Dynamique hors d'équilibre classique et quantique. Formalisme et applications.

Camille ARON

LPTHE, Université Pierre & Marie Curie
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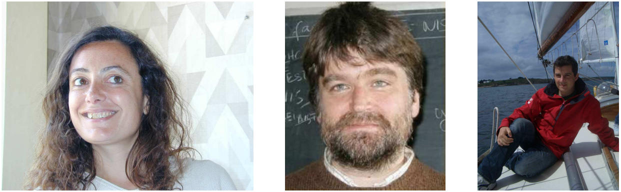
Membres du jury

M.	Denis	BERNARD	examinateur
M.	Federico	CORBERI	examinateur
Mme	Leticia	CUGLIANDOLO	directrice de thèse
M.	Gabriel	KOTLIAR	rapporteur
M.	Marco	PICCO	directeur de thèse
M.	Frédéric	Van WIJLAND	rapporteur



Out-of-equilibrium dynamics

Situations

- Changing a parameter: in the system or the environment (*e.g.* quenching a coupling constant, the temperature, ...)
- Applying a drive: external force or non-equilibrium environment (*e.g.* shear, voltage biais, ...)

Systems of interest

Macroscopic systems exhibiting slow dynamics

- domain growth (*e.g.* ferromagnets, binary liquids, ...)
- disordered interactions
  - weak disorder (*e.g.* random fields)
  - strong disorder (*e.g.* glasses)

General questions

- How does the system relax ?
- What is similar to equilibrium ?
- What are the effects of
  - disorder ?
  - quantum fluctuations ?
- What is universal ?

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What can we do ?

- Equations for the dynamics
  - Classical: stochastic processes (Langevin, Fokker-Planck, ...)
  - Quantum: Schwinger-Keldysh
- Solving the dynamics
  - analytically
    - $1d$ systems
    - mean-field models
  - numerical simulations for small d
- exact statements
  - fluctuation theorems
  - bounds on entropy creation

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Roadmap

- In and out-of-equilibrium dynamics, symmetry approach
- Out-of-equilibrium classical dynamics after a quench
- Driven out-of-equilibrium quantum dynamics

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Part I

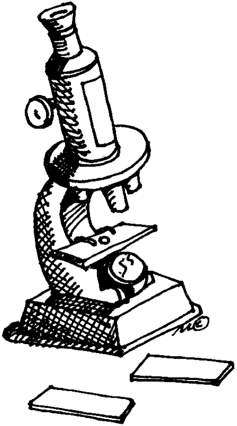
Symmetries of Langevin and Quantum Generating Functionals

`C. A., L. F. Cugliandolo, G. Biroli`

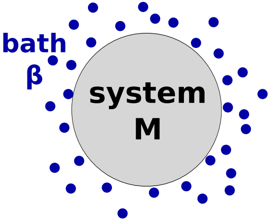
`arXiv:1007.5059 (2010)`

Robert Brown's experiment (1828)

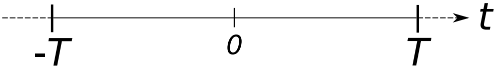
[Video]



Albert Einstein's understanding:



Paul Langevin's equation (1908)



Initial conditions $P_{\rm i}(\psi, \dot\psi)$

Langevin equation

$$m\ddot\psi = F + F_{\rm bath\to system}$$

with the heuristic force

$$F_{\rm bath\to system} = -\eta_0\dot\psi + \xi$$

Gaussian white noise

$$\langle \xi(t)\rangle = 0, \quad \langle \xi(t)\xi(t')\rangle \propto \delta(t-t')$$

Bath equilibrium condition

$$\langle \xi(t)\xi(t')\rangle = 2\beta^{-1}\eta_0\delta(t-t')$$

3 Introduction

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Generalized Langevin equation

- Multiplicative noise

$$F_{\text{bath}\to\text{system}} = -\eta_0 M'(\psi)^2\,\dot{\psi} + M'(\psi)\,\xi$$

