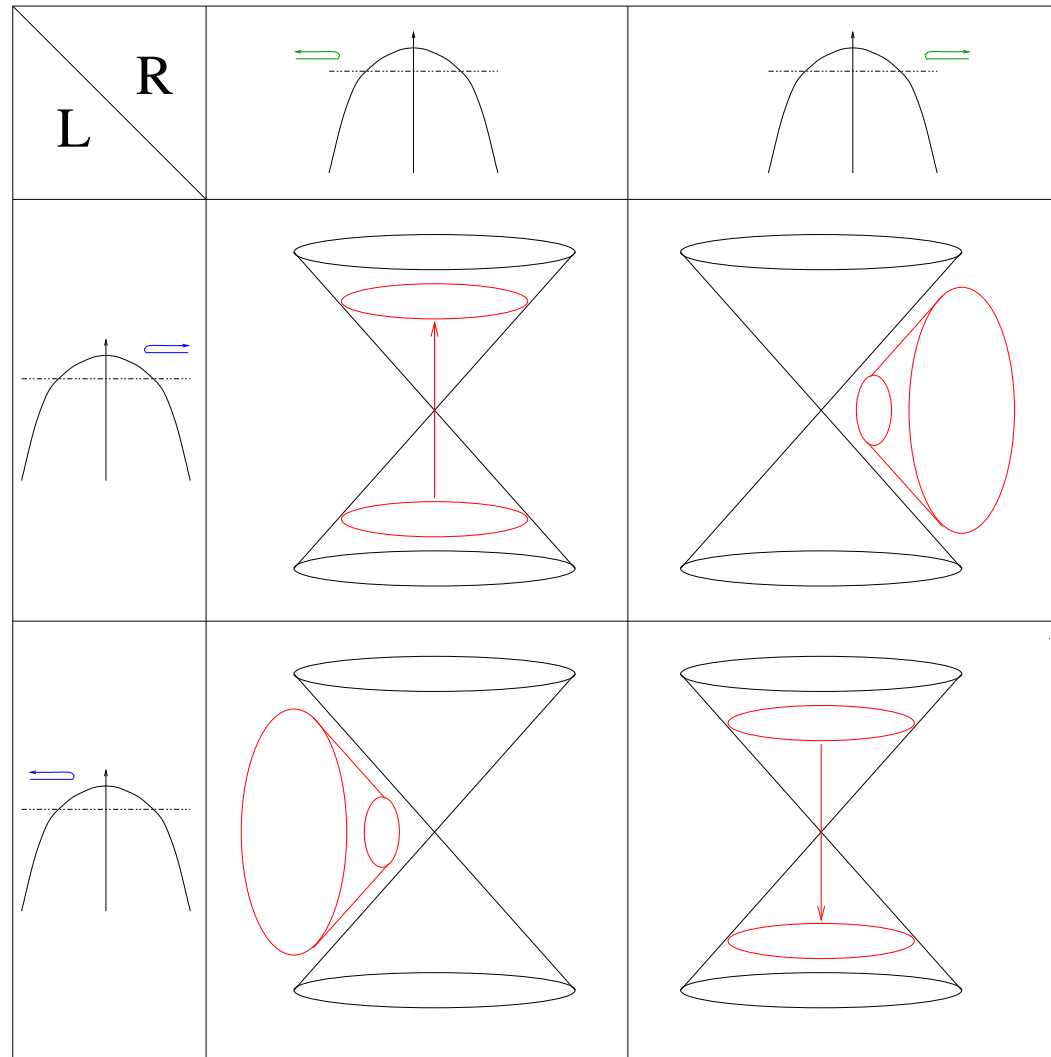
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$\epsilon = -1, \tilde{\epsilon} = 1$ is the analogue in the left Rindler patch.

