Hypermultiplet moduli spaces in type II string theories: a survey

Boris Pioline

CERN & LPTHE

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based on work with Alexandrov, Saueressig, Vandoren, Persson, Manschot

Introduction

- In $D = 4$ string vacua with $N = 2$ supersymmetries , the moduli space splits into a product $\mathcal{M} = VM_4 \times HM_4$ corresponding to vector multiplets and hypermultiplets.

$$\mathrm{IIA}/\mathcal{X} \;\Big|\; \mathrm{IIB}/\hat{\mathcal{X}} \;\Big|\; \mathrm{Het}/K_3 \times T^2 \;\Big|\; \dots$$

- The study of VM_4 and of the BPS spectrum has had tremendous applications in mathematics and physics: classical mirror symmetry, Gromov-Witten invariants, Donaldson-Thomas invariants, black hole precision counting, etc...
- Understanding HM_4 may be even more rewarding: quantum extension of mirror symmetry, new geometric invariants, new checks of Het/II duality, richer automorphic properties...

Hypers = Vectors

- Upon circle compactification to $D = 3$,the VM and HM moduli spaces become two sides of the same coin, exchanged by T-duality along the circle.

  Cecotti Ferrara Girardello

- VM_3 includes VM_4, the electric and magnetic holonomies of the $D = 4$ Maxwell fields, the radius R of the circle and the NUT potential σ, dual to the Kaluza-Klein gauge field in $D = 3$:

$$\begin{aligned} \mathrm{VM}_3 &\approx \text{c-map}(\mathrm{VM}_4) + \text{1-loop} + \mathcal{O}(e^{-R}) + \mathcal{O}(e^{-R^2}) \\ \mathrm{HM}_3 &= \mathrm{HM}_4 \end{aligned}$$

- SUSY requires that both VM_3 and HM_4 are quaternion-Kähler manifolds, i.e. have restricted holonomy $SU(2) \times Sp(n) \subset SO(4n)$.

Instantons = Black holes + KKM

- The $\mathcal{O}(e^{-R})$ corrections come from BPS black holes in $D = 4$, whose Euclidean wordline winds around the circle: thus VM_3 encodes the $D = 4$ spectrum, with chemical potentials for every electric and magnetic charges, and naturally incorporates wall-crossing.

Seiberg Witten; Shenker; Gaiotto Moore Neitzke

- The $\mathcal{O}(e^{-R^2})$ corrections come from Kaluza-Klein monopoles, i.e. gravitational instantons of the form $\mathrm{TN}_k \times \mathcal{Y}$ ($\mathcal{Y} = \hat{\mathcal{X}}, \mathcal{X}, K_3 \times T^2$).
- KK monopoles may resolve the ambiguity of the BH instanton series, and hopefully lead to enhanced automorphic properties, analogous to the $SL(2,\mathbb{Z}) \to Sp(2,\mathbb{Z})$ enhancement in $N = 4$ dyon counting.

BP Vandoren; Gunaydin Neitzke BP Waldron

Back to HM

- On the flip side of the coin, $R = 1/g_{(4)}$ is the inverse string coupling. The $\mathcal{O}(e^{-R})$ and $\mathcal{O}(e^{-R^2})$ corrections to HM_4 now originate from Euclidean D-branes and NS5-branes, respectively.

Becker Becker Strominger

- When $\mathcal{X}$ is K3-fibered, HM_4 can in principle be computed exactly using Het/type II duality: since the heterotic string coupling is a VM, HM_4 is determined by the $(0,4)$ heterotic SCFT at tree level (still non-trivial due to non-perturbative α' corrections)

Aspinwall, Plesser, Louis, ...

- Recent progress has mainly occurred on the type II side, by combining S-duality and mirror symmetry with an improved understanding of twistor techniques.

Robles-Llana Rocek Saueressig Theis Vandoren

Alexandrov BP Saueressig Vandoren

SYM vs. SUGRA and QK/HK correspondence

- A similar version of this problem occurs in (Seiberg-Witten) $\mathcal{N}=2$ SYM field theories on $\mathbb{R}^3 \times S^1$. VM_3 is then hyperkähler,

$$\mathrm{VM}_3^{\mathrm{FT}} \approx \text{rigid c-map}(\mathrm{VM}_4^{\mathrm{FT}}) + \mathcal{O}(e^{-R})$$

- The $\mathcal{O}(e^{-R})$ corrections similarly come from BPS dyons in $D=4$. Understanding their effect in terms of the twistor space $\mathcal{Z}$ of VM_3 has lead to a physical derivation of the KS wall-crossing formula.

Gaiotto Moore Neitzke, Kontsevich Soibelman

- In the absence of KKM (resp. NS5), the QK metric on VM_3 (resp. HM_4) is formally related to the HK metric on $\mathrm{VM}_3^{\mathrm{FT}}$ by the QK/HK correspondence.

Haydys; Alexandrov Persson BP; Neitzke

- Still, many interesting questions remain, e.g. how is S-duality realized in the sector with D1-F1-D3 corrections ?

Outline

1. Introduction
2. Perturbative HM metric and topology
3. D-instantons in twistor space
4. Mirror symmetry and S-duality
5. Conclusion

Outline

The perturbative metric I

- The HM moduli space in type IIA compactified on a CY 3-fold (family) $\mathcal{X}$ is a quaternion-Kähler manifold $\mathcal{M}$ of real dimension $2b_3(\mathcal{X}) = 4(h_{2,1} + 1)$.
- $\mathcal{M} \equiv \mathcal{Q}_c(\mathcal{X})$ encodes
  1. the 4D dilaton $R \equiv 1/g_{(4)}$,
  2. the complex structure of the CY family $\mathcal{X}$,
  3. the periods of the RR 3-form C on $\mathcal{X}$,
  4. the NS axion σ, dual to the Kalb-Ramond B-field in 4D
- To write down the metric explicitly, let us choose a symplectic basis $\mathcal{A}^\Lambda, \mathcal{B}_\Lambda$, $\Lambda = 0 \dots h_{2,1}$ of $H_3(\mathcal{X}, \mathbb{Z})$.

