EFFECTIVE TEMPERATURES IN DRIVEN VORTEX LATTICES WITH RANDOM PINNING

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The relationship between correlations and linear response in non-equilibrium driven vortex lattices with random pinning is analyzed numerically.

• Motivation

– A *generalized fluctuation-dissipation-theorem* (FDT) [1] and the concept of *effective temperatures* has been proposed [2,3,4] for describing slow non-equilibrium dynamics in glasses.

– A "shaking temperature" T_{sh} has been proposed before to characterize statistical properties of the fluctuating pinning force on the moving vortex lattice in superconductors [5,6].

– Is the slow stationary non-equilibrium dynamics in vortex lattices with pinning fundamentally different from "stirred" glasses [7] in stationary states? .

– If an effective temperature from a generalized fluctuation-disipation relation can be defined: Which are its properties? Can be measured? How is it related with T_{sh}?

- **Main results**:

  - For strong disorder or intermediate driving forces we find a generalized FDT holds and an *effective transverse temperature* T_{eff} can be defined for the long-time *transverse* fluctuations.

  - We calculate T_{eff} as a function of driving force, disorder strenght, vortex density and external temperature. We find it has the following properties:

    * T_{eff} could be measured in transport experiments.

    * T_{eff} is a decreasing function of the driving force.

    * $T_{eff}(T) = T_{eff}(0) + T$.

    * In the dynamical transverse freezing $T_{eff} \sim T_m$, where T_m is the equilibrium melting temperature of the unpinned system.

Model

We perform Langevin particle dynamics simulations. The equation of motion of a vortex in position $\mathbf{r}_i$:

$$\eta \frac{d\mathbf{r}_i}{dt} = -\sum_{j \neq i} \nabla_i U_v(r_{ij}) - \sum_p \nabla_i U_p(r_{ip}) + \mathbf{F} + \xi_i$$

- $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ distance between vortices i, j
- $r_{ip} = |\mathbf{r}_i - \mathbf{r}_p|$ distance between the vortex i a pinning site at $\mathbf{r}_p$
- $\eta = \frac{\Phi_0 H_{c2} d}{c^2 \rho_n}$ Bardeen-Stephen friction
- $\mathbf{F} = \frac{d\Phi_0}{c} \mathbf{J} \times \hat{\mathbf{z}} \qquad \mathbf{E} = \frac{B}{c} (\frac{1}{N_v} \sum_i \mathbf{v}_i) \times \hat{\mathbf{z}}$
- $U_v(r) = -A_v \ln(r/\Lambda)$, $\Lambda = 2\lambda^2/d$, $A_v = \Phi_0^2/8\pi\Lambda$, for 2D thin films with thickness $d \ll \lambda$ (Pearl solution).
- Thermal force $\langle \xi_i^a(t) \xi_j^b(t') \rangle = 2\eta T \delta_{ij} \delta_{ab} \delta(t - t')$
- *random* uniform distribution of attractive pinning centers: $U_p(r) = -A_p e^{-(r/\xi)^2}$.

Method

- **Observables**

$$A(t) = \frac{1}{N_v} \sum_{i=1}^{N_v} s_i x_i(t) \;\; \text{and} \;\; B(t) = \sum_{i=1}^{N_v} s_i x_i(t), \tag{1}$$

where $s_i = -1, 1$ are random numbers with $\overline{s_i} = 0$ and $\overline{s_i s_j} = \delta_{ij}$ (driving force is $\mathbf{F} = F\hat{\mathbf{y}}$).