Short and long strings

Closed string trajectories are thus generated by the motion of **two decoupled particles in inverted harmonic oscillators**:



Relation to open string modes

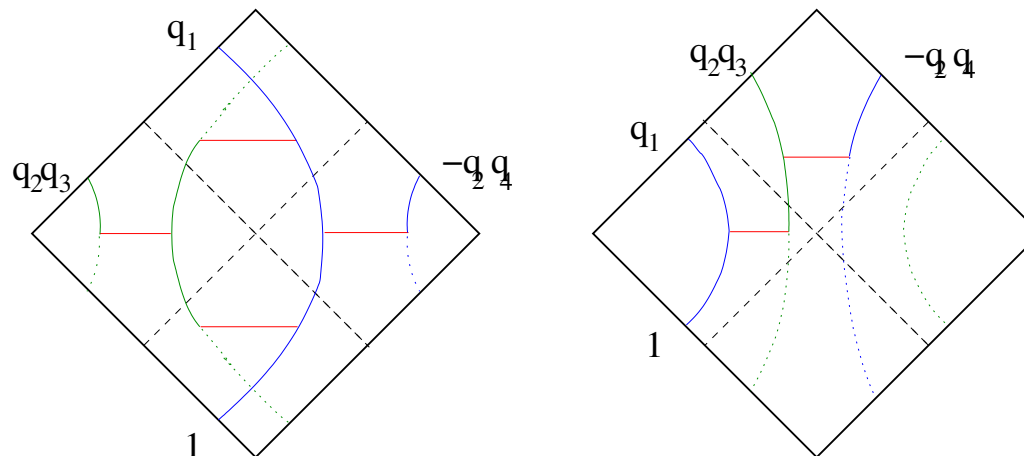
- Instead of following the motion of a point at fixed σ , one may consider instead a point at fixed $\sigma + \tau$: this is precisely the **trajectory of the open string zero-mode**.
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$$\alpha_0^\pm = i\partial_\mp \pm \frac{\nu}{2}x^\pm, \quad \tilde{\alpha}_0^\pm = i\partial_\mp \mp \frac{\nu}{2}x^\pm$$

we observe that x^\pm is the **Heisenberg operator** corresponding to the location of the closed string (at $\sigma = 0$):

$$X_0^\pm(\sigma, \tau) = e^{\mp\nu\sigma} \left[\cosh(\nu\tau) x^\pm + i \sinh(\nu\tau) \partial_\mp \right]$$

- The open string global wave functions...



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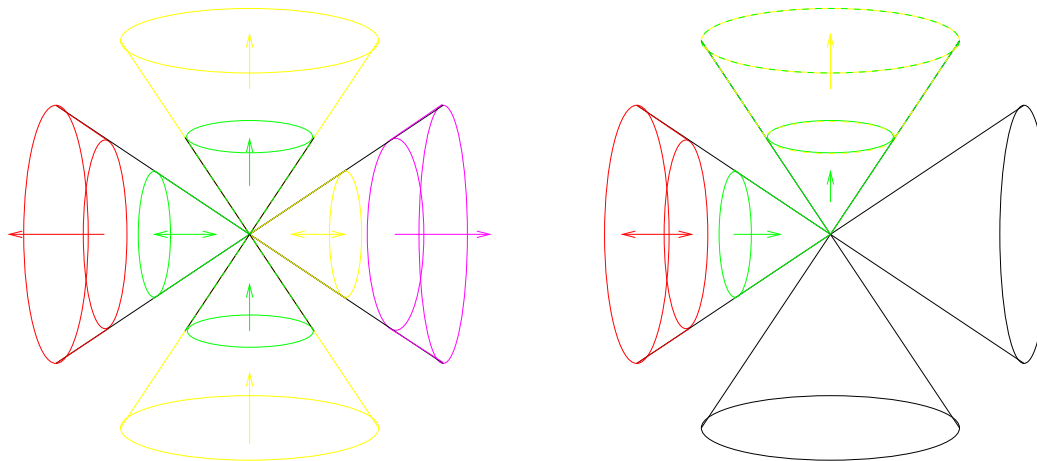
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- The open string global wave functions are also the closed string wave functions...



Comments on winding string production

- The production rate of winding strings can be evaluated by WKB methods: for the Misner Universe,

$$R \sim \exp\left(-\pi \frac{M^2}{\beta w}\right) \rightarrow 1 \text{ as } w \rightarrow \infty$$

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$$p_r^2 + M^2 - \left[wb(r) - \frac{j}{b(r)}\right]^2 = 0$$

with $b(r) = ia(ir)$. Only for $a(T) \propto T$ is the electric analogy precise. **The tunneling rate generically goes to 1 as $w \rightarrow \infty$** , irrespective of the behaviour at late times.