The perturbative metric II

- The complex structure moduli space $\mathcal{M}_c(\mathcal{X})$ may be parametrized by the periods $\Omega(z^a) = (X^\Lambda, F_\Lambda) \in H_3(\mathcal{X}, \mathbb{C})$ of the (3,0) form

$$X^\Lambda = \int_{\mathcal{A}^\Lambda} \Omega_{3,0} \,, \quad F_\Lambda = \int_{\mathcal{B}_\Lambda} \Omega_{3,0} \,,$$

  up to holomorphic rescalings $\Omega \mapsto e^f \Omega$.

- $\mathcal{M}_c(\mathcal{X})$ is endowed with a special Kähler metric

$$\mathrm{d}s^2_{\mathcal{SK}} = \partial\bar\partial\mathcal{K} \,, \qquad \mathcal{K} = -\log[\mathrm{i}(\bar X^\Lambda F_\Lambda - X^\Lambda \bar F_\Lambda)]$$

- Harmonic C-fields on $\mathcal{X}$ may be parametrized by the real periods

$$\zeta^\Lambda = \int_{\mathcal{A}^\Lambda} C, \quad \tilde\zeta_\Lambda = \int_{\mathcal{B}_\Lambda} C \,.$$

The perturbative metric III

- Large gauge transformations require that $C \equiv (\zeta^\Lambda, \tilde{\zeta}_\Lambda)$ takes values in the intermediate Jacobian torus

$$C \in T = H^3(\mathcal{X}, \mathbb{R})/H^3(\mathcal{X}, \mathbb{Z})$$

i.e. that $(\zeta^\Lambda, \tilde{\zeta}_\Lambda)$ have unit periodicities.

- T carries a canonical symplectic form and complex structure induced by the Hodge $\star_\mathcal{X}$, hence a Kähler metric

$$\mathrm{d}s_T^2 = -\frac{1}{2}(\mathrm{d}\tilde{\zeta}_\Lambda - \bar{\mathcal{N}}_{\Lambda\Lambda'}\mathrm{d}\zeta^{\Lambda'})\mathrm{Im}\mathcal{N}^{\Lambda\Sigma}(\mathrm{d}\tilde{\zeta}_\Lambda - \mathcal{N}_{\Sigma\Sigma'}\mathrm{d}\zeta^{\Sigma'})$$

where $\mathcal{N}$ (resp. τ) is the Weil (resp. Griffiths) period matrix,

$$\mathcal{N}_{\Lambda\Lambda'} = \bar{\tau}_{\Lambda\Lambda'} + 2\mathrm{i}\frac{[\mathrm{Im}\tau \cdot X]_\Lambda [\mathrm{Im}\tau \cdot X]_{\Lambda'}}{X^\Sigma\, \mathrm{Im}\tau_{\Sigma\Sigma'} X^{\Sigma'}}\,, \qquad \tau_{\Lambda\Sigma} = \partial_{X^\Lambda}\partial_{X^\Sigma} F$$

The tree-level metric

- At tree level, i.e. in the strict weak coupling limit $R = \infty$, the quaternion-Kähler metric on $\mathcal{M}$ is given by the c-map metric

$$ds^2_{\mathcal{M}} = \frac{4}{R^2}\,\mathrm{d}R^2 + 4\,\mathrm{d}s^2_{\mathcal{SK}} + \frac{\mathrm{d}s^2_T}{R^2} + \frac{1}{16R^4}D\sigma^2\,.$$

  where

$$D\sigma \equiv \mathrm{d}\sigma + \langle C, \mathrm{d}C\rangle = \mathrm{d}\sigma + \tilde{\zeta}_\Lambda \mathrm{d}\zeta^\Lambda - \zeta^\Lambda \mathrm{d}\tilde{\zeta}_\Lambda$$

  Cecotti Girardello Ferrara; Ferrara Sabharwal

- The c-map (aka semi-flat) metric admits continuous isometries

$$T_{H,\kappa} : (C, \sigma) \mapsto (C + H, \sigma + 2\kappa + \langle C, H\rangle)$$

  where $H \in H^3(\mathcal{X}, \mathbb{R})$ and $\kappa \in \mathbb{R}$, satisfying the Heisenberg algebra

$$T_{H_1,\kappa_1} T_{H_2,\kappa_2} = T_{H_1+H_2,\kappa_1+\kappa_2+\frac{1}{2}\langle H_1,H_2\rangle}$$

The one-loop corrected metric I

- The one-loop correction deforms the metric on $\mathcal{M}$ into

$$ds^2_{\mathcal{M}} = 4\frac{R^2+2c}{R^2(R^2+c)}\,\mathrm{d}R^2 + \frac{4(R^2+c)}{R^2}\,\mathrm{d}s^2_{\mathcal{SK}} + \frac{\mathrm{d}s^2_T}{R^2} + \frac{2\,c}{R^4}\,e^{\mathcal{K}}\,|X^\Lambda \mathrm{d}\tilde{\zeta}_\Lambda - F_\Lambda \mathrm{d}\zeta^\Lambda|^2 + \frac{R^2+c}{16R^4(R^2+2c)}D\sigma^2\,.$$

where $D\sigma = \mathrm{d}\sigma + \langle C, \mathrm{d}C\rangle + 8c\mathcal{A}_K$,

$$\mathcal{A}_K = \tfrac{\mathrm{i}}{2}(\mathcal{K}_a \mathrm{d}z^a - \mathcal{K}_{\bar{a}}\mathrm{d}\bar{z}^{\bar{a}})\,, \qquad c = -\frac{\chi(\mathcal{X})}{192\pi}$$

Antoniadis Minasian Theisen Vanhove; Gunther Hermann Louis; Robles-Llana Saueressig Vandoren

- The one-loop correction to g_{rr} was computed by reducing the CP-even R^4 coupling in 10D on $\mathcal{X}$. The correction to $D\sigma$ can be obtained with less effort by reducing CP-odd couplings in 10D.