Generalized Langevin equation

- Multiplicative noise

$$F_{\rm bath\to system} = -\eta_0 M'(\psi)^2\,\dot\psi + M'(\psi)\,\xi$$

- Colored noise

$$\langle \xi(t)\xi(t')\rangle = \beta^{-1}\aleph(t-t')$$

ex: Ornstein-Uhlenbeck process: $\aleph(t-t') = \eta_0\tau^{-1}{\rm e}^{-|t-t'|/\tau}$

Bath equilibrium condition

$$F_{\rm bath\to system} = -\int_{-T}^{t} {\rm d}t'\;\aleph(t-t')\dot\psi(t') + \xi(t)$$

Generalized Langevin equation

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- Multiplicative & Colored noise

Classical field theory

Martin-Siggia-Rose-Janssen-deDominicis path-integral formalism

$$\psi_{\rm sol}(t) \Longrightarrow P[\psi(t)] \propto \mathcal{J} \int \mathcal{D}[\hat\psi] \;\; {\rm e}^{S[\psi,\hat\psi]}$$

Action

$$S = S^{\rm det} + S^{\rm diss}$$

$$S^{\rm det}[\psi,\hat\psi] \equiv \ln P_{\rm i}\left(\psi(-T),\dot\psi(-T)\right) - \int {\rm d}u \; {\rm i}\hat\psi(u)\left[m\ddot\psi(u) - F([\psi],u)\right]$$

$$S^{\rm diss}[\psi,\hat\psi] \equiv \eta_0 \int {\rm d}u \; {\rm i}\hat\psi(u)\left[\beta^{-1}{\rm i}\hat\psi(u) - \dot\psi(u)\right]$$

Additive white noise

Classical field theory

Martin-Siggia-Rose-Janssen-deDominicis path-integral formalism

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Action

$$S = S^{\mathrm{det}} + S^{\mathrm{diss}}$$

$$S^{\mathrm{det}}[\psi,\hat{\psi}] \equiv \ln P_{\mathrm{i}} \left(\psi(-T), \dot{\psi}(-T)\right) - \int \mathrm{d}u \; \mathrm{i}\hat{\psi}(u) \left[m\ddot{\psi}(u) - F([\psi], u)\right]$$

$$S^{\mathrm{diss}}[\psi,\hat{\psi}] \equiv \int \mathrm{d}u \int^{u} \mathrm{d}v \; \mathrm{i}\hat{\psi}(u) M'(\psi(u)) \aleph(u-v) M'(\psi(v)) \left[\beta^{-1}\mathrm{i}\hat{\psi}(v) - \dot{\psi}(v)\right]$$

Multiplicative & colored noise

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Conditions for equilibrium dynamics

- preparation: Gibbs-Boltzmann initial distribution $P_{\mathrm{i}}(\psi(-T),\dot{\psi}(-T)) \propto \mathrm{e}^{-\beta_{\mathrm{i}}\mathcal{H}_{\mathrm{i}}[\psi(-T)]}$, $\mathcal{H}_{\mathrm{i}}[\psi] \equiv \frac{1}{2}m\dot{\psi}^2 + V_{\mathrm{i}}(\psi)$
- evolution: same potential and time-independent forces $F = -V_{\mathrm{i}}'(\psi)$
- equilibrium bath at temperature $\beta = \beta_{\mathrm{i}}$

Action

$$S = S^{\mathrm{det}} + S^{\mathrm{diss}}$$

$$S^{\mathrm{det}} = -\beta\mathcal{H}[\psi(-T)] + \iint \mathrm{d}u\,\mathrm{d}v\ \mathrm{i}\hat{\psi}(u)\frac{\delta\mathcal{L}[\psi(v)]}{\delta\psi(u)}$$

$$S^{\mathrm{diss}} = \int \mathrm{d}u \int^{u} \mathrm{d}v\ \mathrm{i}\hat{\psi}(u) M'(\psi(u)) \aleph(u-v) M'(\psi(v)) \Big[\beta^{-1}\mathrm{i}\hat{\psi}(v) - \dot{\psi}(v)\Big]$$