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- One may instead deform the profile $a(T) \rightarrow \sqrt{\beta^2 T^2 + \epsilon^2}$, such that $a(T)$ does not vanish anymore. The twisted wave equation is well defined without continuation to Rindler space. **What happens to the Rindler patches ?**

Effective gravity analysis

- Consider a general Kasner ansatz

$$ds^2 = -dt^2 + \sum_{i=1}^D a_i^2(t) dx_i^2, \quad T_{\mu\nu} = \text{diag}(\rho, a_i^2 p_i \delta_{ij})$$

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- A bounce in dimension i requires $H'_i > 0$ when $H_i = 0$, hence

$$(D-2)p_i + \rho \geq \sum_{j \neq i} p_j$$

- In the presence of strings wrapped around dimension i ,

$$\rho = \frac{T}{V}, \quad p_i = -\rho, \quad p_{j \neq i} = 0, \quad V = \prod_{j \neq i} a_j \quad \Rightarrow \quad D \leq 3$$

Effective gravity analysis (cont.)

- Modelling the dilaton as the radius of the \sharp th direction, the strings become membranes wrapped around (i, \sharp) :

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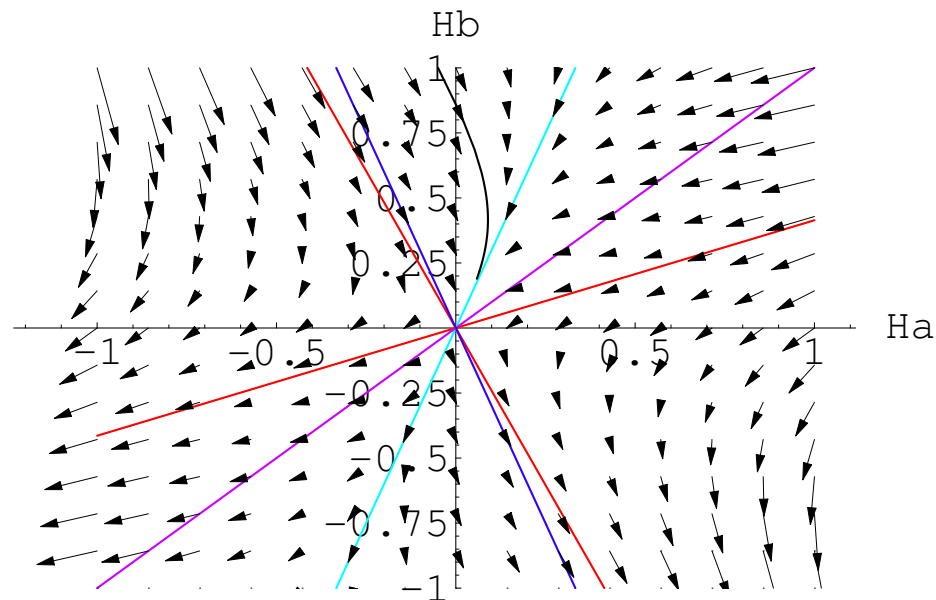
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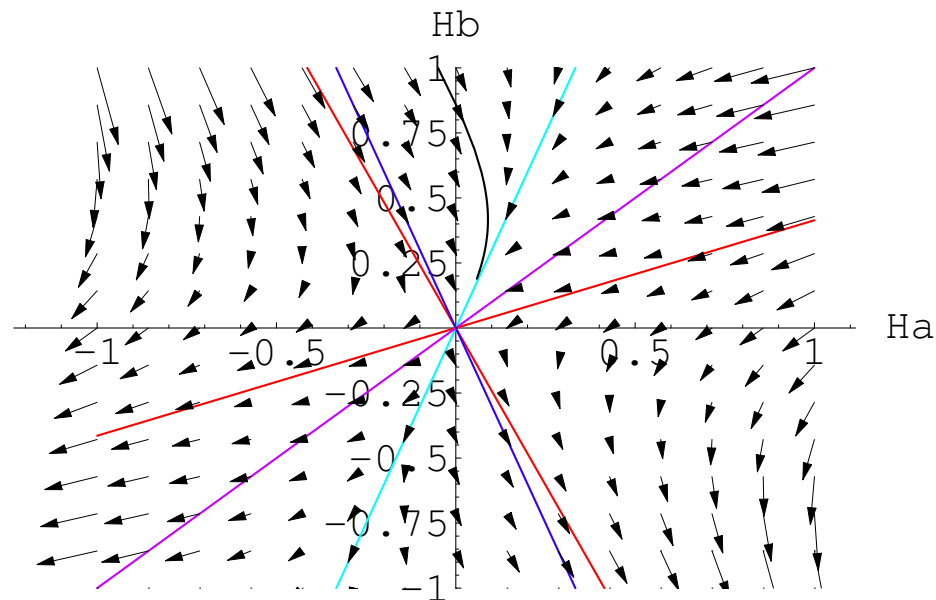


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- This discussion assumed a constant number of wound strings: one should incorporate the dependence on the contraction rate.