The one-loop corrected metric II

- The one-loop correction to $D\sigma$ has important implications for the topology of the HM moduli space, as we shall discuss later.
- The one-loop corrected metric is presumably exact to all orders in $1/R$. It will receive $\mathcal{O}(e^{-R})$ and $\mathcal{O}(e^{-R^2})$ corrections from D-instantons and NS5-brane instantons, eventually breaking all continuous isometries.
- Note the curvature singularity at finite distance $R^2 = -2c$ when $\chi(\mathcal{X}) > 0$! This should hopefully be resolved by instanton corrections.

Topology of the HM moduli space

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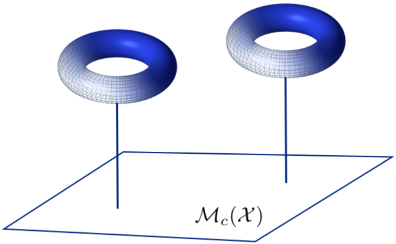
- Quotienting by translations along the NS axion σ, $\mathcal{C}(R)/\partial_\sigma$ reduces to the Weil intermediate Jacobian $T \to \mathcal{J}_c(\mathcal{X}) \to \mathcal{M}_c(\mathcal{X})$.



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- Quotienting by translations along the NS axion σ, $\mathcal{C}(R)/\partial_\sigma$ reduces to the Weil intermediate Jacobian $T \to \mathcal{J}_c(\mathcal{X}) \to \mathcal{M}_c(\mathcal{X})$.
- This is consistent with the fact that Euclidean D2-branes wrapping a special Lagrangian submanifold in integer homology class $\gamma = q_\Lambda \mathcal{A}^\Lambda - p^\Lambda \mathcal{B}_\Lambda \in H_3(\mathcal{X}, \mathbb{Z})$ induce corrections roughly of the form

$$\delta \mathrm{d}s^2|_{D2} \sim \exp\left(-8\pi \frac{|Z_\gamma|}{g_{(4)}} - 2\pi \mathrm{i} \langle \gamma, C \rangle\right) .$$

Here $Z_\gamma \equiv e^{\mathcal{K}/2}(q_\Lambda X^\Lambda - p^\Lambda F_\Lambda)$ is the central charge.

Five-brane instantons I

- NS5-brane instantons with charge $k \in \mathbb{Z}$ are expected to produce corrections to the metric of the form

$$\delta \mathrm{d}s^2|_{\mathrm{NS5}} \sim \exp\left(-4\pi \frac{|k|}{g_{(4)}^2} - \mathrm{i}k\pi\sigma\right) \mathcal{Z}^{(k)}(z^a, C) ,$$

  where $\mathcal{Z}^{(k)} = \mathrm{Tr}[(2J_3)^2(-1)^{2J_3}]$ is the (twisted) partition function of the world-volume theory on a stack of k five-branes.

- Recall that the type IIA NS5-brane supports a self-dual 3-form flux, together with its SUSY partners. The partition function of a self-dual form is known to be a holomorphic section of a non-trivial line bundle $\mathcal{L}_{\mathrm{NS5}}^k$ over the space of metrics and C fields.

Witten; Henningson Nilsson Salomonson; Belov Moore; . . .

Five-brane instantons II

- This means that $\mathcal{Z}^{(k)}(z^a, C)$ satisfies the twisted periodicity condition

$$\mathcal{Z}^{(k)}(z^a, C+H) = \sigma^k(H)\, e^{\mathrm{i}\pi k \langle H, C\rangle}\, \mathcal{Z}^{(k)}(z^a, C)$$

  where $\sigma(H) : H_3(\mathcal{X}, \mathbb{Z}) \to U(1)$ is a quadratic refinement of the symplectic pairing, (here $H = (m_\Lambda, n^\Lambda)$)

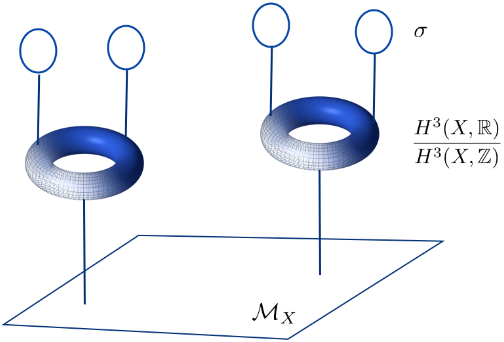
$$\sigma(H+H') = (-1)^{\langle H, H'\rangle}\, \sigma(H)\, \sigma(H')\ , \quad \sigma(H) = e^{-\mathrm{i}\pi m^\Lambda n_\Lambda + 2\pi\mathrm{i}\langle H, \Theta\rangle}$$

  where $\Theta = (\theta, \phi) \in \mathcal{T}$ are a choice of characteristics.

- Holomorphic sections of $(\mathcal{L}_\Theta)^k$ are Siegel theta series of rank $b_3(\mathcal{X})$, level $k/2$, but holomorphy holds only in large volume limit.

Topology of the NS axion

- The coupling $e^{-\mathrm{i}\pi k\sigma}\mathcal{Z}^{(k)}$ is invariant under $\sigma \mapsto \sigma + 2\kappa$, $\kappa \in \mathbb{Z}$ thus $e^{\mathrm{i}\pi\sigma}$ parametrizes a circle bundle over $\mathcal{J}_c(\mathcal{X})$.