Supersymmetric representation

$$\mathcal{J} = \det \frac{\delta\xi}{\delta\psi} = \int \mathcal{D}[c,\bar{c}] \ \ \mathrm{e}^{S^{\mathcal{J}}[\psi,c,\bar{c}]}$$

Superfield:

$$\left\{\psi, \hat{\psi}, c, \bar{c}\right\} \mapsto \Psi(t,\theta,\bar{\theta}) \equiv \psi + \bar{\theta}c + \bar{c}\theta + \bar{\theta}\theta\left(\mathrm{i}\hat{\psi} + \bar{c}c\frac{M''(\psi)}{M'(\psi)}\right)$$

Action

$$S = S^{\mathrm{det}} + S^{\mathrm{diss}}$$

$$S^{\mathrm{det}}[\Psi] \equiv -\beta\mathcal{H}[\Psi(-T,0,0)] + \int \mathrm{d}\Upsilon\ \mathcal{L}[\Psi(\Upsilon)]$$

$$S^{\mathrm{diss}}[\Psi] \equiv \frac{1}{2}\iint \mathrm{d}\Upsilon'\,\mathrm{d}\Upsilon\ M(\Psi(\Upsilon'))\,\mathbf{D}^{(2)}(\Upsilon',\Upsilon)\,M(\Psi(\Upsilon))$$

$$\Upsilon \equiv \{t,\theta,\bar{\theta}\}$$

Supersymmetry of the action

S is invariant under

$$\begin{aligned} \Psi &\longmapsto \Psi + \bar{\epsilon}\,\mathbf{Q}\Psi, \quad \mathbf{Q} \equiv \frac{\partial}{\partial\bar{\theta}} \\ \Psi &\longmapsto \Psi + \epsilon\,\bar{\mathbf{Q}}\Psi, \quad \bar{\mathbf{Q}} \equiv \beta^{-1}\frac{\partial}{\partial\theta} + \bar{\theta}\frac{\partial}{\partial t} \end{aligned}$$

Ward identities ⇒ equilibrium relations

- stationarity, time-translational invariance (TTI)
- fluctuation-dissipation theorem

Symmetry of the action

S is invariant under

$$\mathcal{T}_{\mathrm{eq}} \equiv \begin{cases} \psi(t) & \longmapsto & \psi(-t) \\ \mathrm{i}\hat{\psi}(t) & \longmapsto & \mathrm{i}\hat{\psi}(-t) + \beta \partial_t \psi(-t) \end{cases}$$

Equilibrium relations

Ward identities

$$\langle A[\psi, \hat{\psi}] \rangle_S = \langle A[\mathcal{T}_{\mathrm{eq}}\psi, \mathcal{T}_{\mathrm{eq}}\hat{\psi}] \rangle_S$$

- stationarity, TTI
- equipartition theorem
- fluctuation-dissipation th.
- Onsager relations
- many more...

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Broken symmetry

S is no longer invariant

$$S[\psi,\hat{\psi}] \xmapsto{\mathcal{T}_{\mathrm{eq}}} S_{\mathrm{r}}[\psi,\hat{\psi}] + \mathcal{S}$$

Out-of-equilibrium relations

$$\langle A[\psi,\hat{\psi}]\rangle_S = \langle A[\mathcal{T}_{\mathrm{eq}}\psi, \mathcal{T}_{\mathrm{eq}}\hat{\psi}]\,\mathrm{e}^{\mathcal{S}}\rangle_{S_{\mathrm{r}}}$$

- Kawasaki identity (1967): $\langle \mathrm{e}^{-\mathcal{S}}\rangle_S = 1$
  Jarzynski equality (1997): $\mathrm{e}^{\beta\Delta\mathcal{F}}\langle \mathrm{e}^{-\beta\mathcal{W}}\rangle_S = 1$
- Fluctuation theorem (FT, 1993): $P(\mathcal{S}) = P_{\mathrm{r}}(-\mathcal{S})\,\mathrm{e}^{\mathcal{S}}$
  Crooks FT (1998): $P(\mathcal{W}) = P_{\mathrm{r}}(-\mathcal{W})\mathrm{e}^{\beta(\mathcal{W}-\Delta\mathcal{F})}$
- many more...