Quantization in the Rindler patch

- For **long strings** in conformal gauge, the **worldsheet time** τ is in fact a **spacelike coordinate** wrt to the induced metric. For **short strings**, the induced metric undergoes a **signature flip** as it wanders in the Rindler patch.
- If so we should quantize the string with respect to the **“time” coordinate** σ rather than τ . The canonical generator of time translations

$$E = - \int_{-\infty}^{\infty} d\tau \left(X^+ \partial_\sigma X^- - X^- \partial_\sigma X^+ \right) = \int_{-\infty}^{\infty} d\tau r^2 \partial_\sigma \eta$$

is infinite: **long strings carry an infinite Rindler energy.**

- Introducing a cut-off $-T \leq \tau < T$, the Rindler energy

$$E_T \sim -\frac{e^{2\nu T}}{4\nu^2} \left(\tilde{\alpha}_0^+ \alpha_0^- + \tilde{\alpha}_0^- \alpha_0^+ \right)$$

can be understood as the **tensive energy of the static stretched string.**

- The Rindler energy spectrum is **unbounded** both above and below:

$$E_{short} < -e^{2\nu T} \frac{M\tilde{M}}{4\nu^2} < e^{2\nu T} \frac{M\tilde{M}}{4\nu^2} < E_{long}$$

How can one prevent the decay into short strings ?

Conclusions - speculations

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- If one manages to make sense of the winding production rate, and if the singularity gets resolved, what happens to the whiskers ? Can they provide some time-independent dual description of the cosmological evolution ?

Conclusions - speculations (cont.)

- To demonstrate that the singularity is resolved, one should in principle take into account the production of (an infinite number) of twisted sector states in correlated pairs, i.e. **squeezed states**: **non-local deformations** of the worldsheet ? closed string field theory ?

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- More generally, we still lack a framework to compute the **production of closed strings in cosmological backgrounds**. Those however are likely to lead to large departures from FRW cosmology, and possibly spectacular effects: cosmological bounce, Hagedorn phase transition...

Lawrence Martinec, Gubser

Appendices (not shown during talk)

Vacua of Misner space

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$$H_{-ij}^{(1)}(mT)e^{-ij\theta} \sim e^{-ij\theta - imT} / \sqrt{T}$$

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Closed string one-loop vacuum amplitude

- Independently of this fact, one may compute the one-loop (Euclidean ws, Minkowskian target) free energy using path integral methods:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2 \rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho) \theta_1(i\beta(l + w\rho); \rho)|^2}$$

where θ_1 is the Jacobi theta function,

$$\theta_1(v; \rho) = 2q^{1/8} \sin \pi v \prod_{n=1}^{\infty} (1 - e^{2\pi i v} q^n)(1 - q^n)(1 - e^{-2\pi i v} q^n), \quad q = e^{2\pi i \rho}$$

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- In the **untwisted** sector, this reproduces the integrated vacuum free energy found by the method of images:

$$\int dx^+ dx^- G(x, x) = \sum_{l=-\infty}^{+\infty} \int_0^{\infty} \frac{d\rho}{\rho^{D/2}} \frac{e^{-m^2 \rho}}{\sinh^2(\pi\beta l)}$$

One-loop amplitude and Schwinger pair production

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- Each of the poles at $t = 2k/\nu$ contributes to the imaginary part, yielding the **production rate of charged open strings**,

$$\mathcal{W} = \frac{1}{2(2\pi)^{25}} \frac{(e_0 + e_1)}{\nu} \sum_{k=1}^{\infty} (-)^{k+1} \left(\frac{|\nu|}{k}\right)^{13} \sum_{N=-1}^{\infty} c_b(N) \exp\left(-2\pi k \frac{N}{|\nu|} - 2\pi k |\nu|\right)$$

Bachas Porrati

where $\eta^{-24}(q) = \sum_{N=-1}^{\infty} c_b(N) q^N$. This can be viewed as the **sum of the Schwinger production rates** for each state in the spectrum, of mass $m^2 = 2N + \nu^2$.