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- Under large gauge transformations $e^{\mathrm{i}\pi\sigma}$ must transform as a section of $\mathcal{L}_{\mathrm{NS5}}$, so σ must pick up additional shifts under discrete translations along T,

$$T'_{H,\kappa}:(C,\sigma)\mapsto\left(C+H,\sigma+2\kappa+\langle C,H\rangle-n^{\Lambda}m_{\Lambda}+2\langle H,\Theta\rangle\right)$$

This is needed for the closure of large gauge transformations,

$$T'_{H_1,\kappa_1}T'_{H_2,\kappa_2}=T'_{H_1+H_2,\kappa_1+\kappa_2+\frac{1}{2}\langle H_1,H_2\rangle+\frac{1}{2\pi\mathrm{i}}\log\frac{\sigma(H_1+H_2)}{\sigma(H_1)\sigma(H_2)}}$$

Alexandrov Persson BP

Topology of the NS axion

- Conversely, let's examine the connection along σ in the one-loop corrected metric. Its curvature is given by

$$\mathrm{d}\left(\frac{D\sigma}{2}\right) = \omega_T + \frac{\chi(\mathcal{X})}{24}\,\omega_c\,, \quad \omega_T = \mathrm{d}\tilde{\zeta}_\Lambda \wedge \mathrm{d}\zeta^\Lambda\,, \quad \omega_c = -\frac{1}{2\pi}\mathrm{d}\mathcal{A}_K$$

where ω_T, ω_c are the Kähler forms on T and $\mathcal{M}_c(\mathcal{X})$. Thus, we predict $\mathcal{L}_{\rm NS5} \sim \mathcal{L}_\Theta \times \mathcal{L}^{\chi/24}$.

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- This matches the fact that S-duality maps k NS5 to k D5, which for $k = 1$ are controlled by the topological string amplitude, a section of $\mathcal{L}^{1-\chi/24}$. However, only $\mathcal{L}^{\chi/12}$ is well-defined, the $\mathbb{Z}_2$ ambiguity must cancel between the NS-axion and the five-brane partition function...

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- The consistency of NS5-instantons with one-loop corrected metric is guaranteed by the anomaly inflow mechanism, but remains to be fully understood.

Outline

A lightning review of twistors I

- QK manifolds $\mathcal{M}$ are conveniently described via their twistor space $\mathbb{P}^1 \to \mathcal{Z} \to \mathcal{M}$, a complex contact manifold with real involution. Choosing a stereographic coordinate t on $\mathbb{P}^1$, the contact structure is the kernel of the local (1,0)-form

$$Dt = \mathrm{d}t + p_+ - \mathrm{i}p_3 t + p_- t^2$$

  where $p_3, p_\pm$ are the $SU(2)$ components of the Levi-Civita connection on $\mathcal{M}$. Dt is well-defined modulo rescalings.

- $\mathcal{Z}$ is further equipped with a Kähler-Einstein metric

$$\mathrm{d}s^2_{\mathcal{Z}} = \frac{|Dt|^2}{(1+t\bar{t})^2} + \frac{\nu}{4}\mathrm{d}s^2_{\mathcal{M}} \,, \qquad \nu = \frac{R(\mathcal{M})}{4d(d+2)}$$

  If $\mathcal{M}$ has negative scalar curvature, $\mathcal{Z}$ is pseudo-Kähler with signature $(2, \dim \mathcal{M})$.

Lebrun, Salamon

A lightning review of twistors II

- Rk: complex contact manifolds are projectivizations of complex symplectic cones. The $\mathbb{C}^\times$ bundle over $\mathcal{Z}$ is the hyperkähler cone associated to $\mathcal{M}$. The two approaches are equivalent.

Swann; de Wit Rocek Vandoren

- Locally, there always exist Darboux coordinates $(\Xi, \tilde{\alpha}) = (\xi^\Lambda, \tilde{\xi}_\Lambda, \tilde{\alpha})$ and a "contact potential" Φ such that

$$2\, e^{\Phi} \frac{Dt}{\mathrm{i}t} = \mathrm{d}\tilde{\alpha} + \langle \Xi, \mathrm{d}\Xi \rangle = \mathrm{d}\tilde{\alpha} + \tilde{\xi}_\Lambda \mathrm{d}\xi^\Lambda - \xi^\Lambda \mathrm{d}\tilde{\xi}_\Lambda \ .$$

- The contact potential is independent of $\bar{t}$, and provides a Kähler potential for the Kähler metric on $\mathcal{Z}$ via $e^{K_{\mathcal{Z}}} = (1 + t\bar{t}) e^{Re(\Phi)} / |t|$.

Alexandrov BP Saueressig Vandoren

A lightning review of twistors III

- By the moment map construction, continuous isometries of $\mathcal{M}$ are in 1-1 correspondence with classes in $H^0(\mathcal{Z}, \mathcal{O}(2))$. In particular, any continuous isometry of $\mathcal{M}$ can be lifted to a holomorphic action on $\mathcal{Z}$.

Salamon; Galicki Salamon

- Infinitesimal deformations of $\mathcal{M}$ lift to deformations of the complex contact transformations between Darboux coordinate patches on $\mathcal{Z}$, hence are classified by $H^1(\mathcal{Z}, \mathcal{O}(2))$.