Out-of-equilibrium symmetry

Generalized Langevin equation:

$$\underbrace{m\ddot{\psi}(t) - F([\psi],t) + M'(\psi(t))\int^{t} \mathrm{d}u\, \aleph(t-u) M'(\psi(u))\dot{\psi}(u)}_{\equiv \mathrm{LHS}([\psi],t)} = M'(\psi(t))\xi(t)$$

S is invariant under

$$\mathcal{T}_{\mathrm{eom}} \equiv \begin{cases} \psi(u) & \mapsto \quad \psi(u) \\ \mathrm{i}\hat{\psi}(u) & \mapsto \quad -\mathrm{i}\hat{\psi}(u) + \dfrac{2\beta}{M'(\psi(u))}\displaystyle\int \mathrm{d}v\, \aleph^{-1}(u-v)\, \dfrac{\mathrm{LHS}([\psi],v)}{M'(\psi(v))} \end{cases}$$

Ward identities ⇒ 'Schwinger-Dyson' out-of-equilibrium relations

For additive white noise:

$$m\partial_{t'}^2 C(t,t') + \eta_0 \partial_{t'} C(t,t') - \langle \psi(t) F([\psi],t')\rangle = 2\beta^{-1}\eta_0 R(t,t')$$
$$m\partial_{t}^2 R(t,t') + \eta_0 \partial_{t} R(t,t') - \langle \mathrm{i}\hat{\psi}(t') F([\psi],t)\rangle_S = \delta(t-t')$$

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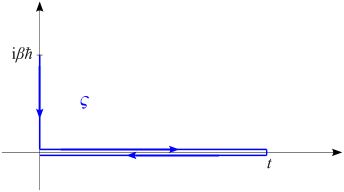
Quantum generalization

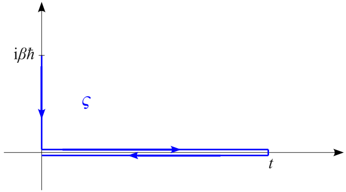
Schwinger-Keldysh approach



$$\begin{aligned}\langle A(t)\rangle &= \mathcal{Z}^{-1}\mathrm{Tr}\left[\mathrm{e}^{\frac{\mathrm{i}}{\hbar}Ht}A(t)\,\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}Ht}\mathrm{e}^{-\beta H}\right] \\ &\propto \int \mathcal{D}[\phi]\,\mathrm{e}^{\frac{\mathrm{i}}{\hbar}\int_{\varsigma}\mathrm{d}u\,\mathcal{L}[\phi(u)]}\,A[\phi(t)]\end{aligned}$$

Quantum generalization

Schwinger-Keldysh approach



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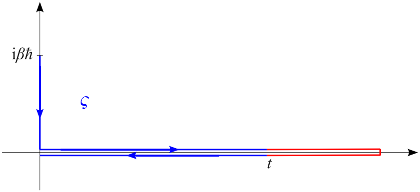

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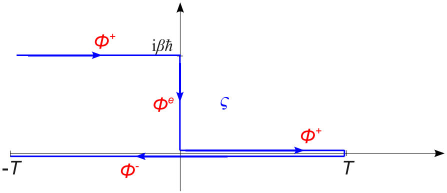
Symmetry



Action

$$S = \int_{-T+i\beta\hbar}^{i\beta\hbar} \mathrm{d}u\ \mathcal{L}[\phi^+(u)] + \int_{i\beta\hbar}^{0} \mathrm{d}u\ \mathcal{L}[\phi^e(u)] + \int_{0}^{T} \mathrm{d}u\ \mathcal{L}[\phi^+(u)] + \int_{T}^{-T} \mathrm{d}u\ \mathcal{L}[\phi^-(u)]$$

S is invariant under

$$\mathcal{T}_{\mathrm{eq}}^{Q} \equiv \begin{cases} \phi^+(u) & \longmapsto & \phi^+(i\beta\hbar - u) \\ \phi^-(u) & \longmapsto & \phi^-(-u) \\ \phi^e(u) & \longmapsto & \phi^e(i\beta\hbar - u) \end{cases}$$