One-loop amplitude and Schwinger pair production

- Using this quantization scheme, the one-loop (Euclidean worldsheet, Minkowskian target) vacuum free energy reads

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- This seems to support the quantization scheme based on a vacuum, hence the absence of physical states. **But physical states do exist classically, how could quantization make them disappear altogether?**

Wick rotation to a rotation orbifold

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- Rotating $\beta = i\mu$, the Rindler region becomes get indeed an Euclidean metric,

$$ds^2 = dr^2 + \mu^2 r^2 d\eta^2 + (dX^i)^2 = 2 dZ d\bar{Z} + (dX^i)^2$$

$$Z = X^+ = r e^{i\mu\eta}, \quad \bar{Z} = -X^- = r e^{-i\mu\eta} \quad (r > 0)$$

- The Milne identification $\eta \equiv \eta + 2\pi$ amounts to a **rotation identification** $Z \rightarrow e^{2\pi i\mu} Z$.

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- By the same token, the left Rindler wedge rotates to another copy of the Euclidean plane with the origin removed: the complete analytic continuation of Misner space is therefore

$$\widetilde{R^2 \setminus \{0\}}_L / e^{i\mu} \setminus \widetilde{R^2 \setminus \{0\}}_R$$

and states of interest are **non-normalizable** !

The one-loop amplitude again

- Recall the (Euclidean ws, Minkowskian target) one-loop amplitude:

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Maldacena Ooguri

- In contrast to the open string case, these poles do not yield an imaginary part: the overall cosmological particle production seems to vanish. *This is not to say that there is no particle production at intermediate stages !*

Physical spectrum at low level

- The ground state **tachyon**

$$|T\rangle = \phi(x^+, x^-) |0_{ex}, k\rangle$$

should satisfy the Virasoro constraint

$$L_0 |T\rangle = \left[-\frac{1}{2} (a_0^+ a_0^- + a_0^- a_0^+) + \frac{1}{2} \nu^2 - 1 + \frac{1}{2} k_i^2 \right] |T\rangle$$

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- The L_1 **Virasoro constraint** eliminates one polarization. Despite the non-vanishing two-dimensional mass $k_i^2 - \nu^2$, the **spurious state** $L_{-1}\phi|0\rangle$ is still physical, eliminating an extra polarization.
- One thus has $D - 2$ **transverse** degrees of freedom, ie a **massless gauge boson** in D dimensions.