Lebrun; Alexandrov BP Saueressig Vandoren

Twistor description of the perturbative metric

- For the one-loop corrected HM metric, the following Darboux coordinates do the job, away from the north and south poles $t = 0, \infty$:

$$\begin{aligned}
\Xi_{\rm sf} &= C + 2\sqrt{R^2 + c}\, e^{\mathcal{K}/2} \left[t^{-1}\Omega - t\,\bar{\Omega} \right] , \quad \Phi_{\rm sf} = 2\log R\,, \\
\tilde{\alpha}_{\rm sf} &= \sigma + 2\sqrt{R^2 + c}\, e^{\mathcal{K}/2} \left[t^{-1}\langle \Omega, C \rangle - t\,\langle \bar{\Omega}, C \rangle \right] - 8\mathrm{i}c \log t
\end{aligned}$$

Neitzke BP Vandoren; Alexandrov

- The isometry $T_{H,\kappa}$ acts holomorphically on $\mathcal{Z}$ by

$$(\Xi, \tilde{\alpha}) \mapsto (\Xi + H, \tilde{\alpha} + 2\kappa + \langle \Xi, H \rangle)$$

- Modding out by large gauge transformations $T'_{H,\kappa}$, $\mathcal{Z}$ becomes a complexified twisted torus $\mathbb{C}^\times \ltimes [H^3(\mathcal{X}, \mathbb{Z}) \otimes_{\mathbb{Z}} \mathbb{C}^\times]$.

- D-instanton corrections to $\mathcal{Z}$ are essentially dictated by wall crossing. Recall that the Kontsevich-Soibelman wall-crossing formula requires that the product

$$\prod_{\gamma} U_\gamma \,, \qquad U_\gamma \equiv \exp\left(\Omega(\gamma; t^a) \sum_{d=1}^{\infty} \frac{e_{d\gamma}}{d^2} \right) \,,$$

ordered such that $\arg(Z_\gamma)$ decreases from left to right, stays invariant across the wall. Here $\Omega(\gamma; t^a)$ are generalized DT invariants, and e_γ satisfy the Lie algebra

$$[e_{\gamma_1}, e_{\gamma_2}] = (-1)^{\langle \gamma_1, \gamma_2 \rangle} \langle \gamma_1, \gamma_2 \rangle \, e_{\gamma_1+\gamma_2} \,.$$

D-instantons in twistor space II

- Using the quadratic refinement σ, one can represent e_γ as a Hamiltonian vector field on $\mathcal{Z}$,

$$\sigma(\gamma)\, e_\gamma = (\partial_{\xi^\Lambda}\mathcal{X}_\gamma)\,\partial_{\tilde\xi_\Lambda} - (\partial_{\tilde\xi_\Lambda}\mathcal{X}_\gamma)\,\partial_{\xi^\Lambda} + 2\mathrm{i}[(2 - \xi^\Lambda\partial_{\xi^\Lambda} - \tilde\xi_\Lambda\partial_{\tilde\xi_\Lambda})\mathcal{X}_\gamma]\,\partial_{\tilde\alpha}$$

where

$$\mathcal{X}_\gamma = \mathrm{E}^{\langle\Xi,\gamma\rangle} = \mathrm{E}^{q_\Lambda\xi^\Lambda - p^\Lambda\tilde\xi_\Lambda}$$

- Exponentiating, U_γ implements the contact transformation

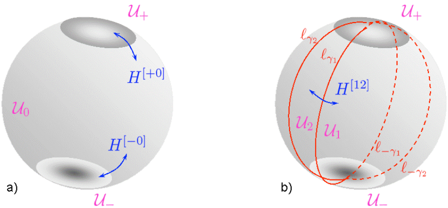
$$\mathcal{X}_{\gamma'} \mapsto \mathcal{X}_{\gamma'}(1-\sigma(\gamma)\mathcal{X}_\gamma)^{\langle\gamma,\gamma'\rangle\,\Omega(\gamma)}\,, \qquad \tilde\alpha \mapsto \tilde\alpha - \frac{1}{2\pi^2}\Omega(\gamma)\,L[\sigma(\gamma)\mathcal{X}_\gamma]$$

where $L(z) = \sum_{n=1}^{\infty}\frac{z^n}{n^2} + \frac{1}{2}\log z\ \log(1-z)$ is Rogers' dilogarithm.

Alexandrov Saueressig BP Vandoren; Alexandrov Persson BP

D-instantons in twistor space III

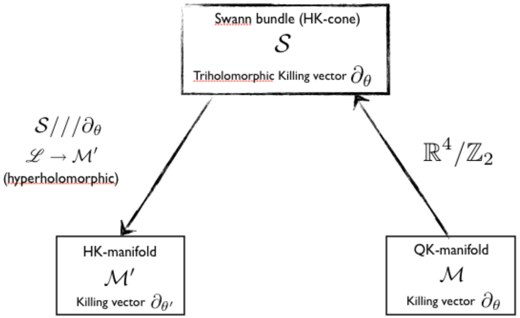
- By analogy with GMN, the D-instanton corrected twistor space is obtained by gluing Darboux coordinate patches along BPS rays $\ell_\pm = \{t \ : \ Z(\gamma; z^a)/t \in \pm \mathrm{i}\mathbb{R}^+\}$, using the contact transf. U_γ:



- The consistency of this prescription across lines of marginal stability is guaranteed by the KS formula. The resulting metric, including multi-instanton corrections, is smooth across the walls.

QK/HK correspondence

- This is more than an analogy: for a given choice of homogeneous prepotential $F(X)$ and BPS indices $\Omega(\gamma)$, the HK manifold $\mathcal{M}'$ constructed by GMN is related to the QK manifold $\mathcal{M}$ constructed as above by the QK/HK correspondence.



Haydys; Alexandrov Persson BP; Hitchin Swann

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- Contact transformations on $\mathcal{Z}_\mathcal{M}$ are interpreted as (complexified) gauge transformations on a line bundle $\mathcal{L}$ on $\mathcal{Z}_{\mathcal{M}'}$, which projects to a hyperholomorphic line bundle on $\mathcal{M}'$. The one-loop correction corresponds to a 1-parameter family of $U(1)_R$ actions on $\mathcal{M}'$.