- **Correlation function**

$$C_{AB}(t, t_w) \;=\; \frac{1}{N_v} \sum_{i,j=1}^{N_v} \langle \overline{s_i x_i(t) s_j x_j(t_w)} \rangle = \frac{1}{N_v} \sum_{i=1}^{N_v} \langle x_i(t) x_i(t_w) \rangle \tag{2}$$

- **Response function**

A perturbative force on each vortex $\mathbf{f}_i = \lambda s_i \hat{\mathbf{x}}$, applied at time t_w, gives the integrated response R_A of the observable A:

$$R_A(t, t_w) = \lim_{\lambda \to 0} \frac{1}{\lambda} [\overline{\langle A(t) \rangle}_\lambda - \overline{\langle A(t) \rangle}_{\lambda=0}] \tag{3}$$

- **Fluctuation-disipation relation**

$$R_A(t, t_w) = \frac{1}{2k_B T_{eff}(t, t_w)} \Delta(t, t_w) \tag{4}$$

where $T_{eff} = T$ in equilibrium. $\Delta(t, t_w)$ is the quadratic mean transverse displacement:

$$\begin{aligned} \Delta(t, t_w) &= \frac{1}{N_v} \sum_{i=1}^{N_v} \langle |x_i(t) - x_i(t_w)|^2 \rangle \\ &= C_{AB}(t, t) + C_{AB}(t_w, t_w) - 2C_{AB}(t, t_w) \end{aligned} \tag{5}$$

- **Effective temperatures**

We are interested in the non-equilibrium *stationary* states:

$$R_A(t) = \frac{1}{2k_B T_{eff}(t)} \Delta(t) \tag{6}$$

where we choose t_w long enough to ensure stationarity. If $R_A(t)$ vs $\Delta(t)$ is approximately linear in a time scale, we then obtain the associated effective temperature T_{eff}:

$$\frac{1}{T_{eff}} = 2k_B \frac{dR_A}{d\Delta} \tag{7}$$

Results

• *Correlations and Response*

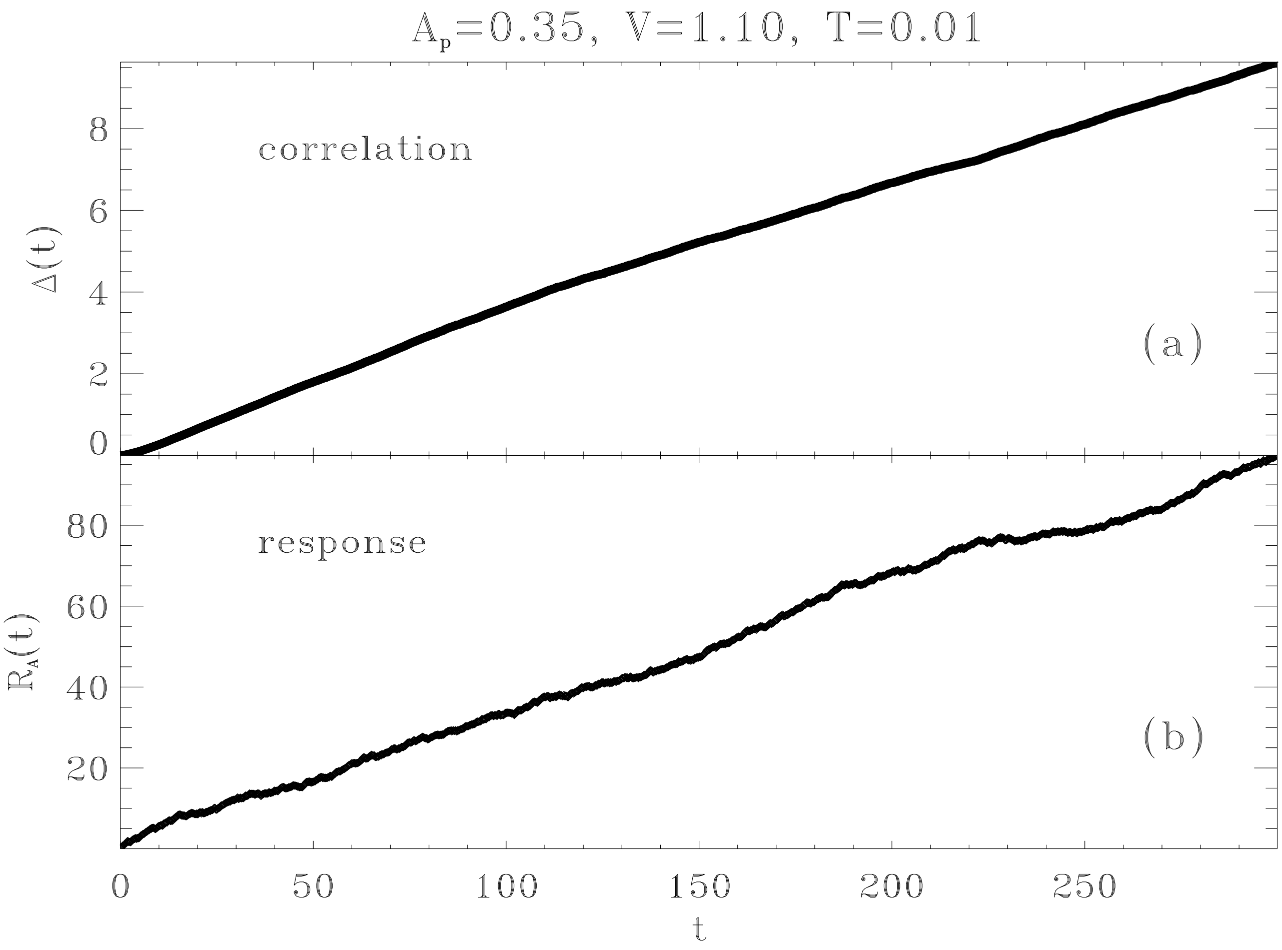

Figure 1: Typical mean quadratic transverse displacement (a) and response (b) in the smectic flow regime.