Part II

Scalings and Super-Universality in Coarsening versus Glassy Dynamics

`C. A., C. Chamon, L. F. Cugliandolo, M. Picco`

`J. Stat. Mech. (2008) P05016`

8 Random Field Ising Model

9 3d Edwards-Anderson Model

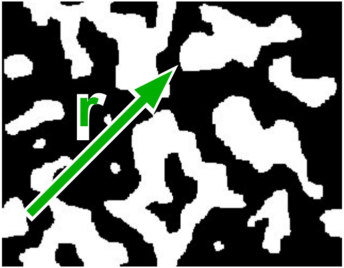
Overview of the 3*d* Random Field Ising Model

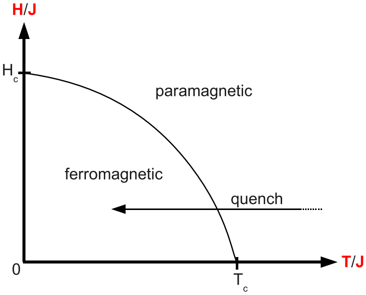
Hamiltonian

$$\mathcal{H} = -\mathbf{J} \sum_{<i,j>} s_i s_j - \sum_i h_i s_i$$

$$h_i = \pm \mathbf{H}$$

Coarsening





Growing length

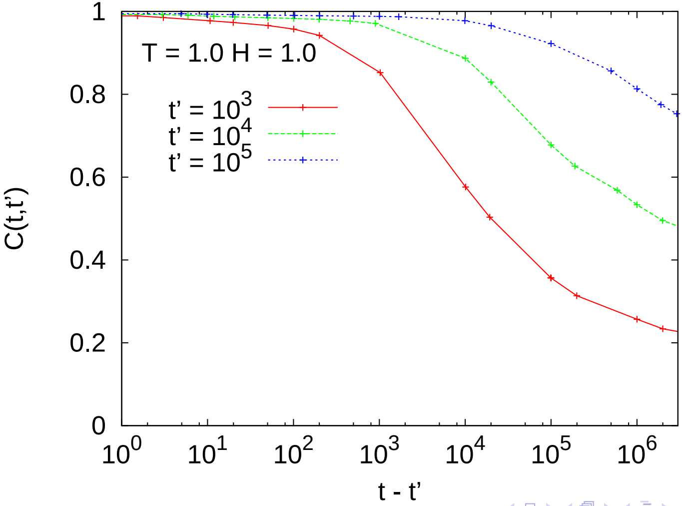
$$C_2(r,t) \equiv \langle s_i(t) s_{i+r}(t) \rangle_i$$

$$\Downarrow$$

We extract $R_{T,H}(t)$

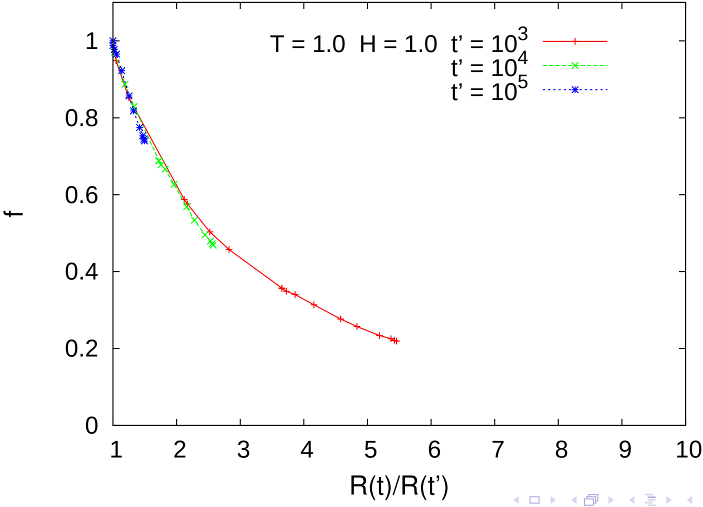
Dynamical scaling

$$C_{\mathbf{T},\mathbf{H}}(t,t') \equiv \langle s_i(t)s_i(t')\rangle_i \qquad C^{ag}_{\mathbf{T},\mathbf{H}}(t,t')$$