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- Of course, there are far fewer BPS instantons in field theory than in string theory ! And KKM / NS5 instanton corrections break this correspondence.

Multi-instanton corrections

- The gluing conditions for $\Xi = (\xi^\Lambda, \tilde{\xi}_\Lambda)$ can be summarized by integral equations

$$\Xi = \Xi_{\rm sf} - \frac{1}{8\pi^2} \sum_\gamma \Omega(\gamma) \langle \cdot, \gamma \rangle \int_{\ell_\gamma} \frac{\mathrm{d}t'}{t'} \frac{t+t'}{t-t'} \mathrm{Li}_1 \left[\sigma(\gamma) \, \mathrm{E}^{-\langle \Xi(t'), \gamma \rangle} \right],$$

where $\mathrm{Li}_1(x) \equiv -\log(1-x)$. Similar eqs allowing to compute $\tilde{\alpha}$, Φ once Ξ is known.

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$$\Xi = \Xi_{\rm sf} - \frac{1}{8\pi^2} \sum_\gamma \Omega(\gamma) \langle \cdot, \gamma \rangle \int_{\ell_\gamma} \frac{dt'}{t'} \frac{t+t'}{t-t'} \mathrm{Li}_1 \left[\sigma(\gamma) \, \mathrm{E}^{-\langle \Xi(t'), \gamma \rangle} \right] ,$$

  where $\mathrm{Li}_1(x) \equiv -\log(1-x)$. Similar eqs allowing to compute $\tilde{\alpha}$, Φ once Ξ is known.

- These eqs are formally identical to Zamolodchikov's Y-system in studies of integrable models.

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  GMN; Alexandrov Roche
- These eqs can be solved iteratively, by first substituting $\Xi \to \Xi_{\rm sf}$ on the rhs, integrating, etc. leading to an infinite (divergent) series of multi-instanton corrections.

Outline

HM moduli space in type IIB I

- The HM moduli space in type IIB compactified on a CY 3-fold $\hat{\mathcal{X}}$ is a QK manifold $\mathcal{M} \equiv \mathcal{Q}_K(\hat{\mathcal{X}})$ of real dimension $4(h_{1,1}+1)$
  1. the 4D dilaton $R \equiv 1/g_{(4)}$,
  2. the complexified Kähler moduli $z^a = b^a + \mathrm{i}t^a = X^a/X^0$
  3. the periods of $C = C^{(0)} + C^{(2)} + C^{(4)} + C^{(6)} \in H^{\mathrm{even}}(\hat{\mathcal{X}}, \mathbb{R})$
  4. the NS axion σ
- Near the infinite volume point, $\mathcal{M}_K(\hat{\mathcal{X}})$ is governed by

$$F(X) = -\frac{1}{6}\kappa_{abc}\frac{X^a X^b X^c}{X^0} + \chi(\hat{\mathcal{X}})\frac{\zeta(3)(X^0)^2}{2(2\pi\mathrm{i})^3} + F_{\mathrm{GW}}(X)$$

where κ_{abc} is the cubic intersection form and F_{GW} are Gromov-Witten instanton corrections:

$$F_{\mathrm{GW}}(X) = -\frac{(X^0)^2}{(2\pi\mathrm{i})^3}\sum_{k_a\gamma^a \in H_2^+(\hat{\mathcal{X}})} n^{(0)}_{k_a}\,\mathrm{Li}_3\left[\mathrm{E}^{k_a\frac{X^a}{X^0}}\right],$$

Quantum mirror symmetry and S-duality

- D-instantons are now Euclidean D5-D3-D1-D(-1), described mathematically by coherent sheaves E on $\mathcal{X}$, with charge

$$\gamma = \mathrm{ch}(E)\,\sqrt{\mathrm{Td}(\hat{\mathcal{X}})}$$

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  - beyond, matching of NS5-instantons.

- While the type IIA description makes symplectic invariance manifest, the type IIB description shows that the exact QK metric on $\mathcal{M}$ should admit an isometric action of $SL(2,\mathbb{Z})$, corresponding to type IIB S-duality in 10 dimensions.

S-duality in twistor space

- For suitable choice of 'type IIB' Darboux coordinates, $\delta = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$ acts holomorphically on $\mathcal{Z}$ via

$$\xi^0 \mapsto \frac{a\xi^0 + b}{c\xi^0 + d}\,, \qquad \xi^a \mapsto \frac{\xi^a}{c\xi^0 + d}\,,$$

$$\tilde\xi_a \mapsto \tilde\xi_a + \frac{c}{2(c\xi^0 + d)}\kappa_{abc}\xi^b\xi^c - c_{2,a}\,\varepsilon(\delta)\,,$$

$$\begin{pmatrix} \tilde\xi_0 \\ \alpha \end{pmatrix} \mapsto \begin{pmatrix} d & -c \\ -b & a \end{pmatrix} \begin{pmatrix} \tilde\xi_0 \\ \alpha \end{pmatrix} + \frac{1}{6}\,\kappa_{abc}\xi^a\xi^b\xi^c \begin{pmatrix} c^2/(c\xi^0 + d) \\ -[c^2(a\xi^0 + b) + 2c]/(c\xi^0 + d)^2 \end{pmatrix}.$$

where $\alpha = -\frac{1}{2}(\tilde\alpha + \xi^\Lambda\tilde\xi_\Lambda)$. Here $\epsilon(\delta)$ is the multiplier system of the Dedekind eta function, required for consistency of D3-instantons,

$$\eta\left(\frac{a\tau + b}{c\tau + d}\right)/\eta(\tau) = e^{2\pi i\epsilon(\delta)}(c\tau + d)^{1/2}\,.$$