Open strings in time dependent backgrounds

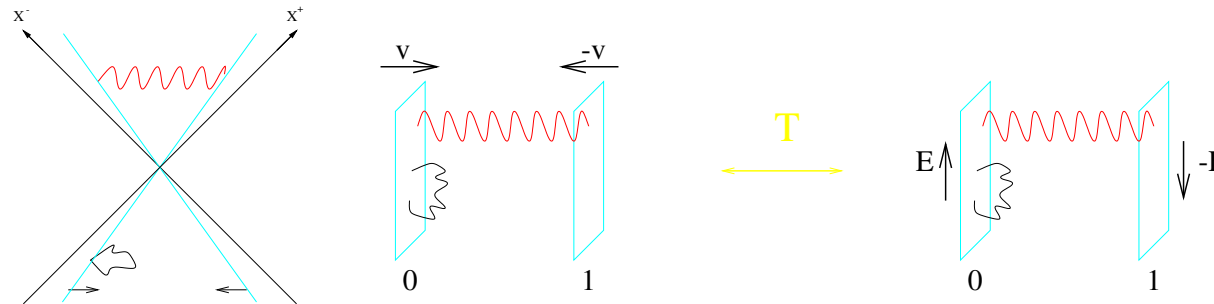
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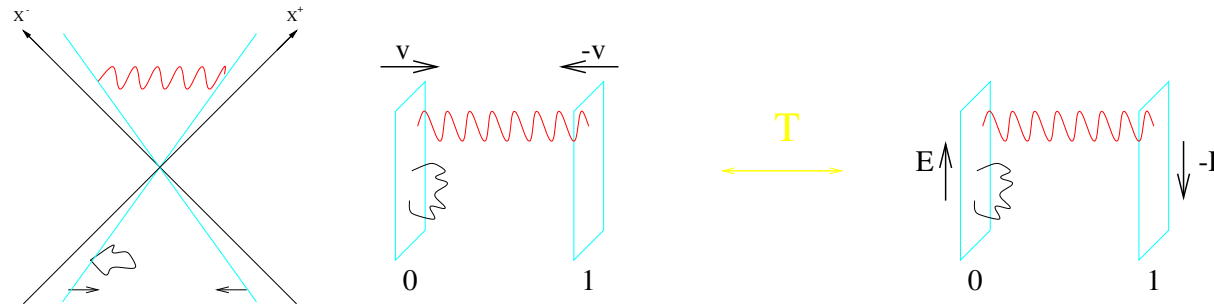
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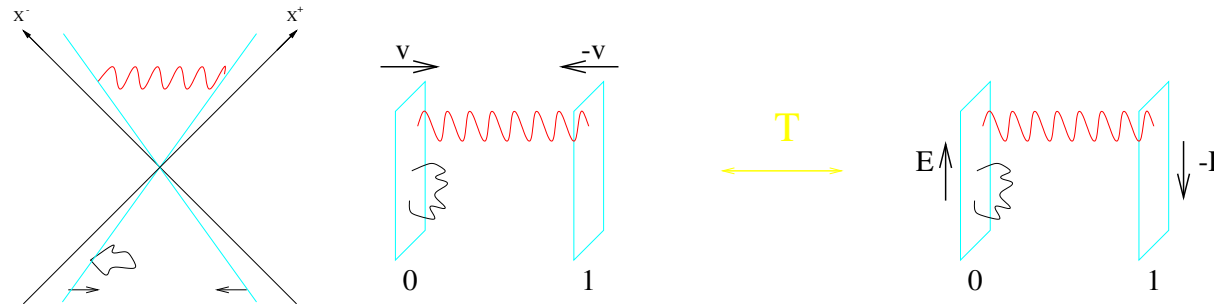
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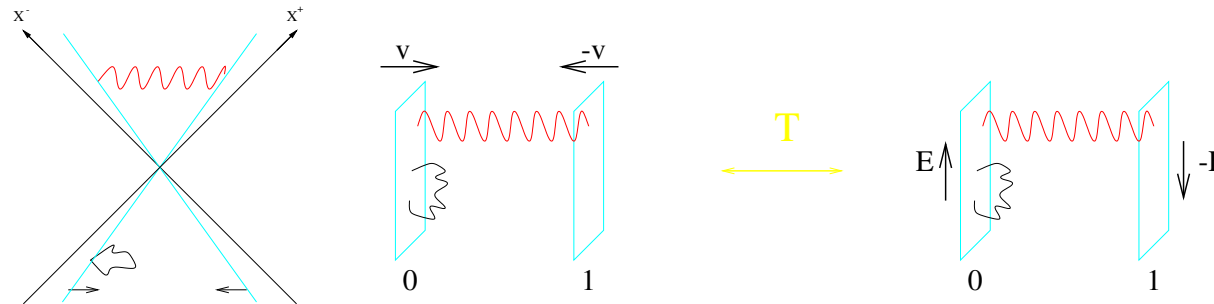


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- Can Schwinger production of twisted closed strings resolve the cosmological singularity of the Lorentzian orbifold ?

The Grant space

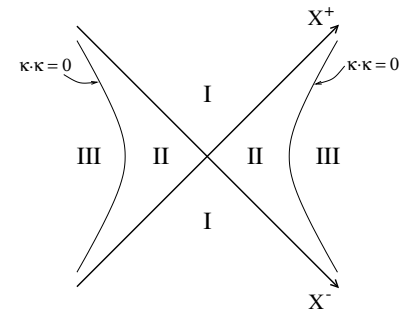
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- Defining $Z^\pm = X^\pm e^{\mp\beta X/R}$, the metric can be written in the Kaluza-Klein form

$$ds^2 = R^2(dX + A)^2 - 2dZ^+dZ^- - \frac{E^2}{2R^2}(Z^+dZ^- - Z^-dZ^+)^2, \quad X \equiv X + 2\pi$$

with radius R and KK electric field

$$R^2 = 1 + 2EZ^+Z^-, \quad dA = \frac{E}{R^4}dZ^+dZ^-, \quad E = \beta/R$$



Cornalba Costa

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- All CTC have to pass into $X^+ X^- < -1/(2E)$, hence may be suppressed by excising this region: *orientifold boundary conditions ?*