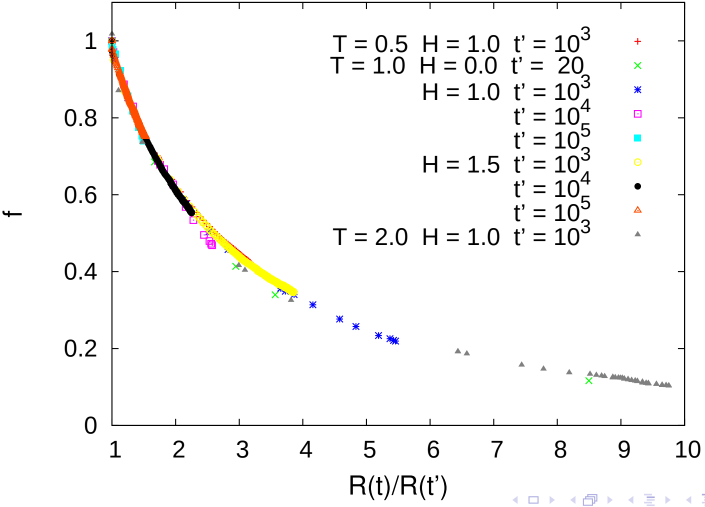
Dynamical scaling

$$C_{\mathbf{T},\mathbf{H}}(t,t') \equiv \langle s_i(t)s_i(t')\rangle_i \qquad C^{ag}_{\mathbf{T},\mathbf{H}}(t,t') = f_{\mathbf{T},\mathbf{H}}\left(\frac{R_{\mathbf{T},\mathbf{H}}(t)}{R_{\mathbf{T},\mathbf{H}}(t')}\right)$$



Dynamical scaling: super-universality

$$C_{\mathbf{T},\mathbf{H}}(t,t') \equiv \langle s_i(t) s_i(t') \rangle_i \qquad C^{ag}_{\mathbf{T},\mathbf{H}}(t,t') = \mathbf{f}\left(\frac{R_{\mathbf{T},\mathbf{H}}(t)}{R_{\mathbf{T},\mathbf{H}}(t')}\right)$$



8. Random Field Ising Model

9. 3*d* Edwards-Anderson Model

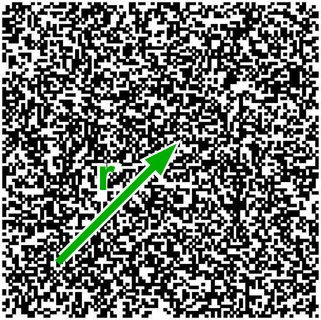
Overview of the 3*d* Edwards-Anderson model