• *Two-temperature - two-time scales*

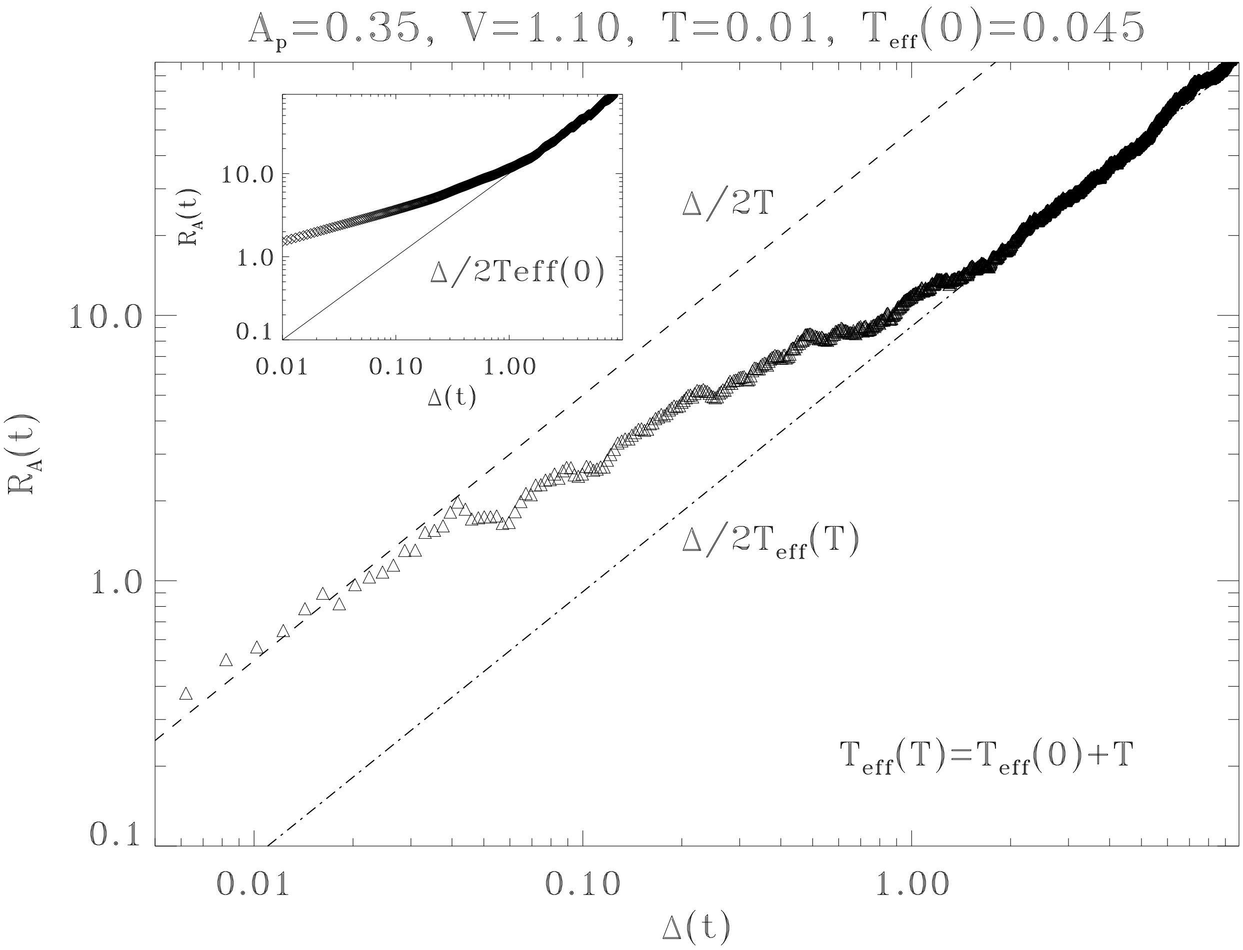

Figure 2: Typical transverse fluctuation-disipation relation for plastic and smectic flow regimes. From the fitting function $\Delta/2T_{eff}(0)$ in the case T=0 we obtain $T_{eff}(0)$ (see inset). In the figure we observe that $\Delta/2T_{eff}(T)$ with $T_{eff}(T) = T_{eff}(0) + T$ fits well when $T \neq 0$. For short times the equilibrium FDT holds.

• *Transverse freezing:* $T_{eff}(V) \longrightarrow T_m$

T_m: equilibrium melting temperature of the unpinned system.