S-duality and D1-F1-D(-1) instantons I

- Continuous S-duality is broken by Gromov-Witten instantons at tree level and by the one-loop correction.
- In the large volume limit, retaining D1-F1-D(-1) instantons, S-duality mixes (one-loop,D(-1)) and (D1,F1). It was shown that $SL(2,\mathbb{Z}) \subset SL(2,\mathbb{R})$ remains unbroken provided

$$\Omega(0,0,0,0) = -\chi(\hat{\mathcal{X}}) \,, \qquad \Omega(0,0,q_a,q_0) = n^{(0)}_{q_a}$$

Robles-Llana Roček Saueressig Theis Vandoren

- The (GMN-type) 'type IIA' Darboux coordinates are not covariant, but they can be mapped by a contact transformation (and Poisson resummation on q_0) to a set of 'type IIB' Darboux coordinates which transform as above.

Alexandrov Saueressig

S-duality and D1-F1-D(-1) instantons II

- In the IIB frame, the twistor space is covered by open sets $U_{m,n}$ centered around $m\xi^0 + n = 0$, with transition functions $U_{0,0} \mapsto U_{m,n}$ generated by

$$G_{m,n}(\xi^0, \xi^a) = -\frac{\mathrm{i}}{(2\pi)^3} \sum_{q_a \geq 0} n^{(0)}_{q_a} \begin{cases} \frac{e^{-2\pi \mathrm{i} m q_a \xi^a}}{m^2 (m\xi^0 + n)} , & m \neq 0 \\ (\xi^0)^2 \frac{e^{2\pi \mathrm{i} n q_a \xi^a / \xi^0}}{n^3} , & m = 0 \end{cases}$$

- Under $SL(2,\mathbb{Z})$, $G_{m,n}$ are mapped into each other,

$$\begin{pmatrix} m' \\ n' \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} m \\ n \end{pmatrix} , \quad G_{m,n} \mapsto \frac{G_{m',n'}}{c\xi^0 + d} + \text{reg.}$$

  UP to a term regular in $U_{m',n'}$.
- The contact potential can be written in terms of Kronecker-Eisenstein series, or elliptic dilogarithm.

S-duality and D3-D1-F1-D(-1) instantons I

- S-duality should further hold upon retaining D3-branes. Since the same BPS invariants govern D3-D1-D(-1) instantons in IIB/$\mathcal{X}$ and D4-D2-D0 black holes in IIA/$\mathcal{X}$, one expects that S-duality should follow from the modularity of the D4-D2-D0 black hole partition
- By lifting the D4 brane to an M5-brane, the BH partition function is given by the elliptic genus of a $(0,4)$ superconformal CFT

$$\mathcal{Z}_{\rm BH}(\tau, y^a) = \mathrm{Tr}'(2J_3)^2(-1)^{2J_3}\, \mathrm{E}^{\left(L_0-\frac{c_L}{24}\right)\tau-\left(\bar{L}_0-\frac{c_R}{24}\right)\bar{\tau}+q_a y^a}$$

Maldacena Strominger Witten

- $\mathcal{Z}_{\rm BH}$ is a multivariate Jacobi form of weight $(-\frac{3}{2},\frac{1}{2})$, index $\kappa_{ab}=\kappa_{abc}p^c$ and multiplier system $\mathrm{E}^{c_{2a}p^a\varepsilon\delta}$.

Denef Moore; Gaiotto Strominger Yin; de Boer Cheng Dijkgraaf Manschot Verlinde

S-duality and D3-D1-F1-D(-1) instantons II

- By spectral flow invariance,

$$\mathcal{Z}_{\rm BH}(\tau, y^a) = \sum_{\mu \in \Lambda^*/\Lambda + \frac{1}{2}p} h_{p^a,\mu_a}(\tau)\, \overline{\theta_{p^a,\mu_a}(\tau, y^a, p^a)}\,,$$

  where h_{p^a,μ_a} is a weight $(-\frac{b_2}{2} - 1, 0)$ vector-valued modular form, and θ_{p^a,μ_a} is a signature $(1, b_2(Y) - 1)$ Siegel-Narain theta series,

$$\theta_{p^a,\mu_a}(\tau, y^a, t^a) = \sum_{k \in \Lambda + \mu + \frac{1}{2}p} (-1)^{p \cdot k}\, \mathrm{E}^{\frac{1}{2}(k_+)^2 \tau + \frac{1}{2}(k_-)^2 \bar\tau + k \cdot y}$$

- There is some tension between the fact that the lattice Λ has indefinite signature $(1, b_2(Y) - 1)$, yet the theta series should be holomorphic in twistor space.

S-duality and D3-D1-F1-D(-1) instantons III

- Again, the GMN-type IIA Darboux coordinates are not covariant under S-duality, but have a modular anomaly.
- The modular anomaly can be absorbed by a contact transformation generated by Zwegers' indefinite theta series

$$H = \sum_{\mu \in \Lambda^*/\Lambda + \frac{1}{2}p} h_{p^a,\mu_a}(\xi_0)\, \Theta_{p^a,\mu_a}(\tilde{\xi}^0, \xi^a)$$

$$\theta_{p^a,\mu_a}(\tilde{\xi}^0, \xi^a) = \sum_{k \in \Lambda + \mu + p/2} (\mathrm{sign}[(k+b)\cdot t] - \mathrm{sign}[(k+b)\cdot t_1]) \times (-1)^{p\cdot k}\, \mathrm{E}^{-k_a \xi^a - \frac{1}{2}\xi^0\, k_a \kappa^{ab} k_b}$$

where t_1 lies on the boundary of the Kahler cone.

- The Penrose transform produces the non-holomorphic completion for free !