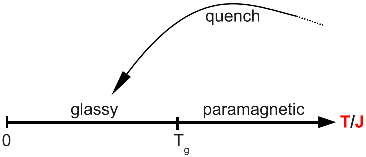
Hamiltonian

$$\mathcal{H} = -\sum_{<i,j>} \mathbf{J_{ij}} s_i s_j$$

$$\mathbf{J_{ij}} = \pm \mathbf{J}$$

Frustration





Results

Dynamical scalings: yes

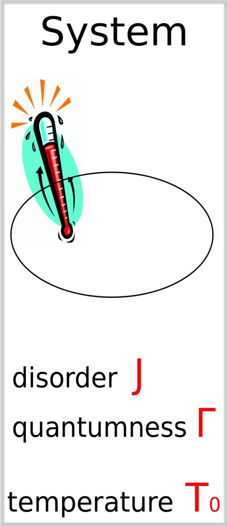
Super-universality: no

Part III

Driven Quantum Coarsening

C. A., G. Biroli, L. F. Cugliandolo
Phys. Rev. Lett. **102**, 050404 (2009)
arXiv:1005.2414 (2010)

Quench



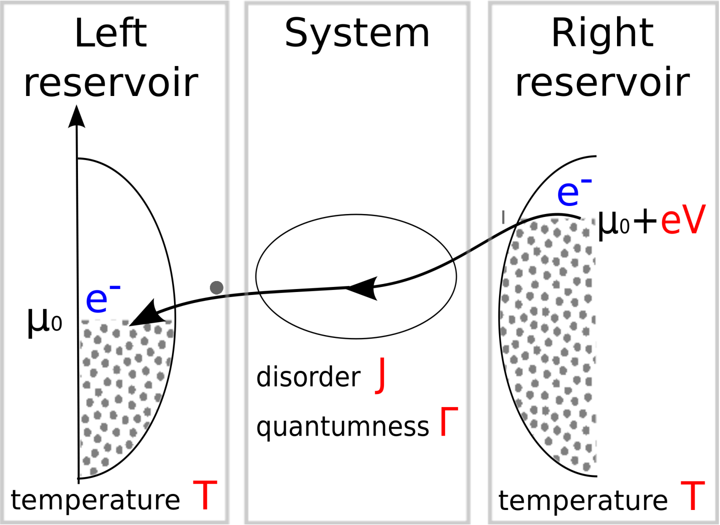
System

disorder J

quantumness Γ

temperature T_0

Quench



Left reservoir

temperature T

System

disorder J

quantumness Γ

Right reservoir

temperature T

10 Setup

11 Model

12 Environment

13 Dynamical phase diagram

14 Dynamics

System

- N n-component quantum rotors: $\mathbf{n}_i \in \mathbb{R}^n, i = 1 \dots N$
- unit length: $\mathbf{n}_i^2 = 1$
- mass $\propto 1/\mathbf{\Gamma}$
- fully connected via random couplings: $\mathbf{J_{ij}} \leftarrow \text{Gauss}(0, \mathbf{J})$

Hamiltonian

$$\mathcal{H}_S = \frac{\mathbf{\Gamma}}{2n} \sum_{i=1}^{N} \mathbf{L}_i^2 - \frac{n}{\sqrt{N}} \sum_{i<j} \mathbf{J_{ij}}\, \mathbf{n}_i \cdot \mathbf{n}_j$$

$\mathbf{L}_i^2 = \sum_{\mu<\nu} (L_i^{\mu\nu})^2$ with $L_i^{\mu\nu} = -\mathrm{i}\hbar \left(n_i^\mu \frac{\partial}{\partial n_i^\nu} - n_i^\nu \frac{\partial}{\partial n_i^\mu} \right)$

Reservoirs

- Free fermions 'ψ_L' and 'ψ_R' in equilibrium at temperature **T**
- applied voltage **eV** between 'L' and 'R' reservoirs.

Coupling System/Reservoirs

$$\mathcal{H}_{SB} = -\mathbf{g}\frac{\sqrt{n}}{N_s}\sum_{i=1}^{N}\sum_{k,k'=1}^{N_s}\sum_{l,l'=1}^{M} \mathbf{n}_i \cdot [\psi^{\dagger}_{Likl}\, \boldsymbol{\sigma}_{ll'}\, \psi_{Rik'l'} + L \leftrightarrow R]$$

$$M^2 - 1 = n$$

Treatment

- Integration over the reservoirs: 2^{nd} order in $\mathbf{g}$
- Average over disorder

$$[\ldots]_J \equiv \int \prod_{i<j} \mathrm{d}J_{ij}\ P(J_{ij}) \ldots$$

$$\Downarrow$$

Quartic terms in $\mathbf{n_i}$

- large n limit

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Effect of the environment

Non-equilibrium environment: $\mathbf{eV} \neq 0$

The effect of the reservoirs on the low frequency dynamics is expected to be the one of an equilibrium bath at

$$\mathbf{T}^* \equiv \frac{\mathbf{eV}}{2} \coth\left(\mathbf{eV}/2\mathbf{T}\right)$$

- Equilibrium ($\mathbf{eV} = 0$): $\mathbf{T}^* = \mathbf{T}$
- Zero temperature ($\mathbf{T} = 0$): $\mathbf{T}^* = \mathbf{eV}/2$