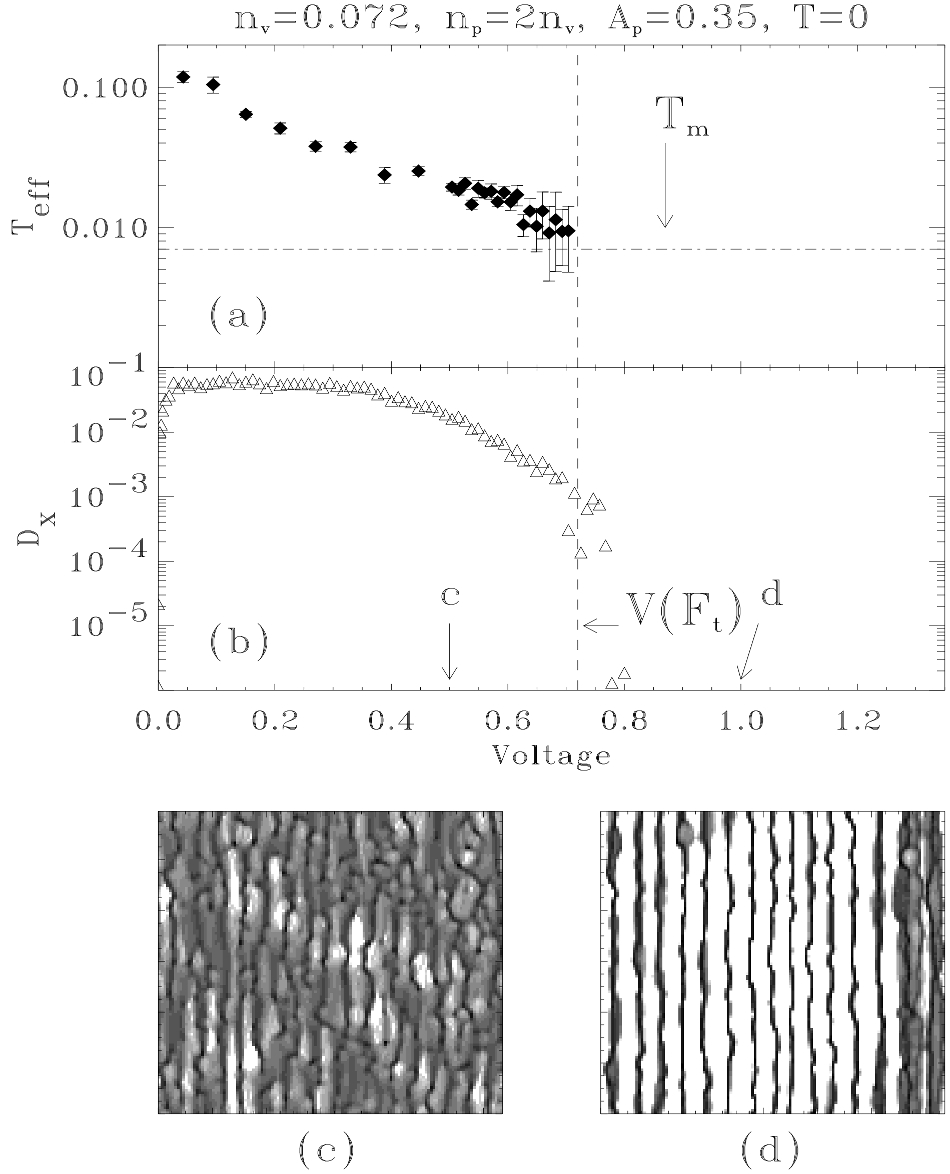

Figure 3: (a) T_{eff} vs voltage V. When $V = V(F_t)$ a sudden drop to zero of the transverse diffusion coeficient D_x is observed (b). In (c) and (d) we show typical trajectories for $V < V(F_t)$ and $V > V(F_t)$ respectivily.