S-duality and D5-D3-D1-F1-D(-1) instantons

- Finally, S-duality relates D5 and NS5. Starting from the known form of D5-D3-D1-D(-1) corrections, one may construct a Poincaré-type series to obtain the contributions from k five branes in one-instanton approximation.
- This leads to a non-Gaussian, non-Abelian generalization of the five-brane partition function

$$H^{(k)}_{\mathrm{NS5}}(\xi,\tilde{\xi},\tilde{\alpha}) = \frac{1}{4\pi^2} \sum_{\substack{\mu\in(\Gamma_m/|k|)/\Gamma_m \\ n\in\Gamma_m+\mu+\theta}} H^{(k,\mu)}_{\mathrm{NS5}}\left(\xi^\Lambda - n^\Lambda\right) \mathrm{E}^{kn^\Lambda(\tilde{\xi}_\Lambda-\phi_\Lambda)-\frac{k}{2}(\tilde{\alpha}+\xi^\Lambda\tilde{\xi}_\Lambda)}.$$

- For $k=1$, one recovers the topological string amplitude:

$$H^{(1,0)}_{\mathrm{NS5}}(\xi^\Lambda) = \left(\xi^0\right)^{-1-\frac{\chi(\hat{\mathcal{X}})}{24}} \left[M(e^{2\pi\mathrm{i}/\xi^0})\right]^{-\chi(\hat{\mathcal{X}})/2} \Psi^{\mathrm{top}}_{\mathbb{R}}(\xi^\Lambda)\,.$$

Alexandrov Persson Pioline

Outline

1. Introduction
2. Perturbative HM metric and topology
3. D-instantons in twistor space
4. Mirror symmetry and S-duality
5. **Conclusion**

Conclusion I

- The hypermultiplet moduli space of $\mathcal{N} = 2$ string vacua is a fascinating subject, which combines many different trades in mathematics (algebraic geometry, symplectic geometry, number theory, etc).
- Physically, it should be the framework of choice for precision counting of black hole microstates, if one can make sense of the divergent series.
- Combining twistor techniques with S-duality and mirror symmetry has lead to a complete and beautiful picture for D-instanton corrections. NS5-brane instantons are still mysterious beyond linear order, yet they are in principle determined by S-duality...

Conclusion II

- So far, we have parametrized the metric in terms of the BPS invariants $\Omega(\gamma)$. It would be very interesting if one could compute those, e.g. by postulating additional automorphic properties (e.g. $SL(3, \mathbb{Z})$ or $SU(2, 1)$)

BP Persson; Bao Kleinschmidt Nilsson Persson BP

- Our understanding of the HM moduli space in heterotic remains rudimentary, yet it is where computations should be easiest...

Aspinwall, Plesser, Louis Valandro

Back up slides: Automorphy

Symmetries

- Monodromies

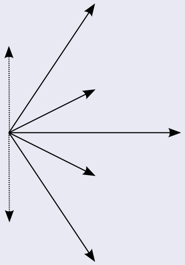
Root diagram (2D projection)

Back up slides: Automorphy

Symmetries

- Monodromies
- Large gauge transf.

Root diagram (2D projection)



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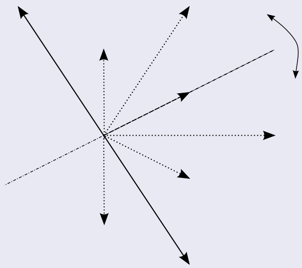

Back up slides: Automorphy

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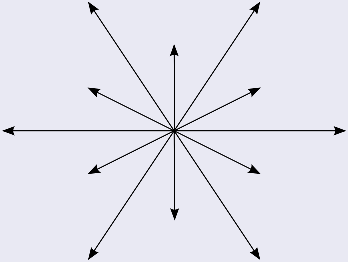

Alexeevsky; Gunaydin Koepsell Nicolai;Gunaydin Neitzke Pavlyk BP

Back up slides: Automorphy

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- Quasiconformal sym.
- 3-step nilpotent

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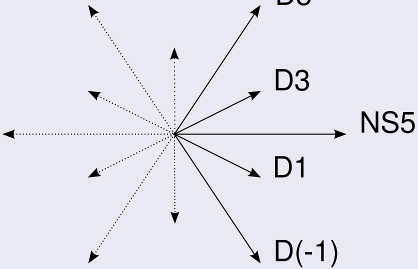

Back up slides: Automorphy

Symmetries

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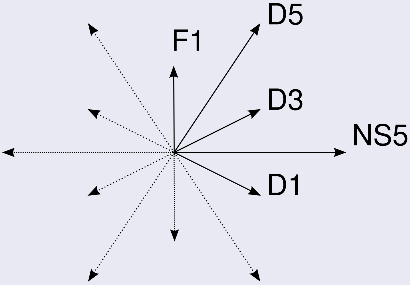
- 3-step nilpotent
- 5-step nilpotent

Root diagram (2D projection)



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Symmetries

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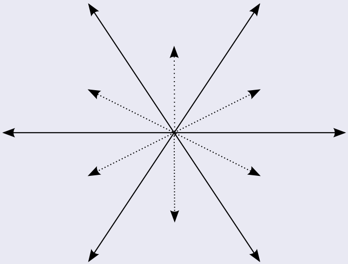
- Long roots: SL(3)

Root diagram (2D projection)



Persson BP

Back up slides: Automorphy

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- Long roots: SL(3)
- Rigid case: SU(2,1)

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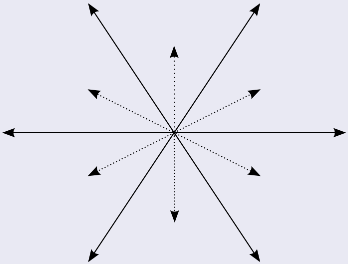

Bao Kleinschmidt Nilsson Persson BP