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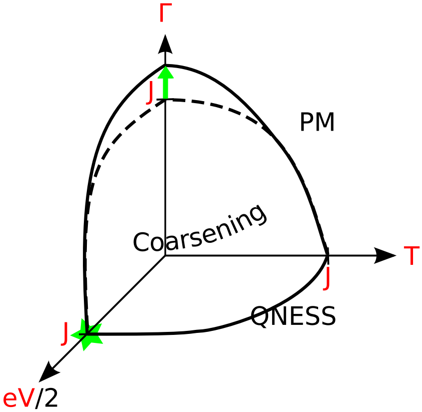
Critical manifold: with drive ($eV \neq 0$)



New 'drive induced' critical point

$$eV_c/2 \propto J$$

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Long-time dynamics

Long-time dynamics described by a classical Langevin equation

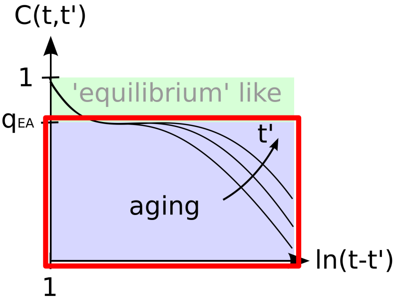
$$\eta_0 \dot{\mathbf{n}} = \ldots + \xi(\mathbf{t})$$

white noise statistics: $\langle \xi(t)\xi(t') \rangle = 2\eta_0 \mathbf{T}^* \delta(t - t')$

temperature $\mathbf{T}^* = \frac{\mathbf{eV}}{2} \coth\left(\mathbf{eV}/2\mathbf{T}\right)$

Long-time dynamics

Classical scenario



$$C_{\text{aging}}(t,t') \propto \ \mathbf{f}(t/t') \quad \mathbf{f}(x) \equiv 2\sqrt{2}\frac{x^{3/4}}{(1+x)^{3/2}}$$

Quantum driven scenario: we expect universal dynamics !

Same t/t' scaling, same $\mathbf{f}(x)$

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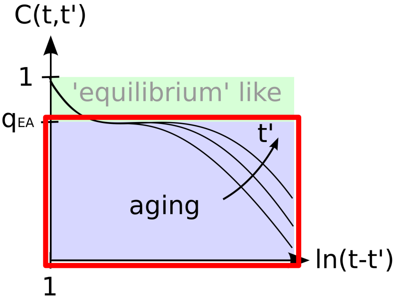
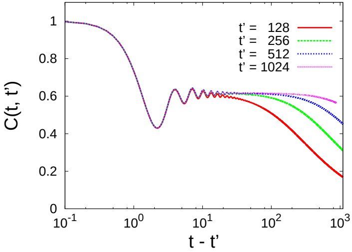
Classical scenario | Quantum scenario





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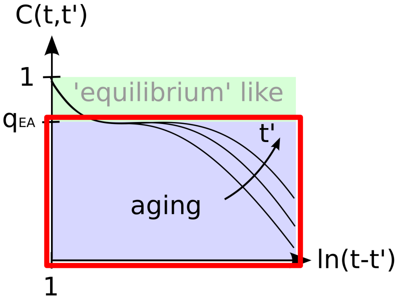

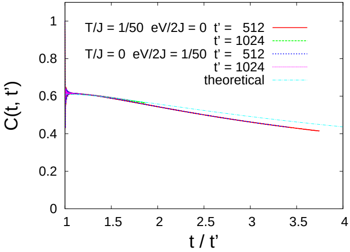

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Conclusions & Outlook

- - Symmetry associated to equilibrium in the MSRJD formalism
  - another symmetry is also valid out of equilibrium
  - handy formalism to obtain relations for non-Markovian systems and multiplicative noise
  - quantum generalization ?
- check of the super-universality hypothesis in the ordering dynamics of the 3*d* RFIM Vs 3*d*EA
- Driven quantum coarsening:
  - dynamical scalings and universality in **J**, **T**, **Γ**, **eV**
  - does this picture survive for non-quadratic models, other couplings ?
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