• *Parameter dependence*

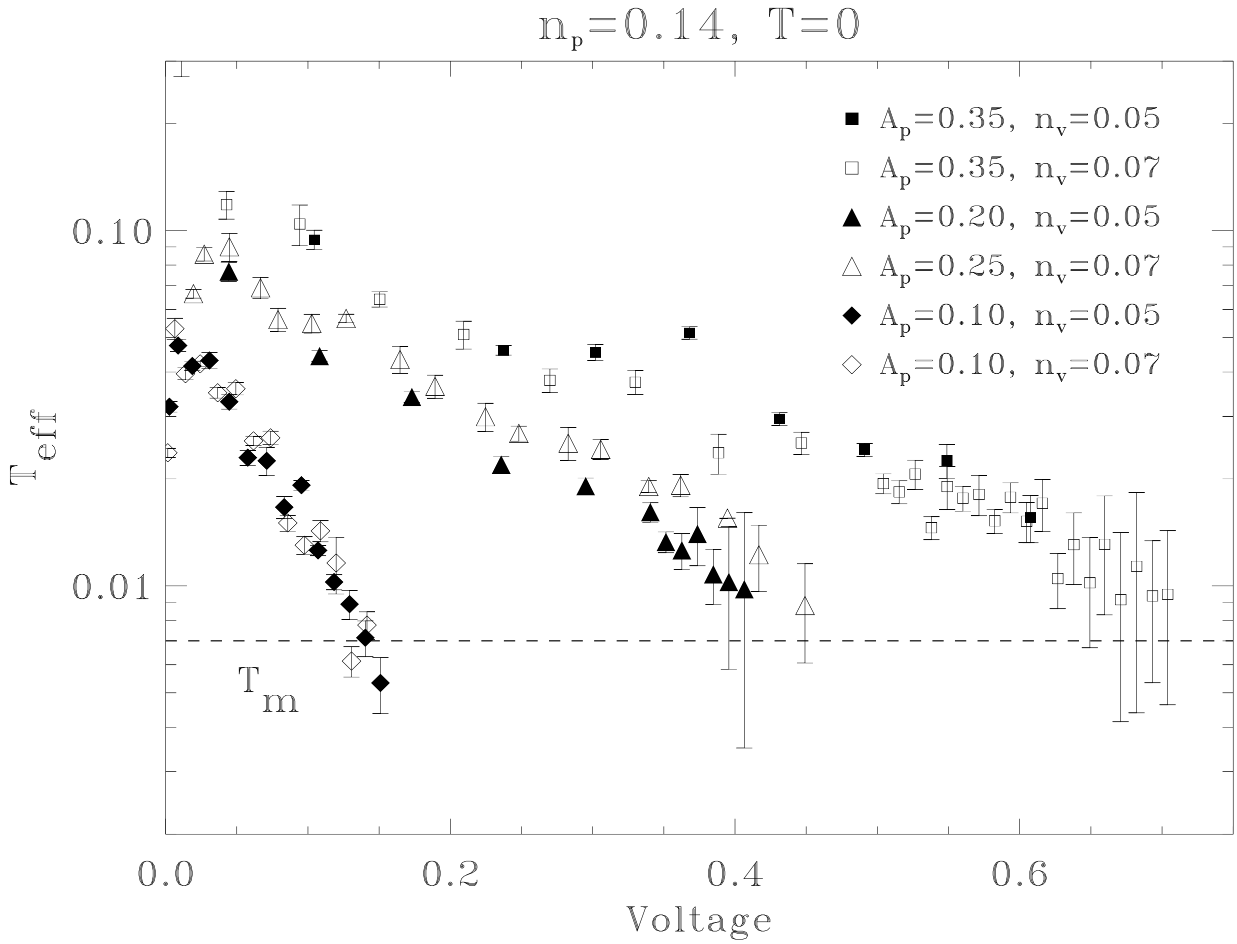

Figure 4: T_{eff} vs voltage for different pinning amplitudes A_p and vortex densities n_v at $T = 0$ and fixed pinning density $np = 0.14$. In all the cases we find $T_{eff} \rightarrow T_m$ in the corresponding transverse freezing voltage.

• *How to measure T_{eff} experimentally*

Kubo relation [8]

$$\langle \frac{dV_x(t)}{d\lambda} \rangle_{\lambda=0} = \langle \frac{dV_x(0)}{d\lambda} \rangle_{\lambda=0} - \beta_{eff} \int_0^t dt' \langle V_x(t) V_x(t') \rangle_{\lambda=0} \tag{8}$$

$\langle V_x(t) V_x(t') \rangle_{\lambda=0}$ **voltage fluctuations (transverse noise)**.

$\langle \frac{dV_x(t)}{d\lambda} \rangle_{\lambda=0} = R_x(t)$ **transverse resistivity**.

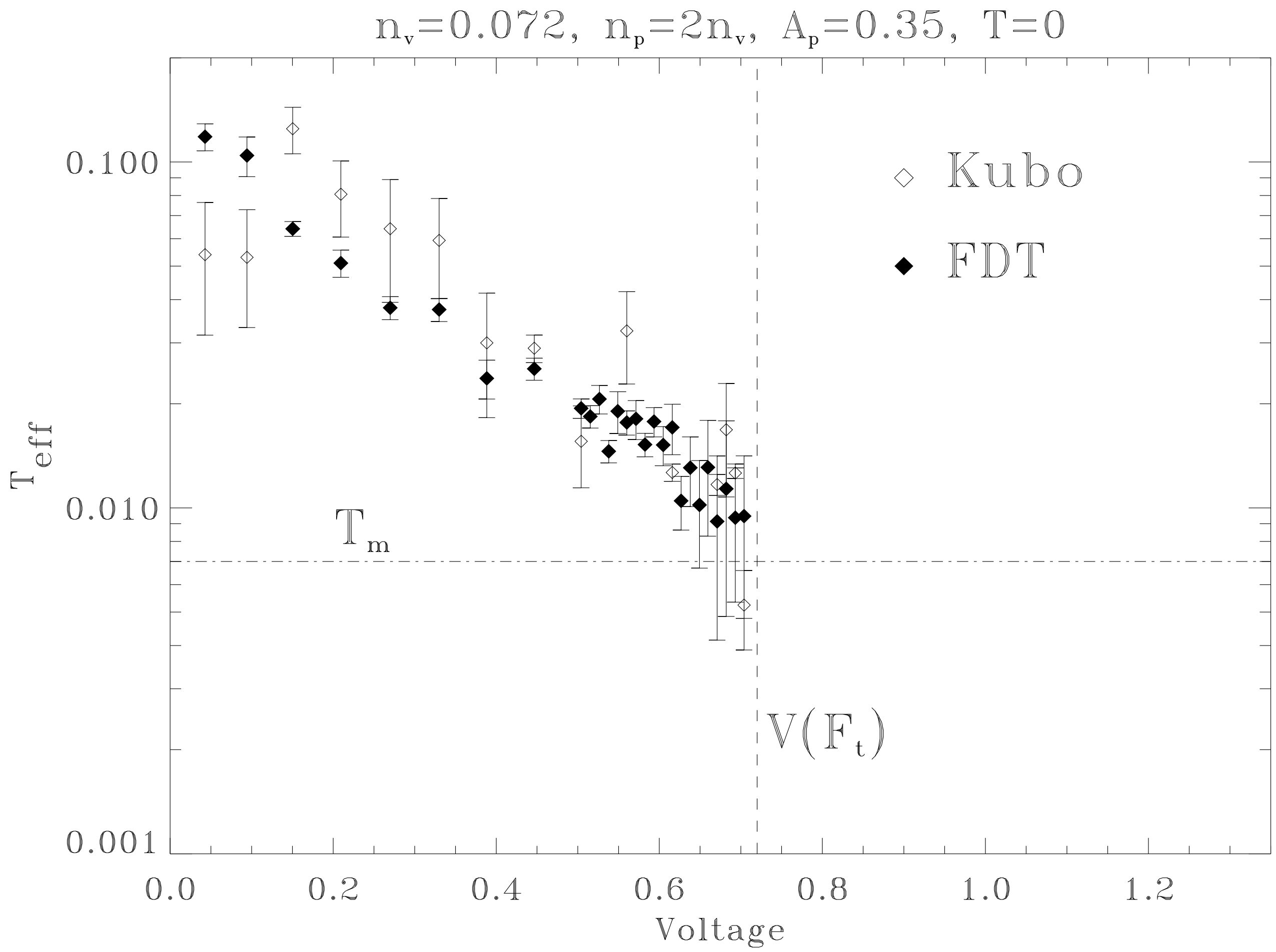

Figure 5: We compare T_{eff} obtained using the fluctuation disipation relation (equation 7) and the Kubo relation for the total transverse voltage (equation 8). T_{eff} could be determined in transport experiments using Kubo relation.

Conclusions

- We find a two-temperature two time scales behaviour in fluctuation disipation relation in the plastic and smectic flow regimes.
- We calculated an *effective transverse temperature* from simulated correlation and response, taking account the actual viscoelastic response of the lattice in the plastic and smectic flow regimes.
- The *effective transverse temperature* could be measured in transport experiments since it was found to coincide with the one defined by a generalized Kubo relation for tranverse voltage fluctuations.
- Much more to be investigated......

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